

Mathematics and Statistics of Algorithmic Trading¹

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February 25, 2016

¹Notes based on textbook “Algorithmic and High-Frequency Trading” with Sebastian Jaimungal and Jose Penalva.

Algorithmic and High Frequency Trading

Main aim is to learn about

- ▶ Algorithmic Trading (AT): The use of computer algorithms that make trading decisions, submit orders, and manage those orders after submission.
 - ▶ For example, the SEC reports that in the NYSE between 2005 and 2009: consolidated average daily share volume increased 181%; average speed of execution for small, immediately executable (marketable) orders shrunk from 10.1 to 0.7 seconds; consolidated average daily trades increased 662%; and consolidated average trade size decreased from 724 to 268 shares, SEC (2010).
- ▶ High Frequency (HF) Trading: refers to the subset of AT trading strategies that are characterised by their reliance on speed differences relative to other traders to make profits based on short-term predictions and also by the objective to hold essentially no asset inventories for more than a very short period of time.

Algorithmic and High Frequency Trading

We will need tools from

- ▶ Market Microstructure theory
- ▶ Stochastic Optimal Control

and learn about

- ▶ market design
- ▶ market regulation
- ▶ how electronic markets function
- ▶ the role of different market players

Mini Course outline

Some mathematical background I assume you already know

- ▶ Basic stochastic calculus tools
 - ▶ Ito's Lemma
 - ▶ Jump processes
 - ▶ OU processes

Financial economics and stats (I assume you do not know)

- ▶ Electronic markets
- ▶ Market Players
- ▶ Basics of the Limit Order Book (LOB)

Why AT? One example

- ▶ Institutional or large players need to trade (buy and sell) large amounts of securities. These quantities are too large for the market to process without prices moving in the 'wrong direction' (slippage).
- ▶ Thus, large orders are broken up in small ones and these are traded over time (minutes, hours, days, weeks, or even months) and across different venues.
- ▶ Deciding how to break up and execute a large order can mean saving millions of pounds for large players

Exchanges

Exchanges

An exchange is a 'place' where 'people' meet to buy/sell securities: shares, commodities, derivatives, etc

- ▶ Quote Driven Market:
 - ▶ Designated market makers and specialists display bids and asks for a specific security.
- ▶ Order Driven Market:
 - ▶ All buyers and sellers display the prices and quantities at which they wish to buy or sell a particular security. can post limit buy or sell orders
 - ▶ Limit orders show an intention to buy or sell and must indicate the amount of shares and price at which the agent is willing to trade
 - ▶ The limit buy order with the highest price tag is known as the best bid
 - ▶ The limit sell order with the lowest price tag is known as the best offer

Evolution of markets

- ▶ Old days brokerage model: Ring a broker, broker sends order to the pit and after screaming and hand signalling the order is executed.
- ▶ Electronic market: Ring or use internet to contact broker who sends the order to the electronic exchange (no screaming)
- ▶ Direct Access Market: clients send orders directly to market

But to which market? ARCA-NYSE: electronic platform of NYSE, BATS (Kansas), BEX: Boston Equity Exchange, CBSX (CBOE Stock Exchange), CSXZ (Chicago Stock Exchange), DRCTEDGE (Direct Edge, Jersey City, NJ), ISE (International Securities Exchange), ISLAND (Acquired by Nasdaq in 2003), LAVA (Citigroup), NSX (National Stock Exchange, Chicago) TRACKECN (Track ECN), ChiX, LSE (London Stock Exchange), etc, etc

Limit Order Book

Limit Order Book

- ▶ During the trading day, all **limit orders** are accumulated in the limit order book (LOB) until they find a counterparty for execution or are cancelled by the agent who posted them.
- ▶ The counterparty is a market order which is an order to buy or sell an amount of shares, regardless of the price, which is immediately executed against limit orders resting in the LOB at the best execution prices.

LOB (part I)

Price/time prioritised collection of buy and sell quotes

Buy Orders		Sell Orders	
Price	Volume	Price	Volume
60	80	60.1	75
59.90	100	60.2	75
59.80	90	60.3	50

- ▶ The best bid is
- ▶ The best offer is
- ▶ Hence the spread is
- ▶ The midquote is
- ▶ Hence the halfspread is
- ▶ The microprice is the volume weighted midprice

$$mp = \frac{80}{80 + 75}60 + \frac{75}{80 + 75}60.1.$$

LOB (part II)

Price/time prioritised collection of buy and sell quotes

Buy Orders		Sell Orders	
Price	Volume	Price	Volume
60	80	60.1	75
59.90	100	60.2	75
59.80	90	60.3	50

What happens if a **market buy order** for 100 shares comes in?

What happens if a **market sell order** for 100 shares comes in?

LOB (part III)

A **market buy order** for 100 shares comes in

Buy Orders		Sell Orders	
Price	Volume	Price	Volume
60	80	60.1	75
59.90	100	60.2	75
59.80	90	60.3	50

- ▶ The first 75 shares are bought at 60.1
- ▶ The rest of the shares (25) are bought at 60.2

Buy Orders		Sell Orders	
Price	Volume	Price	Volume
60	80	60.2	50
59.90	100	60.3	50
59.80	90	-	-

LOB (part IV)

A **market sell order** for 100 shares comes in

Buy Orders		Sell Orders	
Price	Volume	Price	Volume
60	80	60.1	75
59.90	100	60.2	75
59.80	90	60.3	50

- ▶ The first 80 shares are sold at 60
- ▶ The rest of the shares (20) are sold at 59.50

Buy Orders		Sell Orders	
Price	Volume	Price	Volume
59.90	80	60.1	75
59.80	90	60.2	75
-	-	60.3	50

LOB shape and metrics

What happens to the LOB immediately after the buy (sell) market order is filled?

- ▶ Best bid: Stays the same (drops 10 pence)
- ▶ Best offer: Increases 10 pence (stays the same)
- ▶ Spread: Widens from 10 to 20 pence in both cases
- ▶ Depth: Decreases on the sell (buy) side

But more importantly, what happens next in the LOB?

Who are you trading with? Is she better informed?

Imagine that you are a market maker... (part I)

- ▶ How do you make money in this business?
- ▶ What information do you use?
- ▶ How can you devise a strategy to make money?
- ▶ Are there any events that make you withdraw from the market?

Imagine you are a market maker... (part II)

- ▶ You can observe the entire LOB
- ▶ You do not know who you trade with
- ▶ Make money from roundtrip trades by posting limit buy and limit sell orders (maybe you also send market orders)
- ▶ Must be prepared to hold inventories

What is a market making strategy?

It may mean many things, for instance:

- ▶ Where to place your limit orders
- ▶ Are you posting in many places in the book?
- ▶ How do you react to **order flow** information?
- ▶ When is it good to cancel an existing limit order?
- ▶ How do you manage inventory risk?
 - ▶ How much is too much or too little?
 - ▶ When do you aggressively reduce your inventory risk?

Imagine you are a market maker... (part III)

Would your market making strategy change depending on

- ▶ your level of inventories?
- ▶ the volatility of the stock you are buying and selling?
- ▶ the liquidity of the stock you are buying and selling?
- ▶ who you think you are trading with? (Adverse selection)
- ▶ your knowledge about the industry or particular firm?
- ▶ fundamental macroeconomic forecasts and news?

Now imagine you are a **large fund**...

You are a **large** equity fund and must rebalance your portfolio.
What do you fear?

- ▶ being 'frontrun' or detected by other clever algorithms
- ▶ your order is too large and will move the market against you
- ▶ etc

Some standard measures

Let b_t be the best bid and a_t the best ask at time t . Then we define

- ▶ $m_t = \frac{a_t + b_t}{2}$, the **midquote** at time t
- ▶ $s_t = a_t - b_t$, the quoted **spread** at time t
- ▶

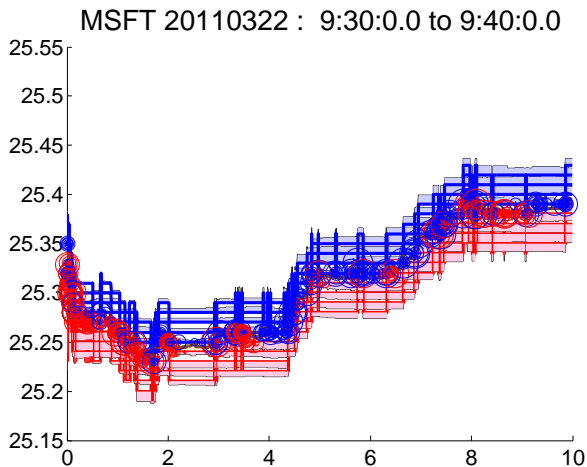
$$effspr_t = 2 |P_t - m_t| ,$$

the **effective spread** where P_t is the price at which a market order was executed. For example, a market order is submitted when the quote is $b_t = 10.00$ and $a_t = 10.20$. The order is executed at $P_t = 10.15$. The effective spread is $2(10.15 - 10.10) = 0.10$. Some people define the effective spread as

$$effspr_t = 2 \left| \frac{P_t - m_t}{m_t} \right| .$$

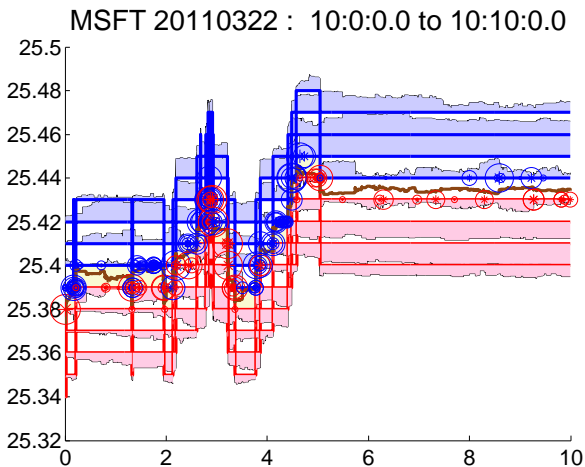
Limit Order Book

Trade Activity



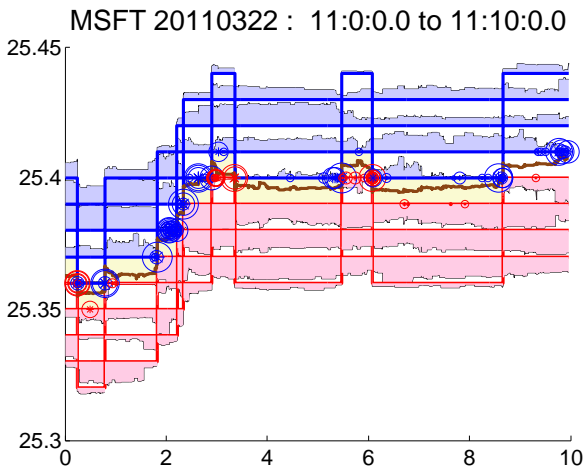
Limit Order Book

Trade Activity



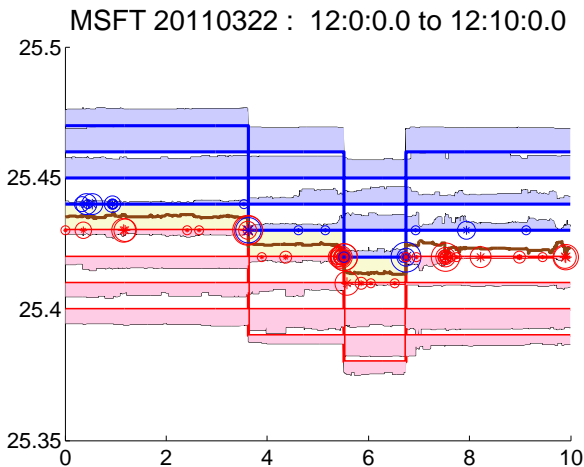
Limit Order Book

Trade Activity



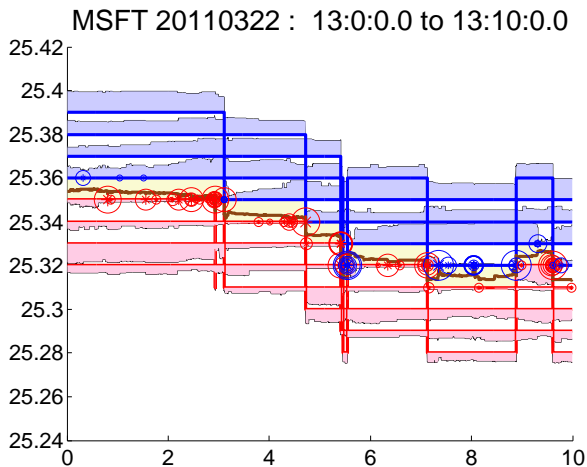
Limit Order Book

Trade Activity



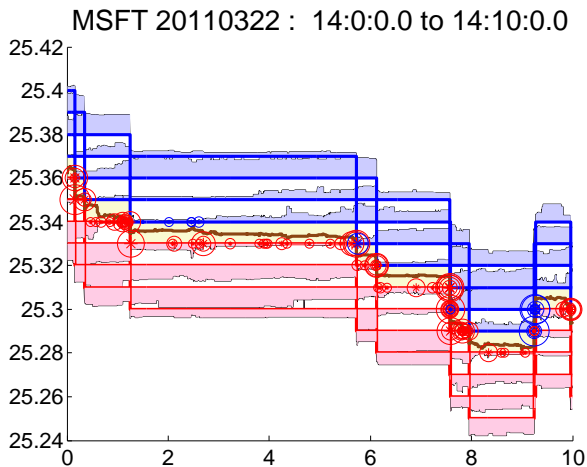
Limit Order Book

Trade Activity



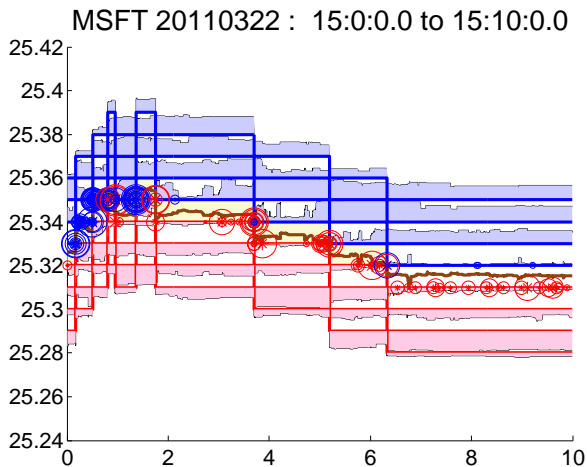
Limit Order Book

Trade Activity



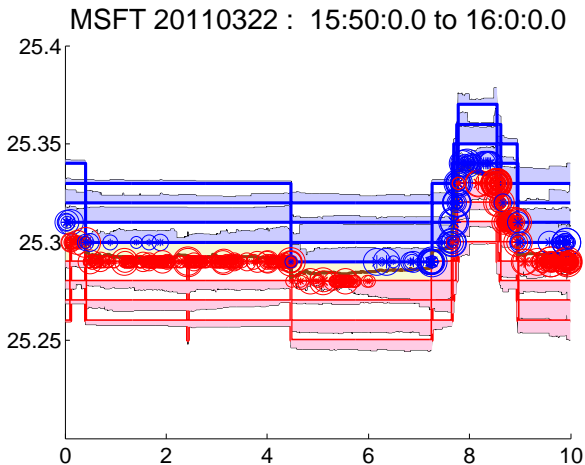
Limit Order Book

Trade Activity



Limit Order Book

Trade Activity



Order Imbalance

Order Imbalance

- Order Imbalance $\rho \in [0, 1]$

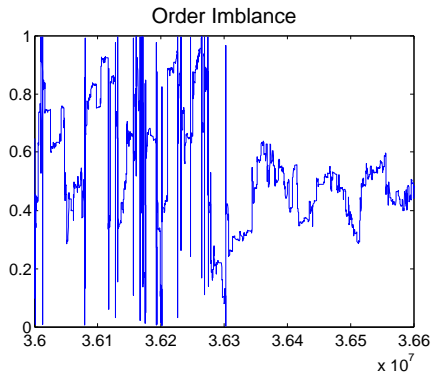
$$\rho_t = \frac{V_t^b}{V_t^a + V_t^b}$$

- It is a good predictor of trade direction
(ORIT June 21, 2011)

ρ	# Buy Orders	# Sell Orders
All	756 (67%)	396 (33 %)
> 0.5	568 (79%)	155 (21%)
> 0.75	320 (84%)	60 (16%)
< 0.5	168 (43%)	225 (57%)
< 0.25	39 (25%)	116 (75%)

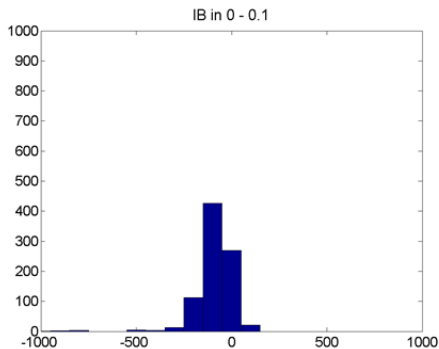
Order Imbalance

A slice of imbalance for MSFT 10:00am to 10:10am on 22 Mar 2011



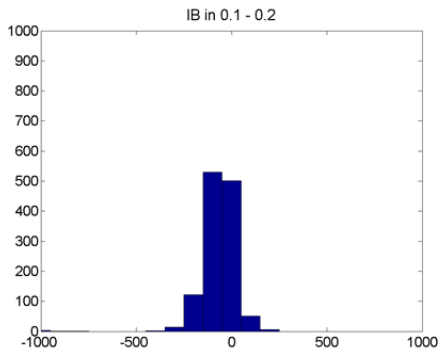
Order Imbalance

Midprice change pre/post MO event with $\rho \in [0, 0.1)$



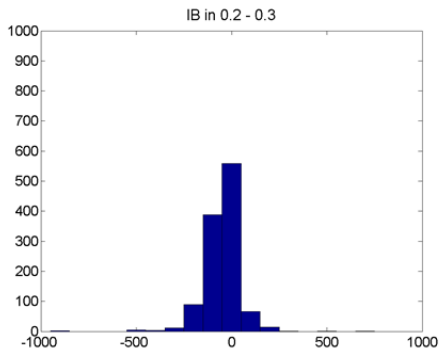
Order Imbalance

Midprice change pre/post MO event with $\rho \in [0.1, 0.2)$



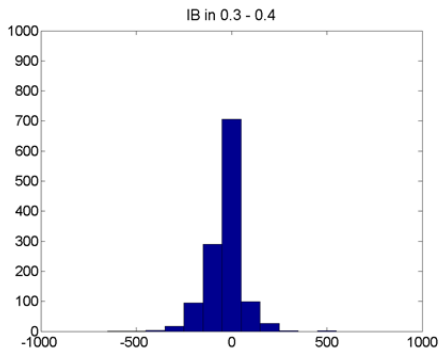
Order Imbalance

Midprice change pre/post MO event with $\rho \in [0.2, 0.3)$



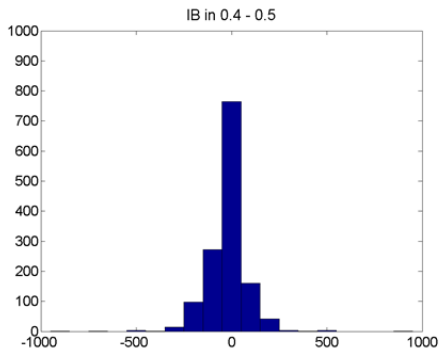
Order Imbalance

Midprice change pre/post MO event with $\rho \in [0.3, 0.4)$



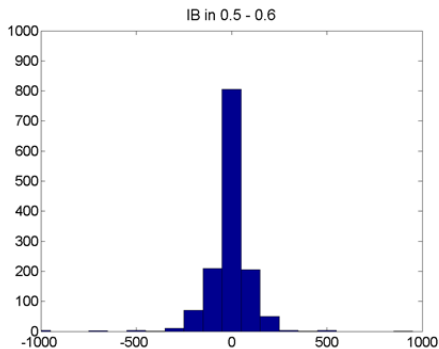
Order Imbalance

Midprice change pre/post MO event with $\rho \in [0.4, 0.5)$



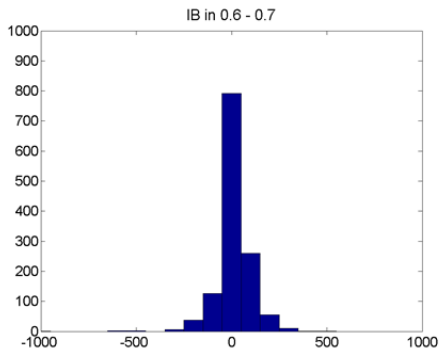
Order Imbalance

Midprice change pre/post MO event with $\rho \in [0.5, 0.6)$



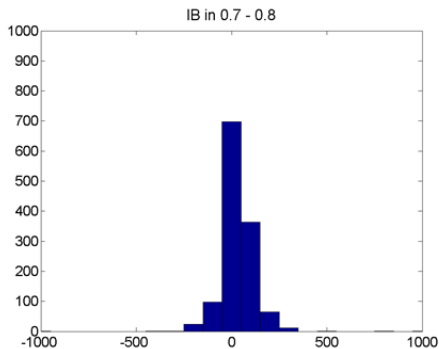
Order Imbalance

Midprice change pre/post MO event with $\rho \in [0.6, 0.7)$



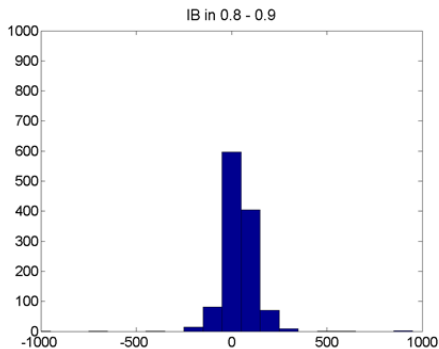
Order Imbalance

Midprice change pre/post MO event with $\rho \in [0.7, 0.8)$



Order Imbalance

Midprice change pre/post MO event with $\rho \in [0.8, 0.9)$



Order Imbalance

Midprice change pre/post MO event with $\rho \in [0.9, 1.0]$

