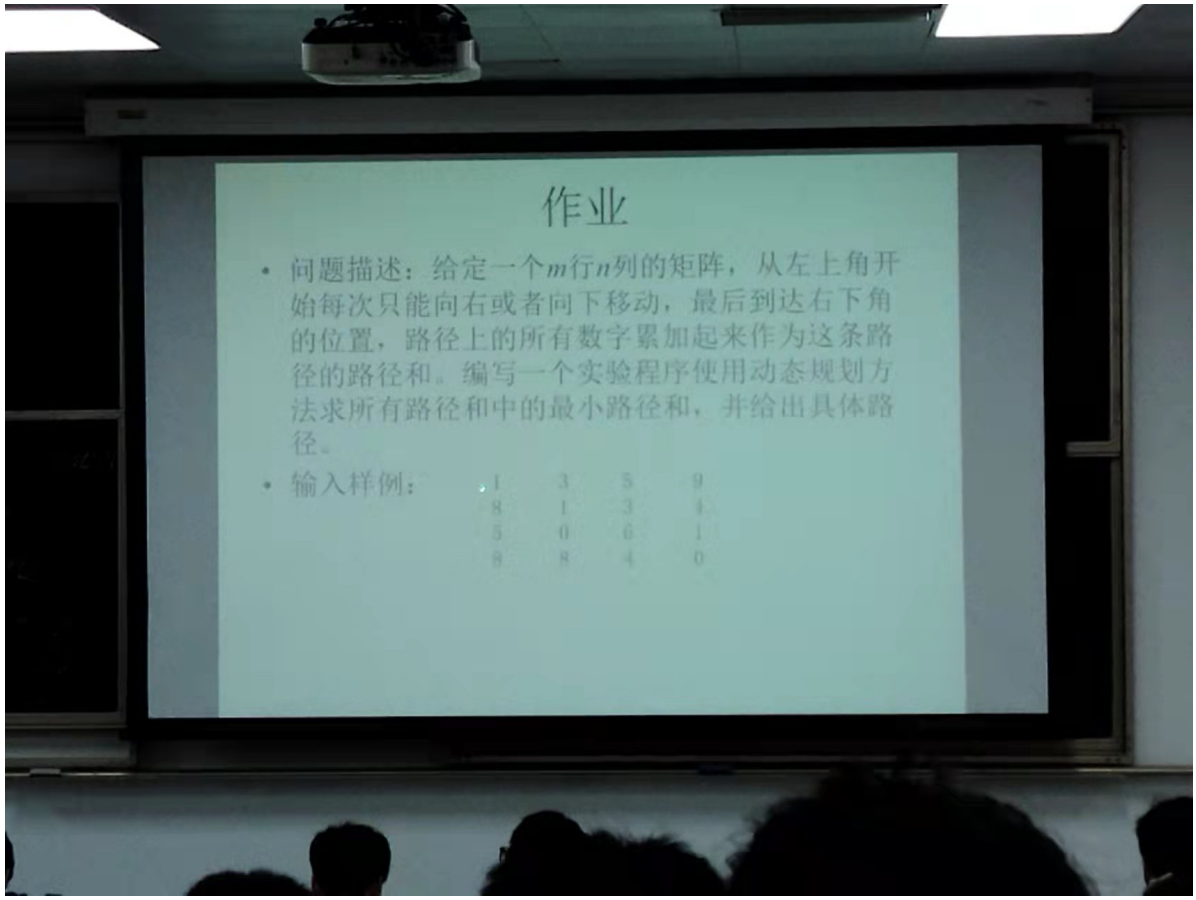
高级算法设计与分析作业_03

1.问题描述：



2.思路分析：

根据给出矩阵M，计算形状，

```
col , row = Matrix.shape
```

初始化全零矩阵dp，

```
dp = np.zeros((col,row))
```

同时为了输出最短路径，我们可以新设同等形状大小的列表min_id，

```
min_id = [[0 for _ in range(col)] for _ in range(row)]
```

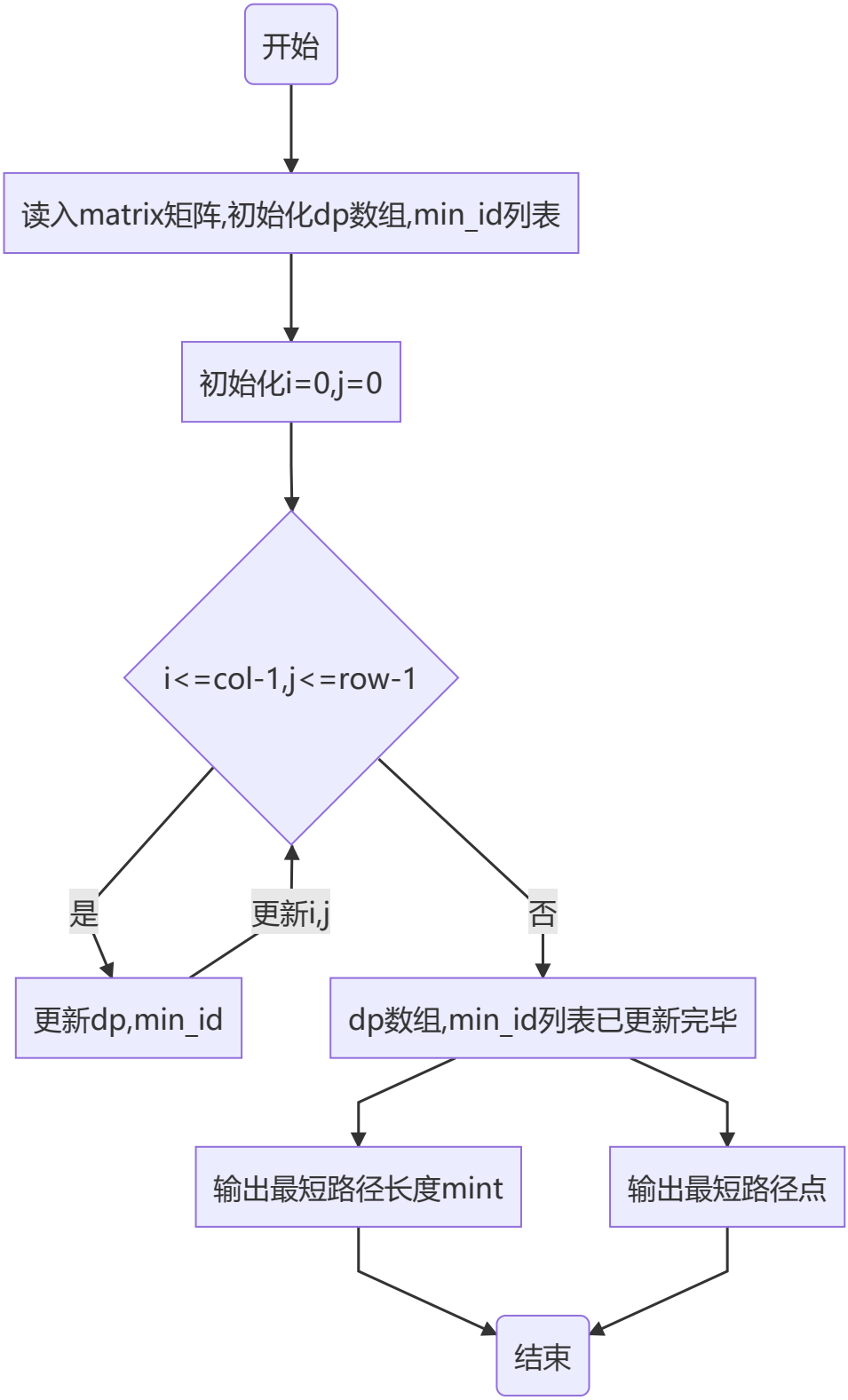
在规定了只能向下和向右移动寻找最短路径的情况下，dp数组转移方程可以定义为比较$dp[i-1,j]$和$dp[i,j-1]$的大小. 先更新dp的第0行和第0列边界值,dp数组更新规则：

$$\begin{cases} dp[i,0] = dp[i-1,0] + Matrix[i,0] & 0 \le i \le col \\ dp[0,j] = dp[0,j-1] + Matrix[0,j] & 0 \le i \le row \\ dp[i,j] = min(dp[i-1,j] + Matrix[i,j], dp[i,j-1] + Matrix[i,j]) & 0 \le i \le col,\ 0 \le i \le row \end{cases}$$

并将每次更新dp时选取的上一个点$(i-1,j)$或$(i,j-1)$放入$min_id[i,j]$中，在最后输出路径的时候使用while循环输出完整路径：

```
id = [col-1,row-1]                                  #根据最后的点坐标，从min_id
列表中一直查找上一个点坐标直到[0,0]
traj.append([col-1,row-1])
while(id!=[0,0]):                                   #逐一输出最短路径上的点
          id = min_id[id[0]][id[1]]
          traj.append(id)
traj.reverse()                                      #转置列表即为正序输出
```

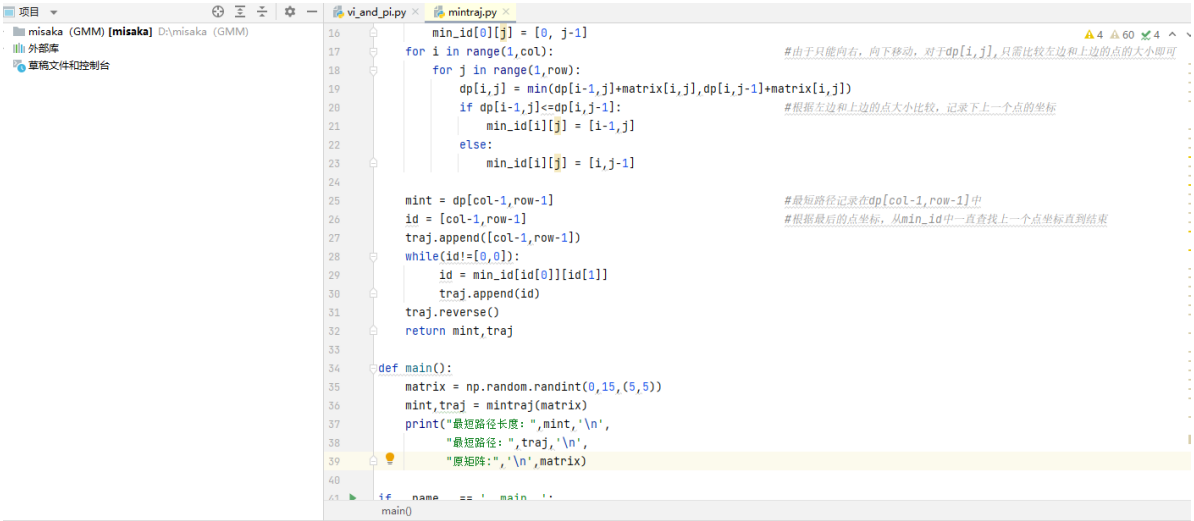
3.流程图



4.完整代码如下

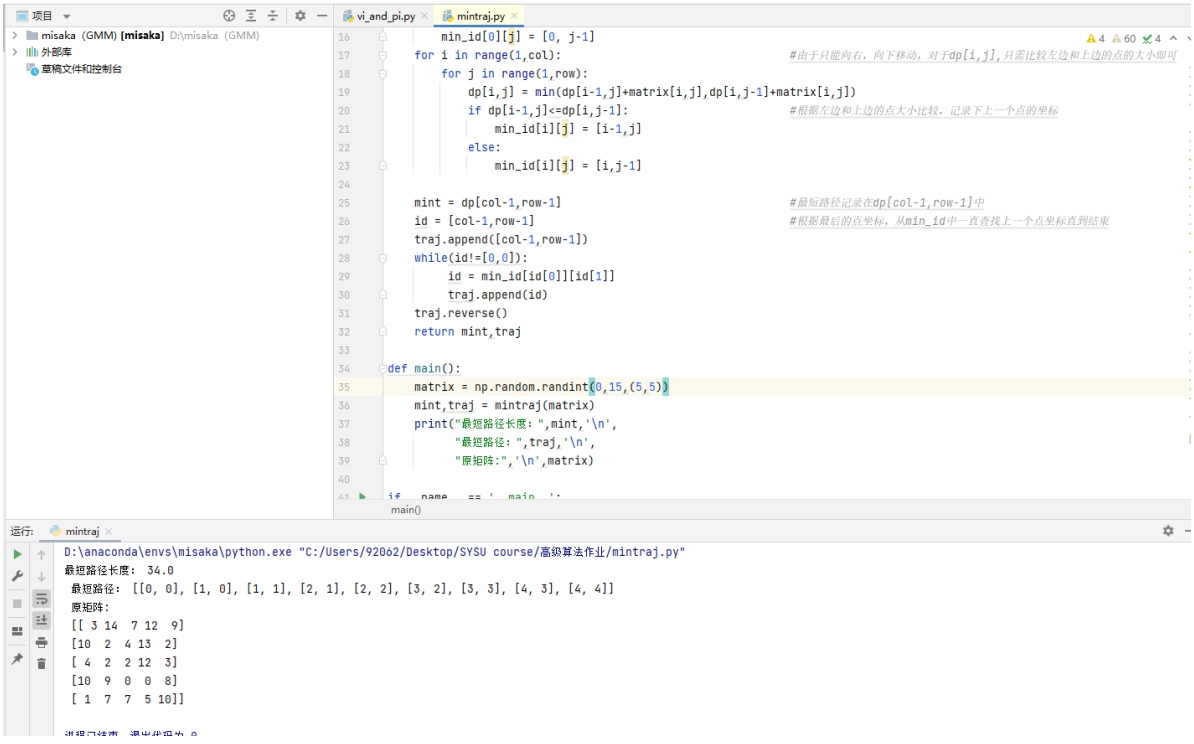
```
import numpy as np

def mintraj(matrix):
    col,row = matrix.shape
    dp = np.zeros((col,row))                                    #dp数组
    min_id = [[0 for _ in range(col)] for _ in range(row)]  #列表，记录达到本点的最短路径的上一个点
    traj = []                                                   #存储最短路径

    # dp updatad
    dp[0,0] = matrix[0,0]                                       #首先更新dp[0,0]

    #更新dp[:,0]和dp[0,:]
    for i in range(1,col):
        dp[i,0] = dp[i-1,0] + matrix[i, 0]
        min_id[i][0] = [i-1, 0]                                 #记录到达该点的上一个点
    for j in range(1,row):
        dp[0,j] = dp[0,j-1] + matrix[0,j]
        min_id[0][j] = [0, j-1]
    #更新dp[i,j]
    for i in range(1,col):                                      #由于只能向右，向下移动，对于dp[i,j],只需比较左边和上边的点的大小即可
        for j in range(1,row):
            dp[i,j] = min(dp[i-1,j] + matrix[i,j],
                          dp[i,j-1] + matrix[i,j])              #更短的路径长度计入dp[i,j]中
            if dp[i-1,j]<=dp[i,j-1]:                            #根据左边和上边的点大小比较，记录下上一个点的坐标
                min_id[i][j] = [i-1,j]
            else:
                min_id[i][j] = [i,j-1]

    mint = dp[col-1,row-1]                                      #最短路径记录在dp[col-1,row-1]中
    id = [col-1,row-1]                                          #根据最后的点坐标，从min_id中一直查找上一个点坐标直到结束
    traj.append([col-1,row-1])
    while(id!=[0,0]):                                           #逐一输出路径上的点
         id = min_id[id[0]][id[1]]
         traj.append(id)
    traj.reverse()
    return mint,traj                                            #返回最短路径长度，轨迹

def main():
    matrix = np.random.randint(0,15,(5,5))                      #建立随机矩阵matrix
    mint,traj = mintraj(matrix)
    print("最短路径长度：",mint,'\n',
          "最短路径：",traj,'\n',
          "原矩阵",matrix)

if __name__ == '__main__':
    main()
```

5.运行结果



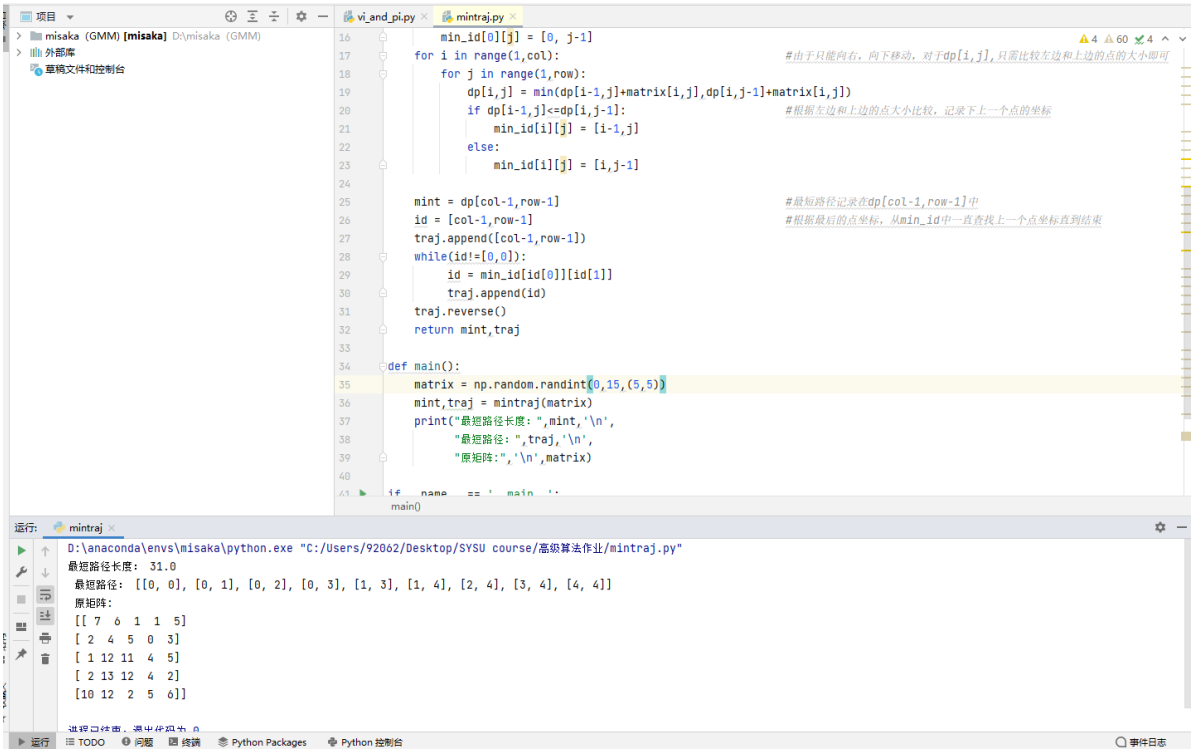
```
        min_id[0][j] = [0, j-1]
    for i in range(1,col):                          #由于只能向右，向下移动，对于dp[i,j],只需比较左边和上边的点的大小即可
        for j in range(1,row):
            dp[i,j] = min(dp[i-1,j]+matrix[i,j],dp[i,j-1]+matrix[i,j])
            if dp[i-1,j]<=dp[i,j-1]:                #根据左边和上边的点大小比较，记录下上一个点的坐标
                min_id[i][j] = [i-1,j]
            else:
                min_id[i][j] = [i,j-1]

    mint = dp[col-1,row-1]                          #最短路径记录在dp[col-1,row-1]中
    id = [col-1,row-1]                              #根据最后的点坐标，从min_id中一直查找上一个点坐标直到结束
    traj.append([col-1,row-1])
    while(id!=[0,0]):
        id = min_id[id[0]][id[1]]
        traj.append(id)
    traj.reverse()
    return mint,traj

def main():
    matrix = np.random.randint(0,15,(5,5))
    mint,traj = mintraj(matrix)
    print("最短路径长度:",mint,'\n',
          "最短路径:",traj,'\n',
          "原矩阵:",'\n',matrix)

if __name__ == '__main__':
    main()
```

```
最短路径长度:  41.0
最短路径:  [[0, 0], [1, 0], [2, 0], [2, 1], [3, 1], [3, 2], [3, 3], [4, 3], [4, 4]]
原矩阵:
[[ 0  6 11  2  0]
 [ 9 13  0 13  8]
 [ 3  1 13 13  2]
 [ 9  1  6  1 14]
 [12  1 12 11  9]]

进程已结束，退出代码为 0
```



```
        min_id[0][j] = [0, j-1]
    for i in range(1,col):                          #由于只能向右，向下移动，对于dp[i,j],只需比较左边和上边的点的大小即可
        for j in range(1,row):
            dp[i,j] = min(dp[i-1,j]+matrix[i,j],dp[i,j-1]+matrix[i,j])
            if dp[i-1,j]<=dp[i,j-1]:                #根据左边和上边的点大小比较，记录下上一个点的坐标
                min_id[i][j] = [i-1,j]
            else:
                min_id[i][j] = [i,j-1]

    mint = dp[col-1,row-1]                          #最短路径记录在dp[col-1,row-1]中
    id = [col-1,row-1]                              #根据最后的点坐标，从min_id中一直查找上一个点坐标直到结束
    traj.append([col-1,row-1])
    while(id!=[0,0]):
        id = min_id[id[0]][id[1]]
        traj.append(id)
    traj.reverse()
    return mint,traj

def main():
    matrix = np.random.randint(0,15,(5,5))
    mint,traj = mintraj(matrix)
    print("最短路径长度:",mint,'\n',
          "最短路径:",traj,'\n',
          "原矩阵:",'\n',matrix)

if __name__ == '__main__':
    main()
```

```
D:\anaconda\envs\misaka\python.exe "C:/Users/92062/Desktop/SYSU course/高级算法作业/mintraj.py"
最短路径长度:  34.0
最短路径:  [[0, 0], [1, 0], [1, 1], [2, 1], [2, 2], [3, 2], [3, 3], [4, 3], [4, 4]]
原矩阵:
[[ 3 14  7 12  9]
 [10  2  4 13  2]
 [ 4  2  2 12  3]
 [10  9  0  0  8]
 [ 1  7  7  5 10]]
```



三个用例输出结果均满足问题描述。