$termvar$, x term variable

$index$, i, j, k, n, m, p

$$
\begin{array}{lcl}
T & ::= & \\
 & | & \mathbf{bool} \\
 & | & \mathbf{int} \\
 & | & \langle T_1, \, .. \, , T_n \rangle \\
 & | & T@r \\
 & | & T@(r_1, \, .. \, , r_n) \\
 & | & T[r_1/r'_1, \, .. \, , r_n/r'_n] \\
 & | & \mathbf{coloring}\,(r) \\
 & | & \mathbf{exists}\, r_1, \, .. \, , r_n.(T_1, \, .. \, , T_m), \Phi, Q \rightarrow \ \mathbf{Tr} \\
 & & \\
flist & ::= & \\
 & | & \{fdef_1, \, .. \, , fdef_n\} \\
 & & \\
fdef & ::= & \\
 & | & \mathbf{function} \\
 & & \\
r & ::= & \\
 & & \\
rr & ::= & \\
 & | & [r_1, \, .. \, , r_n] \\
 & & \\
\Gamma & ::= & \\
 & | & \{(e_1 : T_1), \, .. \, , (e_n : T_n)\} \\
 & | & \varnothing \\
 & & \\
\Omega & ::= & \\
 & | & \{\omega_1, \, .. \, , \omega_n\} \\
 & | & \varnothing \\
 & & \\
\Omega^* & ::= & \\
 & | & \{\omega_1, \, .. \, , \omega_n\} \\
 & | & \mathbf{emptyOst} \\
 & & \\
rs & ::= & \\
 & | & \{r_1, \, .. \, , r_n\} \\
 & | & \varnothing \\
 & & \\
\omega & ::= & \\
 & | & r_1 \leq r_2 \\
 & | & r_1 * r_2 \\
 & & \\
\Phi & ::= & \\
 & | & \{\phi_1, \, .. \, , \phi_n\} \\
 & | & \varnothing \\
 & & \\
\Phi^* & ::= & \\
 & | & \{\phi_1, \, .. \, , \phi_n\}
\end{array}
$$

	$\vert$	**emptyPst**	
ϕ	::=		
	$\vert$	**reads** (r)	
	$\vert$	**writes** (r)	
	$\vert$	**reducesid** (r)	
Q	::=		
	$\vert$	$\{q_1, \ldots, q_n\}$	
q	::=		
	$\vert$	**atomic** (r)	
	$\vert$	**simult** (r)	
ρ	::=		
v	::=		
	$\vert$	bv	
	$\vert$	iv	
	$\vert$	$\langle v_1, v_2\rangle$	
	$\vert$	**null**	
	$\vert$	**l**	
	$\vert$	$\langle\langle\rho_1, \ldots, \rho_n, v\rangle\rangle$	
bv	::=		
	$\vert$	**true**	constant true
	$\vert$	**false**	constant false
iv	::=		
	$\vert$	0	
	$\vert$	**S** iv	
ee	::=		
	$\vert$	$(e_1, \ldots, e_n)$	
id	::=		
e	::=		
	$\vert$	x	
	$\vert$	iv	
	$\vert$	bv	
	$\vert$	$\langle e1, .., en\rangle$	
	$\vert$	$e\ iv$	
	$\vert$	id	
	$\vert$	**new** $T@r$	
	$\vert$	**null** $T@r$	
	$\vert$	**isnull** (e)	

	|	**upregion** $(e, r1, .., rn)$		
	|	**downregion** $(e, r1, .., rn)$		
	|	**read** (e)		
	|	**write** (e_1, e_2)		
	|	**reduce** (id, e_1, e_2)		
	|	**newcolor** r		
	|	**color** (e_1, e_2, e_3)		
	|	$e_1 + e_2$		
	|	$e_1 < e_2$		
	|	**let** $id : T = e_1 \in e_2$		
	|	**if b then** $c0$ **else** $c1$		conditional
	|	$id[r_1, .., r_n](e_1, .., e_m)$		
	|	**partition** r_p **using** e_1 **as** $r_1, .., r_n \in e_2$		
	|	**pack** e_1 **as** $T[r_1, .., r_n]$		
	|	**unpack** e_1 **as** $id : T[r_1, .., r_n] \in e_2$		
	|	**function** $id[r_1, .., r_n](e_1, .., e_m)$		
	|	$\{$ **function** $id_1\ rr_1\ ee_1, .., $ **function** $id_n\ rr_n\ ee_n\}$		
formula	::=			
	|	*judgement*		judgement
	|	$\neg formula$	M	negated formula
	|	$(formula)$	M	bracketed
	|	$\forall_i.\phi \in \Phi$	M	for all variables in domain of Φ
	|	$\forall_i.\phi \in \Phi^*$	M	for all variables in domain of Φ^*
	|	$\exists_i.\phi \in \Phi$	M	for all variables in domain of Φ
	|	$\forall_i.\omega \in \Omega$	M	for all variables in domain of Ω
	|	$\forall_i.\omega \in \Omega^*$	M	for all variables in domain of Ω^*
	|	$\exists_i.\omega \in \Omega$	M	for all variables in domain of Ω
	|	$\exists_i.\omega \in \Omega^*$	M	for all variables in domain of Ω^*
	|	$\forall_i.formula$	M	for all variables in i and *formula*
	|	$\exists_{formula_1}.formula_2$	M	for all variables in $formula_1$ and *for*
	|	$\exists_{formula_1}.formula_2$ where $formula_3$	M	exists $formula_1$ and $formula_2$ where
	|	$\Gamma(id)$		lookup
	|	$formula_1 = formula_2$		equality
	|	$formula_1 \wedge formula_2$		equality
	|	$\bigwedge_i .formula$	M	and fold on i and *formula*
	|	$formula_1 \cap formula_2$	M	
	|	$formula_1 \cup formula_2$	M	
	|	$formula_1 \subseteq formula_2$	M	
	|	$\Gamma, \Phi, \Omega \to T$		impl
	|	$\Gamma, \Phi, Q \to T$		impl
	|	$r_1, .., r_n$		region list
	|	ϕ		phi
	|	ω		om
	|	Ω		
	|	Φ		

	$\mid$	Φ^*
	$\mid$	$\Phi[r_1/r'_1, .., r_n/r'_n]$
	$\mid$	$T[r_1/r'_1, .., r_n/r'_n]$
	$\mid$	$\Gamma[r_1/r'_1, .., r_n/r'_n]$
	$\mid$	$\Gamma[e_1/T_1, .., e_n/T_n]$
	$\mid$	Γ
	$\mid$	$\Gamma[T[r_1/r'_1, .., r_n/r'_n]/id]$
	$\mid$	$\Omega[r_1/r'_1, .., r_n/r'_n]$
	$\mid$	$regions\_of(\Gamma, T)$
	$\mid$	$regions\_of(\Gamma, T1, T_2)$
	$\mid$	rs
	$\mid$	T
$terminals$	$::=$	
	$\mid$	$\exists$
	$\mid$	$\forall$
	$\mid$	$\in$
	$\mid$	ω
	$\mid$	ϕ
	$\mid$	ρ
	$\mid$	$\vee$
	$\mid$	$\wedge$
	$\mid$	$\neg$
	$\mid$	$*$
	$\mid$	$\leq$
	$\mid$	$\longrightarrow$
	$\mid$	$\rightarrow$
	$\mid$	$\Rightarrow$
	$\mid$	λ
	$\mid$	$\mapsto$
	$\mid$	$\vdash$
	$\mid$	$\varnothing$
	$\mid$	$\varnothing$
	$\mid$	$\varnothing$
	$\mid$	$\varnothing$
	$\mid$	$\times$
	$\mid$	$<:$
	$\mid$	$\langle$
	$\mid$	$\rangle$
	$\mid$	$<$
	$\mid$	$\Downarrow$
	$\mid$	σ
	$\mid$	Γ
	$\mid$	ε
$Jtype$	$::=$	

	$\vert$	$\Gamma, \Phi, \Omega \vdash e : T$	Typing
judgement	$::=$		
	$\vert$	*Jtype*	
user_syntax	$::=$		
	$\vert$	*termvar*	
	$\vert$	*index*	
	$\vert$	T	
	$\vert$	*flist*	
	$\vert$	*fdef*	
	$\vert$	r	
	$\vert$	rr	
	$\vert$	Γ	
	$\vert$	Ω	
	$\vert$	Ω^*	
	$\vert$	rs	
	$\vert$	ω	
	$\vert$	Φ	
	$\vert$	Φ^*	
	$\vert$	ϕ	
	$\vert$	Q	
	$\vert$	q	
	$\vert$	ρ	
	$\vert$	v	
	$\vert$	bv	
	$\vert$	iv	
	$\vert$	ee	
	$\vert$	id	
	$\vert$	e	
	$\vert$	*formula*	
	$\vert$	*terminals*	

$\boxed{\Gamma, \Phi, \Omega \vdash e : T}$ Typing

$$\frac{\begin{array}{l}\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, .., r_n) \\ \forall_i.\mathbf{reads}\,(r_i) \in \Phi^*\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{read}\,(e_1) : T} \quad \text{T\_READ}$$

$$\frac{\begin{array}{l}\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, .., r_n) \\ \Gamma, \Phi, \Omega \vdash e_2 : T \\ \forall_i.\mathbf{writes}\,(r_i) \in \Phi^*\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{write}\,(e_1, e_2) : T@(r_1, .., r_n)} \quad \text{T\_WRITE}$$

$$\frac{\begin{array}{l}\{(e_1 : T_1), (e_2 : T_2)\}, \varnothing, \varnothing \to T_1 \\ \Gamma, \Phi, \Omega \vdash e_1 : T_1@(r_1, .., r_n) \\ \Gamma, \Phi, \Omega \vdash e_2 : T_2 \\ \forall_i.\mathbf{reducesid}\,(r_i) \in \Phi^*\end{array}}{\Gamma, \Phi, \Omega \vdash \mathbf{reduce}\,(id, e_1, e_2) : T_1@(r_1, .., r_n)} \quad \text{T\_REDUCE}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \mathbf{new}\ T@r : T@r} \quad \text{T\_NEW}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e : T@(r'_1, .., r'_k) \\ \forall_i . \exists_j . r'_i \leq r_j \in \Omega^*\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{upregion}\,(e_1, r1, .., rn) : T@(r_1, .., r_n)} \quad \textsc{T\_UpRgn}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e : T@(r'_1, .., r'_k)}{\Gamma, \Phi, \Omega \vdash \textbf{downregion}\,(e, r1, .., rn) : T@(r_1, .., r_n)} \quad \textsc{T\_DnRgn}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \textbf{newcolor}\ r : \textbf{coloring}\,(r)} \quad \textsc{T\_NewColor}$$

$$\frac{\begin{array}{l}\Gamma, \Phi, \Omega \vdash e_1 : \textbf{coloring}\,(r) \\ \Gamma, \Phi, \Omega \vdash e_2 : T@r \\ \Gamma, \Phi, \Omega \vdash e_3 : \textbf{int}\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{color}\,(e_1, e_2, e_3) : \textbf{coloring}\,(r)} \quad \textsc{T\_Color}$$

$$\frac{\begin{array}{l}\Gamma, \Phi, \Omega \vdash e_1 : \textbf{coloring}\,(r_p) \\ (\Omega' = \Omega) \wedge \left((\bigwedge_i . r_i \leq r_p) \wedge \left(\bigwedge_j . r_i * r_j\right)\right) \\ \Gamma, \Phi, \Omega' \vdash e_2 : T \\ ((\{r_1, .., r_k\}) \cap regions\_of(\Gamma, T)) = \varnothing\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{partition}\ r_p\ \textbf{using}\ e_1\ \textbf{as}\ r_1, .., r_k\ \in\ e_2 : T} \quad \textsc{T\_Partition}$$

$$\frac{\begin{array}{l}T_1 = \exists_{r'_1}, .., r'_k . T_2 \text{ where } \Omega_1 \\ \Omega_1[r_1/r'_1, .., r_k/r'_k] \subseteq \Omega' \\ \Gamma, \Phi, \Omega \vdash e_1 : T_2[r_1/r'_1, .., r_k/r'_k]\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{pack}\ e_1\ \textbf{as}\ T_1[r_1, .., r_k] : T_1} \quad \textsc{T\_Pack}$$

$$\frac{\begin{array}{l}T_1 = \exists_{r'_1}, .., r'_k . T_2 \text{ where } \Omega_1 \\ \Gamma, \Phi, \Omega \vdash e_1 : T_1 \\ \Gamma' = \Gamma[T_2[r_1/r'_1, .., r_k/r'_k]/id] \\ \Omega' = (\Omega \cup \Omega_1[r_1/r'_1, .., r_k/r'_k]) \\ \Gamma', \Phi, \Omega' \vdash e_2 : T_3 \\ (\{r_1, .., r_k\} \cap regions\_of(\Gamma, T1, T_3)) = \varnothing\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{unpack}\ e_1\ \textbf{as}\ id : T_1[r_1, .., r_k]\ \in\ e_2 : T_3} \quad \textsc{T\_UnPack}$$

$$\frac{\begin{array}{l}\Gamma, \Phi', Q' \to T \\ \Gamma, \Phi, \Omega \vdash e_1 : T[r_1/r'_1, .., r_k/r'_k] \\ \Phi'[r_1/r'_1, .., r_k/r'_k] \subseteq \Phi^*\end{array}}{\Gamma, \Phi, \Omega \vdash id[r_1, .., r_k](e_1, .., e_m) : T} \quad \textsc{T\_Call}$$

$$\frac{\begin{array}{l}\texttt{<<no parses (char 1): t***rip1,..,tripp >>} \\ \Gamma, \Phi', Q' \to T \\ \Omega_1[r_1/r'_1, .., r_k/r'_k] \subseteq \Omega' \\ \Gamma = \Gamma[e_1/T_1, .., e_m/T_m]\end{array}}{\Gamma, \Phi, \Omega \vdash \{\textbf{function}\ id_1\,[r_1, .., r_k]\,(e_1, .., e_m), .., \textbf{function}\ id_n\,[r_1, .., r_k]\,(e_1, .., e_m)\} : T} \quad \textsc{T\_Program}$$

```
Definition rules:        12 good    1 bad
Definition rule clauses: 47 good    1 bad
```