$termvar,\ x$ term variable

$index,\ i,\ j,\ k,\ n,\ m,\ p$

$$
\begin{array}{lll}
T & ::= & \\
 & | & \mathbf{bool} \\
 & | & \mathbf{int} \\
 & | & \langle T_1,\ ..,\ T_n \rangle \\
 & | & T@r \\
 & | & T@(r_1,\ ..,\ r_n) \\
 & | & T[r_1,\ ..,\ r_n] \\
 & | & T[r_1/r_1',\ ..,\ r_n/r_n'] \\
 & | & \mathbf{coloring}\ (r) \\
 & | & \mathbf{exists}\ r_1,\ ..,\ r_n.(T_1,\ ..,\ T_m), \Phi, Q \rightarrow \mathbf{Tr} \\
 & & \\
\mathit{fresh} & ::= & \\
 & | & \mathit{iv} \\
 & & \\
\mathit{fns} & ::= & \\
 & | & \mathbf{apply}\ (S, E) \\
 & | & \mathit{valid\_interleave}(S, C, E_1,\ ..,\ E_n) \\
 & | & \mathbf{taskid}\ \mathit{fresh} \\
 & | & \mathit{mark\_coherence}(E, M,\ \mathbf{taskid}\ ) \\
 & & \\
r & ::= & \\
 & & \\
\mathit{rr} & ::= & \\
 & | & [r_1,\ ..,\ r_n] \\
 & & \\
l & ::= & \\
 & | & \mathbf{nil} \\
 & & \\
\Gamma & ::= & \\
 & | & \{(e_1 : T_1),\ ..,\ (e_n : T_n)\} \\
 & | & \varnothing \\
 & & \\
\Omega & ::= & \\
 & | & \{\omega_1,\ ..,\omega_n\} \\
 & | & \varnothing \\
 & & \\
\Omega^* & ::= & \\
 & | & \{\omega_1,\ ..,\omega_n\} \\
 & | & \mathbf{emptyOst} \\
 & & \\
\mathit{rs} & ::= & \\
 & | & \{r_1,\ ..,\ r_n\} \\
 & | & \varnothing \\
 & & \\
\omega & ::= & \\
 & | & r_1 \leq r_2 \\
 & | & r_1 * r_2
\end{array}
$$

$$
\begin{array}{lll}
\Phi & ::= & \\
 & | & \{\phi_1, .., \phi_n\} \\
 & | & \varnothing \\
 & & \\
\Phi^* & ::= & \\
 & | & \{\phi_1, .., \phi_n\} \\
 & | & \textbf{emptyPst} \\
 & & \\
\phi & ::= & \\
 & | & \textbf{reads}\,(r) \\
 & | & \textbf{writes}\,(r) \\
 & | & \textbf{reducesid}\,(r) \\
 & & \\
Q & ::= & \\
 & | & \{q_1, .., q_n\} \\
 & & \\
q & ::= & \\
 & | & \textbf{atomic}\,(r) \\
 & | & \textbf{simult}\,(r) \\
 & & \\
M & ::= & \\
 & | & M[[Q]] \\
 & & \\
L & ::= & \\
 & | & \textbf{nil} \\
 & | & L[(e_1, v_1), .., (e_n, v_n)] \\
 & | & L[v/id] \\
 & & \\
K & ::= & \\
 & & \\
H & ::= & \\
 & | & H(l) \\
 & & \\
S & ::= & \\
 & & \\
C & ::= & \\
 & & \\
E & ::= & \\
 & | & [] \\
 & | & [e] \\
 & | & E++[e] \\
 & & \\
\rho & ::= & \\
 & & \\
v & ::= & \\
 & | & bv \\
 & | & iv
\end{array}
$$

	$\langle v_1, v_2 \rangle$	
	null	
	l	
	$\langle\langle \rho_1, .., \rho_n, v \rangle\rangle$	
	H	
	K	
	place	
bv	::=	
	true	constant true
	false	constant false
iv	::=	
	0	
	$S\ iv$	
ee	::=	
	$(e_1, .., e_n)$	
id	::=	
e	::=	
	x	
	new $T@r$	
	null $T@r$	
	isnull (e)	
	upregion $(e, r_1, .., r_n)$	
	downregion $(e, r_1, .., r_n)$	
	read (e)	
	excl	
	read $(e_1,$ **excl** $, e_2, e_3)$	
	write (e_1, e_2)	
	reduce (id, e_1, e_2)	
	reduceid (l, e_1, e_2, e_3)	
	newcolor r	
	color (e_1, e_2, e_3)	
	$e_1 + e_2$	
	$e_1 < e_2$	
	let $id : T = e_1 \in e_2$	
	if b then $c0$ **else** $c1$	conditional
	$id[r_1, .., r_n](e_1, .., e_m)$	
	partition r_p **using** e_1 **as** $r_1, .., r_n \in e_2$	
	pack e_1 **as** T	
	unpack e_1 **as** $id : T \in e_2$	
	function $id[r_1, .., r_n](e_1, .., e_m)$	
	$\{$ **function** $id_1\ rr_1\ ee_1, ..,$ **function** $id_n\ rr_n\ ee_n\}$	
	L	

	$e : T$		
	place		
	bv		
	iv		
	$\langle v_1, v_2 \rangle$		
	null		
	l		
	$\langle\langle \rho_1, .., \rho_n, v \rangle\rangle$		
	H		
	K		
	true		constant true
	false		constant false
	0		
	$S\ iv$		

formula	::=		
	judgement		judgement
	$\neg$*formula*	M	negated formula
	(*formula*)	M	bracketed
	$\forall_i.\phi \in \Phi$	M	for all variables in domain of Φ
	$\forall_i.\phi \in \Phi^*$	M	for all variables in domain of Φ^*
	$\exists_i.\phi \in \Phi$	M	for all variables in domain of Φ
	$\forall_i.\omega \in \Omega$	M	for all variables in domain of Ω
	$\forall_i.\omega \in \Omega^*$	M	for all variables in domain of Ω^*
	$\exists_i.\omega \in \Omega$	M	for all variables in domain of Ω
	$\exists_i.\omega \in \Omega^*$	M	for all variables in domain of Ω^*
	$\forall_i$.*formula*	M	for all variables in i and *formula*
	$\exists_{formula_1}$.*formula*$_2$	M	for all variables in *formula*$_1$ and *formula*$_2$
	$\exists_{formula_1}$.*formula*$_2$ where *formula*$_3$	M	exists *formula*$_1$ and *formula*$_2$ where *formula*$_3$
	$\Gamma(id)$		lookup
	formula$_1$ = *formula*$_2$		equality
	formula$_1$ $\wedge$ *formula*$_2$		equality
	$\bigwedge_i$.*formula*	M	and fold on i and *formula*
	formula$_1$ $\cap$ *formula*$_2$	M	
	formula$_1$ $\cup$ *formula*$_2$	M	
	formula$_1$ $\subseteq$ *formula*$_2$	M	
	formula$_1$ $\in$ *formula*$_2$	M	
	$\Gamma, \Phi, \Omega \rightarrow T$		impl
	$\Gamma, \Phi, Q \rightarrow T$		impl
	$r_1, .., r_n$		region list
	ϕ		phi
	ω		om
	Ω		
	Φ		
	Φ^*		
	$\Phi[r_1/r'_1, .., r_n/r'_n]$		

| | $T[r_1/r_1', .., r_n/r_n']$
| | $M[\rho_1/r_1', .., \rho_n/r_n']$
| | $M[[T]]$
| | $M[[Q]]$
| | $\mathbf{domain}\,(S)$
| | $M(r)$
| | $\Gamma[r_1/r_1', .., r_n/r_n']$
| | $\Gamma[e_1/\,T_1, .., e_n/\,T_n]$
| | Γ
| | $\Gamma[T[r_1/r_1', .., r_n/r_n']/id]$
| | $\Omega[r_1/r_1', .., r_n/r_n']$
| | $regions\_of(\Gamma, T)$
| | $regions\_of(\Gamma, T1, T_2)$
| | rs
| | T
| | fns
| | S
| | C
| | M
| | v
| | E
| | L
| | $\overline{e_i = l_i}^{\,i<n}$

terminals ::=

| | $\exists$
| | $\forall$
| | $\in$
| | ω
| | ϕ
| | ρ
| | $\vee$
| | $\wedge$
| | $\neg$
| | $*$
| | $\leq$
| | $\longrightarrow$
| | $\rightarrow$
| | $\Rightarrow$
| | λ
| | $\mapsto$
| | $\vdash$
| | $\varnothing$
| | $\varnothing$
| | $\varnothing$
| | $\varnothing$

	$\times$
	$<:$
	$\langle$
	$\rangle$
	$<$
	$\Downarrow$
	σ
	Γ
	ε

Jtype	::=		
	|	$\Gamma, \Phi, \Omega \vdash e : T$	Typing

Jop	::=		
	|	$M, L, H, S, C \vdash e \mapsto v, E$	Evaluation

judgement	::=	
	|	*Jtype*
	|	*Jop*

user_syntax	::=	
	|	*termvar*
	|	*index*
	|	T
	|	*fresh*
	|	*fns*
	|	r
	|	rr
	|	l
	|	Γ
	|	Ω
	|	Ω^*
	|	rs
	|	ω
	|	Φ
	|	Φ^*
	|	ϕ
	|	Q
	|	q
	|	M
	|	L
	|	K
	|	H
	|	S
	|	C
	|	E
	|	ρ

| v
| bv
| iv
| ee
| id
| e
| *formula*
| *terminals*

$\boxed{\Gamma, \Phi, \Omega \vdash e : T}$ Typing

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, .., r_n)}{\Gamma, \Phi, \Omega \vdash \mathbf{read}\,(e_1) : T} \quad \text{T\_Read}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, .., r_n) \\ \Gamma, \Phi, \Omega \vdash e_2 : T\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{write}\,(e_1, e_2) : T@(r_1, .., r_n)} \quad \text{T\_Write}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : T_1@(r_1, .., r_n) \\ \Gamma, \Phi, \Omega \vdash e_2 : T_2\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{reduce}\,(id, e_1, e_2) : T_1@(r_1, .., r_n)} \quad \text{T\_Reduce}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \mathbf{new}\ T@r : T@r} \quad \text{T\_New}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e : T@(r'_1, .., r'_k)}{\Gamma, \Phi, \Omega \vdash \mathbf{upregion}\,(e_1, r_1, .., r_n) : T@(r_1, .., r_n)} \quad \text{T\_UpRgn}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e : T@(r'_1, .., r'_k)}{\Gamma, \Phi, \Omega \vdash \mathbf{downregion}\,(e, r_1, .., r_n) : T@(r_1, .., r_n)} \quad \text{T\_DnRgn}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \mathbf{newcolor}\ r : \mathbf{coloring}\,(r)} \quad \text{T\_NewColor}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : \mathbf{coloring}\,(r) \\ \Gamma, \Phi, \Omega \vdash e_2 : T@r \\ \Gamma, \Phi, \Omega \vdash e_3 : \mathbf{int}\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{color}\,(e_1, e_2, e_3) : \mathbf{coloring}\,(r)} \quad \text{T\_Color}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : \mathbf{coloring}\,(r_p) \\ \Gamma, \Phi, \Omega' \vdash e_2 : T\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{partition}\ r_p\ \mathbf{using}\ e_1\ \mathbf{as}\ r_1, .., r_k\ \in\ e_2 : T} \quad \text{T\_Partition}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T_2[r_1/r'_1, .., r_k/r'_k]}{\Gamma, \Phi, \Omega \vdash \mathbf{pack}\ e_1\ \mathbf{as}\ T_1[r_1, .., r_k] : T_1} \quad \text{T\_Pack}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : T_1 \\ \Gamma', \Phi, \Omega' \vdash e_2 : T_3\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{unpack}\ e_1\ \mathbf{as}\ id : T_1[r_1, .., r_k]\ \in\ e_2 : T_3} \quad \text{T\_UnPack}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash id[r_1, .., r_k](e_1, .., e_n) : T} \quad \text{T\_Call}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \{\,\mathbf{function}\ id_1\,[r_1, .., r_k]\,(e_1, .., e_m), .., \mathbf{function}\ id_n\,[r_1, .., r_k]\,(e_1, .., e_m)\} : T} \quad \text{T\_Program}$$

$\boxed{M, L, H, S, C \vdash e \mapsto v, E}$ Evaluation

$$\frac{}{M, L, H, S, C \vdash x \mapsto v, E} \quad \text{EVa}$$

$$\frac{M, L, H, S, C \vdash e \mapsto l, E}{M, L, H, S, C \vdash \mathbf{read}\,(e) \mapsto v, E}\ \textsc{ERead1}$$

$$\frac{M, L, H, S, C \vdash e \mapsto l, E}{M, L, H, S, C \vdash \mathbf{read}\,(e) \mapsto H(l), E}\ \textsc{ERead2}$$

$$\frac{\begin{array}{c}M, L, H, S, C \vdash e_1 \mapsto l, E_1\\ M, L, H, S', C \vdash e_2 \mapsto v, E_2\end{array}}{M, L, H, S, C \vdash \mathbf{write}\,(e_1, e_2) \mapsto l, E}\ \textsc{EWrite}$$

$$\frac{\begin{array}{c}M, L, H, S, C \vdash e_1 \mapsto l, E_1\\ M, L, H, S', C \vdash e_2 \mapsto v, E_2\end{array}}{M, L, H, S, C \vdash \mathbf{reduce}\,(id, e_1, e_2) \mapsto l, E}\ \textsc{EReduce}$$

$$\frac{}{M, L, H, S, C \vdash \mathbf{new}\ T@r \mapsto l, []}\ \textsc{ENew}$$

$$\frac{}{M, L, H, S, C \vdash \mathbf{null} \mapsto \mathbf{null}, []}\ \textsc{ENull}$$

$$\frac{\begin{array}{l}M, L, H, S, C \vdash e_1 \mapsto l, E_1\\ l = \mathbf{null}\end{array}}{M, L, H, S, C \vdash \mathbf{isnull}\,(l) \mapsto \mathbf{true}, []}\ \textsc{EIsNullTrue}$$

$$\frac{\begin{array}{l}M, L, H, S, C \vdash e_1 \mapsto l, E_1\\ \neg\,(l = \mathbf{null})\end{array}}{M, L, H, S, C \vdash \mathbf{isnull}\,(l) \mapsto \mathbf{false}, []}\ \textsc{EIsNullFalse}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \mathbf{upregion}\,(e, r_1, \ldots, r_n) \mapsto v, E}\ \textsc{EUpRgn}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \mathbf{downregion}\,(e, r_1, \ldots, r_n) \mapsto l, E}\ \textsc{EDnRgn1}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \mathbf{downregion}\,(e, r_1, \ldots, r_n) \mapsto \mathbf{null}, E}\ \textsc{EDnRgn2}$$

$$\frac{\begin{array}{c}M, L, H, S, C \vdash e_1 \mapsto l, E_1\\ M, L, H, S', C \vdash e_2 \mapsto v, E_2\\ M, L, H, S'', C \vdash e_3 \mapsto v, E_3\end{array}}{M, L, H, S, C \vdash \mathbf{write}\,(e_1, e_2) \mapsto l, E}\ \textsc{EColor}$$

$$\frac{\begin{array}{l}M, L, H, S, C \vdash e_1 \mapsto K, E_1\\ M' = M[\rho_1/r_1, \ldots, \rho_k/r_k]\\ M, L, H, S', C \vdash e_2 \mapsto v, E_2\end{array}}{M, L, H, S, C \vdash \mathbf{partition}\ r_p\ \mathbf{using}\ e_1\ \mathbf{as}\ r_1, \ldots, r_k\ \in\ e_2 \mapsto l, E'}\ \textsc{EPartition}$$

$$\frac{M, L, H, S, C \vdash e_1 \mapsto K, E_1}{M, L, H, S, C \vdash \mathbf{pack}\ e_1\ \mathbf{as}\ T_1[r_1, \ldots, r_k] \mapsto v', E}\ \textsc{EPack}$$

$$\frac{\begin{array}{l}M, L, H, S, C \vdash e_1 \mapsto \langle\langle \rho_1, \ldots, \rho_k, v\rangle\rangle, E_1\\ M' = M[\rho_1/r_1, \ldots, \rho_k/r_k]\\ L' = L[v_1/id]\\ M', L', H, S', C \vdash e_2 \mapsto v, E_2\end{array}}{M, L, H, S, C \vdash \mathbf{unpack}\ e_1\ \mathbf{as}\ id : T_1[r_1, \ldots, r_k]\ \in\ e_2 \mapsto v_2, E'}\ \textsc{EUnpack}$$

$$\frac{}{M, L, H, S, C \vdash id[r_1, \ldots, r_k](e_1, \ldots, e_n) \mapsto v_n, E''}\ \textsc{ECall}$$

```
Definition rules:        30 good    0 bad
Definition rule clauses: 69 good    0 bad
```