To prove

$$\frac{\partial C}{\partial b_j^l} = \delta_j^l$$

$$\frac{\partial C}{\partial b_j^l} = \frac{\partial C}{\partial z_j^l} \times \frac{\partial z_j^l}{\partial b_j^l}$$

$$\because \frac{\partial C}{\partial z_j^l} = \delta_j^l$$

$$\frac{\partial C}{\partial z_j^l} \frac{\partial z_j^l}{\partial b_j^l} = \delta_j^l$$

if This true $\delta_j^l \times \frac{\partial z_j^l}{\partial b_j^l} = \delta_j^l$

This must be true $\frac{\partial z_j^l}{\partial b_j^l} = 1$ $\quad z_j^l = w_{jk}^l a_k^{l-1} + b_j^l$

$$\frac{\partial(\sum w_{jk}^l a_k^{l-1} + b_j^l)}{\partial b_j^l} = \frac{\partial(\sum w_{jk}^l a_k^{l-1})}{\partial b_j^l} + \frac{\partial b_j^l}{\partial b_j^l}$$

$$= 0 + 1 = 1$$

~~Hence~~ Hence proved.

$$\boxed{\frac{\partial C}{\partial b_j^l} = \delta_j^l}$$

(4) $$\frac{\partial C}{\partial w^l_{jk}} = a^{l-1}_k \delta^l_j$$

$$\frac{\partial C}{\partial z^l_j} \frac{\partial z^l_j}{\partial w^l_{jk}} = a^{l-1}_k \delta^l_j \qquad \frac{\partial C}{\partial z^l_j} = \delta^l_j$$

so To prove this we have to prove that

$$\frac{\partial z^l_j}{\partial w^l_{jk}} = a^{l-1}_k$$

$$\frac{\partial (w^l_{jk} \cdot a^{l-1}_k + b^l_j)}{\partial w^l_{jk}}$$

$$= \frac{\partial (w^l_{jk} a^{l-1}_k)}{\partial w^l_{jk}} + \frac{\partial (b^l_j)}{\partial w^l_{jk}}$$

$$= a^{l-1}_k + 0 = a^{l-1}_k$$

Hence proved.

28 December 2025 at 9:34 pm

(3)

BP1 $\Rightarrow$ $\frac{\partial C}{\partial z_j^l} = \delta_j^l$

$$\delta_j^L = \frac{\partial C}{\partial a_j^L} \cdot \frac{\partial a_j^L}{\partial z_j^L}$$

$$= \frac{\partial C}{\partial a_j^L} \frac{\partial (f(z_j^L))}{\partial L_j^L}$$

$$\boxed{\delta_j^L = \frac{\partial C}{\partial a_j^L} \cdot f'(z_j^L)}$$

Ily

BP2 $\Rightarrow$

$$\delta^l = ((\omega^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l)$$

BP3 $\Rightarrow$

$$\frac{\partial C}{\partial b_j^l} = \frac{\partial C}{\partial z_j^l} \cdot \frac{\partial z_j^l}{\partial b_j^l}$$

$$= \frac{\partial C}{\partial z_j^l} = \delta_j^l \;//$$

BP4 $\Rightarrow$

$$\frac{\partial C}{\partial \omega_{jk}^l} = \frac{\partial C}{\partial z_j^l} \frac{\partial z_j^l}{\partial \omega_{jk}^l}$$

$$\frac{\partial C}{\partial \omega_{jk}^l} = \delta_j^l \frac{\partial (\omega_{jk}^l a_k^{l-1} + b_j^l)}{\partial \omega_{jk}^l} = \delta_j^l \cdot a_k^{l-1} \;//$$

28 December 2025 at 10:35 pm

(3)

BP1 $\Rightarrow$ $\frac{\partial C}{\partial z^l_j} = \delta^l_j$

$$\partial^L_j = \frac{\partial C}{\partial a^L_j} \cdot \frac{\partial a^L_j}{\partial z^L_j}$$

$$= \frac{\partial C}{\partial a^L_j} \frac{\partial (f(z^L_j))}{\partial L^L_j}$$

$$\boxed{\delta^2_j = \frac{\partial C}{\partial a^L_j} \cdot f'(z^L_j)}$$

Ily

BP2 $\Rightarrow$

$$\delta^l = ((w^{l+1})^T \delta^{l+1}) \odot \sigma'(z^l)$$

BP3 $\Rightarrow$

$$\frac{\partial C}{\partial b^l_j} = \frac{\partial C}{\partial z^l_j} \cdot \frac{\partial z^l_j}{\partial b^l_j}$$

$$= \frac{\partial C}{\partial z^l_j} = \delta^l_j$$ //

BP4 $\Rightarrow$

$$\frac{\partial C}{\partial w^l_{jk}} = \frac{\partial C}{\partial z^l_j} \frac{\partial z^l_j}{\partial w^l_{jk}}$$

$$\frac{\partial C}{\partial w^l_{jk}} = \delta^l_j \frac{\partial (w^l_{jk} a^{l-1}_k + b^l_j)}{\partial w^l_{jk}} = \delta^l_j \cdot a^{l-1}_k$$ //

28 December 20

(4) if $\sigma(z) = z \Rightarrow f(z) = z$

$BP_1 \Rightarrow$

$$\delta_j^L = \frac{\partial C}{\partial a_j^L} f'(z_j^L)$$

$$\delta_j^L = \frac{\partial C}{\partial a_j^L} \cdot \frac{\partial f(z)}{\partial z}$$

$$= \frac{\partial C}{\partial a} \cdot \frac{\partial z}{\partial z} = \frac{\partial C}{\partial a} \cdot 1$$

$$\boxed{\delta_j^L = \frac{\partial C}{\partial a_j^L}}$$

$BP_2 \Rightarrow$

$$\delta_j^l = \frac{\partial C}{\partial z_j^l} = \sum_k \frac{\partial C}{\partial z_k^{l+1}} \frac{\partial z_k^{l+1}}{\partial z_j^l}$$

from previous dan

$$\delta_j^l = \sum_k \left(w_{kj}^{l+1} \frac{\partial(\sigma(z_j^l))}{\partial z_j^l} \right) \cdot \delta_k^{l+1}$$

$$= \sum_k \delta_k^{l+1} w_{kj}^{l+1} \frac{\partial(z_j^l)}{\partial(z_j^l)}$$

$$= \sum_k \delta_k^{l+1} w_{kj}^{l+1} \cdot 1$$

$\partial \sigma$

$$\frac{\partial(\sigma(z))}{\partial(z)} = \sigma'(z)$$ gives the

senitivity of the neuron
so we can know that if it is near the
predicted output.
so the error value does'nt know about the
actual ~~gradient~~ gradient (from slope on σ)

BP_3 =>

$$\frac{\partial C}{\partial b_j^l} = \delta_j^l$$

BP_4 =>

$$\frac{\partial C}{\partial w_{jk}^l} = a_k^{l-1} \delta_j^l$$

In BP_3 and BP_4 the ~~value~~ formula doesn't
change but the actual value is
calculated from BP_1 and BP_2 so It also
affected in the value