```
term variable
termvar, x
index,\ i,\ j,\ k,\ n,\ m,\ p
               ::=
                        bool
                        int
                        \langle T_1, \ldots, T_n \rangle
                        T@r
                        T@(r_1,\ldots,r_n)
                        T[r_1, \ldots, r_n]
                        T[r_1/r_1', \dots, r_n/r_n']
                        \mathbf{coloring}\left(r\right)
                        exists r_1, \ldots, r_n.(T_1, \ldots, T_m), \Phi, Q \to \mathbf{Tr}
fresh
               ::=
                iv
fns
               ::=
                        \mathbf{apply}\left(S,E\right)
                        valid\_interleave(S, C, E_1, ..., E_n)
                        \mathbf{taskid} \, \mathit{fresh}
                        mark\_coherence(E, M, \mathbf{taskid})
               ::=
r
                        nullr
                        not null r
               ::=
rr
                        [r_1, \ldots, r_n]
l
               ::=
                        \mathbf{nil}
Γ
               ::=
                        \{(e_1:T_1),\ldots,(e_n:T_n)\}
\Omega
               ::=
                        \{\omega_1, \ldots, \omega_n\}
\Omega^*
               ::=
                        \{\omega_1, \ldots, \omega_n\}
                        {\bf emptyOst}
rs
                       \{r_1, \ldots, r_n\}
\omega
               ::=
```

$$| r_1 \le r_2$$

$$| r_1 * r_2$$

$$\Phi^* \qquad ::= \\ | \quad \{\phi_1, \dots, \phi_n\} \\ | \quad \mathbf{emptyPst}$$

$$\begin{array}{ccc} \phi & & & & \\ & | & \mathbf{reads}\left(r\right) \\ & | & \mathbf{writes}\left(r\right) \\ & | & \mathbf{reducesid}\left(r\right) \end{array}$$

$$\begin{array}{ccc} Q & & ::= & \\ & | & \{q_1, \dots, q_n\} \end{array}$$

$$\begin{array}{ccc} q & & ::= & \\ & | & \mathbf{atomic}\left(r\right) \\ & | & \mathbf{simult}\left(r\right) \end{array}$$

$$\begin{array}{ccc} M & & ::= & \\ & | & M[[Q]] \end{array}$$

$$\begin{array}{ccc} L & & ::= & & \\ & | & \mathbf{nil} & \\ & | & L[(e_1,v_1),\ldots,(e_n,v_n)] \\ & | & L[v/id] \end{array}$$

$$K$$
 ::=

$$\begin{array}{ccc} H & & ::= & \\ & | & H(l) \end{array}$$

$$S$$
 ::=

$$C$$
 ::=

$$\begin{array}{cccc} E & & ::= & & & \\ & & | & [] & & \\ & | & [e] & & \\ & & E + + [e] & & \end{array}$$

$$\rho$$
 ::=

```
::=
                   bv
                   iv
                    \langle v_1, v_2 \rangle
                   null
                    \langle\langle\rho_1,\ldots,\rho_n,v\rangle\rangle
                   H
                   K
                   place
bv
                   true
                                                                                          constant true
                   false
                                                                                          constant false
iv
          ::=
                   0
                   S iv
ee
          ::=
                   (e_1, ..., e_n)
id
          ::=
          ::=
e
                   \mathbf{new}\ T@r
                   \mathbf{null}\ T@r
                   \mathbf{isnull}(e)
                   upregion (e, r_1, ..., r_n)
                   downregion (e, r_1, ..., r_n)
                   \mathbf{read}(e)
                   excl
                   \mathbf{read}(e_1, \mathbf{excl}, e_2, e_3)
                   write (e_1, e_2)
                   reduce (id, e_1, e_2)
                   reduceid (l, e_1, e_2, e_3)
                   \mathbf{newcolor}\ r
                   \mathbf{color}\left(e_1,e_2,e_3\right)
                    e_1 + e_2
                    e_1 < e_2
                   \mathbf{let}\,id:\,T=e_1\,\in\,e_2
                   if b then c0 else c1
                                                                                          conditional \\
                   id[r_1,\ldots,r_n](e_1,\ldots,e_m)
                   \mathbf{partition}\ r_p\ \mathbf{using}\ e_1\ \mathbf{as}\ r_1,\ ..\ , r_n\ \in\ e_2
                   \mathbf{pack}\ e_1\ \mathbf{as}\ T
                   unpack e_1 as id:T\in e_2
```

```
function id[r_1, ..., r_n](e_1, ..., e_m)
                         { function id_1 \ rr_1 \ ee_1, ..., function \ id_n \ rr_n \ ee_n}
                         e:T
                         place
                         bv
                         iv
                         \langle v_1, v_2 \rangle
                         null
                         l
                         \langle\langle\rho_1,\ldots,\rho_n,v\rangle\rangle
                         K
                         true
                                                                                                     constant true
                         false
                                                                                                     constant false
                         0
                         S iv
formula
                         judgement
                                                                                                     judgement
                         \neg formula
                                                                                           Μ
                                                                                                     negated formula
                         (formula)
                                                                                                     bracketed
                                                                                           Μ
                         \forall_i.\phi \in \Phi
                                                                                                     for all variables in domain of \Phi
                                                                                           M
                         \forall_i.\phi \in \Phi^*
                                                                                           Μ
                                                                                                     for all variables in domain of \Phi^*
                         \exists_i.\phi\in\Phi
                                                                                           Μ
                                                                                                     for all variables in domain of \Phi
                         \forall_i.\omega\in\Omega
                                                                                                     for all variables in domain of \Omega
                                                                                           M
                         \forall_i.\omega \in \Omega^*
                                                                                           Μ
                                                                                                     for all variables in domain of \Omega^*
                         \exists_i.\omega\in\Omega
                                                                                                     for all variables in domain of \Omega
                                                                                           M
                         \exists_i.\omega \in \Omega^*
                                                                                           Μ
                                                                                                     for all variables in domain of \Omega^*
                         \forall_i.formula
                                                                                           Μ
                                                                                                     for all variables in i and formula
                         \exists_{formula_1}.formula_2
                                                                                           Μ
                                                                                                     for all variables in formula_1 and for
                         \exists_{formula_1}.formula_2 where formula_3
                                                                                           M
                                                                                                     exists formula_1 and formula_2 where
                         \Gamma(id)
                                                                                                     lookup
                         formula_1 = formula_2
                                                                                                     equality
                        formula_1 \wedge formula_2
                                                                                                     equality
                         \bigwedge_i .formula
                                                                                           Μ
                                                                                                     and fold on i and formula
                         formula_1 \cap formula_2
                                                                                           M
                         formula_1 \cup formula_2
                                                                                           Μ
                         formula_1 \subseteq formula_2
                                                                                           M
                        formula_1 \in formula_2
                                                                                           M
                        \Gamma, \Phi, \Omega \to T
                                                                                                     impl
                        \Gamma, \Phi, Q \to T
                                                                                                     impl
                                                                                                     region list
                         r_1, \ldots, r_n
                                                                                                     phi
                         \phi
                        \omega
                                                                                                     om
                         \Omega
```

```
Φ
 \Phi^*

\Phi[r_1/r'_1, ..., r_n/r'_n] 

T[r_1/r'_1, ..., r_n/r'_n] 

M[\rho_1/r'_1, ..., \rho_n/r'_n] 

M[[T]]

M[[Q]]
\mathbf{domain}\left(S\right)
M(r)
\Gamma[r_1/r_1', \dots, r_n/r_n']

\Gamma[e_1/T_1, \dots, e_n/T_n]
\Gamma[T[r_1/r_1', \ldots, r_n/r_n']/id]
\Omega[r_1/r_1', \ldots, r_n/r_n']
regions\_of(\Gamma, T)
regions\_of(\Gamma, T1, T_2)
 T
fns
 S
 C
M
v
 E
L
```

terminals

```
Ø
                           Ø
                           Ø
                            \times
                            <:
                           >
                           \Downarrow
                           \sigma
                           Γ
Jtype
                    ::=
                           \Gamma, \Phi, \Omega \vdash e : T
                                                               Typing
Jop
                    ::=
                           M,L,H,S,C \vdash e \mapsto v,E
                                                                   Evaluation
judgement
                    ::=
                           Jtype
                           Jop
user\_syntax
                    ::=
                            termvar
                           index
                            T
                           fresh
                           fns
                           rr
                           Γ
                           \Omega
                           \Omega^*
                           rs
                           \omega
                            Φ
                            \Phi^*
                           Q
                           \dot{M}
                           L
                           K
                           H
                           S
```

$\Gamma, \Phi, \Omega \vdash e : T$ Typing

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, ..., r_n)}{\Gamma, \Phi, \Omega \vdash \operatorname{read}(e_1) : T} \quad \text{$\operatorname{T_-READ}$}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, ..., r_n)}{\Gamma, \Phi, \Omega \vdash e_2 : T} \quad \text{$\operatorname{T_-WRITE}$}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, ..., r_n)}{\Gamma, \Phi, \Omega \vdash \operatorname{read}(id, e_1, e_2) : T@(r_1, ..., r_n)} \quad \text{$\operatorname{T_-WRITE}$}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T_1@(r_1, ..., r_n)}{\Gamma, \Phi, \Omega \vdash e_2 : T_2} \quad \text{$\operatorname{T_-NEW}$}$$

$$\frac{\Gamma, \Phi, \Omega \vdash \operatorname{reduce}(id, e_1, e_2) : T_1@(r_1, ..., r_n)}{\Gamma, \Phi, \Omega \vdash \operatorname{new} T@r : T@r} \quad \text{$\operatorname{T_-NEW}$}$$

$$\frac{\Gamma, \Phi, \Omega \vdash \operatorname{new} T@r : T@r}{\Gamma, \Phi, \Omega \vdash \operatorname{upregion}(e_1, r_1, ..., r_n) : T@(r_1, ..., r_n)} \quad \text{$\operatorname{T_-UPRGN}$}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e : T@(r'_1, ..., r'_k)}{\Gamma, \Phi, \Omega \vdash \operatorname{downregion}(e, r_1, ..., r_n) : T@(r_1, ..., r_n)} \quad \text{$\operatorname{T_-DNRGN}$}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : \operatorname{coloring}(r)}{\Gamma, \Phi, \Omega \vdash \operatorname{newcolor} r : \operatorname{coloring}(r)} \quad \text{$\operatorname{T_-NEWColor}$}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : \operatorname{coloring}(r)}{\Gamma, \Phi, \Omega \vdash e_2 : T@r} \quad \text{$\Gamma, \Phi, \Omega \vdash e_3 : \operatorname{int}$}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : \operatorname{coloring}(r_p)}{\Gamma, \Phi, \Omega \vdash \operatorname{pack}(e_1, e_2, e_3) : \operatorname{coloring}(r)} \quad \text{$\operatorname{T_-PARTITION}$}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T_1}{\Gamma, \Phi, \Omega \vdash \operatorname{pack}(e_1 \operatorname{as} T_1[r_1, ..., r_k] : T_1} \quad \text{$\Gamma_-PACK}$$

$$\frac{\Gamma, \Phi, \Omega \vdash \operatorname{e_1} : T_1}{\Gamma, \Phi, \Omega \vdash \operatorname{unpack}(e_1 \operatorname{as} id : T_1[r_1, ..., r_k] : T_1} \quad \text{$\Gamma_-DACK}$$

$$\frac{\Gamma, \Phi, \Omega \vdash \operatorname{id}[r_1, ..., r_k](e_1, ..., e_n) : T}{\Gamma, \Phi, \Omega \vdash \operatorname{id}[r_1, ..., r_k](e_1, ..., e_n) : T} \quad \text{$\Gamma_-CALL}$$

 $\overline{\Gamma, \Phi, \Omega \vdash \{ \mathbf{function} \ id_1 \left[r_1, \dots, r_k \right] \left(e_1, \dots, e_m \right), \dots, \mathbf{function} \ id_n \left[r_1, \dots, r_k \right] \left(e_1, \dots, e_m \right) \} : T} \quad \text{$\mathbf{T}_{\mathsf{PROGRAM}}$}$

```
M, L, H, S, C \vdash e \mapsto v, E
                                                    Evaluation
                                                                                                      EVA
                                                       \overline{M.L.H.S.C \vdash x \mapsto v.E}
                                             \frac{M, L, H, S, C \vdash e \mapsto l, E}{M, L, H, S, C \vdash \mathbf{read}\left(e\right) \mapsto v, E}
                                                                                                        EREAD1
                                           \frac{M, L, H, S, C \vdash e \mapsto l, E}{M, L, H, S, C \vdash \mathbf{read}(e) \mapsto H(l), E}
                                                                                                           EREAD2
                                                  M, L, H, S, C \vdash e_1 \mapsto l, E_1
                                                  M, L, H, S', C \vdash e_2 \mapsto v, E_2
                                                                                                             EWRITE
                                          \overline{M, L, H, S, C} \vdash \mathbf{write}(e_1, e_2) \mapsto l, E
                                                 M, L, H, S, C \vdash e_1 \mapsto l, E_1
                                                 M, L, H, S', C \vdash e_2 \mapsto v, E_2
                                                                                                               EREDUCE
                                     \overline{M, L, H, S, C \vdash \mathbf{reduce}(id, e_1, e_2) \mapsto l, E}
                                                                                                         ENULL
                                                \overline{M, L, H, S, C \vdash \mathbf{null} \mapsto \mathbf{null}, []}
                                                                                                           ENEW
                                               \overline{M, L, H, S, C \vdash \mathbf{new} \ T@r \mapsto l, []}
                                                                                                            ENULLTRUE
                                   \overline{M, L, H, S, C \vdash \mathbf{null} \ T@\mathbf{nullr} \mapsto \mathbf{true}, []}
                                                   \neg (T@r = T@\mathbf{nullr})
                                                                                                        ENULLFALSE
                                      \overline{M, L, H, S, C} \vdash \mathbf{null} \ T@r \mapsto \mathbf{false}, []
                                                                                                                                                   FAKEAAAAAB
<<no parses (char 27): M,L,H,S,C |- e1 |-> 1, E1 [***[:prod1value]] >>
                                                             l = \mathbf{null}
                                                                                                      EIsNullTrue
                                     \overline{M, L, H, S, C \vdash \mathbf{isnull}(l) \mapsto \mathbf{true}.}
                                                                                                                                                   FAKEAAAAAC
<<no parses (char 27): M,L,H,S,C |- e1 |-> 1, E1 [***[:prod1value]] >>
                                    \frac{\neg \left( l = \mathbf{null} \right)}{M, L, H, S, C \vdash \mathbf{isnull} \left( l \right) \mapsto \mathbf{false}, \left\| \right\|}
                                                                                                     EIsNullFalse
                                                   M, L, H, S, C \vdash e \mapsto v, E
                                 \frac{M, L, H, S, C \vdash \mathbf{upregion}(e, r_1, ..., r_n) \mapsto v, E}{M, L, H, S, C \vdash \mathbf{upregion}(e, r_1, ..., r_n) \mapsto v, E}
                                                                                                                     EUPRGN
                              \frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \mathbf{downregion} (e, r_1, ..., r_n) \mapsto l, E}
                                                                                                                     EDnRgn1
```

 $M, L, H, S, C \vdash \mathbf{downregion} (e, r_1, ..., r_n) \mapsto l, E$ $M, L, H, S, C \vdash e \mapsto v, E$ $< < \mathsf{no parses (char 45):} \quad \mathsf{M,L,H,S,C} \mid - \mathbf{downregion(e,r1,...,rn)} \mid -> \mathsf{nullr***,} \mid E >>$ $M, L, H, S, C \vdash e_1 \mapsto l, E_1$ $M, L, H, S', C \vdash e_2 \mapsto v, E_2$ $M, L, H, S'', C \vdash e_3 \mapsto v, E_3$ $M, L, H, S'', C \vdash e_3 \mapsto v, E_3$ $M, L, H, S, C \vdash \mathbf{write} (e_1, e_2) \mapsto l, E$ EColor

 $M, L, H, S, C \vdash e_1 \mapsto K, E_1$ $M' = M[\rho_1/r_1, ..., \rho_k/r_k]$ $M, L, H, S', C \vdash e_2 \mapsto v, E_2$

 $M, L, H, S, C \vdash \mathbf{partition} \ r_p \ \mathbf{using} \ e_1 \ \mathbf{as} \ r_1, \dots, r_k \in e_2 \mapsto l, E'$ EPARTITION

$$\frac{M,L,H,S,C \vdash e_1 \mapsto K,E_1}{M,L,H,S,C \vdash \mathbf{pack}\ e_1\ \mathbf{as}\ T_1[r_1,\ldots,r_k] \mapsto v',E} \quad \text{EPack}$$

$$\frac{M,L,H,S,C \vdash e_1 \mapsto \langle\langle \rho_1,\ldots,\rho_k,v \rangle\rangle,E_1}{M' = M[\rho_1/r_1,\ldots,\rho_k/r_k]}$$

$$\frac{L' = L[v_1/id]}{M',L',H,S',C \vdash e_2 \mapsto v,E_2}$$

$$\frac{M',L,H,S,C \vdash \mathbf{unpack}\ e_1\ \mathbf{as}\ id:T_1[r_1,\ldots,r_k] \in e_2 \mapsto v_2,E'}{M,L,H,S,C \vdash id[r_1,\ldots,r_k](e_1,\ldots,e_n) \mapsto v_n,E''} \quad \text{ECall}$$

Definition rules: 31 good 3 bad Definition rule clauses: 69 good 3 bad