

Two samples of an AM signal with carrier frequency  $\omega_c$ .

$$\begin{cases} x[0] = f(t_0) \sin(\omega_c t_0 + \alpha) \\ x[1] = f(t_1) \sin(\omega_c t_1 + \alpha) \end{cases}$$

If the samples were taken  $\Delta t$  seconds apart, then the sampling frequency  $f_s$  is  $1/\Delta t$ .

$$\begin{cases} x[0] = f(t_0) \sin(\omega_c t_0 + \alpha) \\ x[1] = f(t_0 + \Delta t) \sin(\omega_c(t_0 + \Delta t) + \alpha) \end{cases}$$

If  $\Delta t$  is quite small,  $f(t_0) = f(t_0 + \Delta t) = A$ :

$$\begin{cases} x[0] = A \sin(\omega_c t_0 + \alpha) \\ x[1] = A \sin(\omega_c(t_0 + \Delta t) + \alpha) \end{cases}$$

If  $t_0 = 0$ :

$$\begin{cases} x[0] = A \sin(\alpha) \\ x[1] = A \sin(\omega_c \Delta t + \alpha) \end{cases}$$

We define  $\phi$  as  $\omega_c \Delta t = \omega_c / f_s = 2\pi f_c / f_s$ .

$$\begin{cases} x[0] = A \sin(\alpha) \\ x[1] = A \sin(\phi + \alpha) \end{cases}$$

$$\begin{cases} \alpha = \arcsin\left(\frac{x[0]}{A}\right) \\ \alpha = \arcsin\left(\frac{x[1]}{A}\right) - \phi \end{cases}$$

$$\arcsin\left(\frac{x[0]}{A}\right) = \arcsin\left(\frac{x[1]}{A}\right) - \phi$$

$$\frac{x[0]}{A} = \sin\left(\arcsin\left(\frac{x[1]}{A}\right) - \phi\right)$$

$$\frac{x[0]}{A} = \frac{x[1]}{A} \cos \phi - \cos\left(\arcsin\left(\frac{x[1]}{A}\right)\right) \sin \phi$$

$$\frac{x[0]}{A} = \frac{x[1]}{A} \cos \phi - \sqrt{1 - \frac{x[1]^2}{A^2}} \sin \phi$$

$$x[0] = x[1] \cos \phi - \sqrt{A^2 - x[1]^2} \sin \phi$$

$$A^2 - x[1]^2 = \left( \frac{x[1] \cos \phi - x[0]}{\sin \phi} \right)^2$$

$$A^2 - x[1]^2 = \frac{x[1]^2 \cos^2 \phi + x[0]^2 - 2x[1]x[0] \cos \phi}{\sin^2 \phi}$$

$$A^2 = \frac{x[1]^2 \cos^2 \phi + x[1]^2 \sin^2 \phi + x[0]^2 - 2x[1]x[0] \cos \phi}{\sin^2 \phi}$$

$$A = \frac{\sqrt{x[1]^2 + x[0]^2 - 2x[1]x[0] \cos \phi}}{\sin \phi}$$

Then:

$$y[i] = \frac{\sqrt{x[i]^2 + x[i-1]^2 - 2x[i]x[i-1] \cos \phi}}{\sin \phi}$$

Where  $\phi = 2\pi f_c / f_s$