Notes on the point spread function in ultrasound imaging

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Pulse echo response from a point scatterer in position $\vec{r}_s$

$$h_{pe}(t,\vec{r}_s \mid \vec{r}_f) = p(t) \otimes_t h_{Tx}(t,\vec{r}_s \mid \vec{r}_f) \otimes_t h_{Rx}(t,\vec{r}_s \mid \vec{r}_f)$$

Spatial impulse response for transmit aperture, focused in point $\vec{r}_f$: $h_{Tx}(t,\vec{r}_s \mid \vec{r}_f)$
Spatial impulse response for receive aperture, focused in point $\vec{r}_f$: $h_{Rx}(t,\vec{r}_s \mid \vec{r}_f)$

Image formation by linear scan in x direction, and acquiring pulse echo signal according to time of flight in z direction, t= 2 z/c.
Image of point scatterer in position $\vec{r}_s = (x_s, z_s)$

$$p_{sf}(\vec{r} \mid \vec{r}_s) = h_{pe}(\tfrac{2}{c} z, \vec{r}_s \mid \vec{r}\,)$$

For small displacements in z-direction, the pulse echo response is assumed to be spatially invariant, when we correct for time of flight

$$h_{pe}(t,\vec{r}_s \mid \vec{r}\,) \approx h'_{pe}(t - \tfrac{2}{c} z_s, x_s - x)$$

The point spread function will now be spatially invariant

$$p_{sf}(\vec{r} \mid \vec{r}_s) = h'_{pe}(-\tfrac{2}{c}(z_s - z), x_s - x) \equiv p'_{sf}(x_s - x, z_s - z)$$

And the imaging process can be described by a 2D convolution in space, or a product in the spatial Fourier domain

$$i(x,z) = p'_{sf} \otimes_{x,z} o(x,z)$$
$$I(f_x, f_z) = P'_{sf}(f_x, f_z) \cdot O(f_x, f_z)$$

A similar assumption of spatial invariance for the spatial impulse response on transmit and receive gives

$$h_{Tx}(t,\vec{r}_s \mid \vec{r}_f) \approx h'_{Tx}(t - \tfrac{1}{c} z_s, x_s - x_f)$$
$$h_{Rx}(t,\vec{r}_s \mid \vec{r}_f) \approx h'_{Rx}(t - \tfrac{1}{c} z_s, x_s - x_f)$$
$$h'_{pe}(t,x) = p(t) \otimes_t h'_{Tx}(t,x) \otimes_t h'_{Rx}(t,x)$$

In spatial Fourier domain

$$H'_{Pe}(f_t, f_x) = P(f_t) \cdot H'_{Tx}(f_t, f_x) \otimes_x H'_{Rx}(f_t, f_x)$$

And the point spread function in spatial Fourier domain

$$P'_{sf}(f_x, f_z) = H'_{Pe}(\tfrac{c}{2} f_z, f_x) = P(\tfrac{c}{2} f_z) \cdot H'_{Tx}(\tfrac{c}{2} f_z, f_x) \otimes_x H'_{Rx}(\tfrac{c}{2} f_z \ , f_x)$$

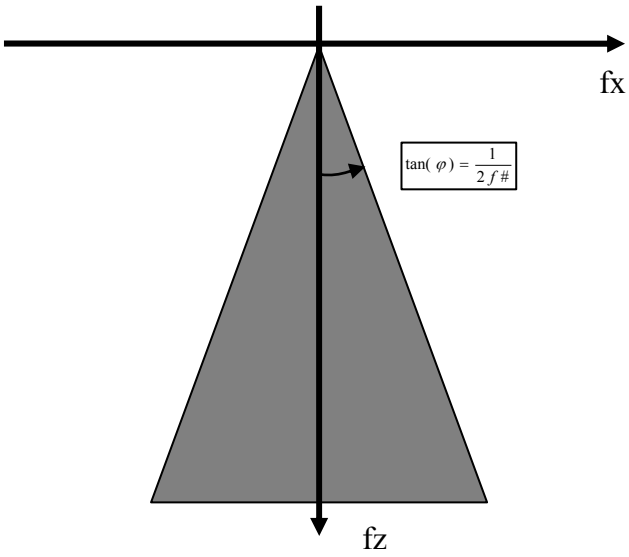

Fig. 1 Aperture function in k-space

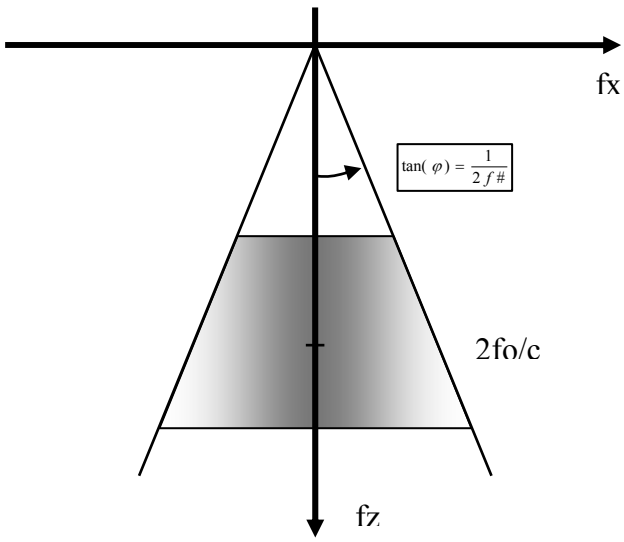

Fig. 2 Point spread function in k-space