$termvar$, x		term variable
$index$, i, j, k, n, m, p		
r	$::=$	
	$\mid$	**nullr**
	$\mid$	**notnullr**
rr	$::=$	
	$\mid$	$[r_1, .., r_n]$
l	$::=$	
	$\mid$	**nil**
C	$::=$	
Γ	$::=$	
	$\mid$	$\{(e_1 : T_1), .., (e_n : T_n)\}$
	$\mid$	$\varnothing$
Ω	$::=$	
	$\mid$	$\{\omega_1, .., \omega_n\}$
	$\mid$	$\varnothing$
Ω^*	$::=$	
	$\mid$	$\{\omega_1, .., \omega_n\}$
	$\mid$	**emptyOst**
rs	$::=$	
	$\mid$	$\{r_1, .., r_n\}$
	$\mid$	$\varnothing$
ω	$::=$	
	$\mid$	$r_1 \leq r_2$
	$\mid$	$r_1 * r_2$
Φ	$::=$	
	$\mid$	$\{\phi_1, .., \phi_n\}$
	$\mid$	$\varnothing$
Φ^*	$::=$	
	$\mid$	$\{\phi_1, .., \phi_n\}$
	$\mid$	**emptyPst**
ϕ	$::=$	
	$\mid$	**reads** (r)
	$\mid$	**writes** (r)
	$\mid$	**reducesid** (r)
Q	$::=$	
	$\mid$	$\{q_1, .., q_n\}$

q	$::=$		
	$\|$	**atomic** (r)	
	$\|$	**simult** (r)	
M	$::=$		
	$\|$	**nullm**	
	$\|$	$M[[Q]]$	
L	$::=$		
	$\|$	**nil**	
	$\|$	$L[(e_1, v_1), .., (e_n, v_n)]$	
	$\|$	$L[e/id]$	
K	$::=$		
	$\|$	**nullk**	
H	$::=$		
	$\|$	$H(l)$	
S	$::=$		
	$\|$	**nullc**	
$emem$	$::=$		
	$\|$	**read** $(e_1, excl, e_2, e_3)$	
	$\|$	**write** $(e_1, excl, e_2, e_3)$	
	$\|$	**reduceid** $(e_1, excl, e_2, e_3)$	
E	$::=$		
	$\|$	$[]$	
	$\|$	$E++[emem]$	
ρ	$::=$		
v	$::=$		
	$\|$	bv	
	$\|$	iv	
	$\|$	$\langle v_1, v_2\rangle$	
	$\|$	**null**	
	$\|$	l	
	$\|$	$\langle\langle \rho_1, .., \rho_n, v\rangle\rangle$	
	$\|$	H	
	$\|$	K	
	$\|$	**place**	
bv	$::=$		
	$\|$	**true**	constant true
	$\|$	**false**	constant false

iv ::=

$fresh$ ::=
| iv

ee ::=
| $(e_1, .., e_n)$

id ::=

T ::=
| **bool**
| **int**
| $\langle T_1, .., T_n \rangle$
| $T@r$
| $T@(r_1, .., r_n)$
| $T[r_1, .., r_n]$
| $T[r_1/r'_1, .., r_n/r'_n]$
| **coloring** (r)
| **exists** $r_1, .., r_n.(T_1, .., T_m), \Phi, Q \rightarrow$ **Tr**

fns ::=
| **apply** (S, E)
| $valid\_interleave(S, C, E_1, .., E_n)$
| **taskid** $fresh$
| $mark\_coherence(E, M,$ **taskid** $)$

$excl$ ::=

e ::=
| x
| **new** $T@r$
| **null** $T@r$
| **isnull** (e)
| **upregion** $(e, r_1, .., r_n)$
| **downregion** $(e, r_1, .., r_n)$
| **read** (e)
| **write** (e_1, e_2)
| **reduce** (id, e_1, e_2)
| **reduceid** (l, e_1, e_2, e_3)
| **newcolor** r
| **color** (e_1, e_2, e_3)
| $e_1 + e_2$
| $e_1 < e_2$
| **let** $id : T = e_1 \in e_2$
| **if** e_1 **then** e_2 **else** e_3 conditional
| $id[r_1, .., r_n](e_1, .., e_m)$

	|	**partition** r_p **using** e_1 **as** $r_1, .., r_n \in e_2$		
	|	**pack** e_1 **as** T		
	|	**unpack** e_1 **as** $id : T \in e_2$		
	|	**function** $id[r_1, .., r_n](e_1, .., e_m)$		
	|	$\{$ **function** $id_1\ rr_1\ ee_1, .., $ **function** $id_n\ rr_n\ ee_n\}$		
	|	$e : T$		
	|	**place**		
	|	bv		
	|	iv		
	|	$\langle v_1, v_2 \rangle$		
	|	**null**		
	|	l		
	|	$\langle\langle \rho_1, .., \rho_n, v \rangle\rangle$		
	|	H		
	|	K		
	|	**true**		constant true
	|	**false**		constant false
	|	0		
	|	$S\ iv$		
formula	::=			
	|	*judgement*		judgement
	|	$\neg formula$	M	negated formula
	|	$(formula)$	M	bracketed
	|	$\forall_i.\phi \in \Phi$	M	for all variables in domain of Φ
	|	$\forall_i.\phi \in \Phi^*$	M	for all variables in domain of Φ^*
	|	$\exists_i.\phi \in \Phi$	M	for all variables in domain of Φ
	|	$\forall_i.\omega \in \Omega$	M	for all variables in domain of Ω
	|	$\forall_i.\omega \in \Omega^*$	M	for all variables in domain of Ω^*
	|	$\exists_i.\omega \in \Omega$	M	for all variables in domain of Ω
	|	$\exists_i.\omega \in \Omega^*$	M	for all variables in domain of Ω^*
	|	$\forall_i.formula$	M	for all variables in i and *formula*
	|	$\exists_{formula_1}.formula_2$	M	for all variables in $formula_1$ and for
	|	$\exists_{formula_1}.formula_2$ where $formula_3$	M	exists $formula_1$ and $formula_2$ where
	|	$formula_1 \in formula_2$		$formula_1$ is in $formula_2$
	|	$\Gamma(id)$		lookup
	|	$formula_1 = formula_2$		equality
	|	$formula_1 \wedge formula_2$		equality
	|	$\bigwedge_i .formula$	M	and fold on i and *formula*
	|	$formula_1 \cap formula_2$	M	
	|	$formula_1 \cup formula_2$	M	
	|	$formula_1 \subseteq formula_2$	M	
	|	$formula_1 \in formula_2$	M	
	|	$\Gamma, \Phi, \Omega \rightarrow T$		impl
	|	$\Gamma, \Phi, Q \rightarrow T$		impl
	|	$r_1, .., r_n$		region list

	$\mid$	ϕ	phi
	$\mid$	ω	om
	$\mid$	Ω	
	$\mid$	Φ	
	$\mid$	Φ^*	
	$\mid$	$\Phi[r_1/r_1', \,..\,, r_n/r_n']$	
	$\mid$	$T[r_1/r_1', \,..\,, r_n/r_n']$	
	$\mid$	$M[\rho_1/r_1', \,..\,, \rho_n/r_n']$	
	$\mid$	$M[[T]]$	
	$\mid$	$M[[Q]]$	
	$\mid$	$\textbf{domain}\,(S)$	
	$\mid$	$M(r)$	
	$\mid$	$\Gamma[r_1/r_1', \,..\,, r_n/r_n']$	
	$\mid$	$\Gamma[e_1/T_1, \,..\,, e_n/T_n]$	
	$\mid$	Γ	
	$\mid$	$\Gamma[T[r_1/r_1', \,..\,, r_n/r_n']/id]$	
	$\mid$	$\Omega[r_1/r_1', \,..\,, r_n/r_n']$	
	$\mid$	$regions\_of(\Gamma, T)$	
	$\mid$	$regions\_of(\Gamma, T1, T_2)$	
	$\mid$	rs	
	$\mid$	T	
	$\mid$	fns	
	$\mid$	S	
	$\mid$	C	
	$\mid$	M	
	$\mid$	e	
	$\mid$	E	
	$\mid$	L	
	$\mid$	$\overline{e_i = l_i}^{\,i<n}$	
$terminals$	$::=$		
	$\mid$	$\exists$	
	$\mid$	$\forall$	
	$\mid$	$\in$	
	$\mid$	ω	
	$\mid$	ϕ	
	$\mid$	ρ	
	$\mid$	$\vee$	
	$\mid$	$\wedge$	
	$\mid$	$\neg$	
	$\mid$	$*$	
	$\mid$	$\leq$	
	$\mid$	$\longrightarrow$	
	$\mid$	$\rightarrow$	
	$\mid$	$\Rightarrow$	
	$\mid$	λ	

	$\|$	$\mapsto$	
	$\|$	$\vdash$	
	$\|$	$\varnothing$	
	$\|$	$\varnothing$	
	$\|$	$\varnothing$	
	$\|$	$\varnothing$	
	$\|$	$\times$	
	$\|$	$<:$	
	$\|$	$\langle$	
	$\|$	$\rangle$	
	$\|$	$<$	
	$\|$	$\Downarrow$	
	$\|$	σ	
	$\|$	Γ	
	$\|$	ε	
Jtype	$::=$		
	$\|$	$\Gamma, \Phi, \Omega \vdash e : T$	Typing
Jop	$::=$		
	$\|$	$M, L, H, S, C \vdash e \mapsto v, E$	Evaluation
judgement	$::=$		
	$\|$	*Jtype*	
	$\|$	*Jop*	
user_syntax	$::=$		
	$\|$	*termvar*	
	$\|$	*index*	
	$\|$	r	
	$\|$	rr	
	$\|$	l	
	$\|$	C	
	$\|$	Γ	
	$\|$	Ω	
	$\|$	Ω^*	
	$\|$	rs	
	$\|$	ω	
	$\|$	Φ	
	$\|$	Φ^*	
	$\|$	ϕ	
	$\|$	Q	
	$\|$	q	
	$\|$	M	
	$\|$	L	
	$\|$	K	
	$\|$	H	

$|$ S
$|$ $emem$
$|$ E
$|$ ρ
$|$ v
$|$ bv
$|$ iv
$|$ $fresh$
$|$ ee
$|$ id
$|$ T
$|$ fns
$|$ $excl$
$|$ e
$|$ $formula$
$|$ $terminals$

$\boxed{\Gamma, \Phi, \Omega \vdash e : T}$ Typing

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, .., r_n)}{\Gamma, \Phi, \Omega \vdash \mathbf{read}\,(e_1) : T} \quad \text{T\_READ}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, .., r_n) \\ \Gamma, \Phi, \Omega \vdash e_2 : T\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{write}\,(e_1, e_2) : T@(r_1, .., r_n)} \quad \text{T\_WRITE}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : T_1@(r_1, .., r_n) \\ \Gamma, \Phi, \Omega \vdash e_2 : T_2\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{reduce}\,(id, e_1, e_2) : T_1@(r_1, .., r_n)} \quad \text{T\_REDUCE}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \mathbf{new}\ T@r : T@r} \quad \text{T\_NEW}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e : T@(r'_1, .., r'_k)}{\Gamma, \Phi, \Omega \vdash \mathbf{upregion}\,(e_1, r_1, .., r_n) : T@(r_1, .., r_n)} \quad \text{T\_UPRGN}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e : T@(r'_1, .., r'_k)}{\Gamma, \Phi, \Omega \vdash \mathbf{downregion}\,(e, r_1, .., r_n) : T@(r_1, .., r_n)} \quad \text{T\_DNRGN}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \mathbf{newcolor}\ r : \mathbf{coloring}\,(r)} \quad \text{T\_NEWCOLOR}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : \mathbf{coloring}\,(r) \\ \Gamma, \Phi, \Omega \vdash e_2 : T@r \\ \Gamma, \Phi, \Omega \vdash e_3 : \mathbf{int}\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{color}\,(e_1, e_2, e_3) : \mathbf{coloring}\,(r)} \quad \text{T\_COLOR}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : \mathbf{coloring}\,(r_p) \\ \Gamma, \Phi, \Omega' \vdash e_2 : T\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{partition}\ r_p\ \mathbf{using}\ e_1\ \mathbf{as}\ r_1, .., r_k \in e_2 : T} \quad \text{T\_PARTITION}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T_2[r_1/r'_1, .., r_k/r'_k]}{\Gamma, \Phi, \Omega \vdash \mathbf{pack}\ e_1\ \mathbf{as}\ T_1 : T_1} \quad \text{T\_PACK}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : T_1 \\ \Gamma', \Phi, \Omega' \vdash e_2 : T_3\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{unpack}\ e_1\ \mathbf{as}\ id : T_1 \in e_2 : T_3} \quad \text{T\_UNPACK}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash id[r_1, .., r_k](e_1, .., e_n) : T} \quad \text{T\_CALL}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \{\, \textbf{function}\; id_1\, [r_1, .., r_k]\, (e_1, .., e_m), .., \textbf{function}\; id_n\, [r_1, .., r_k]\, (e_1, .., e_m)\} : T} \quad \text{T\_PROGRAM}$$

$\boxed{M, L, H, S, C \vdash e \mapsto v, E}$ Evaluation

$$\frac{}{M, L, H, S, C \vdash x \mapsto v, E} \quad \text{EVA}$$

$$\frac{\begin{array}{l} M, L, H, S, C \vdash e \mapsto l, E \\ S' = \textbf{apply}\,(S, E) \\ \neg\,(l \in C) \end{array}}{M, L, H, S, C \vdash \textbf{read}\,(e) \mapsto v, E + +[\textbf{read}\,(l, excl, v, 0)]} \quad \text{EREAD1}$$

$$\frac{\begin{array}{l} M, L, H, S, C \vdash e \mapsto l, E \\ l \in C \end{array}}{M, L, H, S, C \vdash \textbf{read}\,(e) \mapsto H(l), E + +[\textbf{read}\,(l, excl, v, 0)]} \quad \text{EREAD2}$$

$$\frac{\begin{array}{c} M, L, H, S, C \vdash e_1 \mapsto l, E_1 \\ M, L, H, S', C \vdash e_2 \mapsto v, E_2 \end{array}}{M, L, H, S, C \vdash \textbf{write}\,(e_1, e_2) \mapsto l, E} \quad \text{EWRITE}$$

$$\frac{\begin{array}{c} M, L, H, S, C \vdash e_1 \mapsto l, E_1 \\ M, L, H, S', C \vdash e_2 \mapsto v, E_2 \end{array}}{M, L, H, S, C \vdash \textbf{reduce}\,(id, e_1, e_2) \mapsto l, E} \quad \text{EREDUCE}$$

$$\frac{\begin{array}{c} M, L, H, S, C \vdash l \mapsto v_1, E_1 \\ M, L, H, S', C \vdash e_1 \mapsto v_2, E_2 \\ M, L, H, S', C \vdash e_2 \mapsto v_3, E_3 \\ M, L, H, S', C \vdash e_3 \mapsto v_4, E_3 \end{array}}{M, L, H, S, C \vdash \textbf{reduceid}\,(l, e_1, e_2, e_3) \mapsto v, E} \quad \text{EREDUCEID}$$

$$\frac{}{M, L, H, S, C \vdash \textbf{null} \mapsto \textbf{null}, []} \quad \text{ENULL}$$

$$\frac{}{M, L, H, S, C \vdash \textbf{new}\; T@r \mapsto l, []} \quad \text{ENEW}$$

$$\frac{}{M, L, H, S, C \vdash \textbf{null}\; T@\textbf{nullr} \mapsto \textbf{true}, []} \quad \text{ENULLTRUE}$$

$$\frac{\neg\,(T@r = T@\textbf{nullr})}{M, L, H, S, C \vdash \textbf{null}\; T@r \mapsto \textbf{false}, []} \quad \text{ENULLFALSE}$$

$$\frac{\begin{array}{l} M, L, H, S, C \vdash e_1 \mapsto l, E_1 \\ l = \textbf{null} \end{array}}{M, L, H, S, C \vdash \textbf{isnull}\,(l) \mapsto \textbf{true}, []} \quad \text{EISNULLTRUE}$$

$$\frac{\begin{array}{l} M, L, H, S, C \vdash e_1 \mapsto l, E_1 \\ \neg\,(l = \textbf{null}) \end{array}}{M, L, H, S, C \vdash \textbf{isnull}\,(l) \mapsto \textbf{false}, []} \quad \text{EISNULLFALSE}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \textbf{upregion}\,(e, r_1, .., r_n) \mapsto v, E} \quad \text{EUPRGN}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \textbf{downregion}\,(e, r_1, .., r_n) \mapsto l, E} \quad \text{EDNRGN1}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \textbf{downregion}\,(e, r_1, .., r_n) \mapsto \textbf{null}, E} \quad \text{EDNRGN2}$$

$$\frac{\begin{array}{c} M, L, H, S, C \vdash e_1 \mapsto l, E_1 \\ M, L, H, S', C \vdash e_2 \mapsto v, E_2 \\ M, L, H, S'', C \vdash e_3 \mapsto v, E_3 \end{array}}{M, L, H, S, C \vdash \textbf{color}\,(e_1, e_2, e_3) \mapsto K', E'} \quad \textsc{EColor}$$

$$\frac{}{M, L, H, S, C \vdash \textbf{newcolor}\; r \mapsto K, []} \quad \textsc{ENewColor}$$

$$\frac{\begin{array}{c} M, L, H, S, C \vdash e_1 \mapsto K, E_1 \\ M, L, H, S', C \vdash e_2 \mapsto v, E_2 \end{array}}{M, L, H, S, C \vdash \textbf{partition}\; r_p \;\textbf{using}\; e_1 \;\textbf{as}\; r_1, \ldots, r_k \;\in\; e_2 \mapsto l, E'} \quad \textsc{EPartition}$$

$$\frac{M, L, H, S, C \vdash e_1 \mapsto K, E_1}{M, L, H, S, C \vdash \textbf{pack}\; e_1 \;\textbf{as}\; T_1 \mapsto v', E} \quad \textsc{EPack}$$

$$\frac{\begin{array}{l} M, L, H, S, C \vdash e_1 \mapsto \langle\langle \rho_1, \ldots, \rho_k, v\rangle\rangle, E_1 \\ L' = L[v_1/id] \\ M', L', H, S', C \vdash e_2 \mapsto v, E_2 \end{array}}{M, L, H, S, C \vdash \textbf{unpack}\; e_1 \;\textbf{as}\; id : T_1 \;\in\; e_2 \mapsto v_2, E'} \quad \textsc{EUnpack}$$

$$\frac{L' = L[(e_1, v_1), \ldots, (e_n, v_n)]}{M, L, H, S, C \vdash id[r_1, \ldots, r_k](e_1, \ldots, e_n) \mapsto v_n, E''} \quad \textsc{ECall}$$

$$\frac{\begin{array}{c} M, L, H, S, C \vdash e_1 \mapsto v_1, E_1 \\ M, L, H, S, C \vdash e_2 \mapsto v_2, E_2 \end{array}}{M, L, H, S, C \vdash e_1 + e_2 \mapsto v, E} \quad \textsc{EIntOp}$$

$$\frac{\begin{array}{c} M, L, H, S, C \vdash e_1 \mapsto v_1, E_1 \\ M, L, H, S, C \vdash e_2 \mapsto v_2, E_2 \end{array}}{M, L, H, S, C \vdash e_1 \;<\; e_2 \mapsto v, E} \quad \textsc{EComp}$$

$$\frac{\begin{array}{l} M, L, H, S, C \vdash e_1 \mapsto v_1, E_1 \\ M, L, H, S, C \vdash e_2 \mapsto v_2, E_2 \\ L' = L[v_1/id] \end{array}}{M, L, H, S, C \vdash \textbf{let}\; id : T = e_1 \;\in\; e_2 \mapsto v, E} \quad \textsc{ELet}$$

$$\frac{\begin{array}{c} M, L, H, S, C \vdash e_1 \mapsto \textbf{true}, E_1 \\ M, L, H, S, C \vdash e_2 \mapsto v, E_1 \end{array}}{M, L, H, S, C \vdash \textbf{if}\; e_1 \;\textbf{then}\; e_2 \;\textbf{else}\; e_3 \mapsto v, E} \quad \textsc{EIfTrue}$$

$$\frac{\begin{array}{c} M, L, H, S, C \vdash e_1 \mapsto \textbf{false}, E_1 \\ M, L, H, S, C \vdash e_3 \mapsto v, E_2 \end{array}}{M, L, H, S, C \vdash \textbf{if}\; e_1 \;\textbf{then}\; e_2 \;\textbf{else}\; e_3 \mapsto v, E} \quad \textsc{EIfFalse}$$

$$\frac{L' = L[id[r_1, \ldots, r_n](e_1, \ldots, e_m)/id]}{M, L, H, S, C \vdash \textbf{function}\; id[r_1, \ldots, r_n](e_1, \ldots, e_m) \mapsto v, E} \quad \textsc{EFuncDef}$$

$$\frac{}{M, L, H, S, C \vdash \{\,\textbf{function}\; id_1\; rr_1\; ee_1, \ldots, \textbf{function}\; id_n\; rr_n\; ee_n\} \mapsto v, E} \quad \textsc{EFuncDefList}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash e : T \mapsto v, E} \quad \textsc{ETypedExpr}$$

$$\frac{}{M, L, H, S, C \vdash \textbf{place} \mapsto \textbf{place}, E} \quad \textsc{EPlace}$$

$$\frac{}{M, L, H, S, C \vdash bv \mapsto bv, E} \quad \textsc{Ebv}$$

$$\frac{}{M, L, H, S, C \vdash iv \mapsto iv, E} \quad \textsc{Eiv}$$

$$\frac{}{M, L, H, S, C \vdash \langle v_1, v_2 \rangle \mapsto \langle v_1, v_2 \rangle, E} \quad \textsc{ETuple}$$

$$\frac{}{M, L, H, S, C \vdash \mathbf{null} \mapsto \mathbf{null}, E} \quad \textsc{ENullLoc}$$

$$\frac{}{M, L, H, S, C \vdash l \mapsto l, E} \quad \textsc{EMemoryLoc}$$

$$\frac{}{M, L, H, S, C \vdash \langle\langle \rho_1, .., \rho_n, v \rangle\rangle \mapsto \langle\langle \rho_1, .., \rho_n, v \rangle\rangle, E} \quad \textsc{ERegRelInst}$$

$$\frac{}{M, L, H, S, C \vdash H \mapsto H, E} \quad \textsc{EHeapVal}$$

$$\frac{}{M, L, H, S, C \vdash K \mapsto K, E} \quad \textsc{EKthing}$$

```
Definition rules:         51 good    0 bad
Definition rule clauses: 110 good    0 bad
```