

# Fundamentals of Artificial Neural Networks

2/3

## Agenda

Recap last session on perceptrons

Learning in a perceptron

"Normal" ANNs

### Wilco Bonestroo

Researcher at Mechatronics research group at Saxion

Software and AI for robots

Autonomous mobile robots

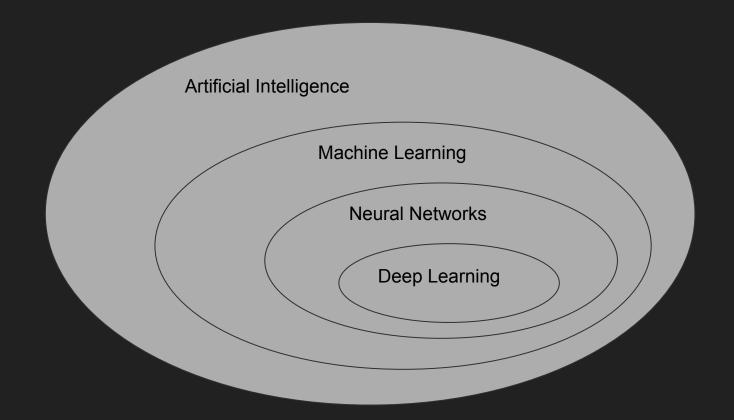




#### Research group

- Unmanned Robotics Systems
- Smart Industrial Systems

## The bigger picture



## **ANN** concepts

Node

Link

Input

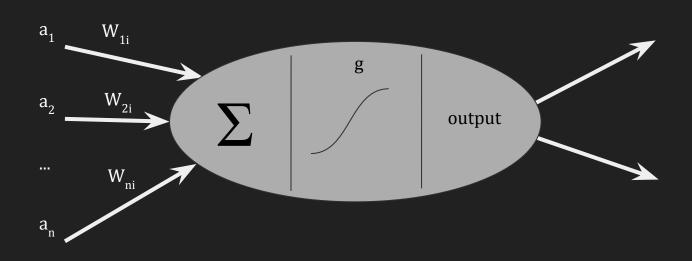
Weight

**Activation function** 

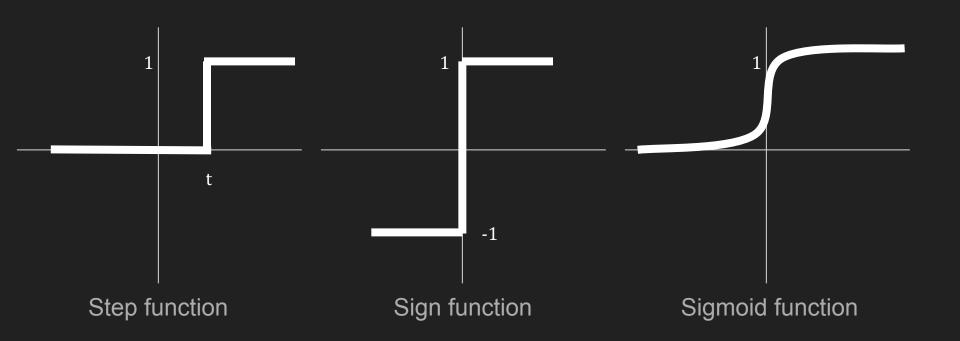
Output

Layer

"Knowledge"

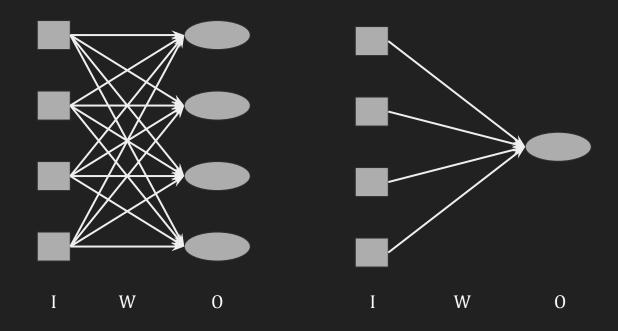


## **Common Activation Functions**



## Perceptrons

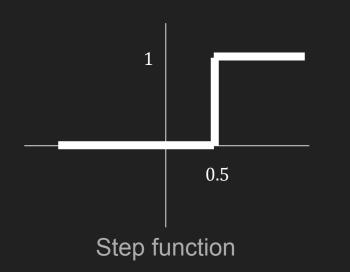
No hidden layers

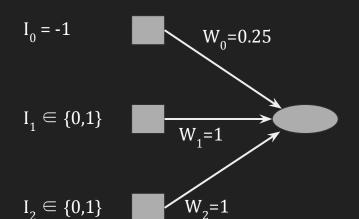


$$O = \operatorname{Step}_{0}(\Sigma \ W_{j}I_{j}) = \operatorname{Step}_{0}(\mathbf{W} \cdot \mathbf{I})$$

$$I_1 * W_1 + I_2 * W_2 + ...$$

## Represent boolean functions

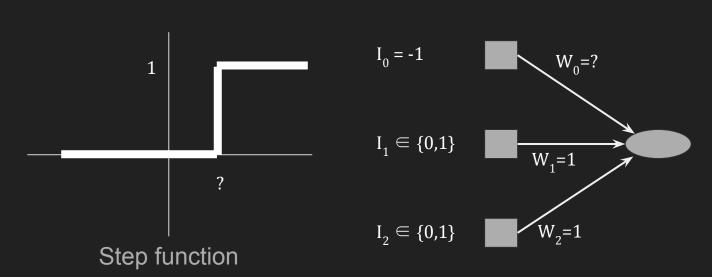




I <sub>1</sub>	I <sub>2</sub>	0
0	0	
0	1	
1	0	
1	1	

More convenient:  $W \times I > 0$ 

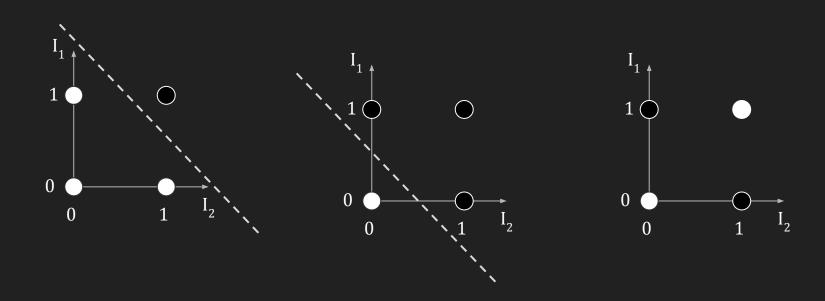
## Represent boolean functions



XOR

I <sub>1</sub>	I <sub>2</sub>	О
0	0	0
0	1	1
1	0	1
1	1	0

## Linear Separable (two inputs / dimensions)



## Learning in a perceptron

Find an algorithm that can "train" a network based on given examples.

Perceptron learning rule by Rosenblat (1960)

$I_1$	$I_2$	0
0	0	0
0	1	0
1	0	0
1	1	1

## Generic Network Learning Algorithm

Initialise network (weights)

#### Repeat

For each example

Determine outcome of current network for example

Compare outcome to expected value for example

Update the network

end

Until all examples are correctly classified

## Generic Network Learning

Initialise network (weights)

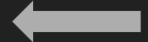
#### Repeat

For each example

Determine outcome of current network for example

Compare outcome to expected value for example

Update the network



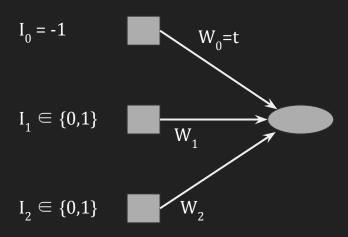
end

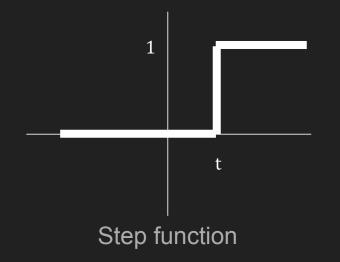
Until all examples are correctly classified

$W_0$	$W_1$	W <sub>2</sub>
0	0	0

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	1	1

Output	Expected

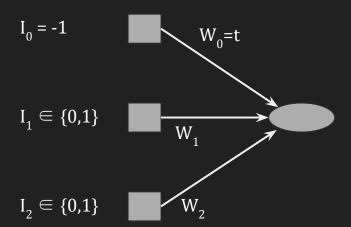




$W_0$	$W_1$	W <sub>2</sub>
0	0	0

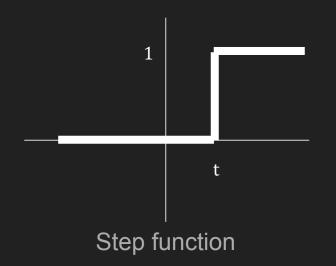
$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	1	1

Output	Expected
0	1



If the output should be **higher**, we should add the values of  $I_i$  to  $W_i$ 

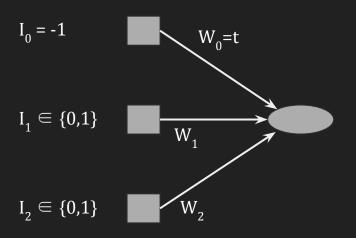
Short notation: W becomes  $W + \alpha \times I$ 

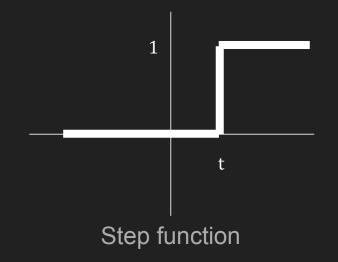


$W_0$	$W_1$	$W_2$
-1	0	0

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	1

Output	Expected

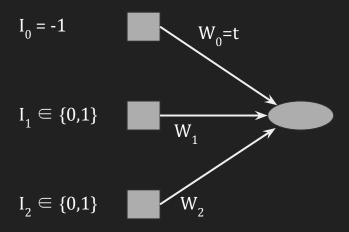




$W_0$	$W_1$	W <sub>2</sub>
-1	0	0

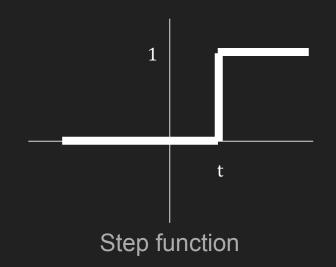
$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	1

Output	Expected
1	0



If the output should be **lower**, we should **subtract** the values of  $I_j$  from  $W_j$ 

Short notation: *W* becomes  $W - \alpha x I$ 



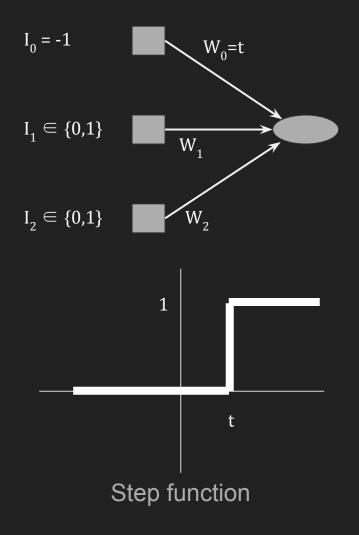
#### The actual output is O

The expected output is T

The error Err = T - O

#### Learning Rule:

 $W_j$  becomes  $W_j + \alpha \times I_j \times Err$ 



$W_0$	$W_1$	W <sub>2</sub>
0	0	0

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0

Output	Expected	

$W_0$	$W_1$	W <sub>2</sub>
0	0	0

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0

Output	Expected	
0	0	4

$W_0$	$W_1$	W <sub>2</sub>
0	0	0
0	0	0

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1

Output	Expected	
0	0	4

$W_0$	$W_1$	$W_2$
0	0	0
0	0	0

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1

Output	Expected	
0	0	4
0	1	<b>⊗</b>

$W_0$	$W_1$	$W_2$
0	0	0
0	0	0
-1	0	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1

Output	Expected	
0	0	4
0	1	<b>8</b>

$W_0$	$W_1$	$W_2$
0	0	0
0	0	0
-1	0	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	0	1

Output	Expected	
0	0	4
0	1	×

$W_0$	$W_1$	$W_2$
0	0	0
0	0	0
-1	0	1

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	0	1

Output	Expected	
0	0	4
0	1	<b>⊗</b>
1	1	4

$W_0$	$W_1$	$W_2$
0	0	0
0	0	0
-1	0	1
-1	0	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	0	1
-1	1	0

Output	Expected	
0	0	4
0	1	<b>⊗</b>
1	1	4

$W_0$	$W_1$	$W_2$
0	0	0
0	0	0
-1	0	1
-1	0	1

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	0	1
-1	1	0

Output	Expected	
0	0	4
0	1	<b>⊗</b>
1	1	4
1	1	4

$W_0$	$W_1$	$W_2$
0	0	0
0	0	0
-1	0	1
-1	0	1
-1	0	1

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	0	1
-1	1	0
-1	1	1

Output	Expected	
0	0	4
0	1	<b>⊗</b>
1	1	4
1	1	4

$W_0$	$W_1$	$W_2$
0	0	0
0	0	0
-1	0	1
-1	0	1
-1	0	1

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	0	1
-1	1	0
-1	1	1

Output	Expected	
0	0	4
0	1	<b>⊗</b>
1	1	4
1	1	4
1	1	4

$W_0$	$W_1$	$W_2$
0	0	0
0	0	0
-1	0	1
-1	0	1
-1	0	1
-1	0	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	0	1
-1	1	0
-1	1	1
-1	0	0

Output	Expected	
0	0	4
0	1	<b>⊗</b>
1	1	4
1	1	4
1	1	4

$W_0$	$W_1$	$W_2$
0	0	0
0	0	0
-1	0	1
-1	0	1
-1	0	1
-1	0	1

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	0	1
-1	1	0
-1	1	1
-1	0	0

Output	Expected	
0	0	4
0	1	<b>⊗</b>
1	1	4
1	1	4
1	1	4
1	0	×

$W_0$	$W_1$	$W_2$
0	0	0
0	0	0
-1	0	1
-1	0	1
-1	0	1
-1	0	1
0	0	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	0	1
-1	1	0
-1	1	1
-1	0	0
-1	0	0

Output	Expected	
0	0	4
0	1	<b>⊗</b>
1	1	4
1	1	4
1	1	<u>4</u>
1	0	<b>⊗</b>

$W_0$	$W_1$	$W_2$
0	0	0
0	0	0
-1	0	1
-1	0	1
-1	0	1
-1	0	1
0	0	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	0	1
-1	1	0
-1	1	1
-1	0	0
-1	0	0

Output	Expected	
0	0	4
0	1	<b>⊗</b>
1	1	4
1	1	<u>4</u>
1	1	<u>4</u>
1	0	<b>⊗</b>
0	0	4

$W_0$	$W_1$	$W_2$
0	0	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0

Output	Expected	
0	0	4

$W_0$	$W_1$	$W_2$
0	0	1
0	0	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1

Output	Expected	
0	0	4

$W_0$	$W_{1}$	$W_2$
0	0	1
0	0	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1

Output	Expected	
0	0	4
1	1	4

$W_0$	$W_1$	W <sub>2</sub>
0	0	1
0	0	1
0	0	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	1	0

Output	Expected	
0	0	4
1	1	4

$W_0$	$W_1$	$W_2$
0	0	1
0	0	1
0	0	1

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	1	0

Output	Expected	
0	0	4
1	1	4
0	1	œ

$W_0$	$W_1$	$W_2$
0	0	1
0	0	1
0	0	1
-1	1	1

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	1	0
-1	1	0

Output	Expected	
0	0	4
1	1	4
0	1	œ

$W_0$	$W_1$	$W_2$
0	0	1
0	0	1
0	0	1
-1	1	1

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	1	0
-1	1	0

Output	Expected	
0	0	4
1	1	4
0	1	<b>⊗</b>
1	1	4

$W_0$	$W_1$	$W_2$
0	0	1
0	0	1
0	0	1
-1	1	1
-1	1	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	1	0
-1	1	0
-1	1	1

Output	Expected	
0	0	4
1	1	4
0	1	œ
1	1	4

$W_0$	$W_1$	$W_2$
0	0	1
0	0	1
0	0	1
-1	1	1
-1	1	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	1	0
-1	1	0
-1	1	1

Output	Expected	
0	0	4
1	1	4
0	1	<b>⊗</b>
1	1	4
1	1	4

$W_0$	$W_1$	$W_2$
0	0	1
0	0	1
0	0	1
-1	1	1
-1	1	1
-1	1	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	1	0
-1	1	0
-1	1	1
-1	0	0

Output	Expected	
0	0	4
1	1	4
0	1	œ
1	1	4
1	1	4

$W_0$	$W_1$	$W_2$
0	0	1
0	0	1
0	0	1
-1	1	1
-1	1	1
-1	1	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	1
-1	1	0
-1	1	0
-1	1	1
-1	0	0

Output	Expected	
0	0	4
1	1	4
0	1	<b>⊗</b>
1	1	4
1	1	4
1	0	<b>⊗</b>

$W_0$	$W_1$	W <sub>2</sub>
-1	1	1

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0

Output	Expected	
1	0	<b>⊗</b>

$W_0$	$W_1$	W <sub>2</sub>
-1	1	1
0	1	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	0

Output	Expected	
1	0	<b>⊗</b>

$W_0$	$W_1$	W <sub>2</sub>
-1	1	1
0	1	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	0

Output	Expected	
1	0	×
0	0	<b>e</b>

$W_0$	$W_1$	$W_2$
-1	1	1
0	1	1
0	1	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	0
-1	0	1

Output	Expected	
1	0	×
0	0	<b>e</b>

$W_0$	$W_1$	W <sub>2</sub>
-1	1	1
0	1	1
0	1	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	0
-1	0	1

Output	Expected	
1	0	×
0	0	<b>e</b>
1	1	<b>\(\theta\)</b>

$W_0$	$W_1$	$W_2$
-1	1	1
0	1	1
0	1	1
0	1	1

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	0
-1	0	1
-1	1	0

Output	Expected	
1	0	<b>⊗</b>
0	0	<b>:</b>
1	1	<b>\(\theta\)</b>

$W_0$	$W_1$	$W_2$
-1	1	1
0	1	1
0	1	1
0	1	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	0
-1	0	1
-1	1	0

Output	Expected	
1	0	<b>⊗</b>
0	0	<b>e</b>
1	1	<b>e</b>
1	1	<b>\(\theta\)</b>

$W_0$	$W_1$	$W_2$
-1	1	1
0	1	1
0	1	1
0	1	1
0	1	1

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	0
-1	0	1
-1	1	0
-1	1	1

Output	Expected	
1	0	<b>⊗</b>
0	0	<b>e</b>
1	1	<b>\(\theta\)</b>
1	1	<b>\(\theta\)</b>

$W_0$	$W_1$	$W_2$
-1	1	1
0	1	1
0	1	1
0	1	1
0	1	1

$I_0$	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	0
-1	0	1
-1	1	0
-1	1	1

Output	Expected	
1	0	<b>⊗</b>
0	0	<b>e</b>
1	1	<b>e</b>
1	1	<b>e</b>
1	1	<b>\(\theta\)</b>

$W_0$	$W_1$	$W_2$
-1	1	1
0	1	1
0	1	1
0	1	1
0	1	1

I <sub>0</sub>	I <sub>1</sub>	I <sub>2</sub>
-1	0	0
-1	0	0
-1	0	1
-1	1	0
-1	1	1

Output	Expected		
1	0	<b>⊗</b>	
0	0	<b>e</b>	
1	1	<b>\(\theta\)</b>	
1	1	<b>\(\theta\)</b>	
1	1	<b>\(\theta\)</b>	

For **linear separable** functions this algorithm **always** converges!

Most functions are not linearly separable :(

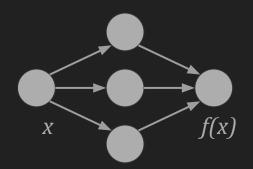
This is a sort of gradient descent, but not backpropagation!

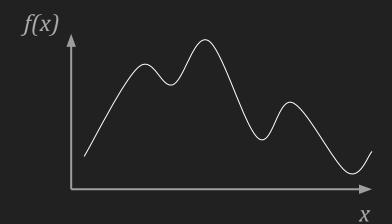
The approach we saw does not work for "normal" ANN with many layers and different activation functions...

## How to represent other functions?

#### Add layers

"With one (sufficiently large) layer of hidden units, it is possible to represent any continuous function of the inputs."





#### Resources

Book: Artificial Intelligence: A Modern Approach

Auteur: Stuart Russell