

$$\begin{cases} x[0] = A \sin(\omega t_0 + \alpha) \\ x[1] = A \sin(\omega t_1 + \alpha) \end{cases}$$

If $t_0 = 0$, then $\omega t_0 = 0$ and $\omega t_1 = \phi$

$$\begin{cases} x[0] = A \sin(\alpha) \\ x[1] = A \sin(\phi + \alpha) \end{cases}$$

$$\begin{cases} \alpha = \arcsin\left(\frac{x[0]}{A}\right) \\ \alpha = \arcsin\left(\frac{x[1]}{A}\right) - \phi \end{cases}$$

$$\arcsin\left(\frac{x[0]}{A}\right) = \arcsin\left(\frac{x[1]}{A}\right) - \phi$$

$$\frac{x[0]}{A} = \sin\left(\arcsin\left(\frac{x[1]}{A}\right) - \phi\right)$$

$$\frac{x[0]}{A} = \frac{x[1]}{A} \cos \phi - \cos\left(\arcsin\left(\frac{x[1]}{A}\right)\right) \sin \phi$$

$$\frac{x[0]}{A} = \frac{x[1]}{A} \cos \phi - \sqrt{1 - \frac{x[1]^2}{A^2}} \sin \phi$$

$$x[0] = x[1] \cos \phi - \sqrt{A^2 - x[1]^2} \sin \phi$$

$$A^2 - x[1]^2 = \left(\frac{x[1] \cos \phi - x[0]}{\sin \phi}\right)^2$$

$$A^2 - x[1]^2 = \frac{x[1]^2 \cos^2 \phi + x[0]^2 - 2x[1]x[0] \cos \phi}{\sin^2 \phi}$$

$$A^2 = \frac{x[1]^2 \cos^2 \phi + x[1]^2 \sin^2 \phi + x[0]^2 - 2x[1]x[0] \cos \phi}{\sin^2 \phi}$$

$$A = \frac{\sqrt{x[1]^2 + x[0]^2 - 2x[1]x[0] \cos \phi}}{\sin \phi}$$

Then:

$$y[i] = \frac{\sqrt{x[i]^2 + x[i-1]^2 - 2x[i]x[i-1] \cos \phi}}{\sin \phi}$$