Sketch of Security Proof for (n+1)Sec Protocol

The (n+1)Sec protocol is composed of following sub protocol:

1. **TDH**: Triple DH deniable Authentication
2. **FAGKE**: Flexible Authenticated Group Key Exchange protocol presented in [ACMP]
3. **SecCom**: Secure (authenticated confidential) Send and Receive.
4. **TCA**: Transcript Consistency Assurance.

The threat model for each of these protocol is described in Section VI. The security of FAGKE is proven in the presented threat model. The SecComm consists of convential "sign" and "encrypt" functions and its security has been studied as a subprotocol to various protocols. We are not aware of any existing proof for TDH and TCA subprotocol.

The sketch of the Sketch goes as follows, Section 3 deals with security of TDH namely its deniability. The authentication of TDH will be proven as parte of AKE security proof. We also prove the TDH protocol as a 2-party secure AKE in model presented in [ACMP]. Section ? prove the security properties of the group key exchange protocol. Section 3 we give proof of the security properties of TCA.

1 General Definition

In this section we introduce the ideas and definition we are using through out the proof.

Definition 1. *Suppos $\mathbb{G}$ is a multiplicative group. Given arbitary $g, g^a, g^b \in \mathbb{G}$, the **Computational Diffie-Hellman (CDH) problem** is to compute g^{ab}.*

Definition 2. *Folliwing the notation in Definition 1, Given arbitary g, g^a, g^b, $g^c \in \mathbb{G}$, the **Decisional Diffie-Hellman (DDH) problem** is to determine if $g^c = g^{ab}$.*

Definition 3. *Folliwing the notation in Definition 2, **Gap Diffie-Hellman probelm** is to compute g^{ab} while having access to a DDH oracle. In other words, **GDH assumption** for group $\mathbb{G}$ asserts that even if DDH is easy in $\mathbb{G}$, computing g^{ab} is hard.*

Definition 4. *A **Gap Diffie-Hellman Solver** or a **GDH solver** $\mathcal{S}$ for group $\mathbb{G}$ is a function $\mathcal{S}$ defined as*

$$\mathcal{S}: (g, g^a, g^b, \mathcal{O}_{\text{DDH}}) \longmapsto g^{ab}$$

Where $\mathcal{O}_{\text{DDH}}$ is a DDH oracle for group $\mathbb{G}$.

Definition 5. *We indicate **the set of all possible participants** (in the universe) by $\mathcal{U}$ such that $|\mathcal{U}| = N$, where each participants. Each participant is represented by a unique identity U_i. Each U_i is verifiably identified by a long term public key LPK_i for which its posses its corresponding long term private key LSK_i.*

In modelling the chat session, in terms of the adversarial models and protocol specifications, the notation of [ACMP10] is followed. This notation is common to other publication on group key exchange such as [GBNM11], and is adherred to for consistency.

Definition 6. *We indicate* ***the set of all possible participants*** *(in the universe) by* $\mathcal{U}$ *such that* $|\mathcal{U}| = N$*. Each participant is represented by a unique identity* U_i*. Each* U_i *is verifiably identified by a long term public key* LPK_i *for which its posses its corresponding long term private key* LSK_i*.*

Definition 7. *A* ***Multi-party chat session*** *is an ordered pair* $\mathcal{S} := (S, \mathrm{sid}^{\mathcal{S}})$*, in which* $S \subseteq \mathcal{U}$ *and* sid *is the unique session id computed as a function of participants id and their ephemeral keys. The* ***Ephemeral key*** *of participant* U_i *is private and public key pair of* (x_i^S, y_i^S) *that is generated by a participant for the purpose of participating in* $\mathcal{S}$ *such that*

$$y_i^S = x_i^S g$$

in addetive notation, where g *is the generator of a group* G *with hard Discrete Logarithm Problem (DLP).We refer to either of* x_i *or* y_i *as the ephemeral key of user* U_i *when there is chance of ambiguity.Without loss of generality we assume:*

$$S := \{U_1, ..., U_n\}$$

The order ***list of participant*** *and order list of their ephemeral keys is defined as:*

$$\mathrm{plist}^{\mathcal{S}} := (U_1, ..., U_n)$$

$$\mathrm{klist}^{\mathcal{S}} := (y_1, ..., y_n)$$

Accordingly we denote the interviewing concatination of these two list as:

$$\mathrm{plist}^{\mathcal{S}} | \mathrm{klist}^{\mathcal{S}} := U_1 | y_1 | U_2 | y_2 | \cdots | U_n | y_n$$

The order is to be uniquely computable by all participants (lexicographically ordered using long term public key of participants, for example).

A subset of participants might want to start a session in which the remaining parties are excluded (for example when those parties leave the chatroom). Following definition formalize such situation:

Definition 8. *For a session* $\mathcal{S} = (S, \mathrm{sid}^{\mathcal{S}})$*,* $\mathcal{T} = (S, \mathrm{sid}_{\mathcal{T}})$ *is called a* ***sub-session*** *of* $\mathcal{S}$ *if* $T \subset S$ *and all participants* $U_i \in T$*, use the same ephemeral key for both* $\mathcal{S}$ *and* $\mathcal{T}$*. In the other words, the same ephemeral keys are use to compute* $\mathrm{sid}_{\mathcal{T}}$*. In such situation we call* $\mathcal{S}$ *the super session of* $\mathcal{T}$*.*

Definition 9. *An* ***authenticated group key exchange (AGKE)*** *is a protocol* Π *each participant executes in order to communicate (by means of hsending, receiving or computing) a cryptographic secret - namely a key - among the other parties of a session. By* $\Pi_i^{\mathcal{S}}$ *we refer to the* ***instance of the protocol run by*** U_i *for session* $\mathcal{S}$*. The* $\mathrm{sid}^{\mathcal{S}}$ *computed by* $\Pi_i^{\mathcal{S}}$ *and denoted by* $\mathrm{sid}_i^{\mathcal{S}}$ *(or* sid_i *when there is no chance of confusion) is called the* ***session id observed by*** U_i*. Similarly,* $\mathrm{plist}_i^{\mathcal{S}}$ *(or* plist_i*) is the list of participants which* U_i *believes are participating in attack and* $\mathrm{klist}_i^{\mathcal{S}}$ *(or* klist_i*) is their preceived set of ephemeral public key.*

Definition 10. *To communicate in a multiparty session, each participant* U_i *need to compute a symmetric* ***session key*** $\mathrm{sk}_i^{\mathcal{S}}$*which should be computable by other parties participating in the chat as or be transmitted confidentially to them. We say a participant enters the* ***accepted state*** *if they have computed* $\mathrm{sk}_i^{\mathcal{S}}$ *and has detected no error in the protocol.*

The essential defining factor is that part of $\mathrm{sk}_i^{\mathcal{S}}$ should become common knowledge for the session participants at the end of AGKE execution, so they can communicated confidentially. Although, it is not necessary that all participants share the same secret $\mathrm{sk}_i^{\mathcal{S}}$ among themselves and they can broadcast their messages encrypted using multiple keys. This decrease the efficiency as well as complicate the security analysis of the protocol. As such we assume that at the end of running a correct AGKE, all participants possess a shared secret:

Definition 11. *Two accepted instances* $\Pi_i^{\mathcal{S}}$ *and* $\Pi_j^{\mathcal{S}}$ *are considered* ***partnered*** *if* $\mathrm{sid}_i = \mathrm{sid}_j$ *and* $\mathrm{plist}_i = \mathrm{plist}_j$.

Definition 12. *A* ***correct*** *AKGE algorithm is an AKGE which, when all* $\Pi_i^{\mathcal{S}}$ *instances of AKE algorithm are initiated with access to a network which correctly forwards all messages without modification, all participants ultimately are partnered and all compute equal* $\mathrm{sk}_i^{\mathcal{S}}$*'s.*

After all instances of a session partnered. They need to use their shared symmetric key to communicate securely. Following sub protocol can guarantee some of the security properties which $(n+1)$sec aims to promise:.

Definition 13. ([BHMS] Definition 3.1) ***A* stateful authenticated encryption with associated data *(stateful AEAD) scheme*** Π *consists of:*

- *A probablistic key generation algorithm (it is the AGKE in our case).*
- *A stateful probablistic encryption* $E(k, \mathrm{ad}, m, \mathrm{st}_E) \rightarrow (c, \mathrm{st}'_E)$.
- *A deterministic decryption algorithm* $D(k, \mathrm{ad}, c, \mathrm{st}_D) \rightarrow (\mathrm{ad}, m, \mathrm{st}'_D)$

  which can output ad *or* $\perp$ *as error and message* m *or* $\perp$ *as error.*

Definition 14. *A* ***correct stateful AEAD scheme*** *is an stateful AEAD scheme* Π*, is scheme which can correctly decrypt cipher text* c *to corresponding message* m *for any sequences of message or output error in case the ciphertext does not correspond to output of* $E(k)$.

2 Adversarial Power

We will re-use these definitions to demonstrate similar routes for other adversaries considered by the threat models in later sections.

2.1 Adversarial power for AKE

The following set of functions model the AKE adversarial threats. The adversary for the authenticated key exchange can mount an attack through a sequence of call to the functions, outlined below. The limitation on the order and condition of calling these functions is defined per adversary.

- Execute(plist): asks all parties in the plist to run (a new) AGKE protocol and $\mathcal{A}$ will receive the execution transcript, i.e. can eavesdrop.
- $\mathrm{Send}_{U_i}(\Pi_i^S)(m)$ sends a message m to the instance Π_i^S as user U_j. We assume that m contains information to identify the sender U_j. U_j will receive the reply transcript. Specifically, by sending plist messages it forces U_i to initiate Π_i^S.
- **SKE(ΠSi,spidi)**: asks ΠSi to compute the subgroup key for the spidi subsession. In response, ΠSi will either send a message or compute the subgroup key kspidi depending on the state of ΠSi. This can be invoked only once per input.

- **RevealGK()**: ΠSi gives ski to Aa if it has accepted (as defined in Definition III.3).
- **RevealSK**: ΠSi gives the subkTi to Aa if it has been computed for subsession T.
- RevealPeer(Π_i^S,U_j): When the $\mathcal{A}$ call this function, it will be provided with the $p2p$ key $k_{i,j}^S$, if it is already computed.
- **Corrupt(Ui)**: Ui gives its long term secret key to Aa (but not the session key).

Definition 15. ***AKE-Security of P2P Keys****, Let $\mathcal{P}$ GKE$+P$ protocol and b a uniformly chosen bit. Adversary $\mathcal{A}_{p2p}$ is allowed to invoke all adversarial queries. At some point the Adversary runs* TestPeer(Π_i^S, U_j) *for some fresh instance,User pair (Π_i^S, U_j) which remains fresh. $\mathcal{A}_{p2p}$ is allowed to continue the adversarial queries provided the test pair remains fresh. Finally $\mathcal{A}_{p2p}$ outputs a bit b'. The adversarial advantage is defined as*

$$\mathrm{Adv}_{\mathcal{A}_{p2p}}(\mathcal{P}) := |2\mathrm{Pr}(b' = b) - 1|$$

We say the $\mathcal{P}$ is secure if the advantage is negligible.

Definition 16. ([ACMP] Definition 5 AKE-Security of group Keys) *Let $\mathcal{P}$ be a correct GKE+P protocol and b a uniformly chosen bit. By* $\mathrm{Game}_{\mathcal{A}_{\mathrm{GKE}}}(\mathcal{P}, \kappa)$*,we define the following adversrial game, which involves a PPT adversary $\mathcal{A}_{\mathrm{GKE}}$ that is given access to all queries:*

– $\mathcal{A}_{\mathrm{GKE}}$ *interacts via queries;*

– *at some point $\mathcal{A}_{\mathrm{GKE}}$ asks a* TestGK$(\Pi_i^{\mathcal{S}})$ *query for some instance $\Pi_i^{\mathcal{S}}$ which is (and remains) fresh;*

– $\mathcal{A}_{\mathrm{GKE}}$ *continues interacting via queries;*

– *when $\mathcal{A}_{\mathrm{GKE}}$ terminates, it outputs a bit, which is set as the output of the game. We define:*

$$\mathrm{Adv}_{\mathcal{A}_{\mathrm{GKE}}}(\mathcal{P}, \kappa) := |2\mathrm{Pr}[\mathrm{Game}_{\mathcal{A}_{\mathrm{GKE}}}(\mathcal{P}, \kappa) = b] - 1|$$

We say that $\mathcal{P}$ provides GKE-security if the maximum of this advantage over all possible PPT adversariesy $\mathcal{A}_{\mathrm{GKE}}$ is negligible.

Definition 17. ([ACMP] Definition 6 AKE Security of subgroup keys) *Let P be a correct GKE+S*

protocol and b a uniformly chosen bit. By Game ake-s,b

A,P (κ) we define the following ad-

versarial game, which involves a PPT adversary A that is given access to all queries:

– *A interacts via queries;*

– *at some point A asks a TestSK(Π i s , spid si) query for some instance-subgroup pair*

(Π i s , spid si) which is (and remains) fresh;

– *A continues interacting via queries;*

– *when A terminates, it outputs a bit, which is set as the output of the game.*

We define

$$\mathrm{Adv}_{\mathcal{A}_{S-\mathrm{GKE}}}(\mathcal{P},\kappa) := |2\Pr[\mathrm{Game}(\kappa)=b]-1|$$

and denote with Adv ake-s(κ) the maximum advantage over all PPT adversaries A. We say that P provides AKE-security of subgroup keys if this advantage is negligible.

2.2 Secure Multiparty Channel Adversary

The desirable way to define an adversary for a multiparty chat session is a secure channel model similar to the two-party secure channels described in [CaKr01] [JKSS]and [KPW13]. As such we set the *authenticated and confidential channel establishment* (ACCE) protocol as our starting point. In this regard, we would like to prove that $(n+1)$sec is an ACCE protocol. It is argued in [JKSS] that if an scheme provides a secure AKE and the symmetric encryption of the session communication satisfies the suitable confidentiality and integrity criteria, then one can conlude that the scheme is a ACCE protocol (although the inverse statement is not true). Following this path, we define the adversary for the communication phase of a secure multiy party chat session. We use the Definition 3.2 from [BHMS] instead of Definition 6 in [JKSS], because hiding the length of the conversation is not considered as a security property of $(n+1)$sec.

2.2.1 Definition of Adversaries and their advantages

Based on Definition 13 the adversay against an AEAD is defined as follows:

Definition 18. ([BHMS] Definition 3.2) *Let* Π *be a stateful AEAD scheme and let A be an PPT adversarial algorithm. Let* $i\in\{1,\ldots,4\}$ *and let* $b\in\{0,1\}$*. The stateful AEAD experiment for* Π *with condition* cond_i *and bit b is given by* $\mathrm{Exp}^{\mathrm{aead}_i-b}(\Pi,\mathcal{A})$ *as defined in [BHMS] Figure 4. The adversaries advantage is defined as*

$$\mathrm{Adv}^{\mathrm{aead}_i}()_{\Pi,\mathcal{A}_{\mathrm{aead}_i}} := \Pr\left[\mathrm{Exp}^{\mathrm{Exp}^{\mathrm{aead}_i-1}}(\Pi,\mathcal{A})=1\right] - \Pr\left[\mathrm{Exp}^{\mathrm{Exp}^{\mathrm{aead}_i-0}}(\Pi,\mathcal{A})=1\right]$$

2.3 Message Origin Authentication Adversary

Any manipulation of data by an outsider is modeled in the AEAD adversary as defined in Defition 18. $(n+1)$sec, however need to also protect insiders from forging messages on behalf of each other. That is why each participant executes a sign and encrypt function before sending their authenticated ephemeral signing key, the message origin adversary model is based on a typical adversary for a signature scheme such as the one presented in [PVY00].

2.3.1 Adversarial power

In addition to adversarial functions defined in Section ?. we must define the following function to allow for the adversary using the chosen-message attack.

- **MakeSend(Π_i^S, Π_j^S, m)** causes the Π_i^S to sign and send a valid message m to instance Π_j^S. $\mathcal{A}_{\mathrm{orig}}$ will receive the transcript including the signature.

2.3.2 Definition of Adversary

Definition 19. ***Message Origin Authentication Adversary****,* $\mathcal{A}_{\mathrm{orig}}$ *a polynomial time algorithm which has access to the* ***Corrupt****,* ***Send****,* ***Reveal*** *and* ***MakeSend*** *functions. The output of the algorithm should be a message m sent to instance* Π_j^S*. The scheme is secure against Message Origin Adversary if the probability in which* Π_j^S *believes that m has originated from an uncorrupted participant* U_i *is negligible under assumption of hardness of Discrete Logarithm Problem.*

3 Security of Triple Diffie-Hellman Authentication

3.1 The Triple Diffie-Hellman Protocol

Assuming that A and B are represeneted by long term public key g^A and g^B respectively:

Round 1	$A \to B: "A", g^a$	$B \to A: "B", g^b$
Key Computation	$k \leftarrow H((g^b)^A \mid (g^B)^a \mid (g^b)^a)$	$k \leftarrow H((g^A)^b \mid (g^a)^B \mid (g^a)^b)$
Round 2	$\mathrm{Enc}_k(H(k, A))$	$\mathrm{Enc}_k(H(k, B))$

Table 1. Triple Diffie-Hellman protocol

3.2 The deniablity of TDH

We will prove a parallel to Theorem 4 [?] which proves the deniability of SKEME. We use the notation which are introduced in Section ?. Following the same notation:

Definition 20. *By* $\mathrm{Adv}^*_{\mathrm{deny}}$ *we represent the party which represent the interaction of the Simulator* Sim *with the adverasy. In other word,* $\mathrm{Adv}^*_{\mathrm{deny}}$ *has access to all information which* $\mathrm{Adv}_{\mathrm{deny}}$ *possess.*

Theorem 21. *If Computational Diffie-Hellman (CDH) is interactable then Triple DH Algorithm is deniable.*

Proof. We build Sim which interacts with $\mathrm{Adv}_{\mathrm{deny}}$. We show that if $\mathcal{J}$ is able to distinguish $\mathrm{Trans}_{\mathrm{Sim}}$ from $\mathrm{Trans}_{\mathrm{Real}}$, ze should be able to solve CDH as well.

Intuitively, when $\mathcal{A}_{\mathrm{deny}}$ sends g^a to $\mathcal{S}_{\mathrm{deny}}$, $\mathcal{S}_{\mathrm{deny}}$ inquire $\mathcal{A}_{\mathrm{deny}}$ for a, in this way $\mathcal{S}_{\mathrm{deny}}$ also can compute the same key k by asking $\mathcal{A}^*_{\mathrm{deny}}$. If $\mathcal{A}_{\mathrm{deny}}$ has chosen $g^a \in \mathrm{Tr}(B)$ or just chosen a random element of the group without knowing its DLP, then $\mathcal{S}_{\mathrm{deny}}$ will choose a random exponent a' and computes the key k based on that and computes the confirmation value using k. Due to hardship of CDH this value is indistinguishable from a k generated by B

Now we suppose that the TDH is not deniable and we build a solver for CDH. First we note that if $\mathcal{A}_{\mathrm{deny}}$ engages in an honest interaction with B there is no way that $\mathcal{J}$ can distinguish between the $T(\mathcal{A}_{\mathrm{deny}}(\mathrm{Aux}))$ and $T(\mathcal{S}_{\mathrm{deny}}(\mathrm{Aux}))$. As $\mathcal{A}_{\mathrm{deny}}$ is able to generate the very exact transcript without help of B. Therefore, logically, the only possibility for $\mathcal{J}$ to distinguish $T(\mathcal{A}_{\mathrm{deny}}(\mathrm{Aux}))$ and $T(\mathcal{S}_{\mathrm{deny}}(\mathrm{Aux}))$ is when $\mathcal{A}_{\mathrm{deny}}$ present $\mathcal{J}$ with a transcript that $\mathcal{A}_{\mathrm{deny}}$ is not able to generate zirself. The only variable that $\mathcal{A}_{\mathrm{deny}}$ has control over in the course of the exchange is g^a and therefore the only way $\mathcal{A}_{\mathrm{deny}}$ is able to claim that ze were unable to generate the geneuine $T(\mathcal{A}_{\mathrm{deny}}(\mathrm{Aux}))$ is by submiting g^a which zirself does not know about its a exponent.

In such case, assuming the undeniability of TDH we have an ε such that

$$\max_{\text{all } \mathcal{J}} |2\Pr(\mathrm{Output}(\mathcal{J}, \mathrm{Aux}) = b) - 1| > \varepsilon$$

The solver $\mathcal{A}_{\mathrm{CDH}}$ receives a triple (g, g^a, g^b) and should compute g^{ab}. To that end, assuming long term identiy g^A for some $\mathcal{A}_{\mathrm{deny}}$, ze engages ,in a TDH key exchange with a hypothetical automated party $\mathcal{A}^*$ with long term private key B who generates g^b as the ephemeral key as well. $\mathcal{A}_{\mathrm{CDH}}$, then toss a coin and based on the result it either choose a random a' and compute $g' = g^{a'}$ or set $g' = g^a$, then ze submits $h_0 = H(g^{bA}, g'^B, g^{ba'})$ along side with (g^B, g^b) to the $\mathcal{J}$ as a proof of engagement with $\mathcal{A}^*$. Due to undeniability assumption

$$\mathrm{Output}(\mathcal{J}, \mathrm{Aux})(h_0, (A, g^a, B, g^b)) = b$$

with significant probablity as means $\mathcal{J}$ is able to distinguish $T(\mathcal{A}_{\text{deny}}(\text{Aux}))$ and $T(\mathcal{S}_{\text{deny}}(\text{Aux}))$ with high probablity. Therefore $\mathcal{J}$ is able to decide if:

$$h_0 \stackrel{?}{=\!=} H(g^{bA}, (g^a)^B, (g^a)^b)$$

Because H is a random oracle the only way that the judge is able to distinguish the second value from the real value is to have knowledge about the exact pre-image: $g^{bA}, (g^a)^B, (g^a)^b$. Using the information in the transcript $\mathcal{J}$ can compute $g^{bA}, (g^a)^B$, but still has to compute g^{ab} using g^a and g^b with high probablity without knowing a or b, at this point $\mathcal{A}_{\text{CDH}}$ is publishing the value of g^{ab}.

□

3.3 Security of TDH as a two party Authenticate Key Exchange

In this section we prove that TDH is a secure two-party authenticated key exchange. we do so in the AKE security model proposed in [Man]. This is because (n+1)Sec key exchange protocol is a varient of the protocol proposed in [ACMP], which is designed to satisifies all three AKE models proposed in [Man] and [ACMP]. Furthermore, based on the security properties required from (n+1)Sec as a secure multiparty chat protocol, we beleive these models provide adequite security for real world threat scenarios.

Theorem 22. *If the GDH problem is hard in $\mathbb{G}$, then TDH protocol explained in Table 1, is secure in AKE model with advantage of the adversary is bounded by*

$$\text{Adv}_{\mathcal{A}_{p2p}(k) \leqslant \mathcal{O}(q^2)} / Q$$

Where q is the maximum number of queries by the adversary

Proof. Suppose that $k_{\text{test}} = H((g^b)^A | (g^B)^a | (g^b)^a)$. Assuming that H is a PRF (SHA-256 in case of (n+1)sec, the only way that adveresy $\mathcal{A}_{p2p}$ can distinguish k_{test} from a random value k' is to compute all element of triplet $(g^b)^A, (g^B)^a, (g^b)^a$.

We show how to construct a GDH solver using an $\mathcal{A}_{p2p}$ who can compute all three above. Assuming that the test session is Fresh, then the adversay can not corrupt neither A or B and can request session reveal on the test session. Therefore it does not have neither access to a or b.

Now suppose simulator $\mathcal{S}$, has access to the Adversary $\mathcal{A}$ oracle which is able to compute the triple diffie hellman inside the paranthesis. $\mathcal{S}$ need to solve g^{ab} for a given g^a and g^b. As such it generates a transcript to set up a session between A and B while inserting g^a and g^b as exchanged keys.

Assuming that the adversary can compute the last token which is the solution to CDH.

□

Theorem 23. *If the GDH problem is hard in $\mathbb{G}$, then (n+1)sec protocol is secure against $\mathcal{A}_{p2p}$ adversary.*

Proof. We argue that the AKE security for the (n+1)Sec $p2p$ keys follows similarly to from the proof of Theorem 8 [ACMP] which proves the security of BD+P protocol.

In fact we follow the same sequence of games for games G_0 and G_1.

For the game G_2 we note that contrary to mBP+P which signs the second round message with LPK_i for authentication, adversary has two ways to forge the authentication and force the other party accept a wrong key. One is to forge the signture generated by ephemeral key. This basically covered by G_2. However, another way is to forge the authentication token we simulate in G_2'.

$\boldsymbol{G_2'}$. In this game, we abort the simulation if $\mathcal{A}_{p2p}$ queries $\text{Send}(U_i, \text{kc}_{i,j})$ with a valid confirmation where neither U_i or U_j is not corrupted. To do so, $\mathcal{A}_{p2p}$ needs to generate $H(k_{ij}|U_i)$. Assuming that H is PRF, this is only possible if $\mathcal{A}_{p2p}$ has successfully computed k_{ij}, which in part necessitates $\mathcal{A}_{p2p}$ computing $g^{b\,\text{LPK}_i}$ to be able to impersonate A to B. Not knowing neither secret b or LPK_i, the advantage of $\mathcal{A}_{p2p}$ is bounded by its advantage in solving GDH. The adversary needs to solve all three GDH problems. Therefore we have:

$$|\Pr[\text{Win}_2] - \Pr[\text{Win}_{2'}]| < q\,(\text{Succ}_{\mathbb{G}}^{\text{GDH}}(\kappa))^3$$

In fact the only difference in the proof is related to G_6. As k_{ij} is computed as $H(g^{Ab}|g^{Ba}|g^{ab})$. Therefore simulator delta will output $H'(g^A|g^B|g^a|g^b)$. However because H is a prefect PRF, this remains indistinguishble, unless the adversary has advantage on computing g^{Ab}, g^{Ba}, g^{ab}.

$$|\Pr[\text{Win}_6] - \Pr[\text{Win}_5]| < qH_p(\text{Succ}_{\mathbb{G}}^{\text{GDH}}(\kappa))^3$$

Consequently, the overal advantage of $\mathcal{A}_{p2p}$ bar its advantage in transition from G_2 to $G_{2'}$, is smaller than their advantage in the original mBD+P protocol:

$$\text{Adv}^{p2p}_{(n+1)\text{sec}}(\kappa) < \text{Adv}^{p2p}_{\text{mBD}+P}(\kappa) + q\,(\text{Succ}_{\mathbb{G}}^{\text{GDH}}(\kappa))^3$$

This proves that $\text{Adv}^{p2p}_{(n+1)\text{sec}}(\kappa)$ is asymtotically the same as $\text{Adv}^{p2p}_{\text{mBD}+P}(\kappa)$.

□

4 Security of (n+1)sec authenticated group key exchange

In this section we prove the security of (n+1)sec group key exchange in the proposed adversarial model. Because the key exchange is essentially FAGKE with only difference is that the traditional DH key exchange replaced by TDH, we prove the security of (n+1)sec GKE based on the security of FAKE.

4.1 Security of GKE

We recall that the GKE protocol in (n+1)Sec is essentially the same as FAGKE protocol except that in (n+1)Sec we have:

$$k_{i,i+1} = H(g^{\text{LS}_i x_{i+1}}, g^{\text{LS}_{i+1} x_i}, g^{x_i x_{i+1}})$$

Where as in FAGKE we have:

$$k_{i,i+1} = g^{x_i x_{i+1}}$$

Therefore, to prove the that $(n+1)$Sec we need to prove Theorem 24:

Theorem 24. *If GDH problem is hard then (n+1)sec key exchange provides AKE-security of group keys.*

Proof. We argue that the AKE security for the (n+1)Sec group key follows similarly to from the proof of Theorem 7 [ACMP] which proves the security of BD+P protocol.

In fact we follow the same sequence of games for games G_0 and G_1.

Similar to the case of $p2p$ argued in Theorem 23, we need to expand game G_2 into two games of G_2 and G_2' to account both for the forgery of the signature and the TDH tocken. With the transitional advantage of

$$|\Pr[\text{Win}_2] - \Pr[\text{Win}_{2'}]| < q\,(\text{Succ}_{\mathbb{G}}^{\text{GDH}}(\kappa))^3$$

We proceesd similarly with game G_3. The difference in the proof is related to G_4. Δ responds with g^a and g^b from the values of the GDH challenge. In this game instead of computing z_i' as $H(H(g^{Ab}|g^{Ba}|g^{ab}),\text{sid})$, simulator Δ will output $H'(g^A|g^B|g^a|g^b)$. However because H is a prefect PRF, this remains indistinguishble, unless the adversary has advantage on computing g^{Ab}, g^{Ba}, g^{ab}. So we have

$$|\Pr[\text{Win}_6] - \Pr[\text{Win}_5]| < qH_p(\text{Succ}_{\mathbb{G}}^{\text{GDH}}(\kappa))^3$$

The remaining argument for game G_4 is the same as $\text{mBD}+P$ proof.

Consequently, the overal advantage of $\mathcal{A}^{\text{ake}-g}_{(n+1)\text{sec}}$ bar its advantage in transition from G_2 to $G_{2'}$, is smaller than their advantage in the original mBD+P protocol:

$$\text{Adv}^{\text{ake}-g}_{(n+1)\text{sec}}(\kappa) < \text{Adv}^{\text{ake}-g}_{\text{mBD}+P}(\kappa) + q\,(\text{Succ}_{\mathbb{G}}^{\text{GDH}}(\kappa))^3$$

This proves that $\text{Adv}^{\text{ake}-g}_{(n+1)\text{sec}}(\kappa)$ is asymtotically the same as $\text{Adv}^{\text{ake}-g}_{\text{mBD}+P}(\kappa)$.

□

5 Security of (n+1)sec authenticated group key exchange

5.1 Security of $(n+1)$sec as a secure channel

In this section we prove the following theorem.

Theorem 25. *(n+1)sec is* authenticated and confidential channel establishment *(ACCE) protocol.*

Proof. Based on [JKSS] an protocol which establish the confidential authentication key using a secure AKE and provides security against stateful AEAD adversary during the secure session using the established key provides a secure (confidential and authenticated) channel. We have already established the GKE security of $(n+1)$sec. Accordingly we only need to prove that the $(n+1)$sec provide stateful AEAD security.

To do so we use [BHMS] Theorem 3.1 to prove that $(n+1)$sec is a secure level-3 AEAD scheme.

First, we note recall the format of (n+1)sec messages:

```
:o3np1sec:Base64EnocodedMessage
```

In which `Base64EnocodedMessage` is encoded as

```
sid (DTHash),  Signature (DTHashx2), Encrypted part of the message
```

Where sid and Signature are associated date and the Encryption is provided by AES-GSM. Using the result of [?] and [?] We know that AES-GSM is both IND-CCA and INT-CTXT. As such $(n+1)$sec is a level -1 AEAD scheme.

Considering the fact that $(n+1)$sec messages has both `own_sender_id` which is strictly increasing for each sender one by one for each message along side with `session_id` and `nonce`, proves that $(n+1)$sec encoding passes TEST4 describe in [BHMS] Figure 3. Therefore based on [BHMS] Theorem 3.1, (n+1)sec resists a level-4 stateful AEAD adversary.

Now using the result of Theorem 24, based on the conclusion of [JKSS], we conclude that (n+1)sec is ACCE protocl.

□

6 Message Origin Authentication Adversary

Using the result of Theorem 25, we know that (n+1)sec session transcript is secure against outsider manipulation. Therefore, it remains to only study the ability of the insiders of the session in forging messages against each other. To prevent such scenario, $(n+1)$sec messages are signed by authenticated ephemeral keys. The authenticity of ephemeral keys are assured based on Theorem 23 and has established before the session starts. Therefore we only need to prove that $(n+1)$sec provide the security against signature forgery.

As each participant executes a sign and encrypt function before sending their authenticated ephemeral signing key, the message origin adversary model is based on a typical adversary for a signature scheme such as the one presented in [PVY00].

Theorem 26. $(n+1)$sec *is secure against* $\mathcal{A}_{\text{orig}}$.

Proof. $(n+1)$sec message is signed using EdDSA signature scheme. Accordingy to [?], EdDSA system is a Schnorr based signature system and it inherits the security properties of Schnorr signature. According to [?] Theorem 4, a chosen message attacker which can break Schnorr scheme, can solve the DLP of the underlying system in polynomial time. This will establish the security of $(n+1)$sec against the adversary defined in Definition 19.

□

7 Security of Transcript Consistency Assurance

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