Work in Progress

Answers to Exercises

Stochastic Calculus for Finance I: The Binomial Asset Pricing Model by *Steven E. Shreve*

Chapter 6 Interest-Rate-Dependent Assets

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Please refer to the book for the exercises themselves. The text that comes before each answer serves only as a recap.

In what follows, I use the notation B_n^m to denote the time-n price of a bond that matures at time-m. This corresponds to the $B_{n,m}$ notation that Shreve uses in the book but is more compact.

Exercise 6.2 Verify that the discounted value of the static hedging portfolio for a forward is a martingale under $\tilde{\mathbb{P}}$.

Answer: To hedge a short position in a forward contract that is initiated at time n with delivery time m, we, at time n, long 1 share of stock and short S_n/B_n^m unit of m-maturity zero-coupon bond. This constructs a static hedging portfolio. The time-n value of the hedging portfolio is

$$X_n = S_n - \Big(\frac{S_n}{B_n^m}\Big)B_n^m \ (=0)$$

To show its discounted value is $\tilde{\mathbb{P}}$-martingale, note that

$$\begin{aligned}
\tilde{\mathbb{E}}_n(D_{n+1}X_{n+1}) &= \tilde{\mathbb{E}}_n\Big[D_{n+1}\Big(S_{n+1} - \Big(\frac{S_n}{B_n^m}\Big)B_{n+1}^m\Big)\Big] \\
&= \tilde{\mathbb{E}}_n(D_{n+1}S_{n+1}) - \Big(\frac{S_n}{B_n^m}\Big)\tilde{\mathbb{E}}_n(D_{n+1}B_{n+1}^m) && \cdots \text{linearity} \\
&= D_nS_n - \Big(\frac{S_n}{B_n^m}\Big)D_nB_n^m && \cdots \text{discounted stock, bond prices are } \tilde{\mathbb{P}}\text{-martingale} \\
&= D_nX_n
\end{aligned}$$

Note: that discounted bond prices are $\tilde{\mathbb{P}}$-martingale can be shown by, for $0 \le k < n \le m$,

$$\tilde{\mathbb{E}}_k(D_nB_n^m) = \tilde{\mathbb{E}}_k\Big[D_n\tilde{\mathbb{E}}_n\big(\frac{D_m}{D_n}\big)\Big] = \tilde{\mathbb{E}}_k[\tilde{\mathbb{E}}_n(D_m)] = \tilde{\mathbb{E}}_k(D_m) = \tilde{\mathbb{E}}_k\big(D_k\cdot\frac{D_m}{D_k}\big) = D_k\tilde{\mathbb{E}}_k\big(\frac{D_m}{D_k}\big) = D_kB_k^m$$

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