state that given proof is true by principle of mathematical induction.

Example 1.3.1 *Prove by induction on n that* $\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$.

AU : May-12, Marks 6

Solution : Initially,

1) Basis of induction -

Assume, $n = 1$. Then,

$$\text{L.H.S} = n, = 1$$

$$\text{R.H.S} = \frac{n(n+1)}{2} = \frac{1(1+1)}{2} = \frac{2}{2}, = 1$$

Now,

2) Induction hypothesis -

Now we will assume n = k and will obtain the result for it. The equation then becomes,

$$1 + 2 + 3 + \ldots + k = \frac{k(k+1)}{2} \qquad \ldots (1)$$

3) Inductive step -

Now we assume that equation is true for n = k. And we will then check if it is also true for n = k + 1 or not.

Consider the equation assuming n = k + 1

$$\text{L.H.S.} = \underbrace{1 + 2 + 3 \ldots + k} + k + 1 \qquad \because \text{equation (1)}$$

$$\frac{k(k+1)}{2} + k + 1$$

$$= \frac{k(k+1) + 2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2} \qquad \because \text{taking common factor}$$

$$\text{i.e.} = \frac{(k+1)(k+1+1)}{2}$$

$$= \text{R.H.S.}$$

Example 1.3.2 *Prove :* $n! >= 2^{n-1}$

Solution : Consider,

Example 1.3.3 *Prove that* $1^2 + 2^2 + 3^2 + ... + n^2 = \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ *using mathematical induction.*

AU : May-07, Marks 6, May-14, Marks 8

Solution : We will first prove it by basis of induction and then will consider induction hypothesis.

1. Basis of induction -

$$\text{Let } n = 1$$

$$\text{L.H.S.} = 1^2 = 1$$

$$\text{R.H.S.} = \frac{1(1+1)(2.1+1)}{6} = \frac{6}{6}$$

$$\text{R.H.S.} = 1$$

As L.H.S. = R.H.S it is proved for n = 1.

2. Induction hypothesis - Let, n = k.

Then

$$1^2 + 2^2 + ... + k^2 = \sum_{i=1}^{k} i^2 = \frac{k(k+1)(2k+1)}{6} \text{ is true} \quad ...(1)$$

Now, we will try to prove it for n = k + 1. Hence we will substitute n by k + 1.

$$\therefore \quad 1^2 + 2^2 + 3^2 + ... + (k+1)^2 = \sum_{i=1}^{K+1} i^2 = \frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$$

$$\text{L.H.S.} = 1^2 + 2^2 + 3^2 + ... + k^2 + (k+1)^2$$

We will substitute equation (1) and we will get,

$$= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 = (k+1)\left[\frac{k(2k+1)}{6} + (k+1)\right]$$

$$= (k+1)\left[\frac{k(2k+1)+6k+6}{6}\right] = (k+1)\left[\frac{2k^2+k+6k+6}{6}\right]$$

$$= (k+1)\left[\frac{2k^2+7k+6}{6}\right] = \frac{(k+1)\ (2k^2+4k+3k+6)}{6}$$

$$= \frac{(k+1)\ (2k(k+2)+3(k+2))}{6} = \frac{(k+1)\ ((2k+3)(k+2))}{6}$$

$$= \frac{(k+1)\ ((k+2)(2k+3))}{6} = \frac{(k+1)\ (k+1+1)(2(k+1)+1)}{6}$$

$$= \text{R.H.S.}$$

As L.H.S. = R.H.S., it is true that using $n = k + 1$ the given expression can be proved.

Example 1.3.4 *Prove* : $1+4+7+\ldots\ldots+(3n-2) = \frac{n(3n-1)}{2}$ *for* $n > 0$

Solution : Consider,

1. Basis of induction -

Let, $n = 1$,

$$\text{L.H.S.} = (3.1-2) = 1$$

$$\text{R.H.S.} = \frac{1(3.1-1)}{2} = \frac{2}{2} = 1$$

As L.H.S. = R.H.S. given expression is true for $n = 1$.

2. Induction hypothesis - We assume that the given expression is true for $n = k$.

i.e.

$$1+4+7+\ldots\ldots+(3k-2) = \frac{k(3k-1)}{2} \text{ is true.} \quad \ldots (1)$$

Now we will prove it for $n = k + 1$.

$$\text{L.H.S.} = 1+4+7+\ldots\ldots+(3k-2)+(3(k+1)-2)$$

We will substitute R.H.S. of equation (1).

$$= \frac{k(3k-1)}{2}+(3(k+1)-2) = \frac{k(3k-1)+2[3(k+1)-2]}{2}$$

$$= \frac{3k^2-k+6k+2}{2} = \frac{3k^2+5k+2}{2}$$

$$= \frac{3k^2+2k+3k+2}{2} = \frac{(2k+2)+(3k^2+3k)}{2}$$

$$= \frac{2(k+1)+3k(k+1)}{2} = \frac{(k+1)(3k+2)}{2} = \frac{(k+1)(3(k+1)-1)}{2}$$

$$= \text{R.H.S.}$$

As L.H.S. = R.H.S. the given expression is true for $n = k + 1$.

Example 1.2.1 *Prove that $\sqrt{2}$ is not rational.* **AU : Dec.-11, Marks 8**

Solution :

Proof : We will assume that, $\sqrt{2}$ is a rational number. That means we can write

$$\sqrt{2} = \frac{a}{b} \quad ...(1)$$

where a and b are integers and $\frac{a}{b}$ is irreducible and can be simplified to lowest term. Squaring on both sides of equation (1),

$$2 = \frac{a^2}{b^2} \qquad \text{i.e.} \qquad 2b^2 = a^2$$

This shows that L.H.S. is even (i.e. multiple of two). Hence R.H.S. is also even. Now if we write $a = 2k$ then,

$$2b^2 = (2k)^2 = 4k^2$$

$$\therefore \quad b^2 = 2k^2$$

This means, even b is an even number. This is contradiction our assumption that $\frac{a}{b}$ is simplified to lowest term because a and b are both even. So $\sqrt{2}$ cannot be rational.

1

Example 1.8.25 *Construct a DFA that accept the following language.*

$\{x \in \{a, b\} : |x|_a = odd \text{ and } |x|_b = even\}$

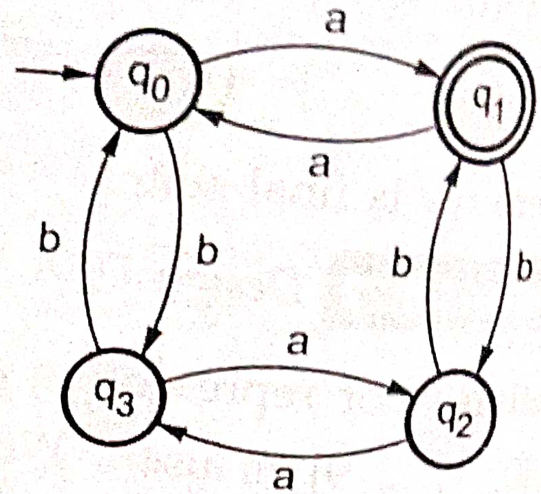
AU : Dec.-12, Marks

Solution : This DFA will consider four different stages for input a and b.

The stages could be -

- even number of a and even number of b
- even number of a and odd number of b
- odd number of a and even number of b
- odd number of a and odd number of b

The DFA will be as shown in figure.



Here q_0 is start state and q_1 is accept state.

q_0 = State of even number of a's and even number of b's

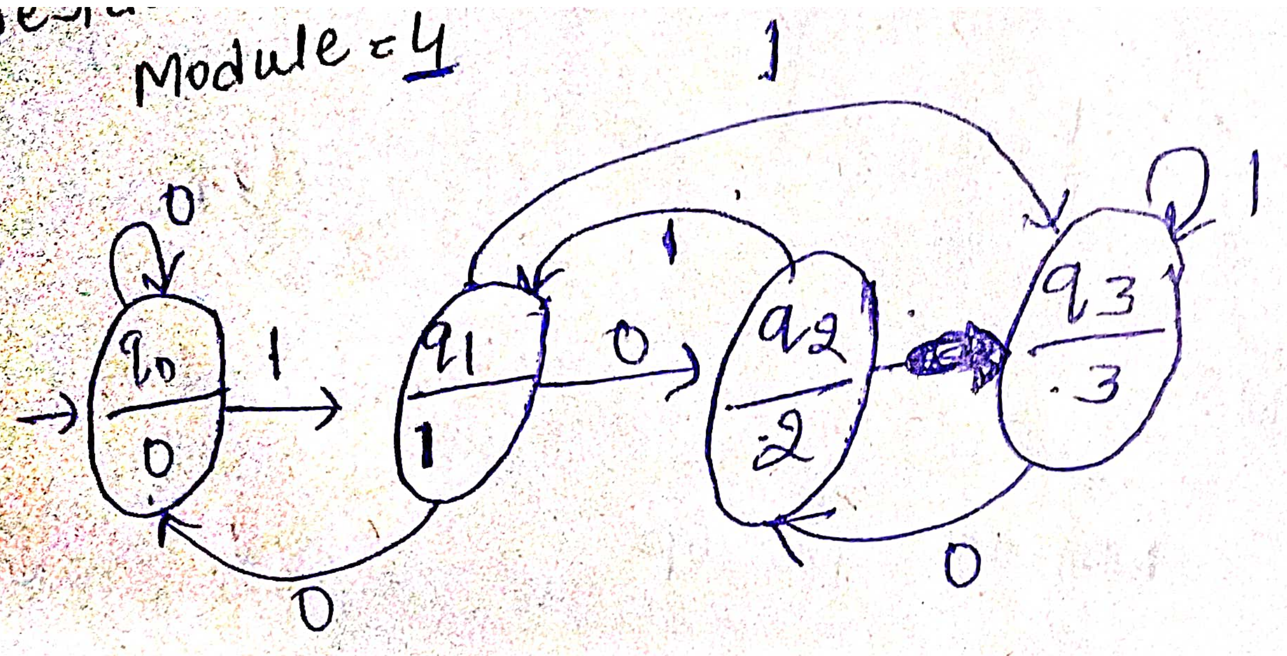
q_1 = State of odd number of a's and even number of b's

q_2 = State of odd number of a's and odd number of b's

q_3 = State of even number of a's and odd number of b's.

Give deterministic finite automata acceptin...

Module = 4



$\Sigma = \{0,1\}$ $\Delta = \{0, 1, 2, 3\}$

0100 = 4 → o/p = 0 Remainder = 0

0101 = 5 → o/p = 1 Remainder = 1

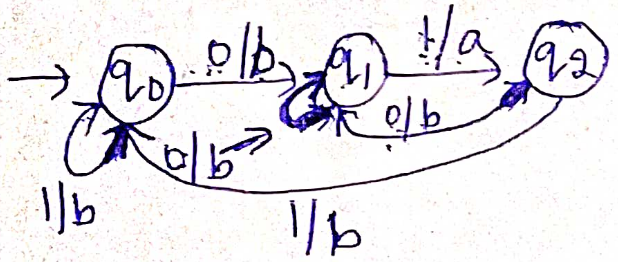
1000 = 8 → o/p = 0 Remainder = 0

8421.
0000 – 0
0001 – 1
0010 – 2
0011 – 3
0100 – 4
0101 – 5
1000 – 8

construct a Melay Machine that print "a" whenever one sequent "01" encountered in any input strings.

$\Sigma = \{0, 1\}$

$\Delta = \{a, b\}$



$1 \ \overbrace{0 \ 1}^{a} \ 1 \ 1 \ \overbrace{0 \ 1}^{a}$

$q_0 \ q_0 \ q_1 \ q_2 \ q_0 \ q_0 \ q_1 \ q_2$

b b b a b b b a

Example 1.8.18 *Draw transition diagram for recognizing the set of all operators in C language.*

AU : Dec.-07, Marks 10

Solution : In C language there are various operators, such as relational operators bitwise operators and arithmetic operators. Various relational operators are <, >, <=, >=, !=, ==. The bitwise operators are &&, ||. Let us draw the transition diagram for these operators.

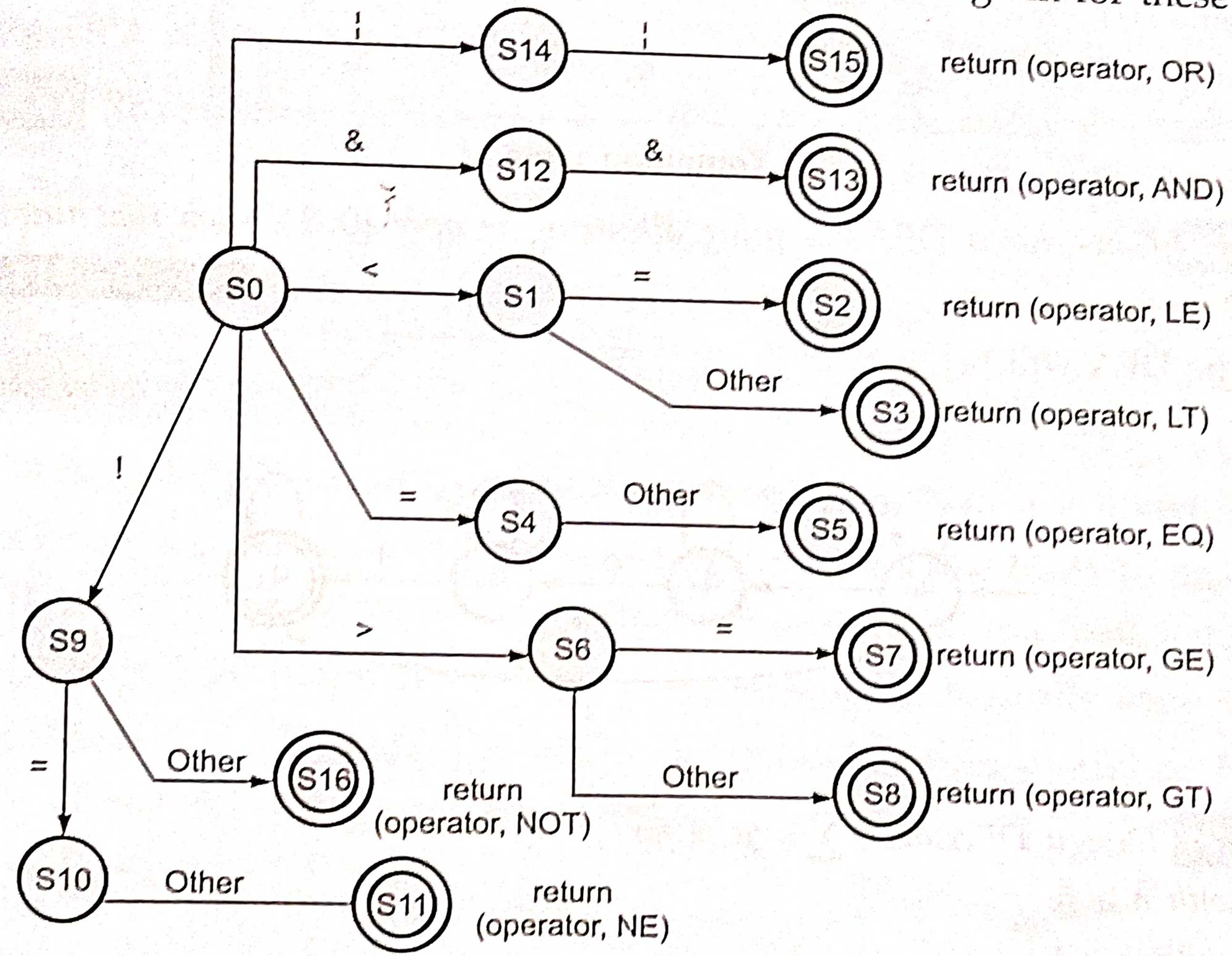


Fig. 1.8.25

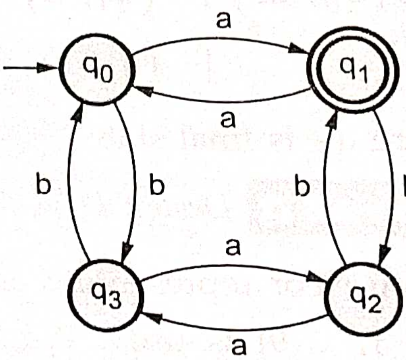
Example 1.8.25 *Construct a DFA that accept the following language.*

$\{x \in \{a, b\} : |x|_a = odd \text{ and } |x|_b = even\}$ **AU : Dec.-12, Marks 10**

Solution : This DFA will consider four different stages for input a and b.

The stages could be -

- even number of a and even number of b
- even number of a and odd number of b
- odd number of a and even number of b
- odd number of a and odd number of b



The DFA will be as shown in figure.

Here q_0 is start state and q_1 is accept state.

q_0 = State of even number of a's and even number of b's

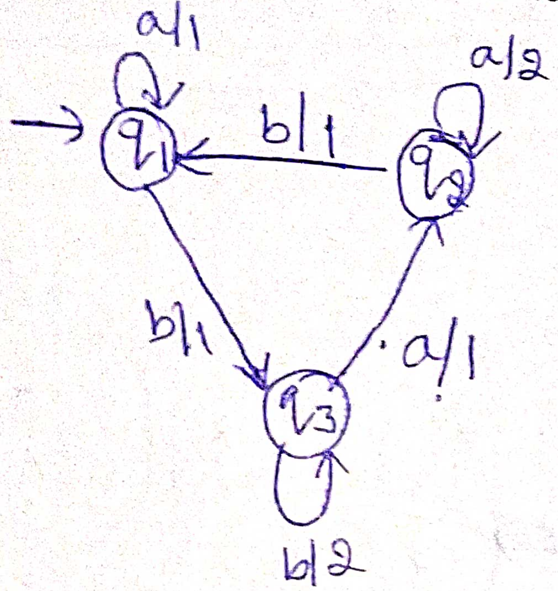
q_1 = State of odd number of a's and even number of b's

q_2 = State of odd number of a's and odd number of b's

q_3 = State of even number of a's and odd number of b's.

0 $\overline{}$ 1

Mealy to Moore Machine



Current state	a NS	a OP	b NS	b OP
q_1	q_1	1	q_3	1
q_2	q_2	2	q_1	1
q_3	q_2	1	q_3	2

$q_1 \dashrightarrow 1$

$q_2 \dashrightarrow 1 = q_{21}$

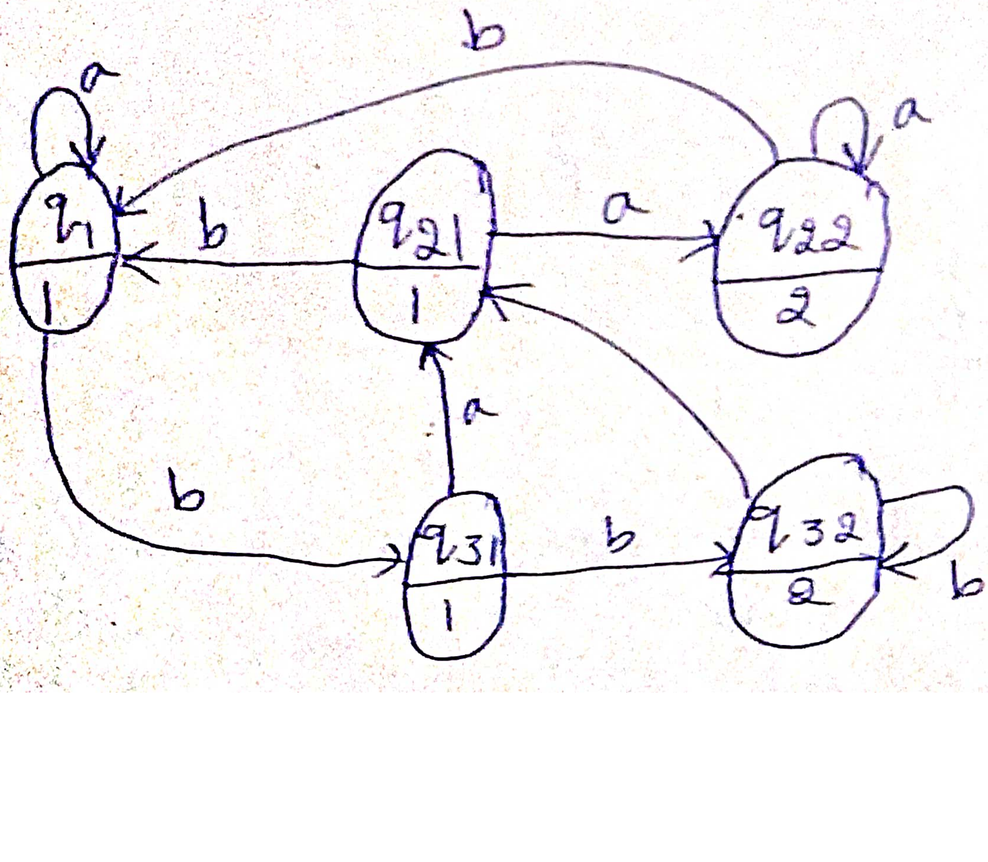
$q_2 \dashrightarrow 2 = q_{22}$

$q_3 \dashrightarrow 1 = q_{31}$

$q_3 \dashrightarrow 2 =$

Moore Machine table

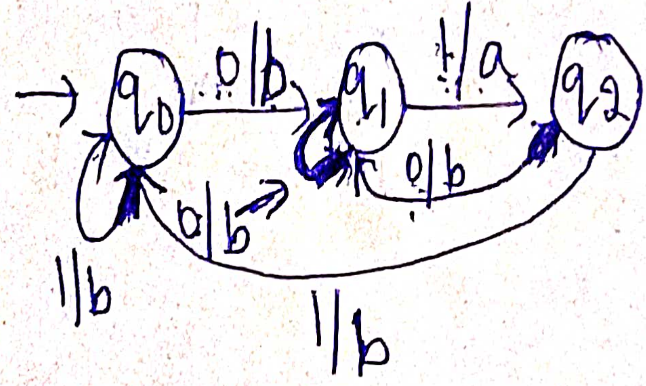
	a	b	O/P
q_1	q_1	q_{31}	1
q_{21}	q_{22}	q_1	1
q_{22}	q_{22}	q_1	2
q_{31}	q_{21}	q_{32}	1
q_{32}	q_{21}	q_{32}	2



construct a Melay Machine that print "a" whenever one sequent "01" encountered in any input strings

$\Sigma = \{0, 1\}$

$\Delta = \{a, b\}$



$$\begin{array}{cccccccc}
 & \overbrace{0\ 1}^{a} & & & & \overbrace{0\ 1}^{a} & \\
1 & 0\ \ 1 & 1 & 1 & & 0\ \ 1 &
\end{array}$$

$q_0 \quad q_0 \quad q_1 \quad q_2 \quad q_0 \quad q_0 \quad q_1 \quad q_2$

b b b a b b b a

Example 1.9.3 *For the finite state machine M given in the following table, test whether the strings 101101, 11111 are accepted by M.* **AU : May-07, Marks 4**

State	Input	
	0	1
$\rightarrow$ (q_0)	q_0	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

Solution : To test whether given strings are accepted by the given FA let us represent it by transition diagram.

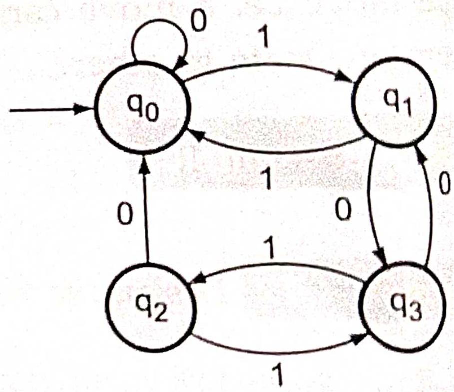


Fig. 1.9.2

Let us trace the string 101101.

$q_0 \vdash 101101$

$1q_1 \vdash 01101$

$10 \vdash q_3\ 1101$

$101 \vdash q_2\ 101$

$1011 \vdash q_3\ 01$

$10110 \vdash q_1 1$

$101101 \vdash q_0$ which is a final state.

Hence the above input is accepted by FA.

Now consider the string 11111.

$q_0 \vdash 11111$

$1\ q_1 \vdash 1111$

$11\ q_0\ 111$

$111\ q_1\ 11$

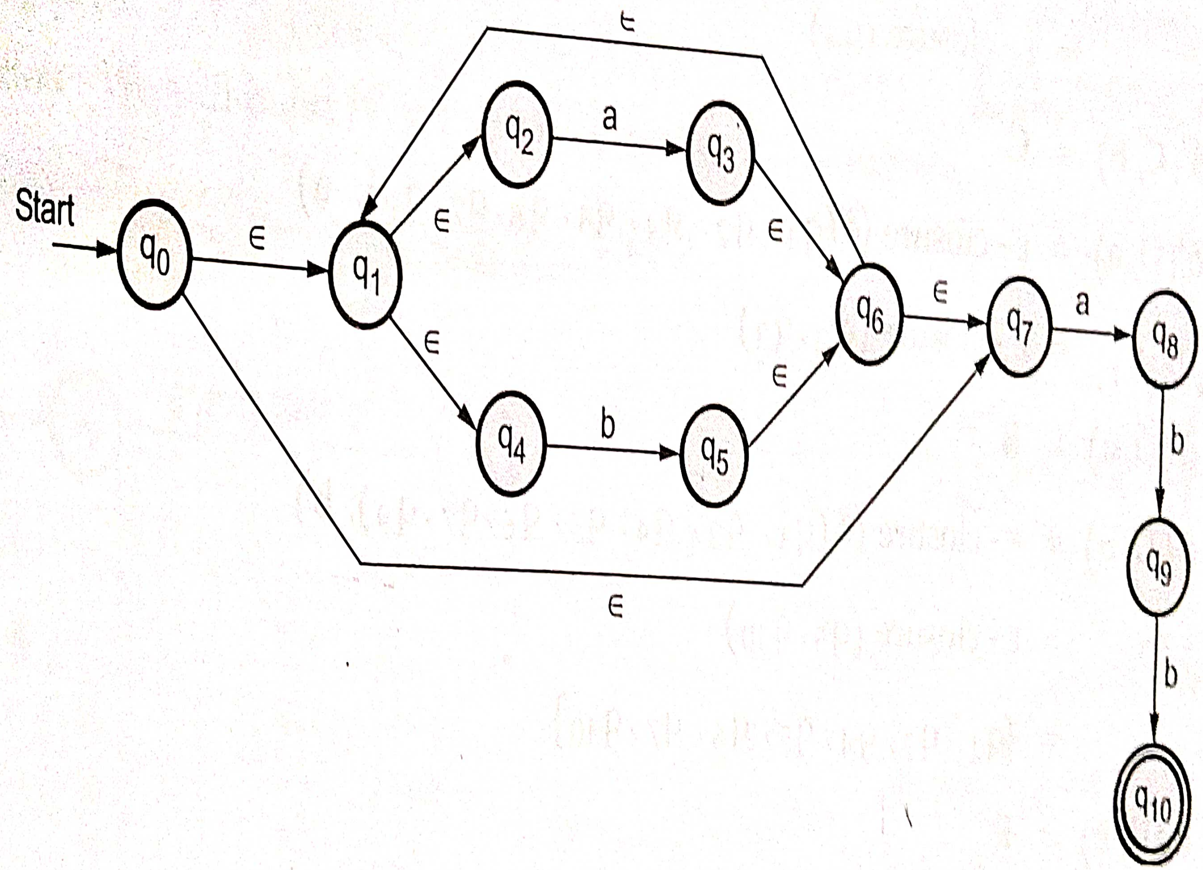
$1111\ q_0\ 1$

$11111\ q_1$ which is not a final state.

Hence this input will not be accepted by given FA.

Example 2.8.16 *Construct a DFA for $(a \mid b)^*\ abb$ and minimize it.*

Solution : We will first construct NFA - ε for given r.e.



$$\varepsilon\text{-closure}(q_0) = \{q_1, q_2, q_4, q_7\} = \text{Call it as state A.}$$

$$\delta'(A, a) = \varepsilon\text{-closure}(\delta(q_1, q_2, q_4, q_7), a)$$

$$= \varepsilon\text{-closure}(q_3, q_8)$$

$$= \{q_1, q_2, q_3, q_4, q_6, q_7, q_8\}$$

= Call it as state B.

$$\therefore \quad \delta'(A, a) = B$$

$$\delta'(A,b) = \varepsilon\text{-closure}\,(\delta(q_1, q_2, q_4, q_7), b)$$

$$= \varepsilon\text{-closure}\,(q_5) = \{q_1, q_2, q_4, q_5, q_6, q_7\}$$

$$\delta'(A, a) = C$$

$$\delta'(B, a) = \varepsilon\text{-closure}\,(\delta(q_1, q_2, q_3, q_4, q_6, q_7, q_8), a)$$

$$= \varepsilon\text{-closure}\,(q_3, q_8)$$

$$\delta'(B, a) = B$$

$$(B, b) = \varepsilon\text{-closure}\,(\delta(q_1, q_2, q_3, q_4, q_6, q_7, q_8), b)$$

$$= \varepsilon\text{-closure}\,(q_5, q_9)$$

$$= \{q_1, q_2, q_4, q_5, q_6, q_7, q_9\}$$

$$\delta'(B, b) = D$$

$$\delta'(C, a) = \varepsilon\text{-closure}\,(\delta(q_1, q_2, q_4, q_5, q_6, q_7,), a)$$

$$= \varepsilon\text{-closure}\,(q_3, q_8)$$

$$\delta'(C, a) = B$$

$$\delta'(c, b) = \varepsilon\text{-closure}\,(\delta(q_1, q_2, q_4, q_5, q_6, q_7), b)$$

$$= \varepsilon\text{-closure}\,(q_5)$$

$$\delta'(C, b) = C$$

$$\delta'(D, a) = \varepsilon\text{-closure}\,(\delta(q_1, q_2, q_4, q_5, q_6, q_7, q_9), a)$$

$$= \varepsilon\text{-closure}\,(q_3, q_8)$$

$$\delta'(D, a) = B$$

$$\delta'(D, b) = \varepsilon\text{-closure}\,(\delta(q_1, q_2, q_4, q_5, q_6, q_7, q_9), b)$$

$$= \varepsilon\text{-closure}\,(q_5, q_{10})$$

$$= \{q_1, q_2, q_4, q_5, q_6, q_7, q_{10}\}$$

$$\delta'(D, b) = E$$

$$\delta'(E, a) = \varepsilon\text{-closure}\,(\delta(q_1, q_2, q_4, q_5, q_6, q_7, q_{10}), a)$$

$$= \varepsilon\text{-closure}\,(\{q_3, q_8\})$$

$$\delta'(E, a) = B$$

$$\delta'(E, b) = \varepsilon\text{-closure}\,(\delta(q_1, q_2, q_4, q_5, q_6, q_7, q_{10})b)$$

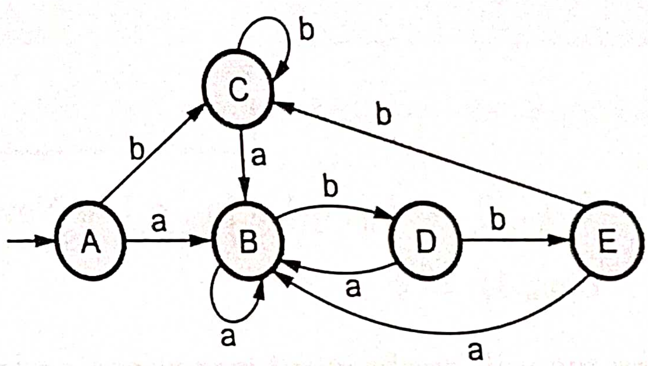
$= \varepsilon\text{ - closure }(q_5)$

$\delta'(E, b) = C$

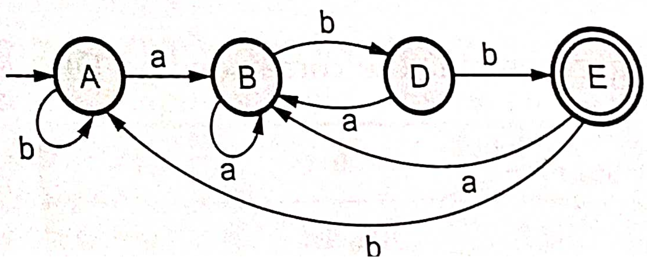
The transition table will be

State \ I/P	a	b
→ A	B	C
B	B	D
C	B	C
D	B	E
(E)	B	C

The transition diagram will be



The states A and C are equivalent. Hence we can have minimized DFA as



Example for Understanding

Example 2.6.3 *Minimize the DFA as given below.*

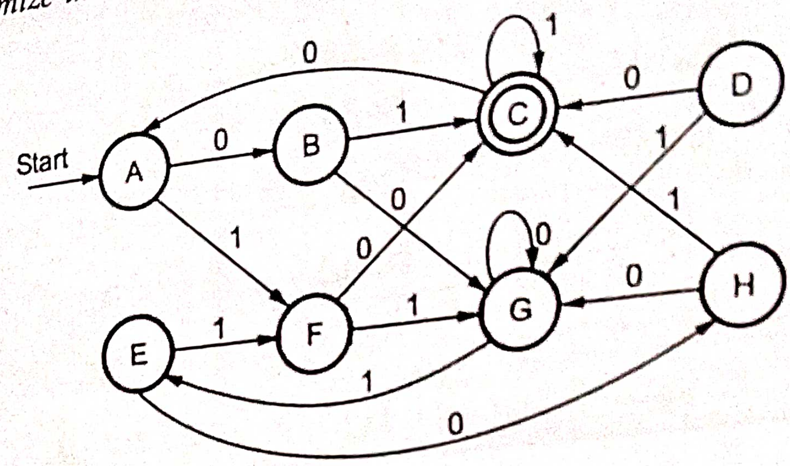


Fig. 2.6.7

Solution : We will build the transitions table for given DFA as follows.

	0	1
→ A	B	F
B	G	C
C	A	C
D	C	G
E	H	F
F	C	G
G	G	E
H	G	C

We will now construct a table for each pair of states.

	A	B	C	D	E	F	G
B							
C							
D							
E							
F							
G							
H							

We will mark X for all the final and non final states. Hence

	A	B	C	D	E	F	G
B							
C	X	X					
D			X				
E			X				
F			X				
G			X				
H			X				

Thus we have marked X for (A, C), (B, C), (C, D), (C, E), (C, F), (C, G), (C, H).

Now we will consider every pair form above table. Consider pair (G, H).

δ (G, 0) = G, δ (G, 1) = E

δ (H, 0) = G, δ (H, 1) = C

As for δ (G, 1) = E and δ (H, 1) = C. We can see X in (C, E) pair. Hence pair (G, H) is not equivalent.

Hence we will mark X in pair (G, H). Thus we will find the equivalent pairs.

	A	B	C	D	E	F	G
B	X						
C	X	X					
D	X	X	X				
E		X	X	X			
F	X	X	X		X		
G	X	X	X	X	X	X	
H	X		X	X	X	X	X

Consider pair (B, H)

δ (B, 0) = G, δ (B, 1) = C

δ (H, 0) = G, δ (H, 1) = C

Thus the pair (B, H) is equivalent. Thus we obtain, Thus we get equivalent pairs as (A, E), (B, H), (D, F). Hence the minimized DFA will be,

	0	1
A	B	D
B	G	C
C	A	C
D	C	G
G	G	A

Solved Examples

Fig. 2.4.2 An equivalent DFA

Example 2.4.2 *Construct DFA equivalent to the NFA*

M = ({p, q, r}, {0, 1}, δ, p {q, s})

Where δ is defined in the following table.

AU : Dec.-04, Marks 8

δ	0	1
p	{q, s}	{q}
q	{r}	{q, r}
r	{s}	{p}
s	–	{p}

Solution : To construct DFA,

$\delta \{p, 0\} = \{q, s\}$ **new state generated**

$\delta \{p, 1\} = \{q\}$

$\delta \{q, 0\} = \{r\}$

$\delta \{q, 1\} = \{q, r\}$ **new state**

$\delta \{r, 0\} = \{s\}$

$\delta \{r, 1\} = \{p\}$

$\delta \{s, 0\} = -$

δ {s, 1} = {p}

δ {{q, s}, 0} = {r}

δ {{q, s}, 1} = {p, q, r} **new state**

δ {{q, r}, 0} = {r, s} **new state**

δ {{q, r}, 1} = {p, q, r}

δ {{p, q, r}, 0} = {q, r, s} **new state**

δ {{p, q, r}, 1} = {p, q, r}

δ {{r, s}, 0} = {s}

δ {{r, s}, 1} = {p}

δ {{q, r, s}, 0} = {r, s}

δ {{q, r, s}, 1} = {p, q, r}

The transition table is as shown below.

δ	0	1
→ p	{q, s}	{q}
q	{r}	{q, r}
r	{s}	{p}
s	–	{p}
{q, s}	{r}	{p, q, r}
{q, r}	{r, s}	{p, q, r}
{p, q, r}	{q, r, s}	{p, q, r}
{r, s}	{s}	{p}
{q, r, s}	{r, s}	{p, q, r}

Example 2.4.9 *Convert the given NFA to DFA.*

Input / State	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
q_1	q_2	q_1
q_2	q_3	q_3
q_3 (final)	ϕ	q_2

Solution : As we know, first we will compute δ' function.

$$\delta(\{q_0\}, 0) = \{q_0, q_1\}$$

Hence $\delta'([q_0], 0) = [q_0, q_1]$

Similarly,

$$\delta(\{q_0\}, 1) = \{q_0\}$$

Hence $\delta'([q_0], 1) = [q_0]$

Thus we have got a new state $[q_0, q_1]$.

Let us check how it behaves on input 0 and 1.

So,

$$\begin{aligned} \delta'([q_0, q_1], 0) &= \delta([q_0], 0) \cup \delta([q_1], 0) \\ &= \{q_0, q_1\} \cup \{q_2\} \\ &= \{q_0, q_1, q_2\} \end{aligned}$$

Hence a new state is generated i.e. $[q_0, q_1, q_2]$

Similarly,

$$\delta'([q_0, q_1], 1) = \delta([q_0], 1) \cup \delta([q_1], 1)$$
$$= \{q_0\} \cup \{q_1\}$$
$$= \{q_0, q_1\}$$

No new state is generated here.

Again δ' function will be computed for $[q_0, q_1, q_2]$, the new state being generated.

State	0	1
$[q_0]$	$[q_0, q_1]$	$[q_0]$
$[q_0, q_1]$	$[q_0, q_1, q_2]$	$[q_0, q_1]$
$[q_0, q_1, q_2]$	$[q_0, q_1, q_2, q_3]$	$[q_0, q_1, q_3]$

As, you have observed in the above table for a new state $[q_0, q_1, q_2]$ the input 0 will give a new state $[q_0, q_1, q_2, q_3]$ and input 1 will give a new state $[q_0, q_1, q_3]$, because

$$\delta'([q_0, q_1, q_2], 0) = \delta'([q_0], 0) \cup \delta'([q_1], 0) \cup \delta'([q_2], 0)$$
$$= \{q_0, q_1\} \cup \{q_2\} \cup \{q_3\}$$
$$= \{q_0, q_1, q_2, q_3\}$$
$$= [q_0, q_1, q_2, q_3]$$

Same procedure for input 1. Thus the final DFA is as given below.

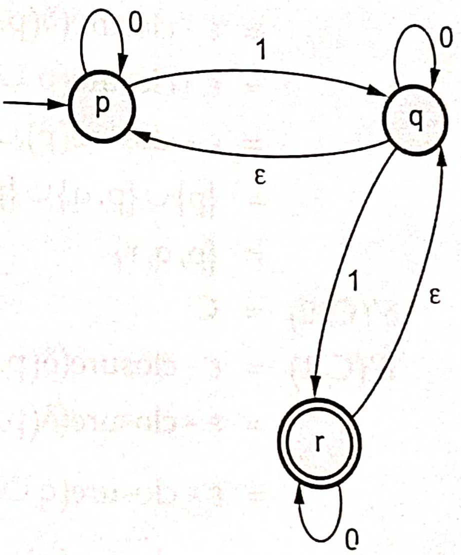
State	0	1
$\rightarrow [q_0]$	$[q_0, q_1]$	$[q_0]$
$[q_1]$	$[q_2]$	$[q_1]$
$[q_2]$	$[q_3]$	$[q_3]$
$[q_3]$	ϕ	$[q_2]$
$[q_0, q_1]$	$[q_0, q_1, q_2]$	$[q_0, q_1]$
$[q_0, q_1, q_2]$	$[q_0, q_1, q_2, q_3]$	$[q_0, q_1, q_3]$
$[q_0, q_1, q_3]$	$[q_0, q_1, q_2]$	$[q_0, q_1, q_2]$
$[q_0, q_1, q_2, q_3]$	$[q_0, q_1, q_2, q_3]$	$[q_0, q_1, q_2, q_3]$

DFA for example 2.4.9

Example 2.5.5 *Construct a DFA from the ε - NFA.*

	ε	0	1
p	-	{p}	{q}
q	{p}	{q}	{r}
*r	{q}	{r}	-

Solution : The transition diagram will be -



ε - closure (p) = {p} = call it as state A.

$$\delta'(A,0) = \varepsilon\text{ - closure}(\delta(A,0))$$
$$= \varepsilon\text{ - closure}(\delta(p,0))$$
$$= \varepsilon\text{ - closure}(p)$$
$$= \{p\}$$
$$\boldsymbol{\delta'(A, 0) = A}$$
$$\delta'(A,1) = \varepsilon\text{ - closure}(\delta(A,1))$$
$$= \varepsilon\text{ - closure}(\delta(p,1))$$
$$= \varepsilon\text{ - closure}(q)$$
$$= \{p, q\}$$
$$\delta\ (A,1) = \text{call it as state B.}$$
$$\therefore \quad \boldsymbol{\delta(A,1) = B}$$
$$\delta'(B,0) = \varepsilon\text{ - closure}(\delta(B,0))$$
$$= \varepsilon\text{ - closure}(\delta(p,q),0)$$
$$= \varepsilon\text{ - closure}(\delta(p,0) \cup \delta(q,0))$$
$$= \varepsilon\text{ - closure}(p, q)$$
$$= \varepsilon\text{ - closure}(p) \cup \varepsilon\text{ - closure}(q)$$
$$= \{p, q\}$$
$$\delta'(B,0) = B$$
$$\delta'(B,1) = \varepsilon\text{ - closure}(\delta((p,q),1))$$

$$= \varepsilon\text{ - closure}(\delta(p,1) \cup \delta(q,1))$$

$$= \varepsilon\text{ - closure}(q \cup r)$$

$$= \varepsilon\text{ - closure}(q) \cup \varepsilon\text{ - closure}(r)$$

$$= \{p, q\} \cup \{p, q, r\}$$

$$= \{p, q, r\} \quad \text{call it as state } \mathbf{C}$$

$$\delta'(B, 1) = C$$

$$\delta'(C, 0) = \varepsilon\text{ - closure}(\delta((p, q, r), 0))$$

$$= \varepsilon\text{ - closure}(\delta(p,0) \cup \delta(q,0) \cup \delta(r,0))$$

$$= \varepsilon\text{ - closure}(p \cup q \cup r)$$

$$= \varepsilon\text{ - closure}(p) \cup \varepsilon\text{ - closure}(q) \cup \varepsilon\text{ - closure}(r)$$

$$= \{p\} \cup \{p, q\} \cup \{p, q, r\}$$

$$= \{p, q, r\}$$

$$\delta'(C, 0) = C$$

$$\delta'(C, 1) = \varepsilon\text{ - closure}(\delta(p, q, r), 1)$$

$$= \varepsilon\text{ - closure}(\delta(p,1) \cup \delta(q,1) \cup \delta(r,1))$$

$$= \varepsilon\text{ - closure}(q \cup r, \phi)$$

$$= \varepsilon\text{ - closure}(q) \cup \varepsilon\text{ - closure}(r)$$

$$= \{p, q\} \cup \{p, q, r\}$$

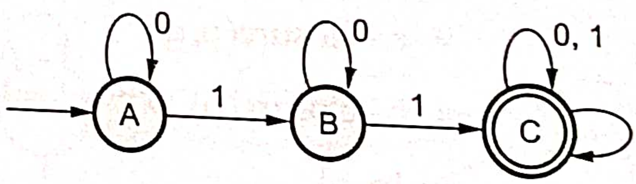
$$= \{p, q, r\}$$

$$\delta'(C, 1) = C$$

The transition table will be,

		0	1
→	A	A	B
	B	B	C
*	C	C	C

The transition diagram will be -



Solution : The regular expression

$$r = b + ba^* \text{ can be broken into } r_1 \text{ and } r_2 \text{ as}$$

$$r_1 = b$$

$$r_2 = ba^*$$

Let us draw the NFA for r_1, which is very simple.

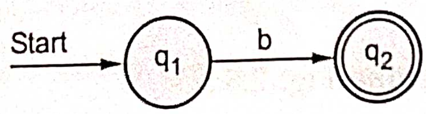


Fig. 2.8.5 For r_1

Now, we will go for $r_2 = ba^*$, this can be broken into r_3 and r_4 where $r_3 = b$ and $r_4 = a^*$. Now the case for concatenation will be applied. The NFA will look like this r_3 will be shown in Fig. 2.8.6.

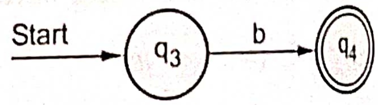


Fig. 2.8.6 For r_3

and r_4 will be shown as

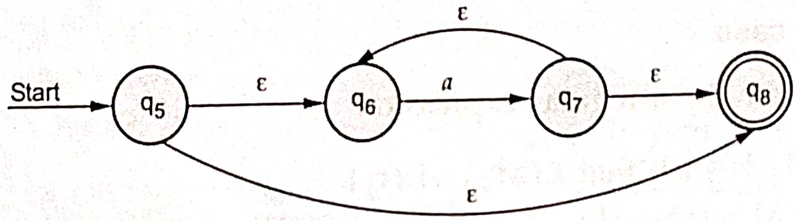


Fig. 2.8.7 For r_4

The r_2 will be $r_2 = r_3 \cdot r_4$

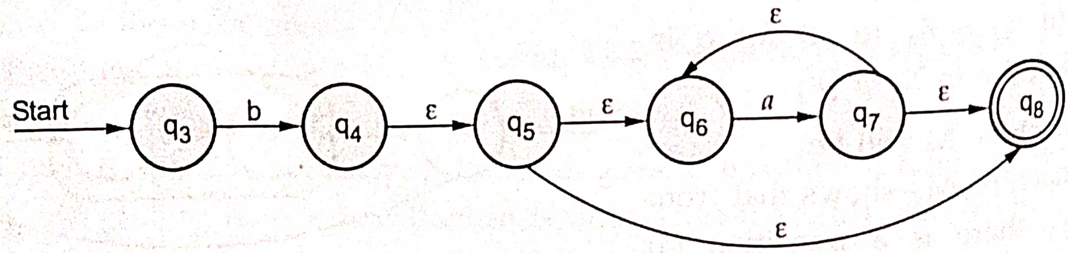


Fig. 2.8.7 (a) For r_2

Now, we will draw NFA for $r = r_1 + r_2$ i.e. $b + ba^*$

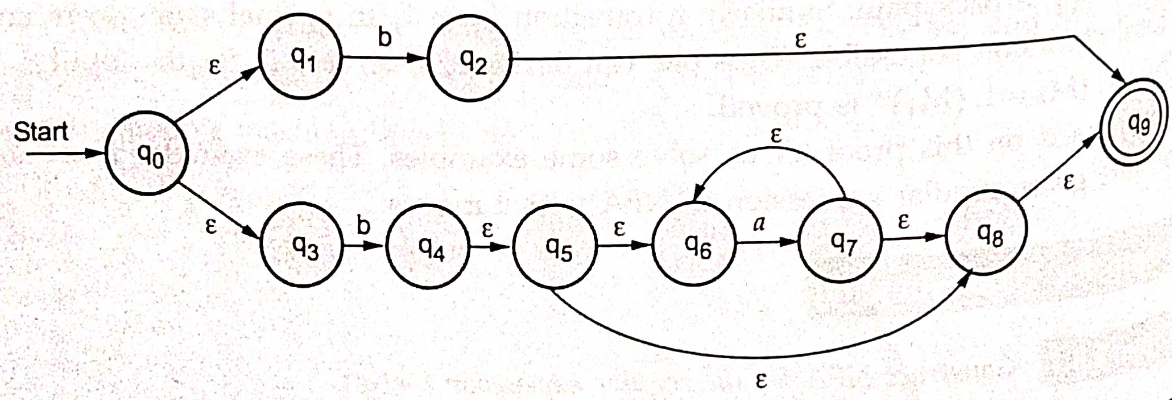


Fig. 2.8.8 For example 2.8.1

Example 2.12.3 *Prove* $L = \{a^p \mid p \text{ is a prime}\}$ *is not regular.* **AU : Dec.-07, Marks 6**

Solution : Let us assume L is a regular and P is a prime number.

$$L = a^P$$

$$|z| = uvw \qquad i = 1$$

Now consider $L = u\,v^i\,w$ where $i = 2$

$$= uv \cdot vw$$

Adding 1 to P we get,

$$P < |uvvw|$$

$$P < P+1$$

But P + 1 is not a prime number. Hence what we have assumed becomes contradictory. Thus L behaves as it is not a regular language.

Example 2.8.9 *Construct the minimal DFA for the regular expression $(b|a)^*$ baa.*

AU : Dec.-10, May-12, Marks 10

Solution : We will first design NFA with ε for the expression $(b|a)^*$ baa

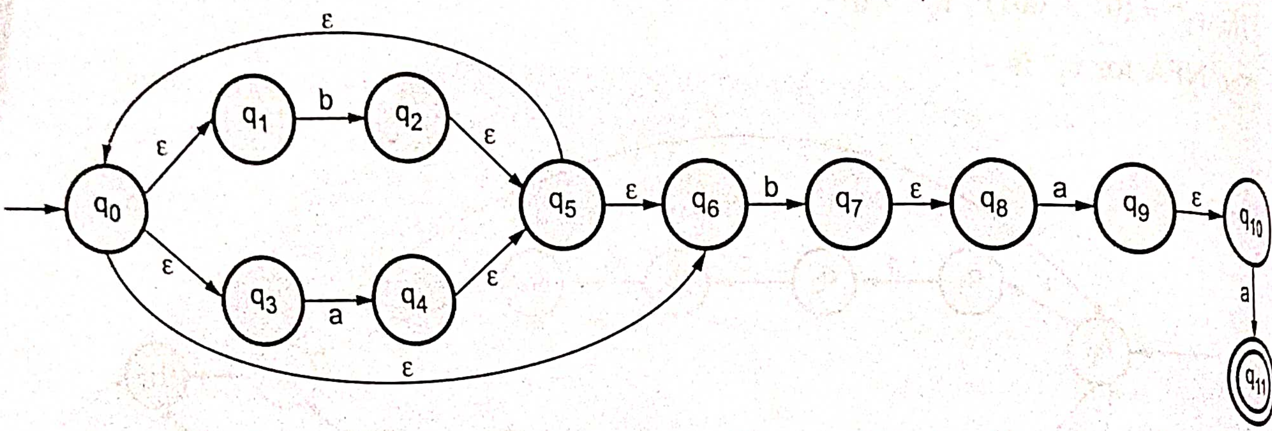


Fig. 2.8.30

Now using ε-closure computation the DFA can be obtained.

ε-closure $(q_0) = \{q_0, q_1, q_3, q_6\}$

ε-closure $(q_1) = \{q_1\}$

ε-closure $(q_2) = \{q_2, q_5, q_0, q_1, q_3, q_6\} = \{q_0, q_1, q_2, q_3, q_5, q_6\}$

ε-closure $(q_3) = \{q_3\}$

Let, ε-closure $(q_0) = \{q_0, q_1, q_3, q_6\}$ we call it as state A.

We will obtain input transitions on state A.

$$\therefore \quad \delta'(A, a) = \varepsilon\text{-closure } \{\delta(q_0, q_1, q_3, q_6), a\}$$

$$= \varepsilon\text{-closure } \{\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_3, a) \cup \delta(q_6, a)\}$$

$$= \varepsilon\text{-closure } \{\phi \cup \phi \cup q_4 \cup \phi\}$$

$$= \varepsilon\text{-closure } \{q_4\}$$

$$= \{q_0, q_1, q_3, q_4, q_5, q_6\} \text{ call it as state B .}$$

$$\therefore \quad \boldsymbol{\delta'(A, a) = B}$$

$$\delta'(A, b) = \varepsilon\text{-closure } \{\delta(q_0, q_1, q_3, q_6), b\}$$

$$= \varepsilon\text{-closure } \{\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_3, b) \cup \delta(q_6, b)\}$$

$$= \varepsilon\text{-closure } \{\phi \cup q_2 \cup \phi \cup q_7\}$$

$$= \varepsilon\text{-closure } \{q_2\} \cup \varepsilon\text{-closure } \{q_7\}$$

$$= \{q_0, q_1, q_2, q_3, q_5, q_6\} \cup \{q_7, q_8\}$$

$$= \{q_0, q_1, q_2, q_3, q_5, q_6, q_7, q_8\}$$

= call it as state C.

$\therefore\ \delta'(A, b) = C$

$$\delta'(B, a) = \varepsilon\text{-closure } \{\delta(q_0, q_1, q_3, q_4, q_5, q_6), a\}$$

$$= \varepsilon\text{-closure } \{\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_3, a) \cup \delta(q_4, a) \cup \delta(q_5, a) \cup \delta(q_6, a)\}$$

$$= \varepsilon\text{-closure } \{\phi \cup \phi \cup q_4 \cup \phi \cup \phi \cup \phi\}$$

$$= \varepsilon\text{-closure } \{q_4\}$$

$$= \text{state B}$$

$\therefore\ \delta'(B, a) = \text{state B}$

$$\delta'(B, b) = \varepsilon\text{-closure } \{\delta(q_0, q_1, q_3, q_4, q_5, q_6), b\}$$

$$= \varepsilon\text{-closure } \{\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_3, b) \cup \delta(q_4, b) \cup \delta(q_5, b) \cup \delta(q_6, b)\}$$

$$= \varepsilon\text{-closure } \{\phi \cup q_2 \cup \phi \cup \phi \cup \phi \cup q_7\}$$

$$= \varepsilon\text{-closure } \{q_2\} \cup \varepsilon\text{-closure } \{q_7\}$$

$$= \text{C state}$$

$\therefore\ \delta'(B, b) = C$

$$\delta'(C, a) = \varepsilon\text{-closure } \{\delta(q_0, q_1, q_2, q_3, q_5, q_6, q_7, q_8), a\}$$

$$= \varepsilon\text{-closure } \{\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_2, a) \cup \delta(q_3, a) \cup \delta(q_5, a) \cup \delta(q_6, a) \cup \delta(q_7, a) \cup \delta(q_8, a)\}$$

$$= \varepsilon\text{-closure } \{\phi \cup \phi \cup \phi \cup q_4 \cup \phi \cup \phi \cup \phi \cup q_9\}$$

$$= \varepsilon\text{-closure } \{q_4\} \cup \varepsilon\text{-closure } \{q_9\}$$

$$= \{q_0, q_1, q_3, q_4, q_5, q_6\} \cup \{q_9, q_{10}\}$$

$$= \{q_0, q_1, q_3, q_4, q_5, q_6, q_9, q_{10}\}$$

= Call it as state D.

$\therefore\ \delta'(C, a) = D$

$$\delta'(C, b) = \varepsilon\text{-closure } \{\delta(q_0, q_1, q_2, q_3, q_5, q_6, q_7, q_8), b\}$$

$$= \varepsilon\text{-closure } \{\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_2, b) \cup \delta(q_3, b), \cup \delta(q_5, b) \cup \delta(q_6, b) \cup \delta(q_7, b) \cup \delta(q_8, b)\}$$

$$= \varepsilon\text{-closure } \{\phi \cup q_2 \cup \phi \cup \phi \cup \phi \cup q_7 \cup \phi \cup \phi\}$$

$$= \varepsilon\text{-closure } \{q_2\} \cup \varepsilon\text{-closure } \{q_7\}$$

$$= C$$

$\therefore \quad \delta'(C, b) = C$

$\delta'(D, a) = \varepsilon\text{-closure } \{\{q_0, q_1, q_3, q_4, q_5, q_6, q_9, q_{10}\}, a\}$

$= \varepsilon\text{-closure } \{\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_3, a) \cup \delta(q_4, a) \cup \delta(q_5, a)$

$\cup\, \delta(q_6, a) \cup \delta(q_9, a) \cup \delta(q_{10}, a)\}$

$= \varepsilon\text{-closure } \{\phi \cup \phi \cup q_4 \cup \phi \cup \phi \cup \phi \cup \phi\ q_{11}\}$

$= \varepsilon\text{-closure } \{q_4\} \cup \varepsilon\text{-closure } \{q_{11}\}$

$= \{q_0, q_1, q_3, q_4, q_5, q_6\} \cup \{q_{11}\}$

$= \{q_0, q_1, q_3, q_4, q_5, q_6, q_{11}\}$

= Call it as state E.

$\therefore \quad \delta'(D, a) = E$

$\delta'(D, b) = \varepsilon\text{-closure } \{\delta(q_0, q_1, q_3, q_4, q_5, q_6, q_9, q_{10}), b\}$

$= \varepsilon\text{-closure } \{\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_3, b) \cup \delta(q_4, b) \cup \delta(q_5, b)$

$\cup\, \delta(q_6, b) \cup \delta(q_9, b) \cup \delta(q_{10}, b)\}$

$= \varepsilon\text{-closure } \{\phi \cup q_2 \cup \phi \cup \phi \cup \phi \cup q_7 \cup \phi \cup \phi\}$

$= \varepsilon\text{-closure } \{q_2\} \cup \varepsilon\text{-closure } \{q_7\}$

= C state

$\therefore \quad \delta'(D, b) = C$

$\delta'(E, a) = \varepsilon\text{-closure } \{\delta(q_0, q_1, q_3, q_4, q_5, q_6, q_{11}), a\}$

$= \varepsilon\text{-closure } \{\delta(q_0, a) \cup \delta(q_1, a) \cup \delta(q_3, a)$

$\cup\, \delta(q_4, a) \cup \delta(q_5, a) \cup \delta(q_6, a) \cup \delta(q_{11}, a)\}$

$= \varepsilon\text{-closure } \{\phi \cup \phi \cup q_4 \cup \phi \cup \phi \cup \phi \cup \phi\}$

$= B$

$\therefore \quad \delta'(E, a) = B$

$\delta'(E, b) = \varepsilon\text{-closure } \{\delta(q_0, q_1, q_3, q_4, q_5, q_6, q_{11}), b\}$

$= \varepsilon\text{-closure } \{\delta(q_0, b) \cup \delta(q_1, b) \cup \delta(q_3, b) \cup \delta(q_4, b)$

$\cup\, \delta(q_5, b) \cup \delta(q_6, b) \cup \delta(q_{11}, b)\}$

$= \varepsilon\text{-closure } \{\phi \cup q_2 \cup \phi \cup \phi \cup \phi \cup q_7 \cup\}$

$= \varepsilon\text{-closure } \{q_2\} \cup \varepsilon\text{-closure } \{q_7\}$

$= C$

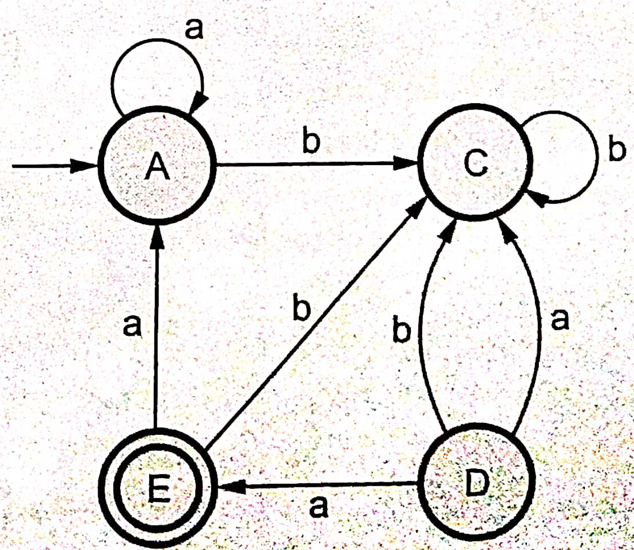
$\therefore \quad \delta'(E, b) = C$

The transition table for above computed DFA is

	a	b
A	B	C
B	B	C
C	D	C
D	E	C
E	B	C

We can say state A = B, because input transitions of both the states is the same and both are non final states.

Hence the transition diagram for minimized DFA will be -



contradictory. Thus L behaves as it is not ...

Example 2.12.4 *Show that* $L = \{0^n 1^{n+1} \mid n > 0\}$ *is not regular.*

Solution : Let us assume that L is a regular language.

$$|z| = |uvw|$$
$$= 0^n \; 1^{n+1}$$

Length of string $|z| = n + n + 1 = 2n + 1$.

That means length is always odd.

By pumping lemma

$$= |uv \cdot vw|$$

That is if we add 2n + 1

$$2n + 1 < (2n + 1) + 2n + 1$$
$$2n + 1 < 4n + 2$$

But if n = 1 then we obtain 4n + 2 = 6 which is no way odd. Hence the language becomes irregular.

Even if we add 1 to the length of $|z|$, then

$$|z| = 2n + 1 + 1$$
$$= 2n + 2$$
$$= \text{even length of the string.}$$

So this is not a regular language.

Solved Examples

Example 2.8.2 *Construct a NFA equivalent to $(0 + 1)^* (00 + 11)$.* **AU : May-04, Marks 8**

Solution : Consider

$$\text{r.e.} = \underbrace{(0+1)^*}_{r_1} \; \underbrace{(00+11)}_{r_2}$$

The NFA for r_1 can be drawn as follows.

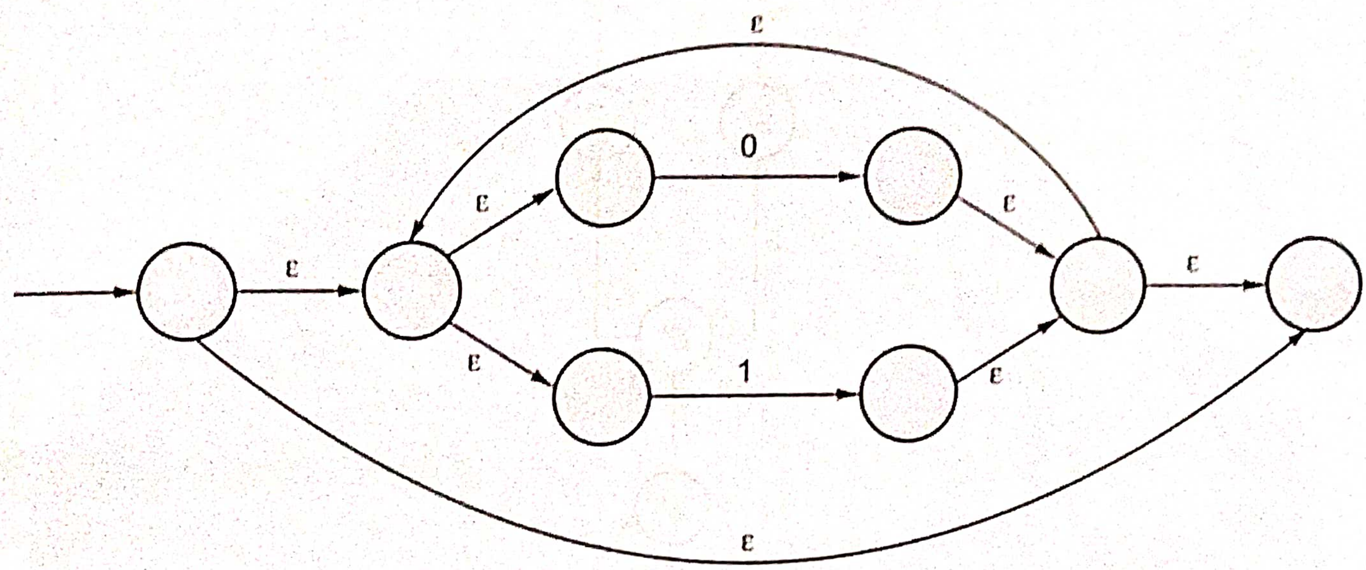


Fig. 2.8.9

The NFA for r_2 can be

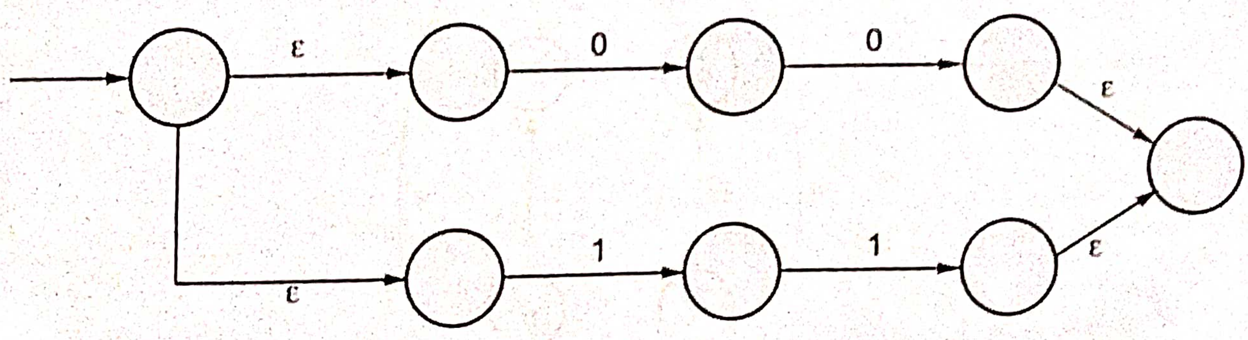


Fig. 2.8.10

The NFA for the given regular expression can be (See Fig. 2.8.11 on next page).

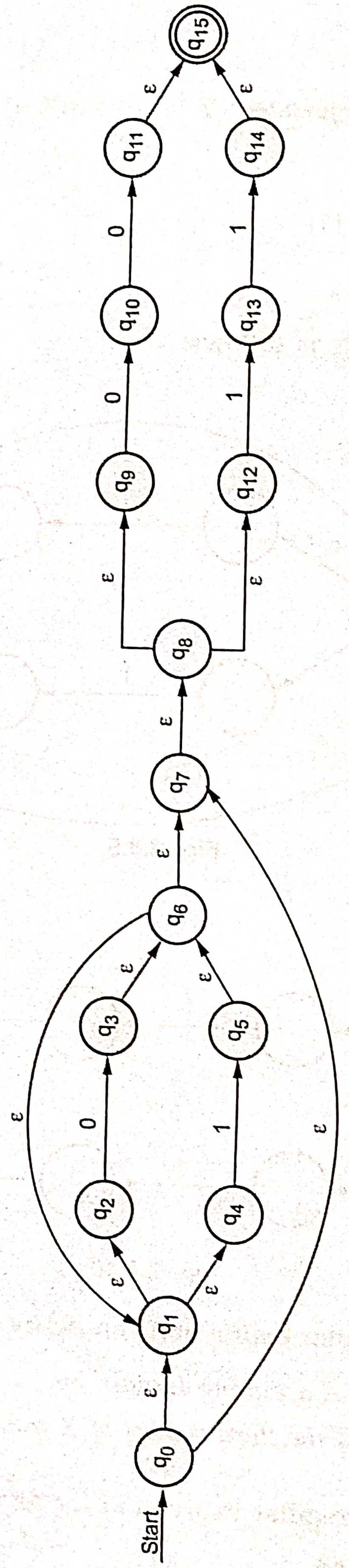


Fig. 2.8.11

Example 2.12.7 *Which of the following languages is regular ? Justify.*

1) $L = \{a^n b^m \mid n, m \geq 1\}$

2) $L = \{a^n b^n \mid n \geq 1\}$

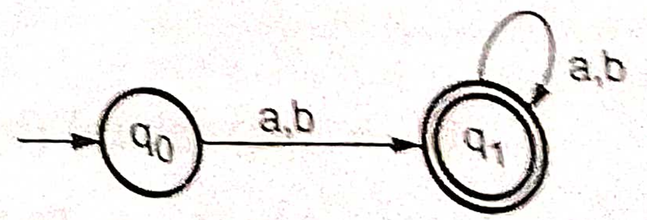
AU : May-12, Marks 8

Solution : 1) Let, $L = \{a^n b^m \mid n, m \geq 1\}$

Here the number of a's and b's can be at least one. We can write the regular expression for given L as

$$\text{r.e.} = a^+ b^+$$

For this regular expression the FA can be



As we can draw the FA for representing given language, the given L is said to be regular.

2) Given $L = \{a^n b^n \mid n \geq 1\}$ is not regular. Refer similar example 2.12.5.

Example 2.12.5 *Show that set* $L = \{0^i 1^i \mid i \geq 1\}$ *is not regular.*

AU : May-04, Dec.-04,05, Marks 6

Solution : Assume that $L = \{0^i 1^i \mid i \geq 1\}$ is regular.

Let, $w = 0^n 1^n$ such that $|w| = 2n$. By pumping lemma we can write

$w = xyz$ such that $|xy| \leq n$ and $|y| \neq 0$.

Now if $xy^iz \in L$ then the language L is said to be regular.

There are many cases – i) y has only 0's ii) y has only 1's iii) y has both 0's and 1's.

i) If y has only 0's then the string

$w = 0^{n-k} 1^n = xz$ since $y = 0^k$ and $i = 0$

Surely $n - k \neq n$. Hence $xz \notin L$.

Hence our assumption of being L regular is wrong.

ii) If y has only 1's then, for $i = 0$ and $y = 1^k$

$$\therefore \quad w = xz = 0^n 1^{n-k}$$

As $n \neq n - k$, $xz = w \notin L$

Again L is not regular.

iii) If y has 0's and 1's then

$$w = 0^{n-k} 0^k 1^j 1^{n-j} = xy^iz$$

If $i = 2$ then

$$w = 0^{n-k} 0^{2k} 1^{2j} 1^{n-j} \notin L$$

Hence from all these 3 cases it is clear that language L is not regular.

Example 2.12.6 Find whether the following languages are regular or not

Example 1.8.8 *Design FA which accepts even number of 0's and even number of 1's.*

Solution : This FA will consider four different stages for input 0 and 1. The stages could be

even number of 0 and even number of 1,

even number of 0 and odd number of 1,

odd number of 0 and even number of 1,

odd number of 0 and odd number of 1.

Let us try to design the machine

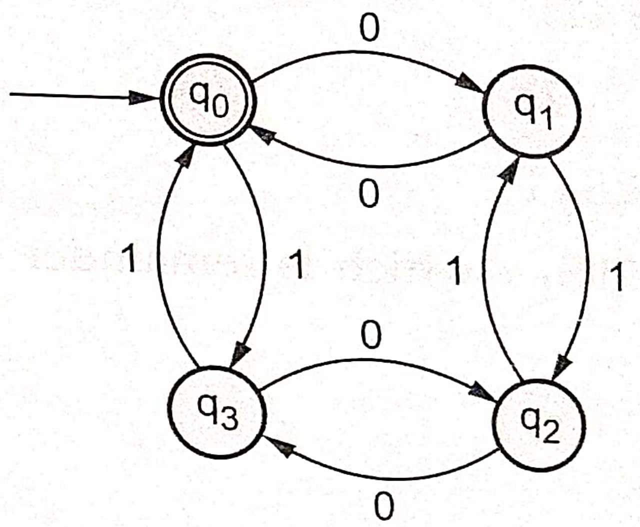


Fig. 1.8.11

Here q_0 is a start state as well as final state. Note carefully that a symmetry of 0's and 1's is maintained. We can associate meanings to each state as :

q_0 : State of even number of 0's and even number of 1's.

q_1 : State of odd number of 0's and even number of 1's.

q_2 : State of odd number of 0's and odd number of 1's.

q_3 : State of even number of 0's and odd number of 1's.

The transition table can be as follows -

	0	1
$\rightarrow q_0$	q_1	q_3
q_1	q_0	q_2
q_2	q_3	q_1
q_3	q_2	q_0