$termvar,\ x$ term variable
$index,\ i,\ j,\ k,\ n,\ m,\ p$

$$
\begin{array}{lcl}
T & ::= & \\
 & | & \mathbf{bool} \\
 & | & \mathbf{int} \\
 & | & \langle T_1,\ ..\ ,T_n \rangle \\
 & | & T@r \\
 & | & T@(r_1,\ ..\ ,r_n) \\
 & | & T[r_1/r'_1,\ ..\ ,r_n/r'_n] \\
 & | & \mathbf{coloring}\,(r) \\
 & | & \mathbf{exists}\, r_1,\ ..\ ,r_n.(T_1,\ ..\ ,T_m), \Phi, Q \rightarrow\ \mathbf{Tr} \\
 & & \\
fresh & ::= & \\
 & | & iv \\
 & & \\
fns & ::= & \\
 & | & \mathbf{apply}\,(S, E) \\
 & | & valid\_interleave(S, C, E_1,\ ..\ ,E_n) \\
 & | & \mathbf{taskid}\, fresh \\
 & | & mark\_coherence(E, M,\ \mathbf{taskid}\,) \\
 & & \\
r & ::= & \\
 & & \\
rr & ::= & \\
 & | & [r_1,\ ..\ ,r_n] \\
 & & \\
\Gamma & ::= & \\
 & | & \{(e_1 : T_1),\ ..\ ,(e_n : T_n)\} \\
 & | & \varnothing \\
 & & \\
\Omega & ::= & \\
 & | & \{\omega_1,\ ..\ ,\omega_n\} \\
 & | & \varnothing \\
 & & \\
\Omega^* & ::= & \\
 & | & \{\omega_1,\ ..\ ,\omega_n\} \\
 & | & \mathbf{emptyOst} \\
 & & \\
rs & ::= & \\
 & | & \{r_1,\ ..\ ,r_n\} \\
 & | & \varnothing \\
 & & \\
\omega & ::= & \\
 & | & r_1 \leq r_2 \\
 & | & r_1 * r_2 \\
 & & \\
\Phi & ::= & \\
 & | & \{\phi_1,\ ..\ ,\phi_n\} \\
 & | & \varnothing
\end{array}
$$

$$
\begin{array}{lcl}
\Phi^* & ::= & \\
& | & \{\phi_1,\,..\,,\phi_n\} \\
& | & \textbf{emptyPst} \\
\\
\phi & ::= & \\
& | & \textbf{reads}\,(r) \\
& | & \textbf{writes}\,(r) \\
& | & \textbf{reducesid}\,(r) \\
\\
Q & ::= & \\
& | & \{q_1,\,..\,,q_n\} \\
\\
q & ::= & \\
& | & \textbf{atomic}\,(r) \\
& | & \textbf{simult}\,(r) \\
\\
M & ::= & \\
& | & M[[Q]] \\
\\
L & ::= & \\
& | & L[(e_1, v_1),\,..\,,(e_n, v_n)] \\
& | & L[v/id] \\
\\
K & ::= & \\
\\
H & ::= & \\
& | & H(l) \\
\\
S & ::= & \\
\\
C & ::= & \\
\\
E & ::= & \\
& | & [] \\
& | & [e] \\
& | & E++[e] \\
\\
\rho & ::= & \\
\\
l & ::= & \\
\\
v & ::= & \\
& | & bv \\
& | & iv \\
& | & \langle v_1, v_2\rangle \\
& | & \textbf{null} \\
& | & l
\end{array}
$$

	$\mid$	$\langle\langle \rho_1, \ldots, \rho_n, v\rangle\rangle$	
	$\mid$	H	
	$\mid$	K	
bv	$::=$		
	$\mid$	**true**	constant true
	$\mid$	**false**	constant false
iv	$::=$		
	$\mid$	0	
	$\mid$	$S\ iv$	
ee	$::=$		
	$\mid$	$(e_1, \ldots, e_n)$	
id	$::=$		
e	$::=$		
	$\mid$	x	
	$\mid$	$\langle e_1, \ldots, e_n\rangle$	
	$\mid$	$e\ iv$	
	$\mid$	id	
	$\mid$	**new** $T@r$	
	$\mid$	**null** $T@r$	
	$\mid$	**isnull** (e)	
	$\mid$	**upregion** $(e, r1, .., rn)$	
	$\mid$	**downregion** $(e, r1, .., rn)$	
	$\mid$	**read** (e)	
	$\mid$	**excl**	
	$\mid$	**read** $(e_1,$ **excl** $, e_2, e_3)$	
	$\mid$	**write** (e_1, e_2)	
	$\mid$	**reduce** (id, e_1, e_2)	
	$\mid$	**reduceid** (l, e_1, e_2, e_3)	
	$\mid$	**newcolor** r	
	$\mid$	**color** (e_1, e_2, e_3)	
	$\mid$	$e_1 + e_2$	
	$\mid$	$e_1 < e_2$	
	$\mid$	**let** $id : T = e_1 \in e_2$	
	$\mid$	**if b then** $c0$ **else** $c1$	conditional
	$\mid$	$id[r_1, \ldots, r_n](e_1, \ldots, e_m)$	
	$\mid$	**partition** r_p **using** e_1 **as** $r_1, \ldots, r_n \in e_2$	
	$\mid$	**pack** e_1 **as** $T[r_1, \ldots, r_n]$	
	$\mid$	**unpack** e_1 **as** $id : T[r_1, \ldots, r_n] \in e_2$	
	$\mid$	**function** $id[r_1, \ldots, r_n](e_1, \ldots, e_m)$	
	$\mid$	$\{$ **function** $id_1\ rr_1\ ee_1, \ldots,$ **function** $id_n\ rr_n\ ee_n\}$	
	$\mid$	v	
	$\mid$	L	

	$\vert$	$e : T$		
	$\vert$	**place**		
formula	::=			
	$\vert$	*judgement*		judgement
	$\vert$	$\neg formula$	M	negated formula
	$\vert$	$(formula)$	M	bracketed
	$\vert$	$\forall_i.\phi \in \Phi$	M	for all variables in domain of Φ
	$\vert$	$\forall_i.\phi \in \Phi^*$	M	for all variables in domain of Φ^*
	$\vert$	$\exists_i.\phi \in \Phi$	M	for all variables in domain of Φ
	$\vert$	$\forall_i.\omega \in \Omega$	M	for all variables in domain of Ω
	$\vert$	$\forall_i.\omega \in \Omega^*$	M	for all variables in domain of Ω^*
	$\vert$	$\exists_i.\omega \in \Omega$	M	for all variables in domain of Ω
	$\vert$	$\exists_i.\omega \in \Omega^*$	M	for all variables in domain of Ω^*
	$\vert$	$\forall_i.formula$	M	for all variables in i and *formula*
	$\vert$	$\exists_{formula_1}.formula_2$	M	for all variables in $formula_1$ and $formula_2$
	$\vert$	$\exists_{formula_1}.formula_2$ where $formula_3$	M	exists $formula_1$ and $formula_2$ where $formula_3$
	$\vert$	$\Gamma(id)$		lookup
	$\vert$	$formula_1 = formula_2$		equality
	$\vert$	$formula_1 \wedge formula_2$		equality
	$\vert$	$\bigwedge_i .formula$	M	and fold on i and *formula*
	$\vert$	$formula_1 \cap formula_2$	M	
	$\vert$	$formula_1 \cup formula_2$	M	
	$\vert$	$formula_1 \subseteq formula_2$	M	
	$\vert$	$formula_1 \in formula_2$	M	
	$\vert$	$\Gamma, \Phi, \Omega \to T$		impl
	$\vert$	$\Gamma, \Phi, Q \to T$		impl
	$\vert$	$r_1, .., r_n$		region list
	$\vert$	ϕ		phi
	$\vert$	ω		om
	$\vert$	Ω		
	$\vert$	Φ		
	$\vert$	Φ^*		
	$\vert$	$\Phi[r_1/r'_1, .., r_n/r'_n]$		
	$\vert$	$T[r_1/r'_1, .., r_n/r'_n]$		
	$\vert$	$M[\rho_1/r'_1, .., \rho_n/r'_n]$		
	$\vert$	$M[[T]]$		
	$\vert$	$M[[Q]]$		
	$\vert$	**domain** (S)		
	$\vert$	$M(r)$		
	$\vert$	$\Gamma[r_1/r'_1, .., r_n/r'_n]$		
	$\vert$	$\Gamma[e_1/T_1, .., e_n/T_n]$		
	$\vert$	Γ		
	$\vert$	$\Gamma[T[r_1/r'_1, .., r_n/r'_n]/id]$		
	$\vert$	$\Omega[r_1/r'_1, .., r_n/r'_n]$		
	$\vert$	$regions\_of(\Gamma, T)$		

$$
\begin{array}{lll}
& | & regions\_of(\Gamma, T1, T_2) \\
& | & rs \\
& | & T \\
& | & fns \\
& | & S \\
& | & C \\
& | & M \\
& | & v \\
& | & E \\
& | & L \\
& | & \overline{e_i = l_i}^{\,i<n} \\
\\
terminals & ::= & \\
& | & \exists \\
& | & \forall \\
& | & \in \\
& | & \omega \\
& | & \phi \\
& | & \rho \\
& | & \vee \\
& | & \wedge \\
& | & \neg \\
& | & * \\
& | & \leq \\
& | & \longrightarrow \\
& | & \rightarrow \\
& | & \Rightarrow \\
& | & \lambda \\
& | & \mapsto \\
& | & \vdash \\
& | & \varnothing \\
& | & \varnothing \\
& | & \varnothing \\
& | & \varnothing \\
& | & \times \\
& | & <: \\
& | & \langle \\
& | & \rangle \\
& | & < \\
& | & \Downarrow \\
& | & \sigma \\
& | & \Gamma \\
& | & \varepsilon \\
\\
Jtype & ::= & \\
& | & \Gamma, \Phi, \Omega \vdash e : T \qquad\qquad \text{Typing}
\end{array}
$$

Jop	::=		
	$\mid$	$M, L, H, S, C \vdash e \mapsto v, E$	Evaluation
judgement	::=		
	$\mid$	*Jtype*	
	$\mid$	*Jop*	
user_syntax	::=		
	$\mid$	*termvar*	
	$\mid$	*index*	
	$\mid$	T	
	$\mid$	*fresh*	
	$\mid$	*fns*	
	$\mid$	r	
	$\mid$	rr	
	$\mid$	Γ	
	$\mid$	Ω	
	$\mid$	Ω^*	
	$\mid$	rs	
	$\mid$	ω	
	$\mid$	Φ	
	$\mid$	Φ^*	
	$\mid$	ϕ	
	$\mid$	Q	
	$\mid$	q	
	$\mid$	M	
	$\mid$	L	
	$\mid$	K	
	$\mid$	H	
	$\mid$	S	
	$\mid$	C	
	$\mid$	E	
	$\mid$	ρ	
	$\mid$	l	
	$\mid$	v	
	$\mid$	bv	
	$\mid$	iv	
	$\mid$	ee	
	$\mid$	id	
	$\mid$	e	
	$\mid$	*formula*	
	$\mid$	*terminals*	

$\boxed{\Gamma, \Phi, \Omega \vdash e : T}$ Typing

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, .., r_n)}{\Gamma, \Phi, \Omega \vdash \mathbf{read}\,(e_1) : T} \quad \text{T\_READ}$$

$$\frac{\begin{array}{l}\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, .., r_n)\\ \Gamma, \Phi, \Omega \vdash e_2 : T\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{write}\,(e_1, e_2) : T@(r_1, .., r_n)} \quad \text{T\_WRITE}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : T_1@(r_1, .., r_n) \\ \Gamma, \Phi, \Omega \vdash e_2 : T_2\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{reduce}\,(id, e_1, e_2) : T_1@(r_1, .., r_n)} \quad \text{T\_Reduce}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \textbf{new}\ T@r : T@r} \quad \text{T\_New}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e : T@(r_1', .., r_k')}{\Gamma, \Phi, \Omega \vdash \textbf{upregion}\,(e_1, r1, .., rn) : T@(r_1, .., r_n)} \quad \text{T\_UpRgn}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e : T@(r_1', .., r_k')}{\Gamma, \Phi, \Omega \vdash \textbf{downregion}\,(e, r1, .., rn) : T@(r_1, .., r_n)} \quad \text{T\_DnRgn}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \textbf{newcolor}\ r : \textbf{coloring}\,(r)} \quad \text{T\_NewColor}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : \textbf{coloring}\,(r) \\ \Gamma, \Phi, \Omega \vdash e_2 : T@r \\ \Gamma, \Phi, \Omega \vdash e_3 : \textbf{int}\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{color}\,(e_1, e_2, e_3) : \textbf{coloring}\,(r)} \quad \text{T\_Color}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : \textbf{coloring}\,(r_p) \\ \Gamma, \Phi, \Omega' \vdash e_2 : T\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{partition}\ r_p\ \textbf{using}\ e_1\ \textbf{as}\ r_1, .., r_k \in e_2 : T} \quad \text{T\_Partition}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T_2[r_1/r_1', .., r_k/r_k']}{\Gamma, \Phi, \Omega \vdash \textbf{pack}\ e_1\ \textbf{as}\ T_1[r_1, .., r_k] : T_1} \quad \text{T\_Pack}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : T_1 \\ \Gamma', \Phi, \Omega' \vdash e_2 : T_3\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{unpack}\ e_1\ \textbf{as}\ id : T_1[r_1, .., r_k] \in e_2 : T_3} \quad \text{T\_UnPack}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash id[r_1, .., r_k](e_1, .., e_n) : T} \quad \text{T\_Call}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \{\textbf{function}\ id_1\,[r_1, .., r_k]\,(e_1, .., e_m), .., \textbf{function}\ id_n\,[r_1, .., r_k]\,(e_1, .., e_m)\} : T} \quad \text{T\_Program}$$

$\boxed{M, L, H, S, C \vdash e \mapsto v, E}$ Evaluation

$$\frac{M, L, H, S, C \vdash e \mapsto l, E}{M, L, H, S, C \vdash \textbf{read}\,(e) \mapsto v, E} \quad \text{ERead1}$$

$$\frac{M, L, H, S, C \vdash e \mapsto l, E}{M, L, H, S, C \vdash \textbf{read}\,(e) \mapsto H(l), E} \quad \text{ERead2}$$

$$\frac{\begin{array}{c}M, L, H, S, C \vdash e_1 \mapsto l, E_1 \\ M, L, H, S', C \vdash e_2 \mapsto v, E_2\end{array}}{M, L, H, S, C \vdash \textbf{write}\,(e_1, e_2) \mapsto l, E} \quad \text{EWrite}$$

$$\frac{\begin{array}{c}M, L, H, S, C \vdash e_1 \mapsto l, E_1 \\ M, L, H, S', C \vdash e_2 \mapsto v, E_2\end{array}}{M, L, H, S, C \vdash \textbf{reduce}\,(id, e_1, e_2) \mapsto l, E} \quad \text{EReduce}$$

$$\frac{}{M, L, H, S, C \vdash \textbf{new}\ T@r \mapsto l, []} \quad \text{ENew}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \textbf{upregion}\,(e, r1, .., rn) \mapsto v, E} \quad \text{EUpRgn}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \textbf{upregion}\,(e, r1, .., rn) \mapsto l, E} \quad \text{EDnRgn1}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \textbf{upregion}\,(e, r1, .., rn) \mapsto \textbf{null}, E} \quad \text{EDnRgn2}$$

$$\frac{\begin{array}{c} M, L, H, S, C \vdash e_1 \mapsto l, E_1 \\ M, L, H, S', C \vdash e_2 \mapsto v, E_2 \\ M, L, H, S'', C \vdash e_3 \mapsto v, E_3 \end{array}}{M, L, H, S, C \vdash \textbf{write}\,(e_1, e_2) \mapsto l, E} \quad \text{EColor}$$

$$\frac{\begin{array}{l} M, L, H, S, C \vdash e_1 \mapsto K, E_1 \\ M' = M[\rho_1/r_1, .., \rho_k/r_k] \\ M, L, H, S', C \vdash e_2 \mapsto v, E_2 \end{array}}{M, L, H, S, C \vdash \textbf{partition}\, r_p\, \textbf{using}\, e_1\, \textbf{as}\, r_1, .., r_k \in e_2 \mapsto l, E'} \quad \text{EPartition}$$

$$\frac{M, L, H, S, C \vdash e_1 \mapsto K, E_1}{M, L, H, S, C \vdash \textbf{pack}\, e_1\, \textbf{as}\, T_1[r_1, .., r_k] \mapsto v', E} \quad \text{EPack}$$

$$\frac{\begin{array}{l} M, L, H, S, C \vdash e_1 \mapsto \langle\langle \rho_1, .., \rho_k, v \rangle\rangle, E_1 \\ M' = M[\rho_1/r_1, .., \rho_k/r_k] \\ L' = L[v_1/id] \\ M', L', H, S', C \vdash e_2 \mapsto v, E_2 \end{array}}{M, L, H, S, C \vdash \textbf{unpack}\, e_1\, \textbf{as}\, id : T_1[r_1, .., r_k] \in e_2 \mapsto v_2, E'} \quad \text{EUnpack}$$

$$\frac{\begin{array}{l} M, L, H, S, C \vdash e_1 \mapsto v_1, E_1 \\ M' = M[\rho_1/r_1, .., \rho_k/r_k] \\ L' = L[(e_1, v_1), .., (e_n, v_n)] \\ M', L', H, S', C' \vdash e_n \mapsto v_n, E_n \end{array}}{M, L, H, S, C \vdash id[r_1, .., r_k](e_1, .., e_n) \mapsto v_n, E''} \quad \text{ECall}$$

```
Definition rules:        26 good    0 bad
Definition rule clauses: 65 good    0 bad
```