Ch-4 Testing of Hypothesis :-

* In many cases we are to take decisions about the population on the basis of sample info i.e we estimate parameter values from the sample data however there are many problems in which rather than estimating the value of Parameter we need to decide whether to accept or reject a statement about the Parameter this statement is called a hypothesis and decision making Process about the hypothesis is called testing of hypothesis

Statistical hypothesis :- It is a statement about the population Parameters.

For eg :- ① Population mean is estimated from the sample mean

② The teaching methods in both the schools are effective

③ The new drug is really effective in curing a desease

④ Hypothesis may be classified into 2 types

* Null hypothesis
* Alternative hypothesis

* Null hypothesis :- It is the statement which asset that there is no significant difference b/w the statistic and Parameter

It is denoted by 'Ho'.

$$H_0 = \mu = \mu_0$$

* Alternative hypothesis :- The hypothesis which contradicts the null hypothesis is called Alternative hypothesis. It is denoted by H_1 (or) H_a

$$H_1 = \mu \neq \mu_0$$ [two tailed alternative H]

$H_1: \mu > \mu_0$ [Right " "]

(or)

$H_1: \mu < \mu_0$ [Left " "]

Errors in testing of hypothesis: The main objective in sampling theory is to draw a valid decisions/conclusions about the population parameter on the basis of sample results in such a way we have two types of errors in testing our hypothesis

decision / statement	Accept H_0	Reject H_0
H_0 true	correct decision	wrong decisions → Type I error
H_0 false	wrong decision ↓ Type II error	correct decisions

Type I error: If the null hypothesis is H_0 is true but is rejected by test procedure then the error is called type I error

Type 2 error: If the null hypothesis is false but it is accepted by the test then the error is called type 2 error

Level of significance: The level of significance is denoted by α is the confidence with which we reject or accept the null hypothesis.

→ level of significance is generally specified before the test procedure in practice we take either 5% or 1% level of significance

Critical region:- It is formed based on the form of an alternative hypothesis. If alternative hypothesis has '≠' then the critical region is divided into equally in the left and right tails of distribution. * If the form of alternative hypothesis has less than sign then the critical region is taken in the left tail. * If the form of alternative hypothesis is greater than sign then the c.r taken on right tail

→ 1 tailed & 2 tailed tests:-

2 tailed tests:- If the alternative hypothesis H_1 in a test of statistical hypothesis be two tailed (is not equal to form) then the test is called two tailed test

1 tailed tests:- If the alternative hypothesis H_1 in a test of statistical hypothesis be 1 tailed then the test is called 1 tailed test for $\mu > \mu_0$ is called right tailed test. & for $\mu < \mu_0$ is called left tailed tests

Procedure for testing of hypothesis:-

(1) Set up the null hypothesis by taking into consideration of the nature of the problem and data involved

(2) Set up the alternative hypothesis so that we could decide whether we can use 1 tailed or 2 tailed test

(3) Select the appropriate level of significance 'α' usually we can choose 5% or 1% level of significance

(3) Calculate the test statistic $|z| = \frac{t - E(t)}{S.E(t)}$

Later we compare the calculated value of 'z' with table value of 'z' at los level of significance then we conclude as follows if calculated value of 'z' is less than table value of 'z' then we accept our null hypothesis otherwise we reject the null hypothesis

The critical value / table value of z for both two tailed & one tailed are given in the following table

α	1%	5%	10%
Two tailed	2.58	1.96	1.645
Right tailed	2.33	1.645	1.28
Left tailed	-2.33	-1.645	-1.28

Test of significance of single mean for large samples

Let $x_1, x_2 ------ x_n$ be the samples of size n drawn from the population with mean μ & variance 'σ^2' now we set the null hypothesis H_0 as there is no significance difference b/w population mean & sample mean

$H_0 - \mu = \mu_0$

$H_1 - \mu \neq \mu_0$ [Two tailed]

(or)

$\mu > \mu_0$ (Right)

(or)

$\mu < \mu_0$ [Left]

→ To test the above null hypothesis the test statistic is given by $|z| = \frac{\bar{x}-\mu}{\sigma/\sqrt{n}}$

Now if the table value of z is greater than the cal value of z then we accept our ho otherwise we reject the null hypothesis

• Test of significance of difference of two means :-

→ Let $x_1, x_2 - - - x_{n_1}$ be the samples of size n_1 drawn from the population with mean μ_1 and variance σ_1^2 & ~~let y~~

→ Let $y_1, y_2 - - - y_{n_2}$ be the samples of size n_2 drawn from the population with mean μ_2 & variance σ_2^2

Now we setup the null hypothesis as $H_0 - \mu_1 = \mu_2$ i.e $\mu_1 - \mu_2 = 0$

$H_1 - \mu_1 \neq \mu_2$ [Two tailed]
(or)

$\mu_1 > \mu_2$ [Right tailed]

(or)

$\mu_1 < \mu_2$ [left]

To test the above null hypothesis the test statistic is given by $|z| = \frac{\bar{x}-\bar{y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$

→ If cal value of z is less than table value of z then we accept our null hypothesis Ho otherwise we reject it

Prob

1. A sample of 64 students have a mean weight of 70 kgs can this be regarded as a sample from a population with mean weight 66 kgs & standard deviation 25 kgs

Sol

$n = 64$

$\bar{x} = 70$

$\sigma = 25$

$H_0 = \mu = 66$

$H_1 = \mu \neq 66$ (Two tailed test)

under H_0 the test statistic is given by

$$|z| = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$\cancel{H \vdash \mu = 66}$

$\cancel{H_1 = \mu \neq 66 \text{ (Two t}}$ $\quad \frac{70-66}{(25/\sqrt{64})} = 1.28$

The table value of z for two tailed test at 5% level of significance is 1.96

Table value of z is greater than cal value of z, so we accept our null hypothesis i.e; $\mu = 66$ or the samples are taken from the population with mean $\mu = 66$

2. The mean lifetime of sample of 100 light [illegible] a company is found to be 1560 hrs with a population standard deviation of 90hrs. Test the hypothesis at 1% level that the mean lifetime of the tubes produced by the company is 1580 hrs

Sol

Given $n = 100$

$\bar{x} = 1560$

$\sigma = 90$

$H_0 - \mu = 1580$

$H_1 - \mu \neq 1580$ (two tailed)

$\alpha = 1\%$

Under the H_0 $|z| = \dfrac{\bar{x} - \mu}{(\sigma/\sqrt{n})} = \dfrac{1560 - 1580}{(90/\sqrt{100})}$

$= 2.2222$

the table value of z for two tailed at 1% level of significance is 2.58

$\therefore$ Cal value of z is less than table value of z so we accept our null hypothesis

. In a random sample of 60 workers the avg time taken by them to get to work is 33.8 min with a S.D of 6.1 min can we test the hypothesis $\mu = 32.6$ min in favour of alternative hypothesis μ is greater than 32.6 at 5% level of significance

Sol

$n = 60$

$\bar{x} = 33.8$

$\sigma = 6.1$

$H_0 : \mu = 32.6$

$H_1 : \mu > 32.6$ (right tailed test)

$\alpha = 5\%$

$$z = \frac{\bar{x} - \mu}{(\sigma / \sqrt{n})} = \frac{33.8 - 32.6}{(6.1 / \sqrt{60})} = 1.5238$$

for right tailed test the table value of z at 5% level of significance is 1.645

∴ As the table value of z is '>' cal value of z so we accept the null hypothesis

4. An ambulance service claims that it takes on the avg less than 10min to reach its destination in emergency calls a sample of 36 calls has a mean of 11 min and the variance of 16 min test the claim at 10% level of significance

Sol

$n = 36$

$\bar{x} = 11$

$\sigma^2 = 16 \Rightarrow \sigma = 4$

$\alpha = 10\%$

$H_0 : \mu = 10$

$H_1 : \mu < 10$

$$z = \frac{\bar{x} - \mu}{(\sigma / \sqrt{n})} = \frac{11 - 10}{\frac{4}{\sqrt{36}}} = 1.5$$

Table value for left tailed z at 10% is -1.28

$\therefore$ The table value of z is less than cal value of z. we reject the null hypothesis

5. The research investigator is intrested in studying whether there is a significant difference in the salaries of MBA grades in 2 metropolitan cities a random sample of size 100 from Mumbai yeilds on avg income of 20,150 rs another random sample of 60 from Chennai results in an avg income of 20250 If the variance of both the population are given as 40000 & 32400 respectively. Test at 5% level of significance.

sol

h_0 = there is no significant difference in the salaries of MBA two cities

h_1 = there is a significant diff in the salaries of MBA grades in two cities

$n_1 = 100$

$n_2 = 60$

$\bar{x} = 20,150$

$\bar{y} = 20,250$

$\sigma_1^2 = 40000 \Rightarrow \sigma_1 = 200$

$\sigma_2^2 = 32400 \Rightarrow \sigma_2 = 180$

To test above h_0 the test statistic is given by (two tailed test)

$$|z| = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{20150 - 20250}{\sqrt{\frac{40000}{100} + \frac{32400}{60}}} = \cancel{0}3.2616$$

∴ The table value of z for two tailed test at 5% level of significance is 1.96

∴ Cal value z is more than table value of z so we reject our null hypothesis

6. The avg marks scored by 32 boys is 72 with S.D of 8 while that for 36 girls is 70 with a S.D of 6 thus this indicates that the boys perform better than the girls at 5% level

Sol H_0 = there is no significant diff in the performance of boys & girls i.e $\mu_1 = \mu_2$

H_1 = The boys performs better than the girls

$\mu_1 > \mu_2$ (right tailed test)

$n_1 = 32$, $n_2 = 36$

$\bar{x} = 72$, $\bar{y} = 70$

$\sigma_1 = 8$, $\sigma_2 = 6$

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{72 - 70}{\sqrt{\frac{64}{32} + \frac{36}{36}}} = 1.1547$$

∴ ~~The~~ For right tailed test 'z' at 5% level of significance is 1.645

∴ Cal value 'z' is ~~more~~ less than table value of z so we accept our H_0

7. A simple sample of the heights of 6400 ... has a mean of 67.85 inches and a S.D of 2.56 Inches while a simple sample of heights of 1600 australians has a mean of 68.55 inches and S.D of 2.52. Do the data indicate the australians are on the avg taller than the avg man use 1% level of significance

Sol

H_0: there is no significant difference on the avg heights of english men and australians i.e $\mu_1 = \mu_2$

H_1: Australians are on the avg taller than the avg men i.e $\mu_1 < \mu_2$ (left tailed test)

$n_1 = 6400$, $n_2 = 1600$

$\bar{x} = 67.85$, $\bar{y} = 68.55$

$\sigma_1 = 2.56$, $\sigma_2 = 2.52$

$$z = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{67.85 - 68.55}{\sqrt{\frac{(2.56)^2}{6400} + \frac{(2.52)^2}{1600}}} \doteq -9.9064$$

∴ For left tailed test 'z' at 1% level significance is -2.33

∴ Cal value of z is ~~more~~ less than table value z so we accept the null hypothesis

The means of two large samples of sizes 1000 & 2000 members are 67.5 inches and 68 inches respectively. Can the samples be regarded as drawn from the same population of S.D 2.5 inches

$H_0 = \mu_1 = \mu_2$

$H_1 = \mu_1 \neq \mu_2$

$\sigma_1 = \sigma_2 = 2.5$

$n_1 = 1000$, $n_2 = 2000$

$\bar{x} = 67.5$, $\bar{y} = 68$

$$|z| = \frac{\bar{x} - \bar{y}}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{67.5 - 68}{\sqrt{\frac{(2.5)^2}{1000} + \frac{(2.5)^2}{1000}}} = 5.1640$$

∴ At 5% level of significance the table value is 1.96

∴ As cal value is more than table value of z we reject the null hypothesis

9. An insurance agent has claimed that the avg age of Policy holder who issue through him is less than the avg for all agents which is 30.5 years. A random sample of 100 Policy holders who had issued through him gave the following age distribution. Test his claim at 5% level of significance

Age	16-20	21-25	26-30	31-35	36-40
No. of persons	12	22	20	30	16

Sol $H_0 = \mu = 30.5$

$H_1 = \mu < 30.5$ (left tailed test)

$n = 100$

class	15.5 - 20.5	20.5 - 25.5	25.5 - 30.5	30.5 - 35.5	35.5 - 40.5
f	12	22	20	30	16
m	18	23	A 28	33	38

$d = \frac{x-A}{c}$ = -2 -1 0 1 2

d^2 = 4 1 0 1 4

fd = -56 -24 0 30 22 = 16

fd^2 = 64 22 0 30 20 = 136

Mean $\bar{x} = \left(\frac{\sum fd}{N}\right) \times c + A = 23.1$

$$S.D. = \sqrt{\frac{\sum fd^2}{N} - \left(\frac{\sum fd}{N}\right)^2} \times c$$

$$= \sqrt{\frac{136}{100} - \left(\frac{12}{100}\right)^2} \times 5$$

$= 6.3530$

$n = 100$

$$z = \frac{\bar{x} - \mu}{(\sigma/\sqrt{n})} = \frac{23.1 - 30.5}{\left(\frac{6.3530}{\sqrt{100}}\right)} = -2.6759$$

∴ 5% level of significance the table value is -1.645

As the calculated value is less than the table value of z we accept the null hypothesis

10. The mean yield of wheat from a district A was 210 pounds with standard deviation 10 pounds per acre from a sample of 100 plots. In another district the mean yield was 220 pounds with a standard deviation 12 from a sample of 150 plots. Assuming that the SD of yield in the entire state was 11 pounds test whether the diff. b/w the mean yield crops in 2 districts is 11 pounds

Sol $H_0: \mu_1 - \mu_2 = \delta$ (δ)

$H_1: \mu_1 - \mu_2 \neq \delta$

$n_1 = 100$, $n_2 = 150$

$\bar{x} = 210$, $\bar{y} = 220$

$\sigma_1 = 10$, $\sigma_2 = 12$

$$Z = \frac{\bar{x} - \bar{y} - \delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{210 - 220 - 13}{\sqrt{\frac{(10)^2}{100} + \frac{(12)^2}{150}}} = 16.42$$

reject H_0

Test of significance for single proportion

let x be an event of n sample units suppose a large sample of size n has a sample proportion 'p' = $\frac{x}{n}$.

To test the hypothesis that the population proportion 'P' has a specified value P_0 we setup the null hypothesis as

H_1: $P \neq P_0$ (Two tailed test)

$P > P_0$ (Right tailed test)

$P < P_0$ (Left " ")

where $Q = 1 - P$

under the H_0, Test statistic is,

$$|Z| = \frac{p - P}{\sqrt{\frac{PQ}{n}}}$$

here p is sample proportion, P = Population proportion

If cal value of z is less than the z table value of z at ... level of significance then we accept our null hypothesis otherwise we reject the hypothesis

Test of significance of diff of two sample proportions -

let p_1 & p_2 be the sample proportions in two larger samples of sizes n_1 & n_2 drawn from the two populations having the proportions P_1 & P_2 to test whether the two samples have been drawn from the same population is setup the null hypothesis as

$H_0 : P_1 = P_2$

$H_1 : P_1 \neq P_2$ (Two tailed test)

$P_1 > P_2$ (Right " ")

$P_1 < P_2$ (left " ")

$Q_1 = 1 - P_1$

$Q_2 = 1 - P_2$

Under H_0,

$$|z| = \frac{p_1 - p_2}{\sqrt{PQ\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

where $P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2}$

$Q = 1 - P$

$p_1 = \frac{x_1}{n_1}$; $p_2 = \frac{x_2}{n_2}$

If cal value of z is less than table value of z then we accept our null hypothesis at α% level of significance

7. A manufacturer claimed that atleast 95% of the equipment which he supply to a factory conformed to specification an examination of a sample of 200 pieces of equipment revealed that 18 were in bad condition. Test his claim at 5% level of significance

Sol

$H_0 = P = 0.95$

$H_1 = P < 0.95$ (left tailed test)

Given $n = 200$ no of equipments in bad conditions are 18

no of pieces (of ~~equipment~~) conforming to specification $= 200 - 18$
$= 182$

$x = 182$

$p =$ Proportion of specification of equipments $= \frac{x}{n} = \frac{182}{200} = 0.91$

$Q = 1 - P = 1 - 0.95$
$= 0.05$

$\alpha = 5\%$ level of significance

under H_0 $\quad Z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.91 - 0.95}{\sqrt{\frac{0.95(0.05)}{200}}} = -2.5955$

for left tailed test the table value of Z at 5% level of significance is $= -1.645$

As the cal value is less than table value we accept the null hypothesis

2. A die was thrown 9000 times at end of these 3220 ... 3 or 4 is this consistent with the hypothesis that the die was unbiased

sol

H_0 = The die is unbiased

H_1 = The die is biased (two tailed)

P = Population proportion of getting 3 or 4 = $\frac{1}{6}+\frac{1}{6}$

$Q = 1-P = 1-\frac{1}{3} = \frac{2}{3}$

$x = 3220 \quad n = 9000$

$p = \frac{x}{n} = \frac{3220}{9000} = 0.3578$

$\alpha = 1\%$

$$|z| = \frac{p-P}{\sqrt{\frac{PQ}{n}}} = \frac{0.3578 - 1/3}{\sqrt{\frac{1/3(2/3)}{9000}}} = 4.9233$$

As 1% level of significance the table value of two tailed test is 2.58

As the cal value of z is more than table value we reject the null hypothesis

3. 20 people were attacked by a disease only 18 were survived will u reject the hypothesis that the survival rate of attacked by the disease is 85% in favour of hypothesis is that is more at 5% level

signi...

$H_0 = P = 85\% = 0.85$

$H_1 = P > .85$ (right tailed)

$n = 20$

$x = 18$ $\qquad Q = 1 - P = 0.15$

$p = \frac{18}{20} = 0.9$

$\alpha = 5\%$

$$z = \frac{p - P}{\sqrt{\frac{PQ}{n}}} = \frac{0.9 - 0.85}{\sqrt{\frac{0.85(0.15)}{20}}} = 0.6262$$

At 5% level of significant table value of z is 1.645

∴ As the cal value is less than the table value of z we accept null hypothesis

2. A manufacturer of electric equipments subjects samples of two completing brands of transistors to an accelerated performance test if 45 of 120 transistors of the 1st kind and 34 of 120 of the 2nd kind fail the test what can conclude that at 5% level about the diff b/w the corresponding sample proportion.

Sol H_0: There is no diff b/w two proportion

$P_1 = P_2$ $\qquad Q = 1 - P = 1 - 0.2633 = 0.7367$

$n_1 = 120$ $\qquad p_1 = \frac{x_1}{n_1} = \frac{45}{180} = 0.25$

$n_2 = 120$

$H_1 = P_1 \neq P_2$

$x_2 = 34$ $\qquad p_2 = \frac{x_2}{n_2} = \frac{34}{120} = 0.2833$

$x_1 = 45$

$\alpha = 5\%$ $\qquad P = \frac{n_1 p_1 + n_2 p_2}{n_1 + n_2} = \frac{180\ 45 + 39.9960}{180 + 120} = 0.2633$

$$|Z| = \frac{P_1 - P_2}{\sqrt{PQ\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{0.25 - 0.2833}{\sqrt{0.2633 \times 0.7367\left(\frac{1}{120}+\frac{1}{120}\right)}} = 0.6616$$

At 5% level of significance the table value of Z at two tailed test is 1.96

As the cal value is less than the table value so we accept the null hypothesis

Test of significance of single mean in case of small samples - the study of test of significance for small samples we can use T-test :

Let A random sample of size n has a sample mean $\bar{x}$ to the the population mean has a specified value μ_0 consider the H

$H_0 = \mu = \mu_0$

$H_1 = \mu \neq \mu_0$ (Two tailed test)

(or)

$\mu > \mu_0$ (Right " ")

(or)

$\mu < \mu_0$ (Left tailed ")

Under H_0,

$$|t| = \frac{\bar{x} - \mu}{S/\sqrt{n-1}} \quad \text{where } S = \sqrt{\frac{\Sigma(x_i - \bar{x})^2}{n-1}}$$

Now compare this cal value of t with table value of t α% level of significance with n-1 degrees of freedom and conclude as. if the cal value of t is less than is less than table value of t then we accept our null hypothesis

Note: For two tailed test at α level of significance the value of $\frac{\alpha}{2}$ is taken for α

Test of significance for diff of two samples in case of small samples :

Let $\bar{x}, \bar{y}$ be the means of samples of sizes n_1 & n_2 drawn from two population having the means μ_1 & μ_2 to test the significance of diff of two means null hypothesis can be stated as

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

(or)

$\mu_1 > \mu_2$

(or)

$\mu_1 < \mu_2$

under H_0,

$$|t| = \frac{\bar{x}-\bar{y}}{s\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \quad \text{where } s = \sqrt{\frac{\Sigma(x_i-\bar{x})^2 + \Sigma(y_i-\bar{y})^2}{n_1+n_2-2}}$$

(or)

$$= \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}}$$

If cal value of t is less than table value of t at α% level of significance with n_1+n_2-2 degrees of freedom then we accept our null hypothesis

Prob

1\. A sample of 26 bulbs given a mean life of 990 hrs with S.D of 20 hrs the manufactures claims that the mean life of bulbs is 1000 hrs is the sample not upto standard.

Sol

$H_0 = \mu = 1000$

$H_1 = \mu < 1000$ (one tailed test)

Given $n = 26$, $\bar{x} = 990$

$s = 20$,

Under H_0, $|t| = \dfrac{\bar{x} - \mu}{s/\sqrt{n-1}}$

$$\frac{990 - 1000}{20/\sqrt{26-1}} = 2.5$$

Let $\alpha = 5\%$ for one tailed test $\dfrac{\alpha}{2} = \dfrac{0.05}{2}$ $= 0.025$

now the table value of t at 0.025 level of significance with $n - 1 = 25$ degrees of freedom is 1.708

The cal value of t is greater than the table value of t so we reject our null hypothesis

2\. A random sample from a companies very extensive files shows that the orders for a certain kind of missionary were filled respectively in 10, 12, 19, 14, 15, 18, 11, 13 days use 1% level of significance to test the claim that on the such orders are filled 10.5 days choose the alternative hypothesis so that the rejection of null hypothesis $\mu = 10.5$ days implies th is ... longer than indicated

$H_0: \mu = 10.5$

$H_1: \mu \neq 10.5$ (two tailed test)

$n = 8$

$\bar{x} = \frac{112}{8} = 14$

$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}} = \sqrt{\frac{(10-14)^2 + (12-14)^2 + (16-14)^2 + (14-14)^2 + \ldots + (11-14)^2 + (13-14)^2}{7}}$

$s = \sqrt{\frac{72}{7}} = 3.2071$

$t = \frac{\bar{x}-\mu}{s/\sqrt{n-1}} = \frac{14-10.5}{3.2071/\sqrt{7}} = 2.8876$

For one tailed test $\alpha = 0.05$ the table value of t at 5% level of significance with $n-1 = 7$ degrees is 2.365

$\therefore$ Cal value of t is less than table value of t so we accept our null hypothesis

2. A machine is designed to produce insulating washers for electrical devices of avg thickness of 0.025cm a random sample of 10 washers was found to be a thickness of 0.024cm with a S.D of 0.002 cm test the significance at 5% level

$H_0: \mu = 0.025$

$H_1: \mu \neq 0.025$ (Two tailed test)

$n = 10$

$\bar{x} = 0.024$

$s = 0.002$

$$|t| = \frac{\bar{x} - \mu}{s / \sqrt{n-1}}$$

$$= \frac{0.024 - 0.025}{0.002 / \sqrt{9}} = 1.5$$

$\alpha = 5\% = 0.05 \quad \frac{\alpha}{2} = \frac{0.05}{2} = 0.025$

for two tailed test we consider $\frac{\alpha}{2}$

∴ The table value of t at 0.025 level of significance with $n-1 = 9$ degrees of freedom 2.262

Cal value of t is less than table value of t so we accept our null hypothesis

Prob:

1. Two independent samples of 8 & 7 items respectively have the following values. is the diff b/w the means of two samples is significant

Sample 1: 11 11 13 11 15 9 12 14

Sample 2: 9 11 10 13 9 8 10

Sol: H_0: There is no significant diff b/w the means of two samples

H_1: There is a significant different b/w the means of two samples

Let x = Sample 1

y = Sample 2

$n_1 = 8, \; n_2 = 7$

$$\bar{x} = \frac{11+11+13+11+15+9+12+14}{8} = 12$$

$\bar{y} = 10$

$(x-\bar{x})^2 = 1 \quad 1 \quad 1 \quad 1 \quad 9 \quad 9 \quad 0 \quad 4 \quad = 26$

$(y-\bar{y})^2 = 1 \quad 1 \quad 0 \quad 9 \quad 1 \quad 4 \quad 0 \quad = 16$

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1+n_2-2}} = \sqrt{\frac{26+16}{8+7-2}} = 1.7974$$

$$|t| = \frac{\bar{x}-\bar{y}}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{12-10}{1.7974\sqrt{\frac{1}{8}+\frac{1}{7}}} = 2.15$$

for two tailed test we can consider $\frac{\alpha}{2}$ = $\alpha = 5\%$ = $\frac{0.05}{2} = 0.025$

∴ The table value of t at 0.025 level of significance with n_1+n_2-2 13 degrees of freedom 2.160

∴ Cal value of t is less than table value of t so we accept our null hypothesis

2. 10 soldiers participated in a shooting competition in 1st week after intensive training they participated in the competetion in the 2nd week follows. Do the data indicate that the soldiers have been benefited by the training

Scores before: 67 24 57 55 63 54 56 68 33 43

Scores after: 70 38 58 58 56 67 68 75 42 38

Sol

H_0 = There is no benefit of training $\mu_1 = \mu_2$

H_1 = There is a benefit of training $\mu_1 \neq \mu_2$

$\bar{x} = 52$, $\bar{y} = 57$

$(x-\bar{x})^2 = 225 \quad 784 \quad 25 \quad 9 \quad 121 \quad 4 \quad 16 \quad 256 \quad 361 \quad 81 = 1882$

$(y-\bar{y})^2 = 169 \quad 361 \quad 1 \quad 1 \quad 1 \quad 100 \quad 121 \quad 324 \quad 225 \quad 361 = 16$

$$S = \sqrt{\frac{\Sigma(x-\bar{x})^2 + \Sigma(y-\bar{y})^2}{n_1+n_2-2}} = \sqrt{\frac{1882+1664}{18}} = 14.0357$$

$$|t| = \frac{\bar{x}-\bar{y}}{S\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} = \frac{52-57}{14.0357\sqrt{\frac{1}{10}+\frac{1}{10}}} = 0.7966$$

~~The table value of t at~~

for one tailed test $\alpha = 5\%$

∴ The table value of t at 0.05 level of significance with 18 degrees of freedom 1.734

∴ Cal value of t is less than table value of t we accept our null hypothesis

3. The intelligence quotients of 16 students from one area of city show a mean of 107 with a S.D of 10 while the IQ's of 14 students from another area of city shows a mean of 112 with a S.D of 8 is there a significance diff b/w the IQ's of 2 groups at 1% level of significance

Sol H_0: There is no significant diff b/w IQ's of two groups

H_1: There is a significant diff b/w IQ's of two groups

given that $n_1 = 16$, $n_2 = 14$

$\bar{x} = 107$, $\bar{y} = 112$

$s_1 = 10$, $s_2 = 8$

$$S = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{1600 + 896}{16+14-2}} = 9.4415$$

$$|t| = \frac{\bar{x} - \bar{y}}{S\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{107 - 112}{9.4415\sqrt{\frac{1}{16} + \frac{1}{14}}} = 1.4471$$

for two tailed test α can be considered as $\frac{\alpha}{2} = \frac{0.01}{2} = 0.005$

∴ table value of t at 0.005 level of significance with $n_1 + n_2 - 2 =$ 28 degrees of freedom 2.763

∴ Cal value is less than table value of t so we accept our null hypothesis