

$termvar, x$	term variable
$index, i, j, k, n, m, p$	
T	$::=$ $ \quad \mathbf{bool}$ $ \quad \mathbf{int}$ $ \quad \langle T_1, \dots, T_n \rangle$ $ \quad T@r$ $ \quad T@(r_1, \dots, r_n)$ $ \quad T[r_1, \dots, r_n]$ $ \quad T[r_1/r'_1, \dots, r_n/r'_n]$ $ \quad \mathbf{coloring}(r)$ $ \quad \mathbf{exists} \, r_1, \dots, r_n. (T_1, \dots, T_m), \Phi, Q \rightarrow \mathbf{Tr}$
$fresh$	$::=$ $ \quad iv$
fns	$::=$ $ \quad \mathbf{apply}(S, E)$ $ \quad \mathbf{valid_interleave}(S, C, E_1, \dots, E_n)$ $ \quad \mathbf{taskid} \, fresh$ $ \quad \mathbf{mark_coherence}(E, M, \mathbf{taskid})$
r	$::=$ $ \quad \mathbf{nullr}$ $ \quad \mathbf{nonnullr}$
rr	$::=$ $ \quad [r_1, \dots, r_n]$
l	$::=$ $ \quad \mathbf{nil}$
Γ	$::=$ $ \quad \{(e_1 : T_1), \dots, (e_n : T_n)\}$ $ \quad \emptyset$
Ω	$::=$ $ \quad \{\omega_1, \dots, \omega_n\}$ $ \quad \emptyset$
Ω^*	$::=$ $ \quad \{\omega_1, \dots, \omega_n\}$ $ \quad \mathbf{emptyOst}$
rs	$::=$ $ \quad \{r_1, \dots, r_n\}$ $ \quad \emptyset$
ω	$::=$

	$\begin{array}{ l} r_1 \leq r_2 \\ r_1 * r_2 \end{array}$
Φ	$\begin{array}{ l} ::= \\ \{ \phi_1, \dots, \phi_n \} \\ \emptyset \end{array}$
Φ^*	$\begin{array}{ l} ::= \\ \{ \phi_1, \dots, \phi_n \} \\ \mathbf{emptyPst} \end{array}$
ϕ	$\begin{array}{ l} ::= \\ \mathbf{reads}(r) \\ \mathbf{writes}(r) \\ \mathbf{reducesid}(r) \end{array}$
Q	$\begin{array}{ l} ::= \\ \{ q_1, \dots, q_n \} \end{array}$
q	$\begin{array}{ l} ::= \\ \mathbf{atomic}(r) \\ \mathbf{simult}(r) \end{array}$
M	$\begin{array}{ l} ::= \\ M[[Q]] \end{array}$
L	$\begin{array}{ l} ::= \\ \mathbf{nil} \\ L[(e_1, v_1), \dots, (e_n, v_n)] \\ L[e/id] \end{array}$
K	$::=$
H	$\begin{array}{ l} ::= \\ H(l) \end{array}$
S	$\begin{array}{ l} ::= \\ \mathbf{nullc} \end{array}$
C	$::=$
E	$\begin{array}{ l} ::= \\ [] \\ [e] \\ E ++ [e] \end{array}$
ρ	$::=$

	$\text{unpack } e_1 \text{ as } id : T \in e_2$ $\text{function } id[r_1, \dots, r_n](e_1, \dots, e_m)$ $\{ \text{function } id_1 rr_1 ee_1, \dots, \text{function } id_n rr_n ee_n \}$ $e : T$ place bv iv $\langle v_1, v_2 \rangle$ null l $\langle \langle \rho_1, \dots, \rho_n, v \rangle \rangle$ H K true false 0 $S \text{ iv}$		
<i>formula</i>	$::=$		
	<i>judgement</i>		judgement
	$\neg \text{formula}$	M	negated formula
	(formula)	M	bracketed
	$\forall_i. \phi \in \Phi$	M	for all variables in domain of Φ
	$\forall_i. \phi \in \Phi^*$	M	for all variables in domain of Φ^*
	$\exists_i. \phi \in \Phi$	M	for all variables in domain of Φ
	$\forall_i. \omega \in \Omega$	M	for all variables in domain of Ω
	$\forall_i. \omega \in \Omega^*$	M	for all variables in domain of Ω^*
	$\exists_i. \omega \in \Omega$	M	for all variables in domain of Ω
	$\exists_i. \omega \in \Omega^*$	M	for all variables in domain of Ω^*
	$\forall_i. \text{formula}$	M	for all variables in i and <i>formula</i>
	$\exists_{\text{formula}_1}. \text{formula}_2$	M	for all variables in <i>formula</i> ₁ and <i>formula</i> ₂
	$\exists_{\text{formula}_1}. \text{formula}_2 \text{ where } \text{formula}_3$	M	exists <i>formula</i> ₁ and <i>formula</i> ₂ where
	$\Gamma(id)$		lookup
	$\text{formula}_1 = \text{formula}_2$		equality
	$\text{formula}_1 \wedge \text{formula}_2$		equality
	$\bigwedge_i. \text{formula}$	M	and fold on i and <i>formula</i>
	$\text{formula}_1 \cap \text{formula}_2$	M	
	$\text{formula}_1 \cup \text{formula}_2$	M	
	$\text{formula}_1 \subseteq \text{formula}_2$	M	
	$\text{formula}_1 \in \text{formula}_2$	M	
	$\Gamma, \Phi, \Omega \rightarrow T$		impl
	$\Gamma, \Phi, Q \rightarrow T$		impl
	r_1, \dots, r_n		region list
	ϕ		phi
	ω		om
	Ω		

	Φ
	Φ^*
	$\Phi[r_1/r'_1, \dots, r_n/r'_n]$
	$T[r_1/r'_1, \dots, r_n/r'_n]$
	$M[\rho_1/r'_1, \dots, \rho_n/r'_n]$
	$M[[T]]$
	$M[[Q]]$
	$\mathbf{domain}(S)$
	$M(r)$
	$\Gamma[r_1/r'_1, \dots, r_n/r'_n]$
	$\Gamma[e_1/T_1, \dots, e_n/T_n]$
	Γ
	$\Gamma[T[r_1/r'_1, \dots, r_n/r'_n]/id]$
	$\Omega[r_1/r'_1, \dots, r_n/r'_n]$
	$regions_of(\Gamma, T)$
	$regions_of(\Gamma, T_1, T_2)$
	rs
	T
	fns
	S
	C
	M
	e
	E
	L
	$\overline{e_i = l_i}^{i < n}$
<i>terminals</i>	$::=$
	\exists
	\forall
	\in
	ω
	ϕ
	ρ
	\vee
	\wedge
	\neg
	$*$
	\leq
	\longrightarrow
	\rightarrow
	\Rightarrow
	λ
	\mapsto
	\vdash
	\emptyset

		\emptyset	
		\emptyset	
		\emptyset	
		\times	
		$<:$	
		\langle	
		\rangle	
		$<$	
		\Downarrow	
		σ	
		Γ	
		ε	
$Jtype$	$::=$		
		$\Gamma, \Phi, \Omega \vdash e : T$	Typing
Jop	$::=$		
		$M, L, H, S, C \vdash e \mapsto v, E$	Evaluation
$judgement$	$::=$		
		$Jtype$	
		Jop	
$user_syntax$	$::=$		
		$termvar$	
		$index$	
		T	
		$fresh$	
		fns	
		r	
		rr	
		l	
		Γ	
		Ω	
		Ω^*	
		rs	
		ω	
		Φ	
		Φ^*	
		ϕ	
		Q	
		q	
		M	
		L	
		K	
		H	
		S	

	C
	E
	ρ
	v
	bv
	iv
	ee
	id
	$excl$
	e
	<i>formula</i>
	<i>terminals</i>

$\boxed{\Gamma, \Phi, \Omega \vdash e : T}$ Typing

$$\begin{array}{c}
\frac{\Gamma, \Phi, \Omega \vdash e_1 : T@ (r_1, \dots, r_n)}{\Gamma, \Phi, \Omega \vdash \mathbf{read} (e_1) : T} \quad \text{T_READ} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e_1 : T@ (r_1, \dots, r_n) \quad \Gamma, \Phi, \Omega \vdash e_2 : T}{\Gamma, \Phi, \Omega \vdash \mathbf{write} (e_1, e_2) : T@ (r_1, \dots, r_n)} \quad \text{T_WRITE} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e_1 : T_1@ (r_1, \dots, r_n) \quad \Gamma, \Phi, \Omega \vdash e_2 : T_2}{\Gamma, \Phi, \Omega \vdash \mathbf{reduce} (id, e_1, e_2) : T_1@ (r_1, \dots, r_n)} \quad \text{T_REDUCE} \\
\\
\frac{}{\Gamma, \Phi, \Omega \vdash \mathbf{new} T@r : T@r} \quad \text{T_NEW} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e : T@ (r'_1, \dots, r'_k)}{\Gamma, \Phi, \Omega \vdash \mathbf{upregion} (e_1, r_1, \dots, r_n) : T@ (r_1, \dots, r_n)} \quad \text{T_UPRGN} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e : T@ (r'_1, \dots, r'_k)}{\Gamma, \Phi, \Omega \vdash \mathbf{downregion} (e, r_1, \dots, r_n) : T@ (r_1, \dots, r_n)} \quad \text{T_DNRGN} \\
\\
\frac{}{\Gamma, \Phi, \Omega \vdash \mathbf{newcolor} r : \mathbf{coloring} (r)} \quad \text{T_NEWCOLOR} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e_1 : \mathbf{coloring} (r) \quad \Gamma, \Phi, \Omega \vdash e_2 : T@r \quad \Gamma, \Phi, \Omega \vdash e_3 : \mathbf{int}}{\Gamma, \Phi, \Omega \vdash \mathbf{color} (e_1, e_2, e_3) : \mathbf{coloring} (r)} \quad \text{T_COLOR} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e_1 : \mathbf{coloring} (r_p) \quad \Gamma, \Phi, \Omega' \vdash e_2 : T}{\Gamma, \Phi, \Omega \vdash \mathbf{partition} r_p \mathbf{using} e_1 \mathbf{as} r_1, \dots, r_k \in e_2 : T} \quad \text{T_PARTITION} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e_1 : T_2[r_1/r'_1, \dots, r_k/r'_k]}{\Gamma, \Phi, \Omega \vdash \mathbf{pack} e_1 \mathbf{as} T_1[r_1, \dots, r_k] : T_1} \quad \text{T_PACK} \\
\\
\frac{\Gamma, \Phi, \Omega \vdash e_1 : T_1 \quad \Gamma', \Phi, \Omega' \vdash e_2 : T_3}{\Gamma, \Phi, \Omega \vdash \mathbf{unpack} e_1 \mathbf{as} id : T_1[r_1, \dots, r_k] \in e_2 : T_3} \quad \text{T_UNPACK} \\
\\
\frac{}{\Gamma, \Phi, \Omega \vdash id[r_1, \dots, r_k](e_1, \dots, e_n) : T} \quad \text{T_CALL} \\
\\
\frac{}{\Gamma, \Phi, \Omega \vdash \{ \mathbf{function} id_1 [r_1, \dots, r_k] (e_1, \dots, e_m), \dots, \mathbf{function} id_n [r_1, \dots, r_k] (e_1, \dots, e_m) \} : T} \quad \text{T_PROGRAM}
\end{array}$$

$$\boxed{M, L, H, S, C \vdash e \mapsto v, E}$$

Evaluation

$\overline{M, L, H, S, C \vdash x \mapsto v, E}$	EVA
$\frac{M, L, H, S, C \vdash e \mapsto l, E}{M, L, H, S, C \vdash \mathbf{read}(e) \mapsto v, E}$	EREAD1
$\frac{M, L, H, S, C \vdash e \mapsto l, E}{M, L, H, S, C \vdash \mathbf{read}(e) \mapsto H(l), E}$	EREAD2
$\frac{M, L, H, S, C \vdash e_1 \mapsto l, E_1 \quad M, L, H, S', C \vdash e_2 \mapsto v, E_2}{M, L, H, S, C \vdash \mathbf{write}(e_1, e_2) \mapsto l, E}$	EWRITE
$\frac{M, L, H, S, C \vdash e_1 \mapsto l, E_1 \quad M, L, H, S', C \vdash e_2 \mapsto v, E_2}{M, L, H, S, C \vdash \mathbf{reduce}(id, e_1, e_2) \mapsto l, E}$	EREDUCE
$\overline{M, L, H, S, C \vdash \mathbf{null} \mapsto \mathbf{null}, []}$	ENULL
$\overline{M, L, H, S, C \vdash \mathbf{new} T@r \mapsto l, []}$	ENew
$\overline{M, L, H, S, C \vdash \mathbf{null} T@\mathbf{nullr} \mapsto \mathbf{true}, []}$	ENULLTRUE
$\frac{\neg(T@r = T@\mathbf{nullr})}{M, L, H, S, C \vdash \mathbf{null} T@r \mapsto \mathbf{false}, []}$	ENULLFALSE
$\frac{M, L, H, S, C \vdash e_1 \mapsto l, E_1 \quad l = \mathbf{null}}{M, L, H, S, C \vdash \mathbf{isnull}(l) \mapsto \mathbf{true}, []}$	EISNULLTRUE
$\frac{M, L, H, S, C \vdash e_1 \mapsto l, E_1 \quad \neg(l = \mathbf{null})}{M, L, H, S, C \vdash \mathbf{isnull}(l) \mapsto \mathbf{false}, []}$	EISNULLFALSE
$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \mathbf{upregion}(e, r_1, \dots, r_n) \mapsto v, E}$	EUPRGN
$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \mathbf{downregion}(e, r_1, \dots, r_n) \mapsto l, E}$	EDNRGN1
$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \mathbf{downregion}(e, r_1, \dots, r_n) \mapsto \mathbf{null}, E}$	EDNRGN2
$\frac{M, L, H, S, C \vdash e_1 \mapsto l, E_1 \quad M, L, H, S', C \vdash e_2 \mapsto v, E_2 \quad M, L, H, S'', C \vdash e_3 \mapsto v, E_3}{M, L, H, S, C \vdash \mathbf{color}(e_1, e_2, e_3) \mapsto K', E'}$	ECOLOR
$\frac{M, L, H, S, C \vdash e_1 \mapsto K, E_1 \quad M' = M[\rho_1/r_1, \dots, \rho_k/r_k] \quad M, L, H, S', C \vdash e_2 \mapsto v, E_2}{M, L, H, S, C \vdash \mathbf{partition} r_p \mathbf{using} e_1 \mathbf{as} r_1, \dots, r_k \in e_2 \mapsto l, E'}$	EPARTITION
$\frac{M, L, H, S, C \vdash e_1 \mapsto K, E_1}{M, L, H, S, C \vdash \mathbf{pack} e_1 \mathbf{as} T_1[r_1, \dots, r_k] \mapsto v', E}$	EPACK

$$\begin{array}{c}
\frac{
\begin{array}{l}
M, L, H, S, C \vdash e_1 \mapsto \langle \langle \rho_1, \dots, \rho_k, v \rangle \rangle, E_1 \\
M' = M[\rho_1/r_1, \dots, \rho_k/r_k] \\
L' = L[v_1/id] \\
M', L', H, S', C \vdash e_2 \mapsto v, E_2
\end{array}
}{M, L, H, S, C \vdash \mathbf{unpack} \, e_1 \mathbf{as} \, id : T_1[r_1, \dots, r_k] \in e_2 \mapsto v_2, E'} \text{EUNPACK} \\
\\
\frac{
\begin{array}{l}
L' = L[(e_1, v_1), \dots, (e_n, v_n)] \\
x = \mathbf{taskid} \, \text{fresh} \\
\text{valid_interleave}(S, C, E', E_1, E_2)
\end{array}
}{M, L, H, S, C \vdash id[r_1, \dots, r_k](e_1, \dots, e_n) \mapsto v_n, E''} \text{ECALL} \\
\\
\frac{
\begin{array}{l}
M, L, H, S, C \vdash e_1 \mapsto v_1, E_1 \\
M, L, H, S, C \vdash e_2 \mapsto v_2, E_2 \\
v = v_1 + v_2
\end{array}
}{M, L, H, S, C \vdash e_1 + e_2 \mapsto v, E} \text{EINTOP} \\
\\
\frac{
\begin{array}{l}
M, L, H, S, C \vdash e_1 \mapsto v_1, E_1 \\
M, L, H, S, C \vdash e_2 \mapsto v_2, E_2 \\
v = v_1 < v_2
\end{array}
}{M, L, H, S, C \vdash e_1 < e_2 \mapsto v, E} \text{ECOMP} \\
\\
\frac{
\begin{array}{l}
M, L, H, S, C \vdash e_1 \mapsto v_1, E_1 \\
M, L, H, S, C \vdash e_2 \mapsto v_2, E_2 \\
L' = L[v_1/id]
\end{array}
}{M, L, H, S, C \vdash \mathbf{let} \, id : T = e_1 \in e_2 \mapsto v, E} \text{ELET} \\
\\
\frac{
\begin{array}{l}
M, L, H, S, C \vdash e_1 \mapsto \mathbf{true}, E_1 \\
M, L, H, S, C \vdash e_2 \mapsto v, E_1
\end{array}
}{M, L, H, S, C \vdash \mathbf{if} \, e_1 \mathbf{then} \, e_2 \mathbf{else} \, e_3 \mapsto v, E} \text{EIFTRUE} \\
\\
\frac{
\begin{array}{l}
M, L, H, S, C \vdash e_1 \mapsto \mathbf{false}, E_1 \\
M, L, H, S, C \vdash e_3 \mapsto v, E_2
\end{array}
}{M, L, H, S, C \vdash \mathbf{if} \, e_1 \mathbf{then} \, e_2 \mathbf{else} \, e_3 \mapsto v, E} \text{EIFFALSE} \\
\\
\frac{
\begin{array}{l}
L' = L[id[r_1, \dots, r_n](e_1, \dots, e_m)/id]
\end{array}
}{M, L, H, S, C \vdash \mathbf{function} \, id[r_1, \dots, r_n](e_1, \dots, e_m) \mapsto v, E} \text{EFUNCDEF} \\
\\
\frac{}{M, L, H, S, C \vdash \{ \mathbf{function} \, id_1 \, rr_1 \, ee_1, \dots, \mathbf{function} \, id_n \, rr_n \, ee_n \} \mapsto v, E} \text{EFUNCDEF LIST} \\
\\
\frac{
\begin{array}{l}
M, L, H, S, C \vdash e \mapsto v, E
\end{array}
}{M, L, H, S, C \vdash e : T \mapsto v, E} \text{ETYPEDEXPR} \\
\\
\frac{}{M, L, H, S, C \vdash \mathbf{place} \mapsto \mathbf{place}, E} \text{EPLACE} \\
\\
\frac{}{M, L, H, S, C \vdash bv \mapsto bv, E} \text{EBV} \\
\\
\frac{}{M, L, H, S, C \vdash iv \mapsto iv, E} \text{EIV} \\
\\
\frac{}{M, L, H, S, C \vdash \langle v_1, v_2 \rangle \mapsto \langle v_1, v_2 \rangle, E} \text{ETUPLE} \\
\\
\frac{}{M, L, H, S, C \vdash \mathbf{null} \mapsto \mathbf{null}, E} \text{ENULLLOC} \\
\\
\frac{}{M, L, H, S, C \vdash l \mapsto l, E} \text{EMEMORYLOC} \\
\\
\frac{}{M, L, H, S, C \vdash \langle \langle \rho_1, \dots, \rho_n, v \rangle \rangle \mapsto \langle \langle \rho_1, \dots, \rho_n, v \rangle \rangle, E} \text{EREGRELINST}
\end{array}$$

$$\frac{}{M, L, H, S, C \vdash H \mapsto H, E} \text{EHEAPVAL}$$

$$\frac{}{M, L, H, S, C \vdash K \mapsto K, E} \text{EKTHING}$$

Definition rules: 49 good 0 bad
Definition rule clauses: 107 good 0 bad