KALE: A High-Degree Algebraic-Resistant Variant of The Advanced Encryption Standard

Spendon Gavekort

c/o Vakiopaine Bar
Kauppakatu 6, 40100 Jyväskylä FINLAND
mjos@iki.fi

Abstract. We have discovered that the S-Box of AES is in fact affine-equivalent to a trivial algebraic transform of low degree and short cycle; inversion operation in the finite field $\mathsf{GF}(2^8)$. This transformation is an involution and hence has an unacceptably low cycle length. Furthermore the algebraic degree of the underlying permutation is extremely low, lower than zero: $\deg x^{-1} = -1$, making it vulnerable to Euclid-Courtois-Gavekort-Schneier (ECGS) algebraic attacks. We propose KALE, a more secure variant that replaces the algebraic component of the S-Box with square root operation $\sqrt{x}$ in the same finite field. All other parameters remain the same. We discuss the obvious security advantages of the new variant and show that it is provably secure.

Keywords: AES, KALE, Algebraic Cryptanalysis, Serpessence.

1 Introduction

The Rijndael algorithm was adopted as Advanced Encryption Standard (AES) by U. S. NIST in 2001 in FIPS-197 [6]. AES is currently widely deployed around the world and frequently used by unsuspecting users. In this note we show that a key component of AES in fact contains a backdoor the allows the Belgian Government and The Catholic Church (the forces behind Rijndael / AES design, who obviously hid the backdoor in the cipher) to secretly eavesdrop on all AES communications. This is why the National Security Agency has been actively promoting the use of AES in public networks [1, 5, 8, 10].

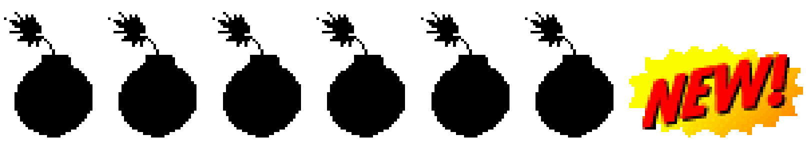


Fig. 1. This paper describes an extremely efficient potential algebraic attack against the U. S. Advanced Encryption Standard (AES).

2 Key Observation

We have discovered that the AES S-Box is in fact affine equivalent to a low-degree algebraic function. Shadowy agents who designed AES have clearly tried to hide this fact by masking the function with a simple convolutional affine transform:

$$\begin{bmatrix} b'_0 \\ b'_1 \\ b'_2 \\ b'_3 \\ b'_4 \\ b'_5 \\ b'_6 \\ b'_7 \end{bmatrix} = \begin{bmatrix} 1\,0\,0\,0\,1\,1\,1\,1 \\ 1\,1\,0\,0\,0\,1\,1\,1 \\ 1\,1\,1\,0\,0\,0\,1\,1 \\ 1\,1\,1\,1\,0\,0\,0\,1 \\ 1\,1\,1\,1\,1\,0\,0\,0 \\ 0\,1\,1\,1\,1\,1\,0\,0 \\ 0\,0\,1\,1\,1\,1\,1\,0 \\ 0\,0\,0\,1\,1\,1\,1\,1 \end{bmatrix} \cdot \begin{bmatrix} b_0 \\ b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \\ b_7 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \tag{1}$$

We may write the AES S-Box as composite function $S = A \circ S'$. When the bitwise affine transform $A : b \mapsto b'$ of Equation 1 is removed, we discovered that the underlying transform is in fact the inverse operation $S' : x \mapsto 1/x$ in the finite field $\mathsf{GF}(2^8)$ defined by irreducible polynomial $p(x) = x^8 + x^4 + x^3 + x + 1$.

3 Security Analysis

We see that the algebraic degree of the underlying transform is $\deg S' = -1$. This degree is very low, in fact lower than zero, the degree recommended by the Chinese Government ("sinkhole transform"). For reference, the U.S. National Security Agency recommends degree 1 ("the identity transform") for all but the most confidential data. AES has been clearly designed to offer even lower security than these proposals against Algebraic Attacks of Courtois [3].

Bruce Schneier (in joint work with Euclid) has developed an algorithm to compute multiplicative inverses in rings mod n, even when the factorization of n is not known. We see that $e = -1$ is clearly unsuitable for modern cryptography [11]. We call this the Euclid-Courtois-Gavekort-Schneier (ECGS) Algebraic Attack on AES. Based to extrapolations from reduced versions, we estimate that attack complexity against AES-128 is $2^{127.92653589793238462643383279502884 2}$ with 2^{127} precomputation.

4 New Technique: Irrational Algebraic Permutations

From RSA cryptanalysis we know that the exponent $e > 1$ must be coprime with $\phi(\mathbb{F})$, in other words $\gcd(e, 255) = 1$. This immediately rules out all smallest primes 3 and 5. However, the security of RSA has never been shown to be even equivalent to factoring, unlike the Rabin cryptosystem. RSA should not be considered to be a secure algorithm since (according to CRYPTO Program Committee) a cryptosystem cannot be secure unless it is provably secure under some arbitrary set of assumptions which can be freely chosen by the author.

Table 1. The S-Box of KALE.

	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9	xA	xB	xC	xD	xE	xF
0x	63	7C	FF	E0	5D	42	C1	DE	6D	72	F1	EE	53	4C	CF	D0
1x	1F	00	83	9C	21	3E	BD	A2	11	0E	8D	92	2F	30	B3	AC
2x	48	57	D4	CB	76	69	EA	F5	46	59	DA	C5	78	67	E4	FB
3x	34	2B	A8	B7	0A	15	96	89	3A	25	A6	B9	04	1B	98	87
4x	9B	84	07	18	A5	BA	39	26	95	8A	09	16	AB	B4	37	28
5x	E7	F8	7B	64	D9	C6	45	5A	E9	F6	75	6A	D7	C8	4B	54
6x	B0	AF	2C	33	8E	91	12	0D	BE	A1	22	3D	80	9F	1C	03
7x	CC	D3	50	4F	F2	ED	6E	71	C2	DD	5E	41	FC	E3	60	7F
8x	02	1D	9E	81	3C	23	A0	BF	0C	13	90	8F	32	2D	AE	B1
9x	7E	61	E2	FD	40	5F	DC	C3	70	6F	EC	F3	4E	51	D2	CD
Ax	29	36	B5	AA	17	08	8B	94	27	38	BB	A4	19	06	85	9A
Bx	55	4A	C9	D6	6B	74	F7	E8	5B	44	C7	D8	65	7A	F9	E6
Cx	FA	E5	66	79	C4	DB	58	47	F4	EB	68	77	CA	D5	56	49
Dx	86	99	1A	05	B8	A7	24	3B	88	97	14	0B	B6	A9	2A	35
Ex	D1	CE	4D	52	EF	F0	73	6C	DF	C0	43	5C	E1	FE	7D	62
Fx	AD	B2	31	2E	93	8C	0F	10	A3	BC	3F	20	9D	82	01	1E

In [9] Rabin shows that computation of square root in a finite ring is actually equivalent to factoring a large composite number. We therefore choose $x \mapsto \sqrt{x}$ as our fundamental transform. The main drawback of the Rabin cryptosystem is that in prime fields each each element x has two square roots $\pm\sqrt{x}$, and effort has to be made in order to identify the correct root. This would also make the S Box non-surjective, and therefore vulnerable to Impossible Differential Attacks such as the Impossible Boomerang Attack (IBA) which are especially dangerous as impossible boomerangs are truly impossible while regular boomerangs are possible boorangs [2].

However, finite fields of characteristic two (such as our $\mathsf{GF}(2^8)$) have an unique $\sqrt{x}$ for each x, including -1. Since $\sqrt{-1}$ is clearly defined, we call the resulting S-Box an **Irrational Permutation** (IP).

5 Improved High-Degree AES Variant KALE

Figure 4 shows the S-Box used by KALE. The S-Box is the only difference between KALE and AES. Appendix A gives a full trace of KALE128 execution that can be used to verify implementation correctness. There are no other modifications to the Key Schedule, number of rounds, etc. Note that the very first elements are unchanged since zero mapped to zero in the AES inversion and $\sqrt{0} = 0$, and furthermore $1^{-1} = \sqrt{1}$. The same masking constant `0x63` is used.

The algebraic degree of $\sqrt{x}$ in real and complex fields is $\frac{1}{2}$, but in a multiplicative subgroup of finite field of size $2n$ it is actually n. Therefore the degree is actually 128. We may write interchangeably $\sqrt{x} = x^{128}$. The cycling properties are also greatly improved for S'.

6 Conclusions and Discussion

We have discovered that the AES S-Box can be decomposed into two affine and algebraic parts: $S = A \circ S'$, where A is an binary affine transform and S' is a low degree inversion transform $S' : x \mapsto x^{-1}$. The degree of hidden algebraic transform is therefore $\deg S' = -1$. Having a degree lower than zero is of course extremely dangerous and makes AES immediately highly vulnerable to potential advanced Euclid-Courtois-Gavekort-Schneier (ECGS) attacks.

We have proposed a minor tweak to AES that replaces the algebraic transform $S' : x \mapsto x^{-1}$ with a secure discrete square root transform $S' : x \mapsto \sqrt{x}$. The square root transform has been proven secure by Rabin. Furthermore its algebraic degree is 128, making it highly resistant to ECGS attacks..

As the ECGA attack is based on Algebraic Cryptanalysis and since the culprits behind the AES backdoor allegedly worked for a Catholic University in Leuven, Belgium, we suspect that Papal and Belgian Secret Service pressure explains why the Courtois' highly effective AES attacks were never fully disclosed in the open literature [3].

The security of KALE against post-quantum [4], neuromorphic and optogenetic [7], and other postmodern attacks is unknown at this point. However, the use of complex numbers should rule out any real or rational quantum attacks.

References

1. Burgin, K., and Peck, M. Suite B Profile for Internet Protocol Security (IPsec). IETF RFC 6380, October 2011.
2. Choy, J., and Yap, H. Impossible boomerang attack for block cipher structures. In *IWSEC 2009* (2009), T. Takagi and M. Mambo, Eds., vol. 5824 of *LNCS*, Springer, pp. 22–37.
3. Courtois, N. How fast can be algebraic attacks on block ciphers? IACR ePrint 2006/168, `eprint.iacr.org/2006/168`, May 2006.
4. Finiasz, M. Syndrome based collision resistant hashing. In *Post-Quantum Cryptography: Second International Workshop, PQCrypto 2008* (2008), J. Buchmann and J. Ding, Eds., vol. 5299 of *LNCS*, Springer, pp. 137–147.
5. Igoe, K. Suite B Cryptographic Suites for Secure Shell (SSH). IETF RFC 6239, May 2011.
6. NIST. Advanced Encryption Standard (AES). FIPS 197, 2001.
7. NSA. Annual report 2014 NSA TC, IEEE CAS society. `http://ewh.ieee.org/soc/icss/committees/nsatc/reports/2014_NSATC_Annual_Report.pdf`, May 2014.
8. NSA. Suite B Cryptography. `www.nsa.gov/ia/programs/suiteb_cryptography`, June 2014.
9. Rabin, M. C. Digitalized signatures and public-key functions as intractable as factorization. Tech. Rep. 212, Massachusetts Institute of Technology / LCS, 1979.
10. Salter, M., and Housley, R. Suite B Profile for Transport Layer Security (TLS). IETF RFC 6460, January 2012.
11. Schneier, B. *Applied Cryptography.* John Wiley & Sons, 1994.

A Trace of Execution for KALE128

The test trace contains intermediate values and all subkeys. The format is similar to that of Appendix C.1 of FIPS 197 [6].

```
PLAINTEXT BLOCK     00 11 22 33  44 55 66 77  88 99 AA BB  CC DD EE FF
SECRET KEY DATA     00 01 02 03  04 05 06 07  08 09 0A 0B  0C 0D 0E 0F

round[ 0].input     00 11 22 33  44 55 66 77  88 99 AA BB  CC DD EE FF
round[ 0].k_sch     00 01 02 03  04 05 06 07  08 09 0A 0B  0C 0D 0E 0F
round[ 1].start     00 10 20 30  40 50 60 70  80 90 A0 B0  C0 D0 E0 F0
round[ 1].s_box     63 1F 48 34  9B E7 B0 CC  02 7E 29 55  FA 86 D1 AD
round[ 1].s_row     63 E7 29 AD  9B 7E D1 34  02 86 48 CC  FA 1F B0 55
round[ 1].s_col     70 60 3A 2A  4A 3B 00 71  11 01 5B 4B  2B 5A 61 10
round[ 1].k_sch     4D CE D2 50  49 CB D4 57  41 C2 DE 5C  4D CF D0 53
round[ 2].start     3D AE E8 7A  03 F0 D4 26  50 C3 85 17  66 95 B1 43
round[ 2].s_box     1B 85 DF 5E  E0 AD B8 EA  E7 79 23 A2  12 5F 4A 18
round[ 2].s_row     1B AD 23 18  E0 79 4A 5E  E7 5F DF EA  12 85 B8 A2
round[ 2].s_col     E1 27 D8 93  44 92 EF B4  01 C9 38 7D  AA 72 01 54
round[ 2].k_sch     06 48 B6 E4  4F 83 62 B3  0E 41 BC EF  43 8E 6C BC
round[ 3].start     E7 6F 6E 77  0B 11 8D 07  0F 88 84 92  E9 FC 6D E8
round[ 3].s_box     6C 03 1C 71  EE 00 2D DE  D0 0C 3C E2  C0 9D 9F DF
round[ 3].s_row     6C 00 3C DF  EE 0C 9F 71  D0 9D 1C DE  C0 03 2D E2
round[ 3].s_col     3B F7 6E 2D  3D 3D 54 58  C5 0B 0C 4D  51 53 A4 AA
round[ 3].k_sch     AC C8 D3 FC  E3 4B B1 4F  ED 0A 0D A0  AE 84 61 1C
round[ 4].start     97 3F BD D1  DE 76 E5 17  28 01 01 ED  FF D7 C5 B6
round[ 4].s_box     C3 87 7A 99  2A 6E F0 A2  46 7C 7C FE  1E 3B DB F7
round[ 4].s_row     C3 6E 7C F7  2A 7C DB 99  46 3B 7A A2  1E 87 F0 FE
round[ 4].s_col     A4 6C 57 B9  92 3D 4B F0  19 1C 74 D4  A0 FE 7B B2
round[ 4].k_sch     98 67 FC 79  7B 2C 4D 36  96 26 40 96  38 A2 21 8A
round[ 5].start     3C 0B AB C0  E9 11 06 C6  8F 3A 34 42  98 5C 5A 38
round[ 5].s_box     04 EE A4 FA  C0 00 C1 58  B1 A6 0A 07  70 D7 75 3A
round[ 5].s_row     04 00 0A 3A  C0 A6 75 FA  B1 D7 A4 58  70 EE C1 07
round[ 5].s_col     38 20 5E 72  E5 F2 99 67  E7 AB DD 0B  0F E8 0E B1
round[ 5].k_sch     3D 30 6C 43  46 1C 21 75  D0 3A 61 E3  E8 98 40 69
round[ 6].start     05 10 32 31  A3 EE B8 12  37 91 BC E8  E7 70 4E D8
round[ 6].s_box     42 1F A8 2B  AA 7D 5B 83  89 61 65 DF  6C CC 37 88
round[ 6].s_row     42 7D 65 88  AA 61 37 2B  89 CC A8 83  6C 1F 5B DF
round[ 6].s_col     EE 9F 76 D5  F0 1A D8 E5  6D 6A 90 F9  7D 60 BF 55
round[ 6].k_sch     6D AB CD 9C  2B B7 EC E9  FB 8D 8D 0A  13 15 CD 63
round[ 7].start     83 34 BB 49  DB AD 34 0C  96 E7 1D F3  6E 75 72 36
round[ 7].s_box     81 0A D8 8A  0B 06 0A 53  DC 6C 30 2E  1C ED 50 96
round[ 7].s_row     81 06 30 96  0B 6C 50 8A  DC ED D8 53  1C 0A 0A 2E
round[ 7].s_col     B5 4B 46 99  78 A9 42 2E  04 3D 6F EC  02 38 70 78
round[ 7].k_sch     13 7E FE 00  38 C9 12 E9  C3 44 9F E3  D0 51 52 80
```

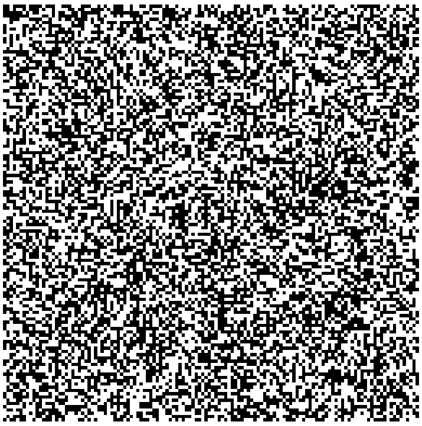
```
round[ 8].start     A6 35 B8 99  40 60 50 C7  C7 79 F0 0F  D2 69 22 F8
round[ 8].s_box     8B 15 5B 6F  9B B0 E7 47  47 DD AD D0  1A A1 D4 A3
round[ 8].s_row     8B B0 AD A3  9B DD D4 6F  47 A1 5B 47  1A 15 E7 D0
round[ 8].s_col     C8 BF 84 C6  EA 32 44 61  6A B4 99 BD  3C D2 B1 67
round[ 8].k_sch     6B 05 FC 86  53 CC EE 6F  90 88 71 8C  40 D9 23 0C
round[ 9].start     A3 BA 78 40  B9 FE AA 0E  FA 3C E8 31  7C 0B 92 6B
round[ 9].s_box     AA C7 C2 9B  44 01 BB CF  3F 04 DF 2B  FC EE E2 3D
round[ 9].s_row     AA 01 DF 3D  44 04 E2 9B  3F EE C2 CF  FC C7 BB 2B
round[ 9].s_col     AE EF 49 41  FD EA 29 07  5A 6A 04 E8  21 94 2B 35
round[ 9].k_sch     E7 CE AF 1D  B4 02 41 72  24 8A 30 FE  64 53 13 F2
round[10].start     49 21 E6 5C  49 E8 68 75  7E E0 34 16  45 C7 38 C7
round[10].s_box     8A 57 73 D7  8A DF BE ED  60 D1 0A BD  BA 47 3A 47
round[10].s_row     8A DF 0A 47  8A D1 3A D7  60 47 73 ED  BA 57 BE BD
round[10].k_sch     B5 52 9E 93  01 50 DF E1  25 DA EF 1F  41 89 FC ED
round[10].output    3F 8D 94 D4  8B 81 E5 36  45 9D 9C F2  FB DE 42 50
```

B Serpessence (Spoiler)

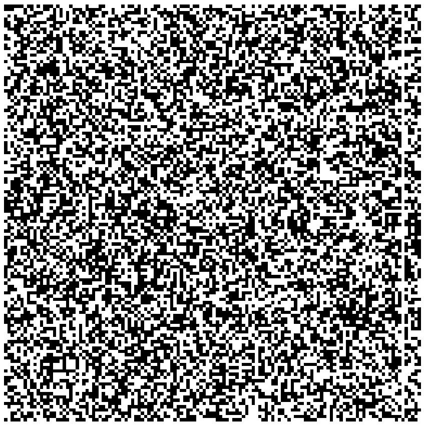
Since the squaring operation $x \mapsto x^2$ in any binary field is bitwise linear, so is its inverse, the "square root" operation. As AES contains no other nonlinear parts apart from its S-Box, the KALE transform is actually fully linear. The encryption of some given plaintext for any number of rounds can be written as a $128 \times k$ matrix M (and constant 128-bit vector addition) where k is the key size in bits. The secret key for (say) KALE128 with ten million rounds can be recovered with high probability via a single known plaintext block and a binary matrix multiplication with M^{-1}.

So unless a code reviewer is capable of checking the properties of a random-looking S-Box lookup table, you're screwed. The actual code is 100 % equivalent to AES. These matrices are really random-looking and would pass even a casual statistical analysis (all LFSRs are fully linear, and they pass most tests). However, this is an easily exploitable key-recovery backdoor.

For 10-round KALE with full key schedule, here's the matrix that can be used to "encrypt" a zero block with any 128-bit key and its inverse which yields the original key from ciphertext with 50 % probability (one bit would not invert in this experiment – random $n \times n$ binary matrices such as this one are fully invertible only with $p = \prod_{i=1}^{n}(1 - 2^{-i}) \approx 0.28879$ probability).



Encryption matrix M:
$\mathsf{KALE128}_K(0) = M \cdot K \oplus C$.



Key recovery matrix M^{-1}:
$M^{-1} \cdot (\mathsf{KALE128}_K(0) \oplus C) = K$.

Document version 20140909135900.