$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$\sigma'(z) = \frac{(-1)1}{(1+e^{-z})^2} \cdot e^{-z}(-1)$$

$$= \frac{+e^{-z}}{(1+e^{-z})^2}$$

$$\sigma(z) = \frac{1}{1+e^{-z}}$$

$$1-\sigma(z) = 1 - \frac{1}{1+e^{-z}} = \frac{1+e^{-z}-1}{1+e^{-z}} = \frac{e^{-z}}{1+e^{-z}}$$

$$\sigma'(z) = \frac{1}{1+e^{-z}} \cdot \frac{e^{-z}}{1+e^{-z}}$$

$$\sigma'(z) = \sigma(z)(1-\sigma(z))$$

Hence proved

(2)

Correct cross entropy.

$C = -[y \ln a + (1-y)\ln(1-a)$

$y \rightarrow$ true or false label

$a \rightarrow$ preticted activation

Correct cross entropy working

if $y = 0$

$C = -\ln(1-a) \Rightarrow$ This means only as long as $a < 1$. ~~cost increase~~ ~~a small~~ $\rightarrow$ a large $\rightarrow$ penalty

if $y = 1$

$C = -\ln a \Rightarrow$ This mean only as long as $a > 1$, ~~[illegible]~~ a small $\rightarrow$ penalty

wrong form

$C = -\left[a \ln y + (1-a)\ln(1-y)\right]$

if $y = 1$

$\ln(1-y) \rightarrow -\infty \quad C \rightarrow +\infty$

if $y = 1$

$\ln(y) \rightarrow -\infty \quad C \rightarrow +\infty$

and $\ln(0)$ is undefined

so It does'nt make sense.

(3)

$C = -\frac{1}{n}$

$C = -\left[y \ln a + (1-y) \ln(1-a) \right]$

↳ for one neuron

we have to find when cost w.r.t to predicted output is minimum.

in the sense $\frac{\partial C}{\partial a} = 0$

$0 = \frac{\partial}{\partial a} - \left[y \ln a + (1-y) \ln(1-a) \right]$

$= -\left[\frac{y}{a} + (-1) \frac{(1-y)}{1-a} \right] = -\left[\frac{y}{a} - \frac{1-y}{1-a} \right]$

$\frac{y}{a} = \frac{1-y}{1-a}$ $\quad y - y/a = a - y/a$

$\boxed{y = a}$

at $\frac{\partial C}{\partial a} = 0$ $\boxed{y = a}$

Hence cost is minimized when predicted equals actual

also $y = a$ mean

$C = -\frac{1}{n} \sum_{n} y \ln y + (1-y) \ln(1-y)]$.

⇓

This also gives its minimum when the a = y. and its Maximum when a is far from y

whole point is to find its Max and Min