

$termvar, x$	term variable
$index, i, j, k, n, m, p$	
T	$::=$ $ \quad \mathbf{bool}$ $ \quad \mathbf{int}$ $ \quad \langle T_1, \dots, T_n \rangle$ $ \quad T@r$ $ \quad T@(r_1, \dots, r_n)$ $ \quad T[r_1, \dots, r_n]$ $ \quad T[r_1/r'_1, \dots, r_n/r'_n]$ $ \quad \mathbf{coloring}(r)$ $ \quad \mathbf{exists} \, r_1, \dots, r_n. (T_1, \dots, T_m), \Phi, Q \rightarrow \mathbf{Tr}$
$fresh$	$::=$ $ \quad iv$
fns	$::=$ $ \quad \mathbf{apply}(S, E)$ $ \quad \mathbf{valid_interleave}(S, C, E_1, \dots, E_n)$ $ \quad \mathbf{taskid} \, fresh$ $ \quad \mathbf{mark_coherence}(E, M, \mathbf{taskid})$
r	$::=$ $ \quad \mathbf{nullr}$ $ \quad \mathbf{nonnullr}$
rr	$::=$ $ \quad [r_1, \dots, r_n]$
l	$::=$ $ \quad \mathbf{nil}$
Γ	$::=$ $ \quad \{(e_1 : T_1), \dots, (e_n : T_n)\}$ $ \quad \emptyset$
Ω	$::=$ $ \quad \{\omega_1, \dots, \omega_n\}$ $ \quad \emptyset$
Ω^*	$::=$ $ \quad \{\omega_1, \dots, \omega_n\}$ $ \quad \mathbf{emptyOst}$
rs	$::=$ $ \quad \{r_1, \dots, r_n\}$ $ \quad \emptyset$
ω	$::=$

		$r_1 \leq r_2$
		$r_1 * r_2$
Φ	::=	
		$\{\phi_1, \dots, \phi_n\}$
		\emptyset
Φ^*	::=	
		$\{\phi_1, \dots, \phi_n\}$
		emptyPst
ϕ	::=	
		reads (r)
		writes (r)
		reducesid (r)
Q	::=	
		$\{q_1, \dots, q_n\}$
q	::=	
		atomic (r)
		simult (r)
M	::=	
		$M[[Q]]$
L	::=	
		nil
		$L[(e_1, v_1), \dots, (e_n, v_n)]$
		$L[v/id]$
K	::=	
H	::=	
		$H(l)$
S	::=	
C	::=	
E	::=	
		\square
		$[e]$
		$E + +[e]$
ρ	::=	

v	$::=$ bv iv $\langle v_1, v_2 \rangle$ null l $\langle \langle \rho_1, \dots, \rho_n, v \rangle \rangle$ H K place	
bv	$::=$ true false	constant true constant false
iv	$::=$ 0 $S\ iv$	
ee	$::=$ (e_1, \dots, e_n)	
id	$::=$	
e	$::=$ x new $T@r$ null $T@r$ isnull (e) upregion (e, r_1, \dots, r_n) downregion (e, r_1, \dots, r_n) read (e) excl read $(e_1, \text{excl}, e_2, e_3)$ write (e_1, e_2) reduce (id, e_1, e_2) reduceid (l, e_1, e_2, e_3) newcolor r color (e_1, e_2, e_3) $e_1 + e_2$ $e_1 < e_2$ let $id : T = e_1 \in e_2$ if b then c_0 else c_1 $id[r_1, \dots, r_n](e_1, \dots, e_m)$ partition r_p using e_1 as $r_1, \dots, r_n \in e_2$ pack e_1 as T unpack e_1 as $id : T \in e_2$	conditional

	function $id[r_1, \dots, r_n](e_1, \dots, e_m)$ $\{ \textbf{function } id_1 \text{ } rr_1 \text{ } ee_1, \dots, \textbf{function } id_n \text{ } rr_n \text{ } ee_n \}$ L $e : T$ place bv iv $\langle v_1, v_2 \rangle$ null l $\langle \langle \rho_1, \dots, \rho_n, v \rangle \rangle$ H K true false 0 $S \text{ } iv$		
<i>formula</i>	$::=$		
	<i>judgement</i>		judgement
	$\neg \text{formula}$	M	negated formula
	(formula)	M	bracketed
	$\forall_i. \phi \in \Phi$	M	for all variables in domain of Φ
	$\forall_i. \phi \in \Phi^*$	M	for all variables in domain of Φ^*
	$\exists_i. \phi \in \Phi$	M	for all variables in domain of Φ
	$\forall_i. \omega \in \Omega$	M	for all variables in domain of Ω
	$\forall_i. \omega \in \Omega^*$	M	for all variables in domain of Ω^*
	$\exists_i. \omega \in \Omega$	M	for all variables in domain of Ω
	$\exists_i. \omega \in \Omega^*$	M	for all variables in domain of Ω^*
	$\forall_i. \text{formula}$	M	for all variables in i and formula
	$\exists_{\text{formula}_1}. \text{formula}_2$	M	for all variables in formula_1 and formula_2
	$\exists_{\text{formula}_1}. \text{formula}_2 \text{ where } \text{formula}_3$	M	exists formula_1 and formula_2 where
	$\Gamma(id)$		lookup
	$\text{formula}_1 = \text{formula}_2$		equality
	$\text{formula}_1 \wedge \text{formula}_2$		equality
	$\bigwedge_i. \text{formula}$	M	and fold on i and formula
	$\text{formula}_1 \cap \text{formula}_2$	M	
	$\text{formula}_1 \cup \text{formula}_2$	M	
	$\text{formula}_1 \subseteq \text{formula}_2$	M	
	$\text{formula}_1 \in \text{formula}_2$	M	
	$\Gamma, \Phi, \Omega \rightarrow T$		impl
	$\Gamma, \Phi, Q \rightarrow T$		impl
	r_1, \dots, r_n		region list
	ϕ		phi
	ω		om
	Ω		

	Φ
	Φ^*
	$\Phi[r_1/r'_1, \dots, r_n/r'_n]$
	$T[r_1/r'_1, \dots, r_n/r'_n]$
	$M[\rho_1/r'_1, \dots, \rho_n/r'_n]$
	$M[[T]]$
	$M[[Q]]$
	$\mathbf{domain}(S)$
	$M(r)$
	$\Gamma[r_1/r'_1, \dots, r_n/r'_n]$
	$\Gamma[e_1/T_1, \dots, e_n/T_n]$
	Γ
	$\Gamma[T[r_1/r'_1, \dots, r_n/r'_n]/id]$
	$\Omega[r_1/r'_1, \dots, r_n/r'_n]$
	$regions_of(\Gamma, T)$
	$regions_of(\Gamma, T_1, T_2)$
	rs
	T
	fns
	S
	C
	M
	v
	E
	L
	$\overline{e_i = l_i}^{i < n}$
<i>terminals</i>	$::=$
	\exists
	\forall
	\in
	ω
	ϕ
	ρ
	\vee
	\wedge
	\neg
	$*$
	\leq
	\longrightarrow
	\rightarrow
	\Rightarrow
	λ
	\mapsto
	\vdash
	\emptyset

		\emptyset	
		\emptyset	
		\emptyset	
		\times	
		$<:$	
		\langle	
		\rangle	
		$<$	
		\Downarrow	
		σ	
		Γ	
		ε	
$Jtype$	$::=$		
		$\Gamma, \Phi, \Omega \vdash e : T$	Typing
Jop	$::=$		
		$M, L, H, S, C \vdash e \mapsto v, E$	Evaluation
$judgement$	$::=$		
		$Jtype$	
		Jop	
$user_syntax$	$::=$		
		$termvar$	
		$index$	
		T	
		$fresh$	
		fns	
		r	
		rr	
		l	
		Γ	
		Ω	
		Ω^*	
		rs	
		ω	
		Φ	
		Φ^*	
		ϕ	
		Q	
		q	
		M	
		L	
		K	
		H	
		S	

	C
	E
	ρ
	v
	bv
	iv
	ee
	id
	e
	<i>formula</i>
	<i>terminals</i>

$\boxed{\Gamma, \Phi, \Omega \vdash e : T}$ Typing

$$\begin{array}{c}
\frac{\Gamma, \Phi, \Omega \vdash e_1 : T@ (r_1, \dots, r_n)}{\Gamma, \Phi, \Omega \vdash \mathbf{read} (e_1) : T} \quad \text{T_READ} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e_1 : T@ (r_1, \dots, r_n) \quad \Gamma, \Phi, \Omega \vdash e_2 : T}{\Gamma, \Phi, \Omega \vdash \mathbf{write} (e_1, e_2) : T@ (r_1, \dots, r_n)} \quad \text{T_WRITE} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e_1 : T_1@ (r_1, \dots, r_n) \quad \Gamma, \Phi, \Omega \vdash e_2 : T_2}{\Gamma, \Phi, \Omega \vdash \mathbf{reduce} (id, e_1, e_2) : T_1@ (r_1, \dots, r_n)} \quad \text{T_REDUCE} \\
\\
\frac{}{\Gamma, \Phi, \Omega \vdash \mathbf{new} T@r : T@r} \quad \text{T_NEW} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e : T@ (r'_1, \dots, r'_k)}{\Gamma, \Phi, \Omega \vdash \mathbf{upregion} (e_1, r_1, \dots, r_n) : T@ (r_1, \dots, r_n)} \quad \text{T_UPRGN} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e : T@ (r'_1, \dots, r'_k)}{\Gamma, \Phi, \Omega \vdash \mathbf{downregion} (e, r_1, \dots, r_n) : T@ (r_1, \dots, r_n)} \quad \text{T_DNRGN} \\
\\
\frac{}{\Gamma, \Phi, \Omega \vdash \mathbf{newcolor} r : \mathbf{coloring} (r)} \quad \text{T_NEWCOLOR} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e_1 : \mathbf{coloring} (r) \quad \Gamma, \Phi, \Omega \vdash e_2 : T@r \quad \Gamma, \Phi, \Omega \vdash e_3 : \mathbf{int}}{\Gamma, \Phi, \Omega \vdash \mathbf{color} (e_1, e_2, e_3) : \mathbf{coloring} (r)} \quad \text{T_COLOR} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e_1 : \mathbf{coloring} (r_p) \quad \Gamma, \Phi, \Omega' \vdash e_2 : T}{\Gamma, \Phi, \Omega \vdash \mathbf{partition} r_p \mathbf{using} e_1 \mathbf{as} r_1, \dots, r_k \in e_2 : T} \quad \text{T_PARTITION} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e_1 : T_2[r_1/r'_1, \dots, r_k/r'_k]}{\Gamma, \Phi, \Omega \vdash \mathbf{pack} e_1 \mathbf{as} T_1[r_1, \dots, r_k] : T_1} \quad \text{T_PACK} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e_1 : T_1 \quad \Gamma', \Phi, \Omega' \vdash e_2 : T_3}{\Gamma, \Phi, \Omega \vdash \mathbf{unpack} e_1 \mathbf{as} id : T_1[r_1, \dots, r_k] \in e_2 : T_3} \quad \text{T_UNPACK} \\
\\
\frac{}{\Gamma, \Phi, \Omega \vdash id[r_1, \dots, r_k](e_1, \dots, e_n) : T} \quad \text{T_CALL} \\
\\
\frac{}{\Gamma, \Phi, \Omega \vdash \{ \mathbf{function} id_1[r_1, \dots, r_k] (e_1, \dots, e_m), \dots, \mathbf{function} id_n[r_1, \dots, r_k] (e_1, \dots, e_m) \} : T} \quad \text{T_PROGRAM}
\end{array}$$

$M, L, H, S, C \vdash e \mapsto v, E$	Evaluation
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$\frac{}{M, L, H, S, C \vdash x \mapsto v, E}$	EVA	
$\frac{M, L, H, S, C \vdash e \mapsto l, E}{M, L, H, S, C \vdash \mathbf{read}(e) \mapsto v, E}$	EREAD1	
$\frac{M, L, H, S, C \vdash e \mapsto l, E}{M, L, H, S, C \vdash \mathbf{read}(e) \mapsto H(l), E}$	EREAD2	
$\frac{M, L, H, S, C \vdash e_1 \mapsto l, E_1 \quad M, L, H, S', C \vdash e_2 \mapsto v, E_2}{M, L, H, S, C \vdash \mathbf{write}(e_1, e_2) \mapsto l, E}$	EWRITE	
$\frac{M, L, H, S, C \vdash e_1 \mapsto l, E_1 \quad M, L, H, S', C \vdash e_2 \mapsto v, E_2}{M, L, H, S, C \vdash \mathbf{reduce}(id, e_1, e_2) \mapsto l, E}$	EREDUCE	
$\frac{}{M, L, H, S, C \vdash \mathbf{null} \mapsto \mathbf{null}, []}$	ENULL	
$\frac{}{M, L, H, S, C \vdash \mathbf{new} T@r \mapsto l, []}$	ENew	
$\frac{}{M, L, H, S, C \vdash \mathbf{null} T@\mathbf{nullr} \mapsto \mathbf{true}, []}$	ENULLTRUE	
$\frac{\neg(T@r = T@\mathbf{nullr})}{M, L, H, S, C \vdash \mathbf{null} T@r \mapsto \mathbf{false}, []}$	ENULLFALSE	
<hr/>		FAKEAAAAAB
$\llcorner \text{no parses (char 27): } M, L, H, S, C \vdash e_1 \mapsto l, E_1 \text{ [***[:prod1value]]} \gg$		
$\frac{l = \mathbf{null}}{M, L, H, S, C \vdash \mathbf{isnull}(l) \mapsto \mathbf{true}, []}$	EISNULLTRUE	
<hr/>		FAKEAAAAAC
$\llcorner \text{no parses (char 27): } M, L, H, S, C \vdash e_1 \mapsto l, E_1 \text{ [***[:prod1value]]} \gg$		
$\frac{\neg(l = \mathbf{null})}{M, L, H, S, C \vdash \mathbf{isnull}(l) \mapsto \mathbf{false}, []}$	EISNULLFALSE	
$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \mathbf{upregion}(e, r_1, \dots, r_n) \mapsto v, E}$	EUPRGN	
$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \mathbf{downregion}(e, r_1, \dots, r_n) \mapsto l, E}$	EDNRGN1	
$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \mathbf{downregion}(e, r_1, \dots, r_n) \mapsto \mathbf{null}, E}$	EDNRGN2	
$\frac{M, L, H, S, C \vdash e_1 \mapsto l, E_1 \quad M, L, H, S', C \vdash e_2 \mapsto v, E_2 \quad M, L, H, S'', C \vdash e_3 \mapsto v, E_3}{M, L, H, S, C \vdash \mathbf{write}(e_1, e_2) \mapsto l, E}$	ECOLOR	
$\frac{M, L, H, S, C \vdash e_1 \mapsto K, E_1 \quad M' = M[\rho_1/r_1, \dots, \rho_k/r_k] \quad M, L, H, S', C \vdash e_2 \mapsto v, E_2}{M, L, H, S, C \vdash \mathbf{partition} r_p \text{ using } e_1 \text{ as } r_1, \dots, r_k \in e_2 \mapsto l, E'}$	EPARTITION	

$$\begin{array}{c}
\frac{M, L, H, S, C \vdash e_1 \mapsto K, E_1}{M, L, H, S, C \vdash \mathbf{pack} \, e_1 \mathbf{as} \, T_1[r_1, \dots, r_k] \mapsto v', E} \quad \text{EPACK} \\
\\
\frac{\begin{array}{l} M, L, H, S, C \vdash e_1 \mapsto \langle \langle \rho_1, \dots, \rho_k, v \rangle \rangle, E_1 \\ M' = M[\rho_1/r_1, \dots, \rho_k/r_k] \\ L' = L[v_1/id] \\ M', L', H, S', C \vdash e_2 \mapsto v, E_2 \end{array}}{M, L, H, S, C \vdash \mathbf{unpack} \, e_1 \mathbf{as} \, id : T_1[r_1, \dots, r_k] \in e_2 \mapsto v_2, E'} \quad \text{EUNPACK} \\
\\
\frac{}{M, L, H, S, C \vdash id[r_1, \dots, r_k](e_1, \dots, e_n) \mapsto v_n, E''} \quad \text{ECALL}
\end{array}$$

Definition rules: 32 good 2 bad
 Definition rule clauses: 70 good 2 bad