

Answers to Exercises

Stochastic Calculus for Finance I: The Binomial Asset Pricing Model by Steven E. Shreve

Chapter 6 Interest-Rate-Dependent Assets

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Please refer to the book for the exercises themselves. The text that comes before each answer serves only as a recap.

In what follows, I use the notation B_n^m to denote the time- n price of a bond that matures at time m . This corresponds to the $B_{n,m}$ notation that Shreve uses in the book but is more compact.

Exercise 6.2 Verify that the discounted value of the static hedging portfolio for a forward is a martingale under $\tilde{\mathbb{P}}$.

Answer: To hedge a short position in a forward contract that is initiated at time n with delivery time m , we, at time n , long 1 share of stock and short S_n/B_n^m unit of m -maturity zero-coupon bond. This constructs a static hedging portfolio. The time- n value of the hedging portfolio is

$$X_n = S_n - \left(\frac{S_n}{B_n^m} \right) B_n^m (= 0)$$

To show its discounted value is $\tilde{\mathbb{P}}$ -martingale, note that

$$\begin{aligned} \tilde{\mathbb{E}}_n(D_{n+1} X_{n+1}) &= \tilde{\mathbb{E}}_n \left[D_{n+1} \left(S_{n+1} - \left(\frac{S_n}{B_n^m} \right) B_{n+1}^m \right) \right] \\ &= \tilde{\mathbb{E}}_n(D_{n+1} S_{n+1}) - \left(\frac{S_n}{B_n^m} \right) \tilde{\mathbb{E}}_n(D_{n+1} B_{n+1}^m) \quad \dots \text{linearity} \\ &= D_n S_n - \left(\frac{S_n}{B_n^m} \right) D_n B_n^m \quad \dots \text{discounted stock, bond prices are } \tilde{\mathbb{P}}\text{-martingale} \\ &= D_n X_n \end{aligned}$$

Note: that discounted bond prices are $\tilde{\mathbb{P}}$ -martingale can be shown by, for $0 \leq k < n \leq m$,

$$\tilde{\mathbb{E}}_k(D_n B_n^m) = \tilde{\mathbb{E}}_k \left[D_n \tilde{\mathbb{E}}_n \left(\frac{D_m}{D_n} \right) \right] = \tilde{\mathbb{E}}_k[\tilde{\mathbb{E}}_n(D_m)] = \tilde{\mathbb{E}}_k(D_m) = \tilde{\mathbb{E}}_k \left(D_k \cdot \frac{D_m}{D_k} \right) = D_k \tilde{\mathbb{E}}_k \left(\frac{D_m}{D_k} \right) = D_k B_k^m$$

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