$termvar$, x		term variable
$index$, i, j, k, n, m, p		
T	$::=$	
	$\mid$	$\mathbf{bool}$
	$\mid$	$\mathbf{int}$
	$\mid$	$\langle T_1, .., T_n \rangle$
	$\mid$	$T@r$
	$\mid$	$T@(r_1, .., r_n)$
	$\mid$	$T[r_1, .., r_n]$
	$\mid$	$T[r_1/r'_1, .., r_n/r'_n]$
	$\mid$	$\mathbf{coloring}\,(r)$
	$\mid$	$\mathbf{exists}\, r_1, .., r_n.(T_1, .., T_m), \Phi, Q \rightarrow \mathbf{Tr}$
$fresh$	$::=$	
	$\mid$	iv
fns	$::=$	
	$\mid$	$\mathbf{apply}\,(S, E)$
	$\mid$	$valid\_interleave(S, C, E_1, .., E_n)$
	$\mid$	$\mathbf{taskid}\, fresh$
	$\mid$	$mark\_coherence(E, M, \mathbf{taskid}\,)$
r	$::=$	
rr	$::=$	
	$\mid$	$[r_1, .., r_n]$
Γ	$::=$	
	$\mid$	$\{(e_1 : T_1), .., (e_n : T_n)\}$
	$\mid$	$\varnothing$
Ω	$::=$	
	$\mid$	$\{\omega_1, .., \omega_n\}$
	$\mid$	$\varnothing$
Ω^*	$::=$	
	$\mid$	$\{\omega_1, .., \omega_n\}$
	$\mid$	$\mathbf{emptyOst}$
rs	$::=$	
	$\mid$	$\{r_1, .., r_n\}$
	$\mid$	$\varnothing$
ω	$::=$	
	$\mid$	$r_1 \leq r_2$
	$\mid$	$r_1 * r_2$
Φ	$::=$	
	$\mid$	$\{\phi_1, .., \phi_n\}$

$$
\begin{array}{lcl}
& | & \varnothing \\
\\
\Phi^* & ::= & \\
& | & \{\phi_1, \,..\,, \phi_n\} \\
& | & \mathbf{emptyPst} \\
\\
\phi & ::= & \\
& | & \mathbf{reads}\,(r) \\
& | & \mathbf{writes}\,(r) \\
& | & \mathbf{reducesid}\,(r) \\
\\
Q & ::= & \\
& | & \{q_1, \,..\,, q_n\} \\
\\
q & ::= & \\
& | & \mathbf{atomic}\,(r) \\
& | & \mathbf{simult}\,(r) \\
\\
M & ::= & \\
& | & M[[Q]] \\
\\
L & ::= & \\
& | & L[(e_1, v_1), \,..\,, (e_n, v_n)] \\
& | & L[v/id] \\
\\
K & ::= & \\
\\
H & ::= & \\
& | & H(l) \\
\\
S & ::= & \\
\\
C & ::= & \\
\\
E & ::= & \\
& | & [] \\
& | & [e] \\
& | & E++[e] \\
\\
\rho & ::= & \\
\\
l & ::= & \\
\\
v & ::= & \\
& | & bv \\
& | & iv \\
& | & \langle v_1, v_2 \rangle
\end{array}
$$

	$\mid$	**null**	
	$\mid$	l	
	$\mid$	$\langle\langle \rho_1, .., \rho_n, v\rangle\rangle$	
	$\mid$	H	
	$\mid$	K	
bv	$::=$		
	$\mid$	**true**	constant true
	$\mid$	**false**	constant false
iv	$::=$		
	$\mid$	0	
	$\mid$	$S\ iv$	
ee	$::=$		
	$\mid$	$(e_1, .., e_n)$	
id	$::=$		
e	$::=$		
	$\mid$	x	
	$\mid$	$\langle e_1, .., e_n\rangle$	
	$\mid$	$e\ iv$	
	$\mid$	id	
	$\mid$	**new** $T@r$	
	$\mid$	**null** $T@r$	
	$\mid$	**isnull** (e)	
	$\mid$	**upregion** $(e, r_1, .., r_n)$	
	$\mid$	**downregion** $(e, r_1, .., r_n)$	
	$\mid$	**read** (e)	
	$\mid$	**excl**	
	$\mid$	**read** $(e_1,$ **excl** $, e_2, e_3)$	
	$\mid$	**write** (e_1, e_2)	
	$\mid$	**reduce** (id, e_1, e_2)	
	$\mid$	**reduceid** (l, e_1, e_2, e_3)	
	$\mid$	**newcolor** r	
	$\mid$	**color** (e_1, e_2, e_3)	
	$\mid$	$e_1 + e_2$	
	$\mid$	$e_1 < e_2$	
	$\mid$	**let** $id : T = e_1 \in e_2$	
	$\mid$	**if b then** $c0$ **else** $c1$	conditional
	$\mid$	$id[r_1, .., r_n](e_1, .., e_m)$	
	$\mid$	**partition** r_p **using** e_1 **as** $r_1, .., r_n \in e_2$	
	$\mid$	**pack** e_1 **as** T	
	$\mid$	**unpack** e_1 **as** $id : T \in e_2$	
	$\mid$	**function** $id[r_1, .., r_n](e_1, .., e_m)$	
	$\mid$	$\{$ **function** $id_1\ rr_1\ ee_1, .., $ **function** $id_n\ rr_n\ ee_n\}$	

	|	v		
	|	L		
	|	$e : T$		
	|	**place**		
formula	::=			
	|	*judgement*		judgement
	|	$\neg formula$	M	negated formula
	|	$(formula)$	M	bracketed
	|	$\forall_i.\phi \in \Phi$	M	for all variables in domain of Φ
	|	$\forall_i.\phi \in \Phi^*$	M	for all variables in domain of Φ^*
	|	$\exists_i.\phi \in \Phi$	M	for all variables in domain of Φ
	|	$\forall_i.\omega \in \Omega$	M	for all variables in domain of Ω
	|	$\forall_i.\omega \in \Omega^*$	M	for all variables in domain of Ω^*
	|	$\exists_i.\omega \in \Omega$	M	for all variables in domain of Ω
	|	$\exists_i.\omega \in \Omega^*$	M	for all variables in domain of Ω^*
	|	$\forall_i.formula$	M	for all variables in i and *formula*
	|	$\exists_{formula_1}.formula_2$	M	for all variables in $formula_1$ and $formula_2$
	|	$\exists_{formula_1}.formula_2$ where $formula_3$	M	exists $formula_1$ and $formula_2$ where $formula_3$
	|	$\Gamma(id)$		lookup
	|	$formula_1 = formula_2$		equality
	|	$formula_1 \wedge formula_2$		equality
	|	$\bigwedge_i.formula$	M	and fold on i and *formula*
	|	$formula_1 \cap formula_2$	M	
	|	$formula_1 \cup formula_2$	M	
	|	$formula_1 \subseteq formula_2$	M	
	|	$formula_1 \in formula_2$	M	
	|	$\Gamma, \Phi, \Omega \rightarrow T$		impl
	|	$\Gamma, \Phi, Q \rightarrow T$		impl
	|	$r_1, .., r_n$		region list
	|	ϕ		phi
	|	ω		om
	|	Ω		
	|	Φ		
	|	Φ^*		
	|	$\Phi[r_1/r'_1, .., r_n/r'_n]$		
	|	$T[r_1/r'_1, .., r_n/r'_n]$		
	|	$M[\rho_1/r'_1, .., \rho_n/r'_n]$		
	|	$M[[T]]$		
	|	$M[[Q]]$		
	|	$\mathbf{domain}\,(S)$		
	|	$M(r)$		
	|	$\Gamma[r_1/r'_1, .., r_n/r'_n]$		
	|	$\Gamma[e_1/T_1, .., e_n/T_n]$		
	|	Γ		
	|	$\Gamma[T[r_1/r'_1, .., r_n/r'_n]/id]$		

	$\|$	$\Omega[r_1/r'_1, .., r_n/r'_n]$
	$\|$	$regions\_of(\Gamma, T)$
	$\|$	$regions\_of(\Gamma, T1, T_2)$
	$\|$	rs
	$\|$	T
	$\|$	fns
	$\|$	S
	$\|$	C
	$\|$	M
	$\|$	v
	$\|$	E
	$\|$	L
	$\|$	$\overline{e_i = l_i}^{\,i<n}$
$terminals$	$::=$	
	$\|$	$\exists$
	$\|$	$\forall$
	$\|$	$\in$
	$\|$	ω
	$\|$	ϕ
	$\|$	ρ
	$\|$	$\vee$
	$\|$	$\wedge$
	$\|$	$\neg$
	$\|$	$*$
	$\|$	$\leq$
	$\|$	$\longrightarrow$
	$\|$	$\rightarrow$
	$\|$	$\Rightarrow$
	$\|$	λ
	$\|$	$\mapsto$
	$\|$	$\vdash$
	$\|$	$\varnothing$
	$\|$	$\varnothing$
	$\|$	$\varnothing$
	$\|$	$\varnothing$
	$\|$	$\times$
	$\|$	$<:$
	$\|$	$\langle$
	$\|$	$\rangle$
	$\|$	$<$
	$\|$	$\Downarrow$
	$\|$	σ
	$\|$	Γ
	$\|$	ε

Jtype	$::=$		
	$\|$	$\Gamma, \Phi, \Omega \vdash e : T$	Typing
Jop	$::=$		
	$\|$	$M, L, H, S, C \vdash e \mapsto v, E$	Evaluation
judgement	$::=$		
	$\|$	*Jtype*	
	$\|$	*Jop*	
user_syntax	$::=$		
	$\|$	*termvar*	
	$\|$	*index*	
	$\|$	T	
	$\|$	*fresh*	
	$\|$	*fns*	
	$\|$	r	
	$\|$	rr	
	$\|$	Γ	
	$\|$	Ω	
	$\|$	Ω^*	
	$\|$	rs	
	$\|$	ω	
	$\|$	Φ	
	$\|$	Φ^*	
	$\|$	ϕ	
	$\|$	Q	
	$\|$	q	
	$\|$	M	
	$\|$	L	
	$\|$	K	
	$\|$	H	
	$\|$	S	
	$\|$	C	
	$\|$	E	
	$\|$	ρ	
	$\|$	l	
	$\|$	v	
	$\|$	bv	
	$\|$	iv	
	$\|$	ee	
	$\|$	id	
	$\|$	e	
	$\|$	*formula*	
	$\|$	*terminals*	

$\boxed{\Gamma, \Phi, \Omega \vdash e : T}$ Typing

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, .., r_n)}{\Gamma, \Phi, \Omega \vdash \textbf{read}\,(e_1) : T} \quad \textsc{T\_Read}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : T@(r_1, .., r_n) \\ \Gamma, \Phi, \Omega \vdash e_2 : T\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{write}\,(e_1, e_2) : T@(r_1, .., r_n)} \quad \text{T\_WRITE}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : T_1@(r_1, .., r_n) \\ \Gamma, \Phi, \Omega \vdash e_2 : T_2\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{reduce}\,(id, e_1, e_2) : T_1@(r_1, .., r_n)} \quad \text{T\_REDUCE}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \textbf{new}\ T@r : T@r} \quad \text{T\_NEW}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e : T@(r'_1, .., r'_k)}{\Gamma, \Phi, \Omega \vdash \textbf{upregion}\,(e_1, r_1, .., r_n) : T@(r_1, .., r_n)} \quad \text{T\_UPRGN}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e : T@(r'_1, .., r'_k)}{\Gamma, \Phi, \Omega \vdash \textbf{downregion}\,(e, r_1, .., r_n) : T@(r_1, .., r_n)} \quad \text{T\_DNRGN}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \textbf{newcolor}\ r : \textbf{coloring}\,(r)} \quad \text{T\_NEWCOLOR}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : \textbf{coloring}\,(r) \\ \Gamma, \Phi, \Omega \vdash e_2 : T@r \\ \Gamma, \Phi, \Omega \vdash e_3 : \textbf{int}\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{color}\,(e_1, e_2, e_3) : \textbf{coloring}\,(r)} \quad \text{T\_COLOR}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : \textbf{coloring}\,(r_p) \\ \Gamma, \Phi, \Omega' \vdash e_2 : T\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{partition}\ r_p\ \textbf{using}\ e_1\ \textbf{as}\ r_1, .., r_k \in e_2 : T} \quad \text{T\_PARTITION}$$

$$\frac{\Gamma, \Phi, \Omega \vdash e_1 : T_2[r_1/r'_1, .., r_k/r'_k]}{\Gamma, \Phi, \Omega \vdash \textbf{pack}\ e_1\ \textbf{as}\ T_1[r_1, .., r_k] : T_1} \quad \text{T\_PACK}$$

$$\frac{\begin{array}{c}\Gamma, \Phi, \Omega \vdash e_1 : T_1 \\ \Gamma', \Phi, \Omega' \vdash e_2 : T_3\end{array}}{\Gamma, \Phi, \Omega \vdash \textbf{unpack}\ e_1\ \textbf{as}\ id : T_1[r_1, .., r_k] \in e_2 : T_3} \quad \text{T\_UNPACK}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash id[r_1, .., r_k](e_1, .., e_n) : T} \quad \text{T\_CALL}$$

$$\frac{}{\Gamma, \Phi, \Omega \vdash \{\,\textbf{function}\ id_1\,[r_1, .., r_k]\,(e_1, .., e_m), .., \textbf{function}\ id_n\,[r_1, .., r_k]\,(e_1, .., e_m)\} : T} \quad \text{T\_PROGRAM}$$

$\boxed{M, L, H, S, C \vdash e \mapsto v, E}$ Evaluation

$$\frac{M, L, H, S, C \vdash e \mapsto l, E}{M, L, H, S, C \vdash \textbf{read}\,(e) \mapsto v, E} \quad \text{EREAD1}$$

$$\frac{M, L, H, S, C \vdash e \mapsto l, E}{M, L, H, S, C \vdash \textbf{read}\,(e) \mapsto H(l), E} \quad \text{EREAD2}$$

$$\frac{\begin{array}{c}M, L, H, S, C \vdash e_1 \mapsto l, E_1 \\ M, L, H, S', C \vdash e_2 \mapsto v, E_2\end{array}}{M, L, H, S, C \vdash \textbf{write}\,(e_1, e_2) \mapsto l, E} \quad \text{EWRITE}$$

$$\frac{\begin{array}{c}M, L, H, S, C \vdash e_1 \mapsto l, E_1 \\ M, L, H, S', C \vdash e_2 \mapsto v, E_2\end{array}}{M, L, H, S, C \vdash \textbf{reduce}\,(id, e_1, e_2) \mapsto l, E} \quad \text{EREDUCE}$$

$$\frac{}{M, L, H, S, C \vdash \textbf{new}\ T@r \mapsto l, []} \quad \text{ENEW}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \textbf{upregion}\,(e, r_1, .., r_n) \mapsto v, E} \quad \text{EUpRgn}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \textbf{downregion}\,(e, r_1, .., r_n) \mapsto l, E} \quad \text{EDnRgn1}$$

$$\frac{M, L, H, S, C \vdash e \mapsto v, E}{M, L, H, S, C \vdash \textbf{downregion}\,(e, r_1, .., r_n) \mapsto \textbf{null}, E} \quad \text{EDnRgn2}$$

$$\frac{\begin{array}{c} M, L, H, S, C \vdash e_1 \mapsto l, E_1 \\ M, L, H, S', C \vdash e_2 \mapsto v, E_2 \\ M, L, H, S'', C \vdash e_3 \mapsto v, E_3 \end{array}}{M, L, H, S, C \vdash \textbf{write}\,(e_1, e_2) \mapsto l, E} \quad \text{EColor}$$

$$\frac{\begin{array}{l} M, L, H, S, C \vdash e_1 \mapsto K, E_1 \\ M' = M[\rho_1/r_1, .., \rho_k/r_k] \\ M, L, H, S', C \vdash e_2 \mapsto v, E_2 \end{array}}{M, L, H, S, C \vdash \textbf{partition}\ r_p\ \textbf{using}\ e_1\ \textbf{as}\ r_1, .., r_k\ \in\ e_2 \mapsto l, E'} \quad \text{EPartition}$$

$$\frac{M, L, H, S, C \vdash e_1 \mapsto K, E_1}{M, L, H, S, C \vdash \textbf{pack}\ e_1\ \textbf{as}\ T_1[r_1, .., r_k] \mapsto v', E} \quad \text{EPack}$$

$$\frac{\begin{array}{l} M, L, H, S, C \vdash e_1 \mapsto \langle\langle \rho_1, .., \rho_k, v \rangle\rangle, E_1 \\ M' = M[\rho_1/r_1, .., \rho_k/r_k] \\ L' = L[v_1/id] \\ M', L', H, S', C \vdash e_2 \mapsto v, E_2 \end{array}}{M, L, H, S, C \vdash \textbf{unpack}\ e_1\ \textbf{as}\ id : T_1[r_1, .., r_k]\ \in\ e_2 \mapsto v_2, E'} \quad \text{EUnpack}$$

$$\frac{\begin{array}{l} M, L, H, S, C \vdash e_1 \mapsto v_1, E_1 \\ M' = M[\rho_1/r_1, .., \rho_k/r_k] \\ L' = L[(e_1, v_1), .., (e_n, v_n)] \\ M', L', H, S', C' \vdash e_n \mapsto v_n, E_n \end{array}}{M, L, H, S, C \vdash id[r_1, .., r_k](e_1, .., e_n) \mapsto v_n, E''} \quad \text{ECall}$$

```
Definition rules:        26 good    0 bad
Definition rule clauses: 65 good    0 bad
```