Bitcoin and Blockchain Technology
Cryptography

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POLITECNICO MILANO 1863

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Hash Functions

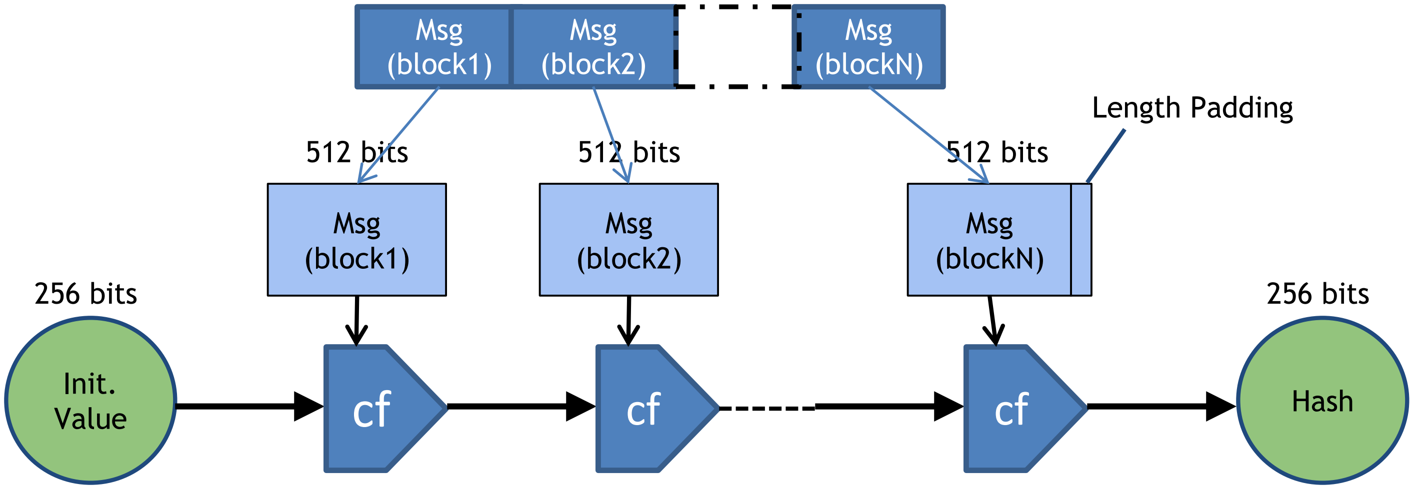
- A hash function is a map from the set of input data (of arbitrary length) to the output set of hash values (bit-string of fixed length)
- Small differences in the input data produce large differences in the result

Properties of Hash Functions

1. Arbitrary message size: h(x) can be applied to message x of any size
2. Fixed output lenghts: h(x) produces a hash value of fixed length (SHA256 uses 256 bits)
3. Efficiency: h(x) is relatively easy to compute

Merkle–Damgård hash construction

Hash function is based on a one-way *compression function*; it is as resistant to collisions as its compression function



Cryptographic Hash Functions

4. One-wayness (*preimage resistance*): Given h(x), it is computationally infeasible to find x.
5. Weak collision resistance (*second pre-image resistance*): Given x it is computationally infeasible to find y!=x such that h(y)=h(x)
6. (Strong) Collision resistance: it is computationally infeasible to find (x, y!=x) such that h(y)=h(x)

- Collision resistance implies second preimage resistance
- Second preimage resistance implies preimage resistance

Hierarchical properties: the converse is not true in general

Computationally Infeasible Collisions

- The size of the possible hash values is smaller than the size of possible input data. Therefore, many input data points will share a single hash value output: collisions do exist...
- Try 2^130 randomly chosen inputs: 99.8% chance that two of them will collide. This works, but it takes *too* long
- Is there a faster way to find collisions? For some hash functions, yes; for the "good" functions, we don't know of one.

Application: Hash as message digest

- If we know $h(x) = h(y)$, it is safe to assume that $x = y$
- Useful because $h(x)$ can be much smaller than x

Puzzle friendlyness

Given x and a target set Y, to find r from high min-entropy distribution such that

$$h(x|r) \in Y$$

there is no solving strategy better than trying random values of r

Min-entropy measures how likely you are to guess a value on your first try. If this probability is p, then the min-entropy is defined as $-\log_2 p$. For example, for a fair coin toss, you'd have p = 0.5, giving a min entropy of 1 bit. A uniformly random 256-bit string would have $-\log_2 2^{256} = 256$ bits of min entropy

Partial Hash Inversion

- Find *nonce* so that h(x|*nonce*) is small, e.g. starts with zero

Output Space

0	1	2	3	4	5	6	7	8	9	a	b	c	d	e	f

↑

target space

- The smaller the target space, the harder to find a solution
- For good hash functions, the problem can only be solved by brute force trials
- It is always trivial to verify the solution: it is just one hash function evaluation

Homework #1

- Find the nonce that appended to your name obtains a hash value starting with 7 zeros
- How many numbers in the range $[0, N]$ result into hash values starting with zeros?
- Produce the histogram with the frequency of hash values starting with 1, 2, 3, 4, 5, 6, and 7 zeros in the $[0, N]$ range

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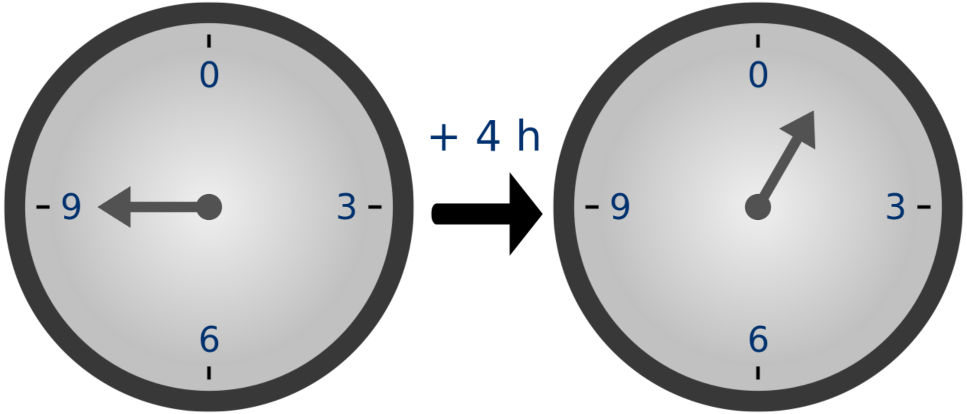
Modular Arithmetic

arithmetic for integers: numbers "wrap around" upon reaching the *modulus* value

$9+4 = 1 \mod 12$



Source: https://commons.wikimedia.org/wiki/File:Clock_group.svg

Congruence and Remainders

If $a_1 \equiv b_1 (\bmod n)$ and $a_2 \equiv b_2 (\bmod n)$ then

- $a_1 + a_2 \equiv b_1 + b_2 (\bmod n)$
- $a_1 - a_2 \equiv b_1 - b_2 (\bmod n)$
- $a_1 a_2 \equiv b_1 b_2 (\bmod n)$

- $(a \bmod n)(\mathrm{b} \bmod n) \equiv ab \ (\bmod n)$
- $\big((a \bmod n)(\mathrm{b} \bmod n)\big) \bmod n = (ab) \bmod n$

Group $(G, +)$

A set G together with an operation $+$ that combines any two elements a and b to form another element $a+b$ is a group if:

- Closure: for all a and b in G, $a+b$ is also in G
- Associativity: for all a, b and c in G, $(a+b)+c=a+(b+c)$
- Identity: there exists an (unique) element 0 in G, such that for every element a in G, the equation $0+a=a+0=a$
- Invertibility: for each a in G, there exists an element b in G, commonly denoted $-a$, such that $a+b=b+a=0$

A group is commutative if for all a and b in G, $a+b=b+a$

e.g. integers under addition $(Z, +)$ are a commutative group, integers under multiplication $(Z, \bullet)$ are not a group (the multiplicative inverse of 2 is not an integer)

Ring and Field *(G, + , ●)*

- A ring is a group with a second operation that is associative and the distributive properties make the two operations "compatible"
- A field is a ring such that the second operation satisfies all the group properties, after throwing out the identity element of the first operation

Modular Addition and Multiplication

- For any modulus *p*, (*[0, p−1], +)* is a commutative group
- 0 is the identity element
- The inverse of any element *a* is *p−a*

- For any **prime number** *p*, (*[1, p-1],*●) is a commutative group
- 1 is the identity element
- For any element *a* there exist its inverse *ab=1 (mod p)*

The Finite Field F_p

e.g. F_7

- ({0, 1, 2, 3, 4, 5, 6}, +) is a commutative group
- 4+3 %7 = 0 --> 3 is the additive opposite of 4

- ({1, 2, 3, 4, 5, 6}, ●) is a commutative group
- 4*2 %7 = 1 --> 2 is the multiplicative inverse of 4

Division must be interpreted as multiplication by the inverse

- 4 = 2*2 %7 --> 2 is a (even) square root of 4
- 4 = 5*5 %7 --> 5 is a (odd) square root of 4

-2 %7 = 5

odd root + even root = 7

The Finite Field F_7

	opposite	inverse	odd sqrt	even sqrt
0	0	#N/A	0	0
1	6	1	1	6
2	5	4	3	4
3	4	5	#N/A	#N/A
4	3	2	5	2
5	2	3	#N/A	#N/A
6	1	6	#N/A	#N/A

Homework #2

- Calculate for F_{19} and F_{23} the table of opposites, inverses, and square roots

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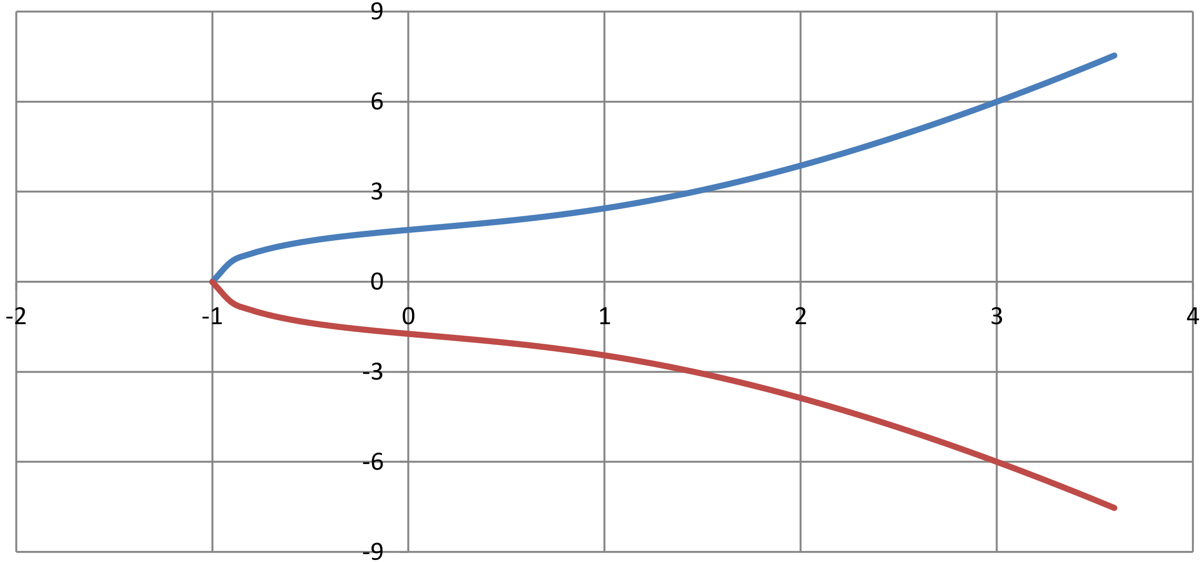
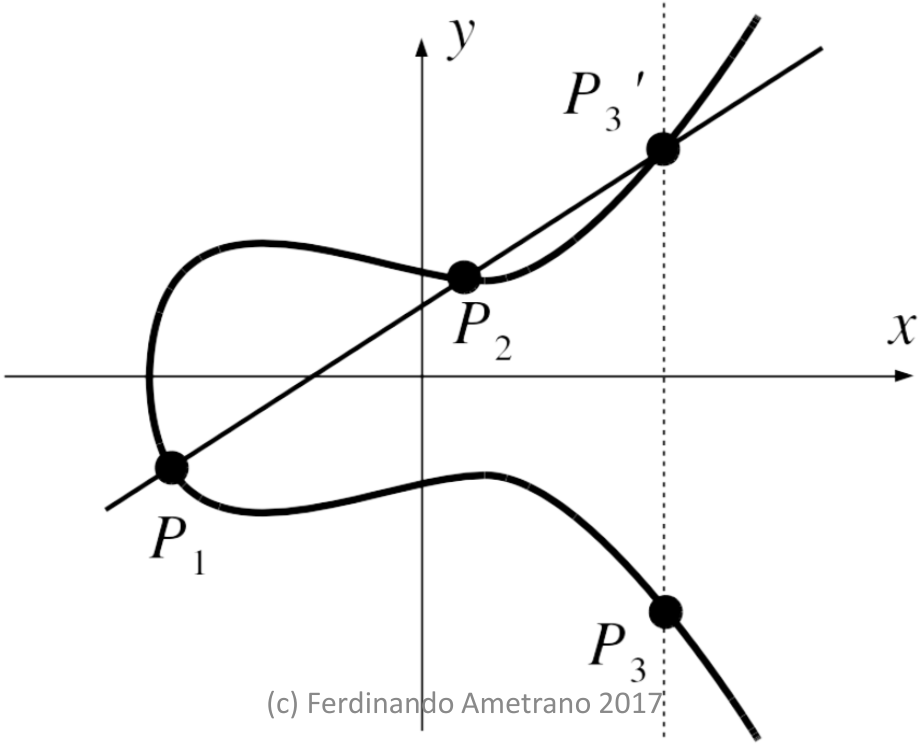
Elliptic Curves $y^2 = x^3 + ax + b$

e.g. $y^2 = x^3 + 2x + 3$



Point Addition $P_1 + P_2 = P_3$

Source: Pedro Franco, “Understanding Bitcoin”, Wiley



Point Addition $P_1 + P_2 = P_3$

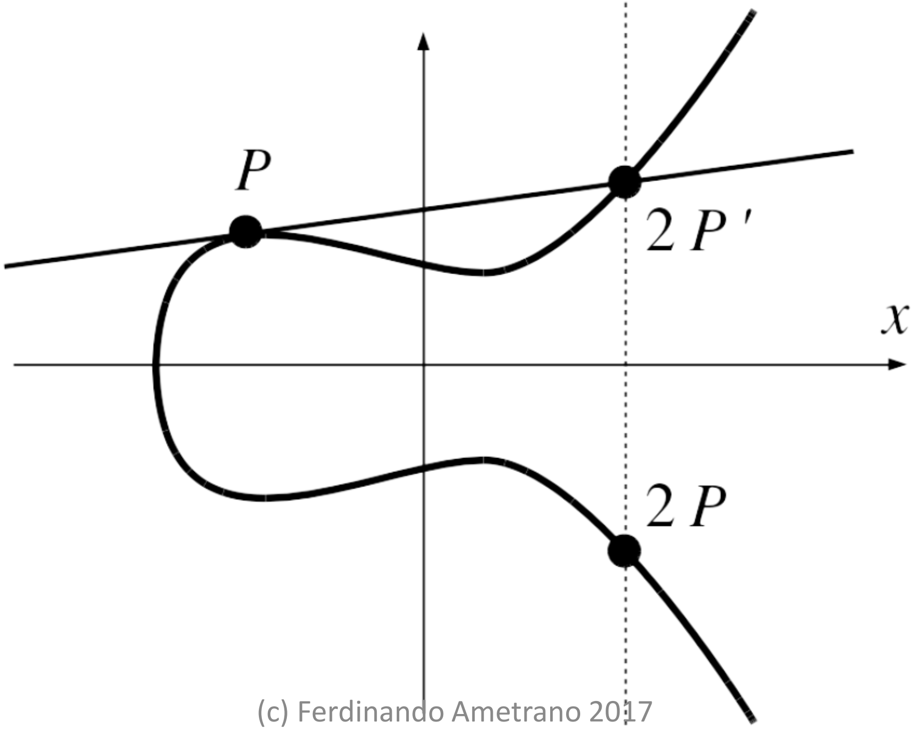
$$P_i = (x_i, y_i)$$

$$x_3 = \left(\frac{y_2 - y_1}{x_2 - x_1}\right)^2 - x_2 - x_1$$

$$y_3 = \frac{y_2 - y_1}{x_2 - x_1}(x_1 - x_3) - y_1$$

Point Doubling

Source: Pedro Franco, “Understanding Bitcoin”, Wiley



Point Doubling $P_1 + P_1 = P_3$

$$P_i = (x_i, y_i)$$

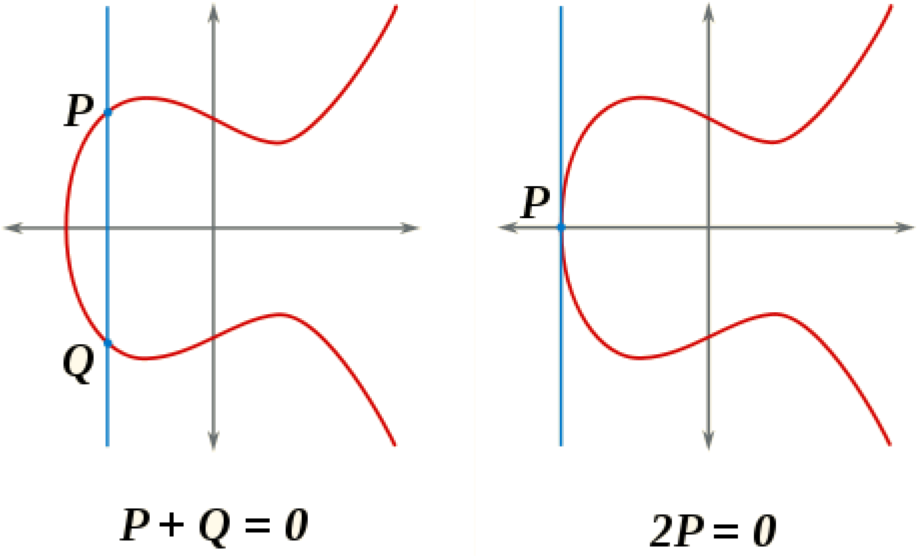
$$x_3 = \left(\frac{3{x_1}^2 + a}{2y_1}\right)^2 - 2x_1$$

$$y_3 = \frac{3{x_1}^2 + a}{2y_1}(x_1 - x_3) - y_1$$

Infinity Point (Group Identity, Neutral Element)

- Zero in additive notation
- Identity in multiplicative notation
- Sometime also indicated as ∞

- $P + Q = 0 \rightarrow Q = -P$



Source:
https://en.wikipedia.org/wiki/Elliptic_curve#/media/File:ECClines.svg

EC Commutative Group

- Points on an elliptic curve with the *addition operation* form a commutative group
- The point at infinity is the neutral element

- Arbitrarily named addition: it is the *group operation* and could have been called multiplication instead
- In multiplicative notation doubling would have been called squaring

Multiplication $nP = Q$
Double and Add Algorithm

e.g. $947 = 2^0 + 2^1 + 2^4 + 2^5 + 2^7 + 2^8 + 2^9$

$947P = P + 2P + 16P + 32P + 128P + 256P + 512P$

9 doublings and 6 additions: polynomial in the number of bits representing *n*. Much better than 946 additions!

```
def pointMultiply(n, P):
  if n==1:
    return P
  elif n%2==1: # addition when n is odd
    return pointAdd(P, pointMultiply(n-1, P))
  else:        # doubling when n is even
    return pointMultiply(n/2, pointDouble(P))
```

https://en.wikipedia.org/wiki/Elliptic_curve_point_multiplication

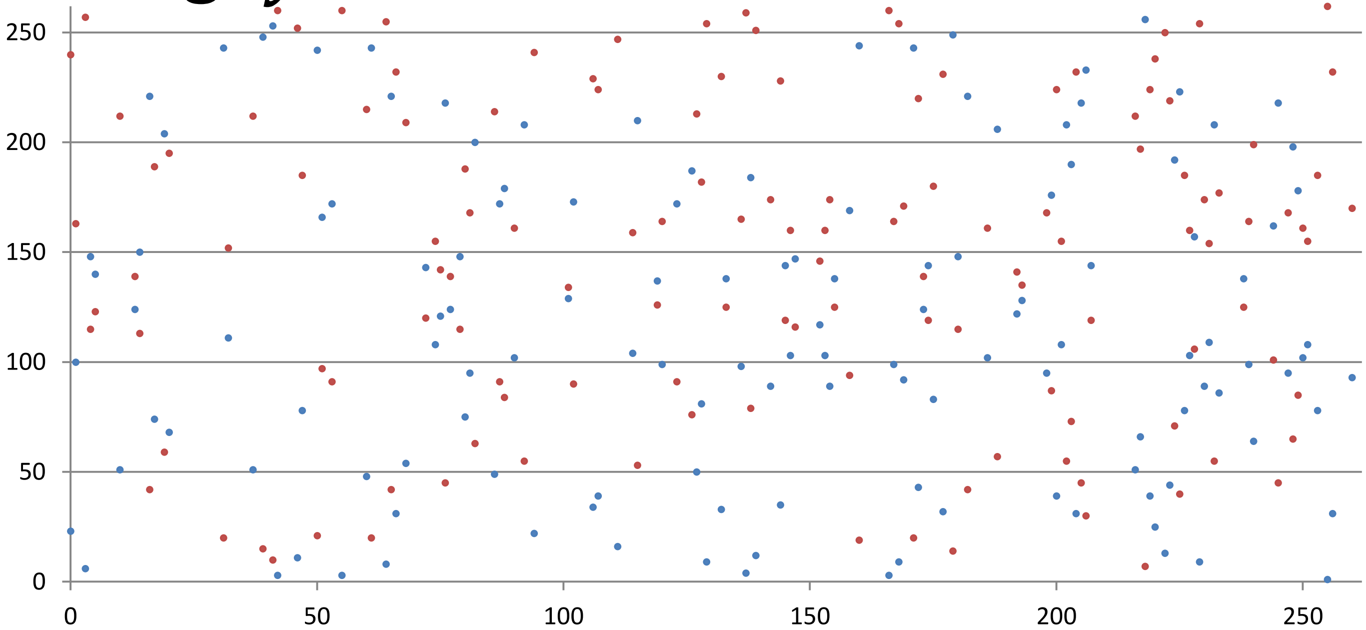
Discrete Logarithm One Way Function

- With a known n it is easy and fast to compute
$$Q = nP = P + \ldots + P$$
- For $n \in [1, N-1]$, to infer n from (P, Q) is exponential in the number of bits representing N
- This inverse problem is known as **discrete logarithm**, computationally unfeasible for large N
- Multiplicative notation would have been $Q = P^n$, leading to usual logarithm definition: $n = \log_P Q$

Elliptic Curves Over a Finite Field F_p

$$y^2 = x^3 + ax + b \text{ mod } p$$

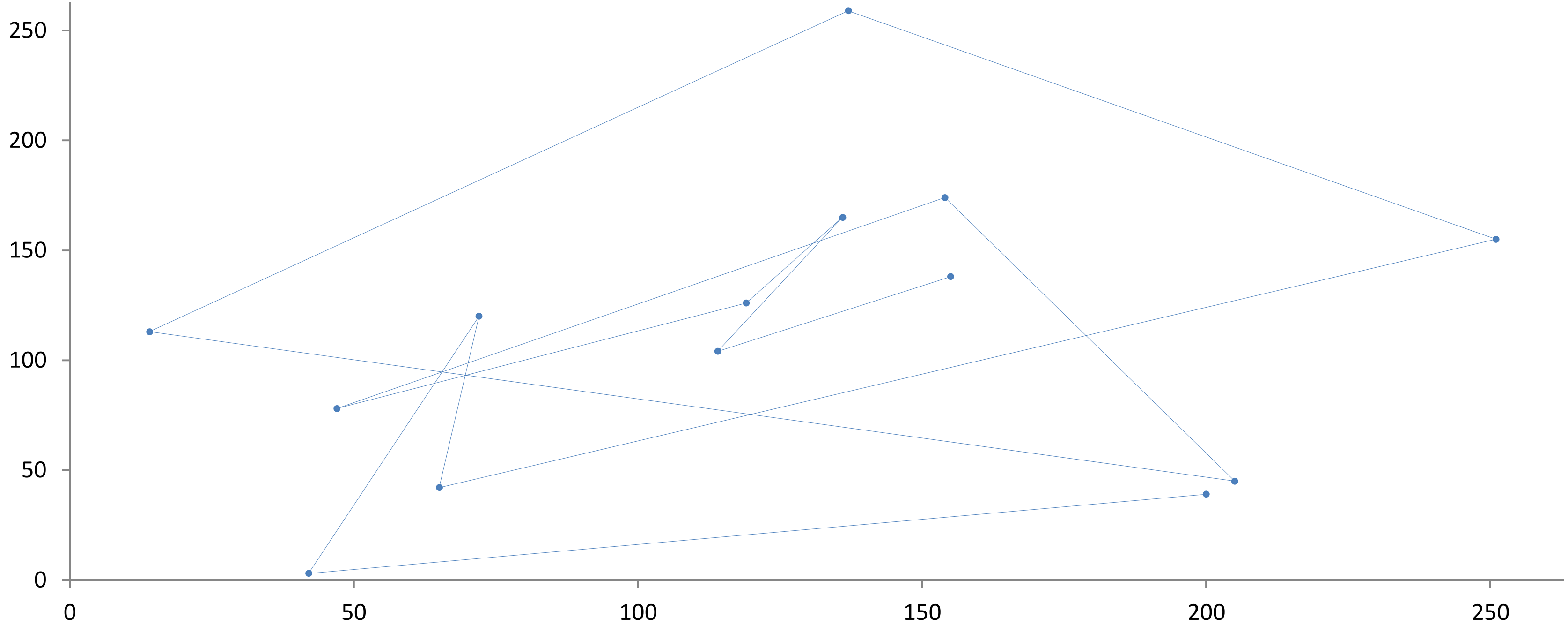
e.g. $y^2 = x^3 + 2x + 3 \quad \text{mod } 263$



Finite Cyclic Group

- An elliptic curve defined *over a finite field* F_p is also a finite cyclic group
- Starting with an initial generator point *G* in the curve and adding this point successively, all the N points in the group are recovered
- Any point can be reached quickly from a predecessor if the number of steps is known
- $y^2 = x^3 + 2x + 3$ over F_{263} has 270 points: 269 affine points including $P = (262, 0)$ and the point at infinity

$y^2 = x^3 + 2x + 3$ over F_{263}
First 14 points starting with (200, 39)



Elliptic Koblitz Curve secp256k1 Domain Parameters

- F_p is defined by p = 2^256 – 2^32 – 2^9 – 2^8 – 2^7 – 2^6 – 2^4 – 1, i.e.
  p = `FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE FFFFFC2F`
- The elliptic curve defined over F_p is $y^2 = x^3 + 7$
- The generation point G =
  `(79BE667E F9DCBBAC 55A06295 CE870B07 029BFCDB 2DCE28D9 59F2815B 16F81798,`
  `483ADA77 26A3C465 5DA4FBFC 0E1108A8 FD17B448 A6855419 9C47D08F FB10D4B8)`
- Finally the order of G:
  `FFFFFFFF FFFFFFFF FFFFFFFF FFFFFFFE BAAEDCE6 AF48A03B BFD25E8C D0364141`

SECG, *SEC 2: Recommended Elliptic Curve Domain Parameters*, http://www.secg.org/sec2-v2.pdf

https://en.bitcoin.it/wiki/Secp256k1

Elliptic Curve Private/Public Key

- A public key is one point $PubKey$ on the elliptic curve
- A private key is the number $prKey$ of additive steps from the generator point G to arrive at point $PubKey$

$$PubKey = prKey\ G$$

- In multiplicative notation $prKey$ is called *secret exponent*

$$PubKey = G^{prKey}$$

https://en.wikipedia.org/wiki/Elliptic_curve_cryptography

Homework #3

- Create a spreadsheet for

$$y^2 = x^3 + 2x + 2 \text{ mod } 17$$

  - List all its points
  - It does not have subgroups, why?
- Create a spreadsheet for

$$y^2 = x^3 + 4x + 20 \text{ mod } 29$$

  - List all its points
  - What is the order of the group with generator (8,10)?

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Asymmetric Cryptography Families Key Generation Algorithms

- Integer factorization (1977), based on the difficulty of factoring large integers (e.g. RSA)
- Discrete Logarithm (1976), based on the intractability of the discrete logarithm problem on finite cyclic groups (e.g. Diffie and Hellman)
- Elliptic Curve (1985), based on the difficulty of computing the generalized logarithm problem on an elliptic curve (e.g. Bitcoin)

Break Elliptic Curve Cryptography

- The best known algorithms to break the EC discrete logarithm problem take steps proportional to $\sqrt{2^m}$ where m is the number of bits of the key
- secp256k1 uses 256bit keys: 2^{128} steps are needed to break it
- An EC computation takes 1 million CPU cycles. A 3GHz CPU is able to process $2^{11.55}$ EC computations per second
- A CPU can break the EC in $2^{116.45}$ seconds, or about $2^{91.54}$ years, i.e. about 3599861590422752583114293248 years
- Throwing a million CPUs at the problem would reduce the time by a million, leaving it at 3599861590422752583114 years, roughly 260,859,535,537 times the age of the universe

Key Size At Comparable Security Levels

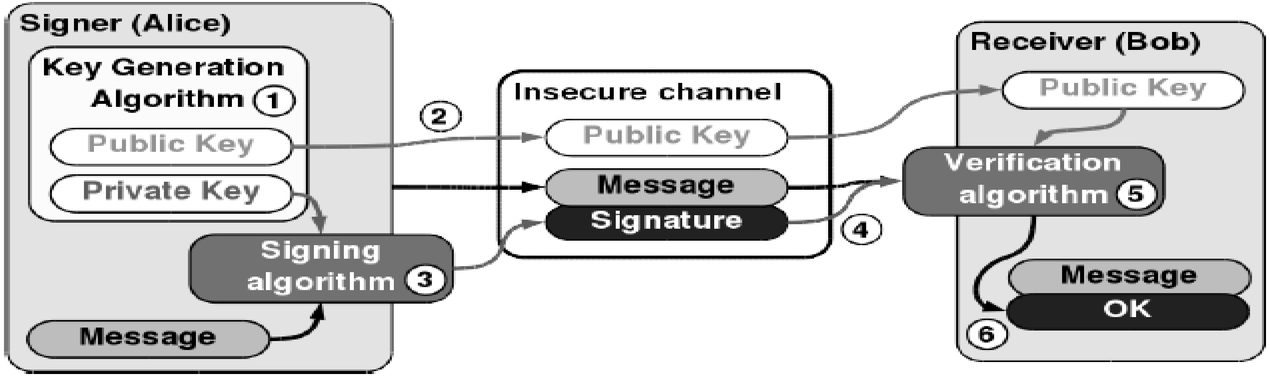
Source: Pedro Franco, “Understanding Bitcoin”, Wiley

	Security level (bits)			
	80	128	192	256
RSA	1024	3072	7680	15360
DL	1024	3072	7680	15360
ECC	160	$\boxed{256}$	384	512
Symmetric	80	128	192	256

Digital Signature Protocol

- Public-key algorithm + digital signature scheme
- Message is only authenticated, not encrypted

Source: Pedro Franco, "Understanding Bitcoin", Wiley

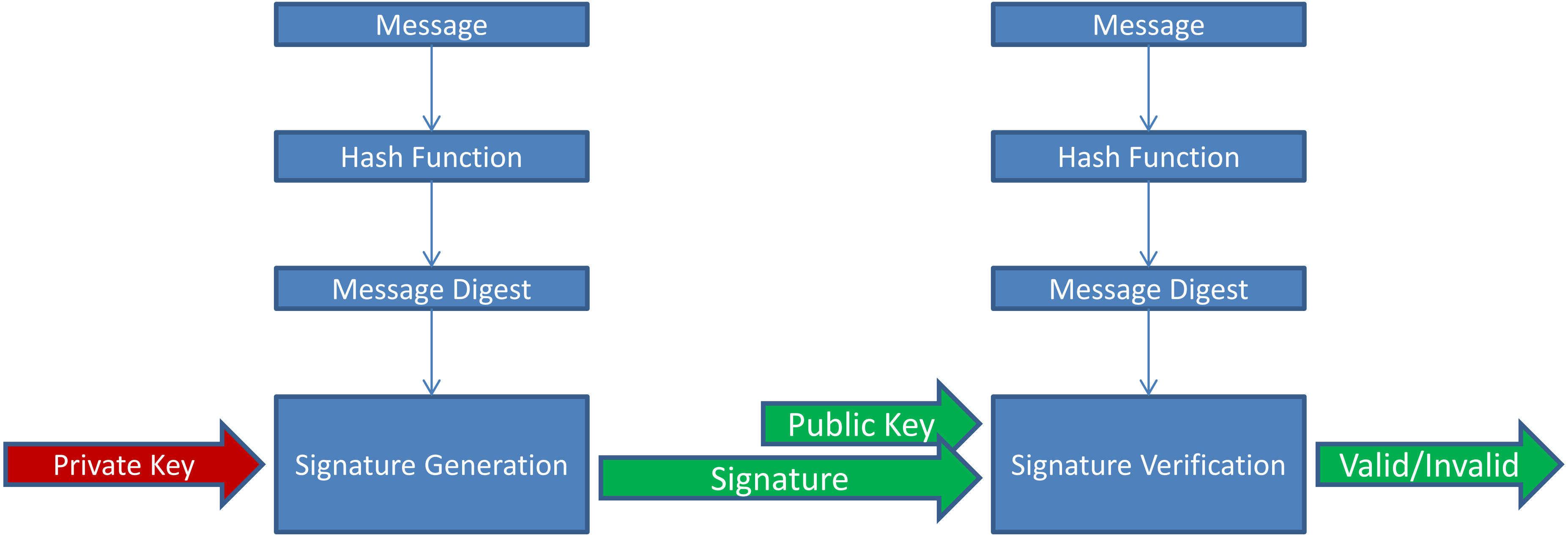


Signing The Message Digest

- Problem: signature generation/verification is quite slow: message length can be a problem
- Solution: sign the hash value of the message $h(msg)$, whose length is independent from the message's size
- This is also useful if, for some reason, the msg is to be revealed only after signature verification
  - If the msg can take only few values (e.g. {tail, head}), it can be concealed using an ephemeral number k: sign and reveal $h(msg|k)$

Digital Signature Process

- Signature Generation
- Signature Verification



Digital Signature Algorithms

- **RSA**, the most widely used
- **Elgamal** signature. It has little use being computationally intensive and having large signature
- Digital Signature Algorithm (**DSA**), quicker and smaller than RSA: widely used
- **Schnorr** signature, the simplest scheme. Signing and verification are computationally efficient, signature is small. Not used so far because of patents

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EC DSA: Generation

1. Choose a nonce as secret ephemeral key
$$0 < k < order$$
2. Compute $\mathrm{K} = (x_K, y_K) = kG$
3. $s = (h + x_K\, prKey)k^{-1} \bmod order$

If $x_K = 0$ or $s = 0$ (extremely unlikely) then restart with another k

The signature of h is (x_K, s)

k **<u>must be secret</u>**: if revealed, $prKey = (sk - h){x_K}^{-1} \bmod order$

EC DSA: Verification

Steps for the (x_K, s) verification of h

1. $u = h\, s^{-1} \bmod order$
2. $v = x_K\, s^{-1} \bmod order$
3. $(x, y) = uG + vPubKey$
4. The signature is valid if $x = x_K \bmod order$

Malleability: if (x_K, s) is a valid signature of h, then also $(x_K, order - s)$ is a valid signature

Note that assuming a valid signature, $PubKey = ((x_K, y_{odd\ or\ even}) - uG)/v$

EC DSA: Correctness Proof

$x = x_K \bmod order$ if $uG + vPubKey = kG$

1. $(u + v\, prKey)G = kG$
   from public key definition
2. $(h\, s^{-1} + x_K\, s^{-1}\, prKey)G = kG$
   from signature verification [2] and [3]
3. $(h + x_K prKey)s^{-1}G = kG$
4. $(h + x_K prKey)(h + x_K prKey)^{-1}kG = kG$
   from signature generation [5]
5. $kG = kG$

Ephemeral Key Used for Signing

The ephemeral key k must remain secret: therefore, it is **to be used only once** per $prKey$. Reusing the nonce k would reveal it:

$$s_1 = (h_1 + x_K prKey)k^{-1} \bmod order$$
$$s_2 = (h_2 + x_K prKey)k^{-1} \bmod order$$
$$k = (h_1 - h_2)/(s_1 - s_2) \bmod order$$

Ask Sony PS3 developers and bitcoin owners using Android Wallet in 2013…

Solution: use a deterministically different k for each msg, with k remaining secret because of $prKey$ salting (RFC6979):

$$k = h(msg|prKey)$$

EC Schnorr SA: Generation

1. Choose a nonce as secret ephemeral key
$$0 < k < order$$
2. Compute $\mathrm{K} = kG$
3. $s = k - h\, prKey \mod order$

If $s = 0$ (extremely unlikely) then restart with another k

The signature of h is (K, s)

k **must be secret**: if revealed, $prKey = (k - s)h^{-1} \mod order$

EC SSA: Verification

Steps for the (K, s) verification of h

1. $V = K - h\ PubKey$
2. The signature is valid if $sG = V$

Schnorr signature is **not** malleable

Note that assuming a valid signature, $PubKey = (K - sG)/h$

EC SSS: Correctness Proof

$$sG = V$$

$$(k - h\, prKey)G = K - h\, PubKey$$

$$(k - h\, prKey)G = kG - h\, prKeyG$$

$$(k - h\, prKey)G = (k - h\, prKey)G$$

Ephemeral Key Used for Signing

For Schnorr too, the ephemeral key k must remain secret: therefore, it is **<u>to be used only once</u>** per $prKey$. Reusing the nonce k would reveal it:

$$s_1 = k - h_1\, prKey \bmod order$$
$$s_2 = k - h_2\, prKey \bmod order$$
$$prKey = (s_1 - s_2)(h_2 - h_1)^{-1} \bmod order$$

Again: for each msg use a deterministically different k that remains secret because of $prKey$ salting (RFC6979):

$$k = h(msg|prKey)$$

Homework #4

Calculate the public key(s) from this valid signature:

```
*** message hash and its signature
h: 9788fd27b3aafd1bd1591a1158ce2d8bdc37ab4040dddb64e64d17616e69ce2b
x: 2ab2a733eae4e67e06611aba01345b85cdd4f5ad44f72e369ef0dd640424dbfa
s: 27598b74fc77ee8aaa6f56d3f976949ac2c2f5849c98412d10ce02c170262be8
```

A second signature is computed in error using the same ephemeral key. Calculate the private key:

```
*** another message hash and its signature
h2: 7adb91982ec03ef87efcae7f0199aefa231d8855e0bd03319460e58c0bd18049
 x: 2ab2a733eae4e67e06611aba01345b85cdd4f5ad44f72e369ef0dd640424dbfa
s2: 691d6fb6b4b90a8358a3b1c241bbc53b5be5bf52196561dbe5270ba1f54815a2
```

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Public Key Compression

$$PubKey: prKeyG = (x, y)$$

- **Un**compressed $PubKey$ representation is $04\ x\ y$

For every x, two y roots are possible:

$$y^2 = x^3 + 7$$

- Even y root, compressed $PubKey$ representation is $02\ x$
- Odd y root, compressed $PubKey$ representation is $03\ x$

Bitcoin Uses Base58 Digits

- Binary digits: 01
- Decimal digits: 0123456789
- Hexadecimal digits: 0123456789ABCDEF

- Base58 digits: all alphanumeric characters (numbers, uppercase, and lowercase) omitting 0 (zero), O (capital o), I (capital i) and l (lower case L)
123456789ABCDEFGHJKLMNPQRSTUVWXYZabcdefghijkmnopqrstuvwxy

Fewer digits to represent large numbers

Base58Check Encoding

includes a checksum algorithm, that helps detecting errors

1. Payload
2. Prefix + Payload
3. Prefix + Payload + Checksum
4. Base58Check encoding

Checksum Calculation

a. Prefix+Payload
b. SHA256(Prefix+Payload)
c. SHA256(SHA256(Prefix+Payload))
d. Checksum = first 4 bytes of the previous step

Base58 encoding can be decoded

Base58 Representation of Public/Private Keys

Base58 is used for compact representation of Bitcoin Public/Private Keys:

- Address: $PubKey$ in Base58 representation
- Wallet Import Format: $prKey$ in Base58 representation

From Uncompressed Public Key to Bitcoin Address (1/4)

Having a private EC key

`18E14A7B6A307F426A94F8114701E7C8E774E7F9A47E2C2035DB29A206321725`

1. Start from the associated public key in **un**compressed representation (65 bytes: 1 byte x04 prefix, 32 bytes corresponding to X coordinate, 32 bytes corresponding to Y coordinate)

   04
   `50863AD64A87AE8A2FE83C1AF1A8403CB53F53E486D8511DAD8A04887E5B2352`
   `2CD470243453A299FA9E77237716103ABC11A1DF38855ED6F2EE187E9C582BA6`

From Uncompressed Public Key to Bitcoin Address (2/4)

2. Perform SHA-256 hashing on the public key
   `600FFE422B4E00731A59557A5CCA46CC183944191006324A447BDB2D98D4B408`
3. Perform RIPEMD-160 hashing on the result of SHA-256
   `010966776006953D5567439E5E39F86A0D273BEE`
4. Add version byte in front of RIPEMD-160 hash (x00 for Main Network) to obtain the extended RIPEMD-160
   00`010966776006953D5567439E5E39F86A0D273BEE`

Note: step 2-3 make the PubKey->Address derivation not invertible

From Uncompressed Public Key to Bitcoin Address (3/4)

Base58**Check** encoding steps:

5. Perform SHA-256 hash on the extended RIPEMD-160
   `445C7A8007A93D8733188288BB320A8FE2DEBD2AE1B47F0F50BC10BAE845C094`
6. Perform SHA-256 hash on the result of the previous step
   `D61967F6``3C7DD183914A4AE452C9F6AD5D462CE3D277798075B107615C1A8A30`
7. Take the first 4 bytes of the second SHA-256 hash. This is the address checksum
   `D61967F6`

From Uncompressed Public Key to Bitcoin Address (4/4)

8. Add the 4 checksum bytes from stage 7 at the end of extended RIPEMD-160
   `00010966776006953D5567439E5E39F86A0D273BEE`**`D61967F6`**
9. Convert into a base58 string using Base58 encoding
   `16UwLL9Risc3QfPqBUvKofHmBQ7wMtjvM`

https://en.bitcoin.it/wiki/Technical_background_of_version_1_Bitcoin_addresses

From Compressed Public Key to Bitcoin Address

Using the same private EC key as in the previous slide
`18E14A7B6A307F426A94F8114701E7C8E774E7F9A47E2C2035DB29A206321725`

- Start from the associated public key in compressed representation (33 bytes: 1 byte x02 or x03, 32 bytes corresponding to X coordinate)
  02
  `50863AD64A87AE8A2FE83C1AF1A8403CB53F53E486D8511DAD8A04887E5B2352`
- Arrive to a Base58 encoded bitcoin address
  `1PMycacnJaSqwwJqjawXBErnLsZ7RkXUAs`

Private Key (Uncompressed) *WIF*

Having a private EC key

0C28FCA386C7A227600B2FE50B7CAE11EC86D3BF1FBE471BE89827E19D72AA1D

- **Un**compressed Extended Key (x80 prefix, no suffix)

  800C28FCA386C7A227600B2FE50B7CAE11EC86D3BF1FBE471BE89827E19D72AA1D

- Base58 encode

  5HueCGU8rMjxEXxiPuD5BDku4MkFqeZyd4dZ1jvhTVqvbTLvyTJ

- When an address will be derived, it will be **un**compressed

Note: the process is invertible

Private Key (Compressed) *WIF*

Using the same private EC key as in the previous slide

`0C28FCA386C7A227600B2FE50B7CAE11EC86D3BF1FBE471BE89827E19D72AA1D`

- Compressed Extended Key (x80 prefix, x01 suffix)

  800C28FCA386C7A227600B2FE50B7CAE11EC86D3BF1FBE471BE89827E19D72AA1D**01**

- Base58 encode

  `KwdMAjGmerYanjeui5SHS7JkmpZvVipYvB2LJGU1ZxJwYvP98617`

- When an address will be derived, it will be compressed

Bitcoin Address Version (main net)

version	usage	leading symbol(s)
x00	pubkey hash (P2PKH address)	1
x05	script hash (P2SH address)	3
x80	prvkey (WIF, uncompressed pubkey)	5
x80	prvkey (WIF, compressed pubkey)	K or L
x0488B21E	BIP32 pubkey	xpub
x0488ADE4	BIP32 private key	xprv

Bitcoin Address Version (test net)

version	usage	leading symbol(s)
6F	pubkey hash	m or n
C4	script hash	2
EF	prvkey (WIF, uncompressed pubkey)	9
EF	prvkey (WIF, compressed pubkey)	c
x043587CF	BIP32 pubkey	tpub
x04358394	BIP32 private key	tprv

Homework #5

With the private key obtained with the previous homework:

1. Calculate the **un**compressed WIF
2. Calculate the compressed WIF
3. Calculate the bitcoin address from **un**compressed public key
4. Calculate the bitcoin address from compressed public key

Check the results at www.bitaddress.org (Wallet Details)

Bibliography

- Christof Paar and Jan Pelzl, “Understanding Cryptography”, Springer, chapter 8, 9, 10, 11
- Pedro Franco, “Understanding Bitcoin”, Wiley, chapter 5, 7 “Public Key Cryptography”
- Andreas Antonopoulos, “Mastering Bitcoin”, O'Reilly, chapter 4 “Keys, Addresses, Wallets” https://goo.gl/tNBu5w
- A. Narayanan et al., “Bitcoin and Cryptocurrencies Technologies”, Princeton, chapter 1
- NIST, Digital Signature Standard, http://nvlpubs.nist.gov/nistpubs/FIPS/NIST.FIPS.186-4.pdf

Homework #6

Please provide few paragraphs of feedback on this lesson: its strongest and weakest points, if and where is would need improvements, what had you struggling the most, what got you most excited, etc.