Unit-III

Distributions

Binomial distribution

Suppose we perform a series of 'n' independent trials such that for each trial, 'p' is the probability of sucess and 'q' that of a failure, then probability of getting exactly 'x' successes in 'n' trials is given by $^nC_x p^x q^{n-x}$ where 'x' takes any integral value from '0' to 'n', the probability of 0,1,2,3,.... trials with sucesses are given by $^nC_0 p^0 q^{n-0}$, $^nC_1 p^1 q^{n-1}$, $^nC_2 p^2 q^{n-2}$.....

→ The probability of the number of successes so obtained is called the binomial probability distribution i.e $(p+q)^n$. This distribution contains 2 independent constants namely 'n' & 'p'. These are called parameters

Defn:- A Random variable 'x' has the binomial Distribution (B.D) with probability function

$$P(X=x) = P(x) = \begin{cases} \binom{n}{x} p^x q^{n-x} & \text{for } x=0,1,2\ldots\ldots \\ 0, & \text{otherwise} \end{cases}$$

Note:- $\sum p(x) = \sum (p+q)^n = \sum_{x=0}^{n} \binom{n}{x} p^x q^{n-x}$

$$= \binom{n}{0} p^0 q^n + \binom{n}{1} p^1 q^{n-1} + \binom{n}{2} p^2 q^{n-2} + \ldots\ldots\ldots$$

$= (p+q)^n$

$= 1$

Conditions of B.D :-

① Trials are repeated under identical conditions for a fixed number of times say 'n' times

② there are only two possible outcomes namely → sucess and failure

③ The probability of sucess in each trial remains constant.

④ The Trials are independent

Properties of B.D :- (constants)

① Mean of B.D is np

Proof:- Mean $= E(x) = \sum x\, p(x)$

$$= \sum_{x=0}^{n} x \, (n_x)\, p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{(n-x)!\,x!} p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \cancel{x} \frac{n(n-1)!}{\cancel{x}(x-1)!\,(n-x)!} p \cdot p^{x-1} \cdot q^{n-x}$$

$$= np \sum_{x=1}^{n} \frac{(n-1)!}{(x-1)!\,(n-x)!} p^{x-1} q^{n-x}$$

$$= np \left[\sum_{x=1}^{n} \left({}^{n-1}C_{x-1}\right) p^{x-1} q^{n-x} \right]$$

$$= np\left[\binom{n-1}{0}p^0q^{n-1}+\binom{n-1}{1}p^1q^{n-2}+\dots\right]$$

$$= np(p+q)^{n-1} = np \quad (\because p+q=1)$$

② Variance of B.D is npq

Proof: $V(x) = E(x^2)-(E(x))^2$

$$E(x^2) = \sum x^2 p(x)$$

$$= \sum_{x=0}^{n} x^2 (n_x) p^x q^{n-x}$$

$$= \sum_{x=0}^{n} (x(x-1)+x)\frac{n!}{(n-x)!\,x!}p^x q^{n-x}$$

$$= \sum_{x=0}^{n} x(x-1)\frac{n!}{x!(n-x)!}p^x q^{n-x} + \sum_{x=0}^{n} x\frac{n!}{x!(n-x)!}p^x q^{n-x}$$

$$= \sum_{x=0}^{n} \frac{\cancel{x(x-1)}\,n(n-1)(n-2)!}{\cancel{x(x-1)}(x-2)!\,(n-x)!}p^2p^{x-2}q^{n-x} + np \quad (\because ① \text{ property})$$

$$= n(n-1)p^2\sum_{x=2}^{n}\frac{(n-2)!}{(x-2)!\,(n-x)!}p^{x-2}q^{n-x}+np$$

$$= n(n-1)p^2\sum_{x=2}^{n}\binom{n-2}{x-2}p^{x-2}q^{n-x}+np$$

$$= n(n-1)p^2\left[\binom{n-2}{0}q^{n-2}+\binom{n-2}{1}p^1q^{n-3}+\dots\right]+np$$

$$= n(n-1)p^2\left[[p+q]^{n-2}\right]+np \quad [\because p+q=1]$$

$$= n(n-1)p^2(1)+np = n^2p^2-np^2+np$$

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$$V(x) = n^2p^2 - np^2 + np - (np)^2$$
$$= \cancel{n^2p^2} - np^2 + np - \cancel{n^2p^2}$$
$$= np - np^2 = \cancel{np(n-p)}$$
$$= np(1-p) = npq \quad [\because p+q=1]$$

③ Mode of the B.D

→ Mode of the B.D is the value of x at which $P(x)$ has maximum value and it is given by integral part of $(n+1)p$

If $(n+1)p$ is not an integer i.e

Mode = Integral part of $(n+1)p$ if $(n+1)$ is not an integer.

$= (n+1)p - 1, (n+1)p$ if $(n+1)p$ is an integer

[not integral means one mode, otherwise 2 modes]

→ Recurrence relation for probabilities :- (RR)

* Recurrence relation is used to fit B.D

$$P(x) = \binom{n}{x} p^x q^{n-x} \rightarrow ①$$

$$P(x+1) = \binom{n}{x+1} p^{x+1} q^{n-(x+1)} \rightarrow ②$$

①/②

$$\frac{P(x+1)}{P(x)} = \frac{\binom{n}{x+1} p^{x+1} q^{n-x-1}}{\binom{n}{x} p^x q^{n-x}}$$

$= \cancel{n!}$

$$= \frac{\dfrac{n!}{(x+1)!\,(n-x-1)!}\, p^{x+1}\, q^{n-x-1}}{\dfrac{n!}{x!\,(n-x)!}\, p^{x}\, q^{n-x}}$$

$$= \frac{\cancel{n!}\,\cancel{p^{x}}\, p \cdot \cancel{q^{n-x}}\, q^{-1}}{\cancel{x!}\,(x+1)\cancel{(n-x-1)!}} \times \frac{\cancel{x!}\,\cancel{(n-x-1)!}\,(n-x)}{\cancel{n!}\,\cancel{p^{x}}\,\cancel{q^{n-x}}}$$

$$\Rightarrow \frac{p(x+1)}{p(x)} = \left(\frac{n-x}{x+1}\right)\frac{p}{q}$$

$$p(x+1) = \left(\frac{n-x}{x+1}\,\frac{p}{q}\right) p(x)$$

P:- In tossing 10 coins what is the probability of having or getting exactly 4 heads.

Sol Given $n = 10$ — small (B.D) $\boxed{\text{If large} - \text{Poisson} \quad \text{cont} - \text{Normal}}$

$p = \frac{1}{2}$, $q = 1/2$

$P(x = x) = p(x) = (n\,x)\, p^{x} q^{n-x}$

$P(x = 4) = (10\,4)\left(\frac{1}{2}\right)^{4}\left(\frac{1}{2}\right)^{10-4}$

$= (10\,4)\left(\frac{1}{2}\right)^{4}\left(\frac{1}{2}\right)^{6}$

$= 0.2051$

P:- If 10% of the bolts by the machine are defective Find the probability that out of 10 bolts choosen at random

i) None will be defective

ii) 1 will be defective.
iii) atmost 2 will be defective.
iv) atleast 2 will be defective

Sol $n=10$ $\quad P(X=x)=\binom{n}{x}p^x q^{n-x}$

$p=10\% = \cancel{0.01}\ 0.1$

$q=1-p=1-0.1=0.9$

i) none will be defective

$P(X=0)=\binom{10}{0}\cancel{p}(0.1)^0(0.9)^{10}$

$=(0.9)^{10}=0.3487$

ii) 1 will be defective

$P(X=1)=\binom{10}{1}(0.1)^1(0.9)^9$

$=0.3874$

iii) atmost 2 will be defective

$P(X\le 2)=P(X=0)+P(X=1)+P(X=2)$

$=0.3487+0.3874+\binom{10}{2}(0.1)^2(0.9)^8$

$=0.3487+0.3874+0.1937$

$=0.9298$

iv) atleast two :-

$P(X\ge 2)=1-P(X<2)$

$=1-[P(X=0)+P(X=1)]$

$=1-[0.3487+0.3874]$

$=1-0.7361=0.2639$

P:- If the probability of defective item is $1/8$ find the mean & variances of the distribution out of 640 defective items.

Sol

$n = 640$

$p = 1/8 \qquad q = 1 - \frac{1}{8} = \frac{7}{8}$

$p(x) = \binom{n}{x} p^x q^{n-x}$

Mean of B.D $= np = 640 \times \frac{1}{8} = 80$

variance of BD $= npq = 640 \times \frac{1}{8} \times \frac{7}{8} = 70$

P:- If has been claimed that 60% of all solar heat installations, the utility bill is reduced by atleast $1/3$rd of. Find the probability that the utility bill will be reduced by atleast $1/3$rd in 4 of 5 installations

Sol $n = 5 \qquad p = 60\% = 0.6$ (The probability that in the solar heat installations the utility bill will be reduced by $1/3$rd)

$q = 1 - p$

$q = 1 - 0.6 = 0.4$

$P(x=4) = \binom{5}{4} (0.6)^4 (0.4)^1 = 0.2592$

P:- The mean and variances of binomial Distribution with parameters n & p are given by 16 & 8 respectively Find i) $P(x \geq 1)$

ii) $P(x > 2)$

Sol $~~n = 16~~ \qquad ~~p = 8~~ \qquad q = ~~1 - p = 1 - 8~~ =$

Mean of B.D $= np = 16 \qquad$ variance $= npq = 8$

$\frac{npq}{np} = \frac{8}{16}$ $\Rightarrow$ $q = 1/2$ $p = 1 - q = \frac{1}{2}$

i) $P(X \geq 1) = \cancel{P(X \geq 1)}$

$np = 16$

$n \times \frac{1}{2} = 16 \Rightarrow 32$

$= 1 - P(X < 1) = 1 - P(X = 0)$

$= 1 - \binom{32}{0}\left(\frac{1}{2}\right)^{32} = 0.99$

ii) $P(X > 2) = 1 - P(X \leq 2)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[\binom{32}{0}\left(\frac{1}{2}\right)^{32} + \binom{32}{1}\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^{31} + \binom{32}{2}\left(\frac{1}{2}\right)^{2}\left(\frac{1}{2}\right)^{30}\right]$

$= 1 - \left(\frac{1}{2}\right)^{32}\left[1 + 32 + 496\right]$

$= 0.99$

Pr In a binomial distribution consisting of 5 independent trials probability of 1 & 2 successes are given by 0.4096 & 0.2048 respectively. Find the parameter 'p' of the distribution.

Sol $n = 5$

$P(X=1) = 0.4096$

$P(X=2) = 0.2048$

$P(X=x) = \binom{n}{x} p^x q^{n-x}$

$P(X=1) = \binom{5}{1} p q^4 = 0.4096$

$P(x=2) = \binom{5}{2} p^2 q^3 = 0.2048$

$$\frac{P(x=1)}{P(x=2)} = \frac{5 \times \cancel{p} \times q^{4}}{10 \times p^{\cancel{2}} \times \cancel{q^3}} = \frac{\cancel{5} q}{\cancel{10}_2 p} = \frac{1}{2} \frac{q}{p} = \frac{0.4096}{0.2048}$$

$$\frac{1}{2} \frac{q}{p} = 2$$

$q = 4p \Rightarrow 1-p = 4p \Rightarrow 5p = 1 \Rightarrow p = 1/5$

Q: The probability of a man hitting the target is 1/3
(i) If he fires 5 times what is the probability of his hitting the target atleast twice.
(ii) How many times must he fire so that the probability of his hitting the ~~tra~~ target atleast once is more than 90%.

Sol $p = \frac{1}{3}$ (probability of hitting the target)

$q = 1 - p = 1 - \frac{1}{3} = \frac{2}{3}$ ~~n = 5~~

(i) $n = 5$

$P(x \geq 2) = 1 - P(x < 2)$

$= 1 - [P(x=0) + P(x=1)]$

$= 1 - \binom{5}{0} \left(\frac{1}{3}\right)^0 \left(\frac{2}{3}\right)^5 + \binom{5}{1} \left(\frac{1}{3}\right)^1 \left(\frac{2}{3}\right)^4$

$= 1 - \left(\frac{2}{3}\right)^5 + 0.3292 = 1 - 0.4609$

$= 0.5391$

(ii) P(atleast one) $> 90\%$

$P(x \geq 1) > 90\%$ or 0.9

$1 - P(x < 1) > 0.9$

$$[1 - P(x=0)] > 0.9$$

$$\left(1 - \binom{n}{0}\left(\frac{2}{3}\right)^n\right) > 0.9$$

$$1 - \left(\frac{2}{3}\right)^n > 0.9$$

$$-1 + 1 - \left(\frac{2}{3}\right)^n > 0.9 - 1$$

$$-\left(\frac{2}{3}\right)^n > -0.1 \quad \text{(sign change)}$$

$$\left(\frac{2}{3}\right)^n < 0.1$$

put $n=1$ $0.6 \nless 0.1$

$n=2$ $0.4 \nless 0.1$

$n=3$ $0.2 \nless 0.1$

$n=4$ $0.19 \nless 0.1$

$n=5$ $0.13 \nless 0.1$

$n=6$ $0.08 < 0.1$

So he must fire 6 times to hit the target

Fit a B.D to the following data:

x	0	1	2	3	4	5	6
f	6	28	56	60	36	12	2

x	f	fx	$\frac{n-x}{x+1}$	$\left(\frac{n-x}{x+1}\right)\frac{p}{q}$	P(x) Exp freq
0	6	0	$\frac{6}{1}$	4.8438	5.7384
1	28	28	$\frac{5}{2}$	2.0183	27.7957
2	56	112	$\frac{4}{3}$	1.0764	56.1
3	60	180	$\frac{3}{4}$	0.6055	60.38

4	36	144	$\frac{2}{5}$	0.3229	36.564
5	12	60	$\frac{1}{6}$	0.1346	12.0625
6	2	12	0	0	1.6236
	$N=\sum f=200$	536		X	200.2642

RR of probabilities

$$p(x+1) = \left(\frac{n-x}{x+1}\right)\frac{p}{q}\, p(x)$$

$n=6$

Mean of B.D $= np = \frac{\sum fx}{\sum f} = \frac{536}{200}$

$$6p = 2.68$$

$$p = \frac{2.68}{6} = 0.4467$$

$$q = 1-p = 1-0.4467 = 0.5533$$

$$p/q = \frac{0.4467}{0.5533} = 0.8073$$

$$P(x) = (n_x)\, p^x q^{n-x}$$

$$P(0) = (6_0)(0.4467)^0(0.5533)^6 = 0.0287$$

~~$P(1) = (6_1)(0.4467)^1(0.5533)^5 = 2.7321$~~

$$F(0) = N \times P(0) = 200 \times 0.0287 = 5.7384$$

Sum of the given frequencies is approximately equal to the sum of the expected frequencies. Therefore, the given data is fitted.

P:- 7 coins are tossed 128 times the no of heads observed and recorded and the results are given below. Fit a binomial distribution by assuming that coins are

i) unbiased

ii) biased

Sol

x	0	1	2	3	4	5	6	7
f	7	6	19	35	30	23	7	1

— coins (n)

i) unbiased

when coins are unbiased then probability of getting head is $p = \frac{1}{2}$ $q = \frac{1}{2}$

$n = 7$

x	f	$\frac{n-x}{x+1}$	$\left(\frac{n-x}{x+1}\right)\frac{p}{q}$	Expected freq $F(x)$
0	7	7	7	1
1	6	3	6/2	7
2	19	5/3	5/3	21
3	35	4/4	4/4	35
4	30	3/5	3/5	35
5	23	2/6	2/6	21
6	7	1/7	1/7	7
7	1	0	0	1
	$\Sigma f = 128$			128

RR of probabilities in BD $= \left(\frac{n-x}{x+1}\right)\frac{p}{q}\, p(x) = P(x+1)$

$\frac{p}{q} = 1$

$P(x) = (n_x)\, p^x\, q^{n-x}$

$P(0) = (7_0)\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^7 = \frac{1}{128}$

$p(1) = \binom{7}{1}\left(\frac{1}{2}\right)^1\left(\frac{1}{2}\right)^6 = \frac{7}{128}$.

$F(0) = N \times P(0) = 128 \times \frac{1}{128} = 1$

Sum of the given frequencies = Sum of the Expected frequencies

Hence the data is fitted.

ii) biased.

$n = 7$

x	f	fx	$\frac{n-x}{x+1}$	$\left(\frac{n-x}{x+1}\right)\frac{p}{q}$	$F(x)$
0	7	0	7/1	6.5478	1.2586
1	6	6	6/2	2.8062	8.2411
2	19	38	5/3	1.5590	23.1261
3	35	105	4/4	0.9354	36.0535
4	30	120	3/5	0.5612	33.7245
5	23	115	2/6	0.3118	18.9262
6	7	42	1/7	0.1336	5.9012
7	1	7	1	0.9354	0.7884
	$\Sigma f = 128$	433			128.0196

RR of BD

$$p(x+1) = \left(\frac{n-x}{x+1}\right)\frac{p}{q}\,p(x)$$

Mean of BD is np

Mean of any data $= \frac{\Sigma fx}{f}$

$\therefore np = \frac{\Sigma fx}{f}$

$7 \times p = \frac{433}{128}$ $\quad p = 0.4833$

$q = 1 - p = 1 - 0.4833 = 0.5167$

$\frac{p}{q} = \frac{0.4833}{0.5167} = 0.9354$

$P(x) = (^n x) p^x q^{n-x}$

$P(0) = (^7 0) p^0 (0.4833)^e (0.5167)^7$

$= 0.0098$

$F(0) = N \times P(0) = 128 \times 0.0098$

$= 1.2586 \simeq 44$

Poisson Distribution :- n-large

The Poisson Distribution (PD) can be desined as limiting case of the B.D under the following condition

(i) 'p' is the probability of occurance of an event is very small

(ii) 'n' is very large where 'n' is the no of trials

(iii) 'np' is the finite quantity say $np = \lambda$ where 'λ' is the parameter of PD

Theorem :- Limiting case of B.D is P.D

Proof $(^n x) p^x q^{n-x} = \frac{n(n-1)(n-2) \ldots (n-(x-1))}{x!} p^x q^{n-x}$

$np = \lambda \Rightarrow p = \frac{\lambda}{n} \Rightarrow q = 1 - \frac{\lambda}{n}$

$n = \frac{\lambda}{p}$

$$= \frac{\frac{\lambda}{p}\left(\frac{\lambda}{p}-1\right)\left(\frac{\lambda}{p}-2\right)\cdots\left(\frac{\lambda}{p}-(x-1)\right)}{x!} p^x q^{n-x}$$

$$= \frac{\lambda(\lambda-p)(\lambda-2p)\cdots(\lambda-(x-1)p)}{\cancel{p^x}\, x!} \cancel{p^x} q^{n-x}$$

$p \to 0$, $n \to \infty$ $\frac{1}{n} \to 0$

$$= \frac{\lambda \cdot \lambda \cdots \lambda}{x!} \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^x}$$

$$= \frac{\lambda^x}{x!} \lim_{n\to\infty} \left[\left(1-\frac{\lambda}{n}\right)^{-n/\lambda}\right]^{-\lambda}$$

$$\left[\lim_{n\to\infty}\left(1-\frac{\lambda}{n}\right)^x = 1\right]$$

$$= \frac{\lambda^x e^{-\lambda}}{x!} = \frac{e^{-\lambda}\lambda^x}{x!}$$

Note:-

Defn:- Random variable 'x' is said to follow poisson distribution with probability function given by $p(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $x = 0, 1, 2 \ldots$

Note:- $\Sigma p(x) = 1$

$$\sum_{x=0}^{n} \frac{e^{-\lambda}\lambda^x}{x!} = e^{-\lambda}\sum_{x=0}^{n}\frac{\lambda^x}{x!}$$

$$= e^{-\lambda}\left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \ldots\right]$$

$$= e^{-\lambda} \times e^{\lambda}$$

$$= 1$$

Properties

i) Mean of P.D is λ

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\text{Mean} = E(x) = \sum x\, p(x)$$

$$= \sum_{x=0}^{n} x \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= \sum_{x=0}^{n} \cancel{x} \frac{e^{-\lambda} \lambda^{x-1} \lambda}{\cancel{x}(x-1)!}$$

$$= \lambda e^{-\lambda} \sum_{x=1}^{n} \frac{\lambda^{x-1}}{(x-1)!}$$

$$= \lambda e^{-\lambda} \left[1 + \frac{\lambda}{1!} + \frac{\lambda^2}{2!} + \dots\right]$$

$$= \lambda e^{-\lambda} \times e^{\lambda}$$

$$= \lambda$$

ii) Variance of PD is λ

$$V(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \sum_{x=0}^{n} x^2 p(x)$$

$$= \sum_{x=0}^{n} x^2 \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= \sum_{x=0}^{n} (x(x-1)+x) \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{n} x(x-1) \frac{e^{-\lambda}\lambda^x}{x!} + \sum_{x=0}^{n} x \frac{e^{-\lambda}\lambda^x}{x!}$$

$$= \sum_{x=0}^{n} \frac{\cancel{x(x-1)} \lambda^{x-2} \times \lambda^2 e^{-\lambda}}{\cancel{x(x-1)}(x-2)!} + \lambda$$

$$= \lambda^2 e^{-\lambda} \sum_{x=2}^{n} \frac{\lambda^{x-2}}{(x-2)!} + \lambda$$

$$= \lambda^2 e^{-\lambda} \left[1 + \frac{\lambda}{1} + \frac{\lambda^2}{2!} + \cdots \right] + \lambda$$

$$= \lambda^2 e^{-\lambda} [e^{\lambda}] + \lambda$$

$E(x^2) = \lambda^2 + \lambda$

$V(x) = E(x^2) - (E(x))^2$

$= \cancel{\lambda^2} + \lambda - \cancel{\lambda^2} = \lambda$

iii) Mode of PD

→ Mode of a PD lies between $\lambda - 1$ and λ i.e if 'λ' is an integer then '$\lambda - 1$' is also an integer so we have two maximum values of PD and it is said to be bimodal distribution with $\lambda-1, \lambda$ modes

→ If 'λ' is not an integer then mode of PD is its integral part. (Eg- 4.32 → 4)

Recurrence Relation of Probabilities :- (PD)

$P(x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $\quad$ $P(x+1) = \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!}$

$$\frac{P(x+1)}{P(x)} = \frac{\cancel{e^{-\lambda}}\lambda^{x+1}}{(x+1)!} \times \frac{x!}{\cancel{e^{-\lambda}}\lambda^{x}}$$

$$\frac{P(x+1)}{P(x)} = \frac{\cancel{\lambda^{x}} \times \lambda \times \cancel{x!}}{\cancel{x!}\,(x+1)!\,\cancel{\lambda^{x}}}$$

$$P(x+1) = \frac{\lambda}{x+1} P(x)$$

P:- A distributor determined from extensive tests with 5% of large bag of seeds will not germinate. He sells the seeds in packets of 200 and guarantees that 90% germination. Find the probability that the particular packet will violate the guarantees quality.

Sol

$n = 200$

$p = 5\% = 0.05$ (will not germinate) $q = 1 - 0.05 = 0.95$ (will germinate)

$np = \lambda$

$200 \times 0.05 = \lambda \Rightarrow \lambda = 10$

$\% \rightarrow 200 \rightarrow 200 \times \frac{90}{100} = 180$

90% of $200 = 180$

$\therefore 10\% = 20$

A packet will violate guarantees quality more than 20 germination seeds that is

$$P(X > 20) = 1 - P(X \le 20)$$

$$= 1 - \left[e^{-10} + e^{-10}10 + \frac{e^{-10}10^{2}}{2!} + \cdots + \frac{e^{-10} \times 10^{20}}{20!}\right]$$

P: The Average no of phone calls per minute coming into a switch board between 2pm to 4pm is 2.5. Find the probability that during one particular minute

(i) four or fewer

(ii) more than six calls

$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

Sol $\lambda = 2.5$

Average no of phone calls per minute is 2.5 (λ)

i) P(four or fewer calls) = $P(X \le 4)$

$= P(X=0) + P(X=1) + P(X=2) + P(X=3) + P(X=4)$

$$= \frac{e^{-2.5} \times 2.5^{0}}{} + e^{-2.5}(2.5) + \frac{e^{-2.5}(2.5)^2}{2} + \frac{e^{-2.5}(2.5)^3}{6} + \frac{e^{-2.5}(2.5)^4}{24} = 0.8912$$

ii) P(more than 6 calls) = $P(X>6)$

$= 1 - P(X \le 6)$

$$= 1 - e^{-2.5}\left[1 + 2.5 + \frac{(2.5)^2}{2} + \frac{(2.5)^3}{6} + \frac{(2.5)^4}{24} + \frac{(2.5)^5}{120} + \frac{(2.5)^6}{720}\right]$$

$= 0.0153$

P: If X and Y are independent poisson variates such that $P(X=1) = P(X=2)$ and $P(Y=2) = P(Y=3)$

Find i) mean of X, mean of Y

~~(ii) variance~~ ii) find variance of $X+2Y$

Sol $P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$

$P(X=1) = P(X=2)$

$\frac{\cancel{e^{-\lambda}}\,\lambda^{1}}{1!} = \frac{\cancel{e^{-\lambda}}\,\lambda^{2}}{2!}$

$1 = \frac{\lambda}{2} \Rightarrow \boxed{\lambda = 2}$

i) Mean of X is 2 $\qquad P(Y=y) = \frac{e^{-\lambda}\lambda^{y}}{y!}$

$P(Y=2) = P(Y=3)$

$\frac{\cancel{e^{-\lambda}}\,\cancel{\lambda^{2}}}{2!} = \frac{\cancel{e^{-\lambda}}\,\lambda^{3}}{3!}$

$\frac{\lambda}{6} = \frac{1}{2} \qquad \boxed{\lambda = 3}$

Mean of Y is 3

ii) $V(X+2Y) = V(X) + 2^2 V(Y)$

$= 2 + 2^2(3) = 2 + 4(3) = 14$

P:- X is a poisson variate with $3P(X=4) = \frac{1}{2}P(X=2) + P(X=0)$ then find

i) $P(X \leq 2)$ ii) Mean of X

Sol $3P(X=4) = \frac{1}{2}P(X=2) + P(X=0)$

$3 \times \frac{\cancel{e^{-\lambda}}\lambda^{4}}{4!} = \frac{1}{2} \times \frac{\cancel{e^{-\lambda}}\lambda^{2}}{2!} + \cancel{e^{-\lambda}}\lambda^{0}$

$\frac{\cancel{3}\lambda^{4}}{\cancel{24}_{8}} = \frac{\lambda^{2}}{4} + 1$

$\frac{\lambda^4}{8} = \frac{\lambda^2}{4} + 1 \Rightarrow \frac{\lambda^4}{8} - \frac{\lambda^2}{4} - 1 = 0$

$\lambda^4 - 2\lambda^2 - 8 = 0$ $\qquad \lambda^2 = x$

$(\lambda^2)^2 - 2\lambda^2 - 8 = 0$

$x^2 - 2x - 8 = 0$

$x^2 - 4x + 2x - 8 = 0$

$x(x-4) + 2(x-4) = 0$

$x = 4 \qquad x = -2$

$\lambda^2 = 4 \qquad \lambda^2 = -2$

$\lambda = \pm 2 \qquad \lambda = \pm\sqrt{2}\, i$

Mean of x is $\lambda = \pm 2$

if $\lambda = 2$

$P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$= e^{-2} + e^{-2} \times 2 + \frac{e^{-2} \times 2^2}{2!}$

$= 0.6767$ ✓

if $\lambda = -2$

$P(x \leq 2) = P(x=0) + P(x=1) + P(x=2)$

$= e^{2} + e^{2} \times 2 + \frac{e^{2} \times 2^2}{2} = 36.9453$ ✗

P:- Average no of accidents on a national highway is 1.6. Find the probability that the no of accidents are i) atleast one

ii) atmost one

Sol Average no of accidents on national highway per day $= \lambda = 1.6$

i) P(atleast 1) $= P(X \geq 1)$ $\quad P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$

$= 1 - P(X<1)$

$= 1-(P(X=0))$

$= 1-\left(\frac{e^{-1.6}(1.6)^0}{0!}\right)$

$= 1 - e^{-1.6} = 0.7981$

ii) P(atmost 1) $= P(X \leq 1)$

$= P(X=0) + P(X=1)$

$= e^{-\lambda} + e^{-\lambda}\lambda$

$= e^{-1.6} + e^{-1.6} \times 1.6 = 0.5249$

Q:- 2% of the items of a factory are defective. The items are packed in boxes. what is the probability that

i) two defective items

ii) atleast 3 defective items in a box of 100 items

Sol $p = 2\% = 0.02$ $\quad n=100$

$np = \lambda = 100 \times 0.02 = 2.$

i) P(2 defective items) $= P(X=2)$

$= \frac{e^{-2}2^2}{2!} = 0.2707$

ii) P(atleast 3 defective item) $= P(X \geq 3)$

$= 1 - P(X<3)$

$= 1-(P(X=0)+P(X=1)+P(X=2))$

$= 1 - \left[e^{-2} + e^{-2} \times 2 + \frac{e^{-2} \times 2^2}{2!} \right]$

$= 0.3233$

Fit a P.D to the following data

x	0	1	2	3	4	5	6	7
f	305	365	210	80	28	9	2	1

x	f	fx	$\frac{\lambda}{x+1}$	$F(x)$ Expected freq
0	305	0	1.201	300.9
1	365	365	$\frac{1.201}{2} = 0.6005$	361.3809
2	210	420	$\frac{1.201}{3} = 0.4003$	217.0092
3	80	240	$\frac{1.201}{4} = 0.3003$	86.8688
4	28	112	$\frac{1.201}{5} = 0.2402$	26.0867
5	9	45	$\frac{1.201}{6} = 0.2002$	6.2660
6	2	12	$\frac{1.201}{7} = 0.1716$	1.2545
7	1	7	$\frac{1.201}{8} = 0.1501$	0.2153
	$\Sigma f = N = 1000$	1201		999.9754 or 999.9814

RR for probabilities under PD :-

$$P(x+1) = \frac{\lambda}{x+1} P(x)$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

Mean of PD $= \lambda$

$$\text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{1201}{1000} = 1.2010 = \lambda$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$P(0) = e^{-1.201} = 0.3009$$

$$F(0) = N \times P(0) = 1000 \times 0.3009 = 300.9$$

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Q: Using Recurrence Relation find the probabilities when $x = 0,1,2,3,4,5$. If the mean of the PP is 3 ($\lambda = 3$)

Sol: RR of PD is

$$P(x+1) = \frac{\lambda}{x+1} P(x) \longrightarrow ①$$

$$P(x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

when $x = 0$ $P(0) = e^{-3} = 0.0498$

when $x = 0$ sub in ①

$$P(1) = \frac{\lambda}{x+1} P(0)$$

$$= \frac{3}{0+1} 0.0498 = \frac{3}{1} \times 0.0498 = \cancel{0.0747}$$

$$= 0.1494$$

when $x = 1$ sub in ①

$$P(2) = \frac{3}{2} \times P(1) = \frac{3}{2} \times 0.1494 = 0.2241$$

when $x = 2$ sub in ①

$$P(3) = \frac{3}{3} \times P(2) = 0.2241$$

When $x = 3$ sub in ①

$$P(4) = \frac{3}{4} \times P(3) = \frac{3}{4} \times 0.2241 = 0.1681$$

when $x = 4$ sub in ①

$$P(5) = \frac{3}{5} \times P(4) = \frac{3}{5} \times 0.1681 = 0.1009$$

Q: The distribution of typing mistakes committed by a typist is given below. Assuming the distribution is poisson find Expected ~~to~~ frequencies.

x	f	fx	Expected fre $\frac{\lambda}{x+1}$	Expect freq
0	125	0	~~110.3100~~	110.3700
1	95	95	$1/2$	110.3700
2	49	98	$1/3$	55.1850
3	20	60	$1/4$	18.3950
4	8	32	$1/5$	4.5988
5	3	15	$1/6$	0.9198
	$\Sigma f = 300$	300		299.8356

Mean of PD $= \lambda = \frac{\Sigma f x}{\Sigma f} = \frac{300}{300} = 1 = \lambda$

$p(x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$ $\quad p(0) = e^{-1} = 0.3679$

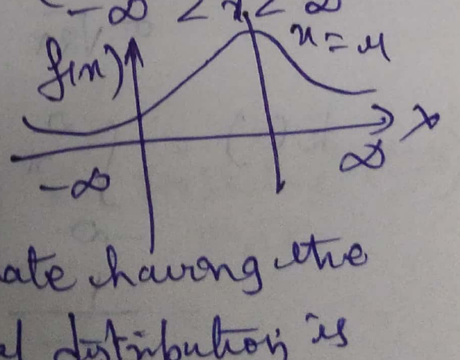
$F(0) = N \times p(0) = 300 \times 0.3679 = \cancel{110.3678}$ 110.3700

Normal distribution :- (ND)

A Random variable 'x' is said to have ND if it density function is given by $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\frac{1}{2}\left(\frac{x-\mu}{2}\right)^2}$

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \quad \begin{cases} -\infty < \mu < \infty \\ \sigma > 0 \\ -\infty < x < \infty \end{cases}$$

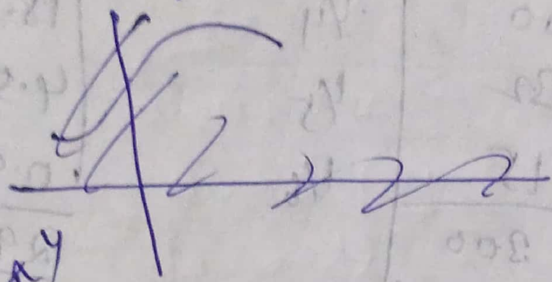
Normal probable curve:-

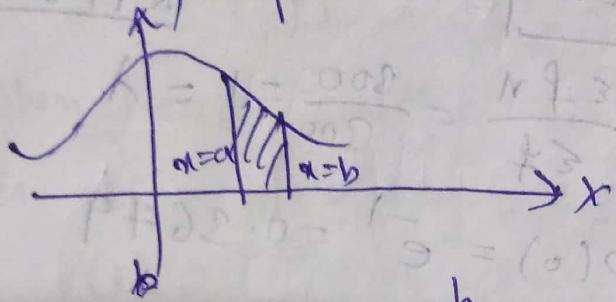


A Random variable x is said to have ND (or) Normal Variate having the curve representing the normal distribution is called a Normal probable curve. and the total area bounded by the curve and x-axis is unity

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

→ The area under the curve between the ordinates $x=a$ & $x=b$ above the x-axis represents the probability lies between a and b





$$P(a<x<b) = \int_a^b f(x)\,dx$$

<u>constants of N.D</u> :- (properties)

1) Mean of N.D is μ

<u>Proof</u> :- $E(x) = \text{Mean} = \int_{-\infty}^{\infty} x \cdot f(x)\,dx$

where $f(x)$ is pdf of N.D

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$E(x) = \int_{-\infty}^{\infty} x \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x\, e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

put $\frac{x-\mu}{\sigma} = z$

$$x-\mu = \sigma z$$

$$x = \sigma z + \mu$$

$dx = \sigma dz$ $= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z + \mu) e^{-z^2/2} \cancel{\sigma} dz$

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \sigma \underset{\text{odd}}{z} \underset{\text{even (always)}}{e^{-z^2/2}} dz + \frac{\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2} dz$$

WKT if $f(x)$ is even

$$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

if $f(x)$ is odd

$$\int_{-a}^{a} f(x)\,dx = 0.$$

And also

$E \times O = O$

$O \times E = O$

$E \times E = E$

$O \times O = E$

For to check odd

$f(-x) = -f(x)$

to check even

$f(-x) = f(x)$

$$= \frac{\sigma}{\sqrt{2\pi}} \times 0 + \frac{2\mu}{\sqrt{2\pi}} \int_{0}^{\infty} \underbrace{e^{-z^2/2}}_{} dz$$

standard values is $\sqrt{\frac{\pi}{2}}$

$$= \frac{2\mu}{\sqrt{2\pi}} \times \sqrt{\frac{\pi}{2}}$$

$= \mu$

2) Variance of ND is σ^2

WKT $V(x) = E(x^2) - (E(x))^2$ WKT $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$$

$$= \int_{-\infty}^{\infty} x^2 \times \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} x^2 e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$$

put $\frac{x-\mu}{\sigma} = z$

$$x-\mu = \sigma z$$
$$x = \sigma z + \mu$$
$$dx = \sigma dz$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{\infty} (\sigma z+\mu)^2 e^{-z^2/2} \cancel{\sigma} dz$$

$$= \frac{1}{\sqrt{2\pi}} \left[\int_{-\infty}^{\infty} \sigma^2 z^2 e^{-z^2/2} dz + \int_{-\infty}^{\infty} \mu^2 e^{-z^2/2} dz + \int_{-\infty}^{\infty} 2\sigma\mu z e^{-z^2/2} dz \right]$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z^2 e^{-z^2/2} dz + \frac{\mu^2}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-z^2/2} dz + \frac{2\sigma\mu}{\sqrt{2\pi}} \int_{-\infty}^{\infty} z e^{-z^2/2} dz$$

$$= \frac{\sigma^2}{\sqrt{2\pi}} 2 \int_{0}^{\infty} z^2 e^{-z^2/2} dz + \frac{2\mu^2}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-z^2/2} dz$$

put $\frac{z^2}{2} = t$ $\quad z = \sqrt{2t}$

$$\cancel{2} z\, dz = dt$$
$$dz = \frac{dt}{\sqrt{2t}}$$

$z=0 \quad t=0$

$z \to \infty \quad t \to \infty$

$$= \cancel{\frac{2\sigma^2}{\sqrt{2\pi}} \cancel{2} \int_0^\infty \sqrt{2t}\, e^{-t} \frac{dt}{\cancel{2}\sqrt{\cancel{t}}} \cancel{\sqrt{t}} + \frac{2\mu^2}{\sqrt{2\pi}}\sqrt{\frac{\pi}{2}}}$$

$$= \frac{2\sigma^2}{\sqrt{2\pi}} \cancel{2} \int_0^\infty \sqrt{2t}\, e^{-t}\, dt + \frac{2\mu^2}{\sqrt{2\pi}}\sqrt{\frac{\pi}{2}}$$

$$= \frac{2\sqrt{2}\,\sigma^2}{\sqrt{2\pi}} \int_0^\infty e^{-t}\, t^{1/2}\, dt + \mu^2$$ gama.

$$= \frac{2\sqrt{2}\,\sigma^2}{\sqrt{2\pi}} \int_0^\infty e^{-t}\, t^{3/2-1}\, dt + \mu^2$$

$$\Gamma(n) = \int_0^\infty e^{-x} x^{n-1}\, dx$$

$$\Gamma(n+1) = n\Gamma(n)$$

$$= \frac{2\sqrt{2}\,\sigma^2}{\sqrt{2\pi}} \Gamma(3/2) + \mu^2$$

$$\Gamma(3/2) = \Gamma(1/2 + 1) = \frac{1}{2}\Gamma(1/2) = \frac{1}{2}\sqrt{\pi}$$

$$= \frac{2\sqrt{2}\,\sigma^2}{\sqrt{2\pi}} \sigma^2 \times \frac{1}{2}\sqrt{\pi} + \mu^2$$

$$= \sigma^2 + \mu^2$$

$$E(x^2) = \sigma^2 + \mu^2$$

$$V(x) = E(x^2) - (E(x))^2$$

$$= \sigma^2 + \cancel{\mu^2} - \cancel{\mu^2} = \sigma^2$$

3) Mode of ND is μ.

→ Mode :- Mode is the value of x for which $f(x)$ is maximum

to find max $f(x)$
< less than - max
more than min
2 time differentiate
$f''(x)$

~~For~~ pdf of ND is

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}$$

$$\Rightarrow f'(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \frac{d}{dx}\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \times -\frac{1}{\cancel{2}} \times \cancel{2}\left(\frac{x-\mu}{\sigma}\right) \times \frac{1}{\sigma}$$

$$f'(x) = -\frac{1}{\sigma^3\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2}(x-\mu)$$

at $x=\mu$ $f'(x)=0$

$$f''(x) = -\frac{1}{\sigma^3\sqrt{2\pi}}\left[1 \times e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} + (x-\mu)\left(e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} \times -\frac{1}{2} \times 2\left(\frac{x-\mu}{\sigma}\right) \times \frac{1}{\sigma}\right)\right]$$

at $x=\mu$

$f''(x) < 0$

$\therefore$ $x = \mu$ is the mode of ND

iv) Median of ND is μ.

<u>Median</u>:- If 'μ' is the median of the distribution then it divides the complete probability into two parts. Therefore

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{\mu} f(x)\,dx + \int_{\mu}^{\infty} f(x)\,dx$$

i.e $\quad 1 = \frac{1}{2} + \frac{1}{2}$

consider $\int_{-\infty}^{\mu} f(x)\,dx = 1/2$

$$\int_{-\infty}^{\mu} f(x)dx + \int_{\mu}^{M} f(x)dx = \frac{1}{2} \rightarrow (1)$$

consider

~~again consi~~

$$\int_{-\infty}^{\mu} f(x)dx$$

where $f(x)$ is pdf of N.D

$$= \int_{-\infty}^{\mu} \frac{1}{\sigma\sqrt{2\pi}} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\mu} e^{-1/2\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

put $\frac{x-\mu}{\sigma} = z$

~~$x = \mu + z$~~

$dx = \sigma dz$

$x \rightarrow -\infty$

$z \rightarrow -\infty$

$x \rightarrow \mu$

$z \rightarrow 0$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{-\infty}^{0} e^{-z^2/2} \cancel{\sigma} dz \text{ (can be written as)}$$

$$= \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} e^{-z^2/2} dz$$

$$= \frac{1}{\sqrt{2\pi}} \sqrt{\frac{\pi}{2}} = \frac{1}{2}$$

sub $\int_{-\infty}^{\mu} f(x)dx = \frac{1}{2}$ in (1)

$$= \frac{1}{2} + \int_{\mu}^{M} f(x)dx = 1/2$$

$$= \int_{\mu}^{M} f(x)dx = 0$$

$M = \mu$ Median $= \mu$

Note :- Since mean, median, mode of N.D are same, hence ND is a symmetrical distribution

5) Area property of ND :-

$$P(a \le x \le b) = \int_a^b f(x)\,dx = \int_a^b \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

put $z = \frac{x-\mu}{\sigma}$ $\quad dx = \sigma\, dz$

where $x = a$ $\quad z = \frac{a-\mu}{\sigma} = c$

$x = b$ $\quad z = \frac{b-\mu}{\sigma} = d$

$$P(a \le x \le b) = \frac{1}{\sigma\sqrt{2\pi}} \int_a^b e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_c^d e^{-z^2/2}\, \cancel{\sigma}\, dz$$

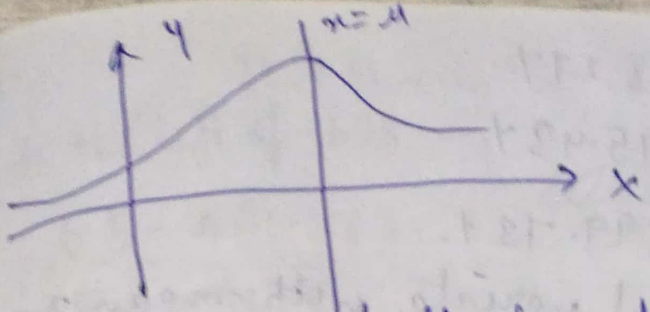
$$= \frac{1}{\sqrt{2\pi}} \int_c^d e^{-z^2/2}\, dz = P(c \le z \le d)$$

Note :- ① $z = \frac{x-\mu}{\sigma}$ is called standard normal variate and $\phi(z) = \frac{1}{\sqrt{2\pi}} \int_0^d e^{-z^2/2}\, dz$ is called the standard Normal probable integral.

② Area under normal curve is same as area under standard normal curve.

Chief characteristics of ND :

① The graph of ND $y = f(x)$ in x-y plane is known to be a normal curve.



(2) The curve is bell shaped curve and is symmetrical about the line $x = \mu$

(3) Area under normal curve represents the total population

(4) Mean, Median & Mode of the distribution coincide

(5) x-axis is an asymptote to the curve.

(6) Probability i.e

$P(a \le X \le b) = \int_a^b f(x)\, dx$

(7) Area under normal curve is distributed as follows

(i) Area of normal curve between $\mu - \sigma$ & $\mu + \sigma$ is 68.27%.

(ii) Area of normal curve between $\mu - 2\sigma$ & $\mu + 2\sigma$ is 95.43%.

(iii) Area of normal curve between $\mu - 3\sigma$ & $\mu + 3\sigma$ is 99.73%.

Chief characteristics of standard N D :-

(1) The N D for $\mu = 0$ & $\sigma = 1$ is known as standard N D

(2) It is having the same shape as that of Normal curve

(3) It is symmetrical about the line $x = 0$. (y-axis)

(4) Area under the standard normal curve is unity

(5) The Area under standard normal curve is distributed as follows:-

(i) -1 to +1 is 68.27%.

(ii) -2 to +2 is 95.43%.

(iii) -3 to +3 is 99.73%.

Pr If x is Normal variate with mean 30 & S.D 5 find the probabilities that

i) $26 \le X \le 40$ ii) $X \ge 45$

Sol Mean $\mu = 30$ $\sigma = 5$ (SD)

i) $P(26 \le X \le 40) =$

when $x_1 = 26$ then $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{26-30}{5} = -0.8$

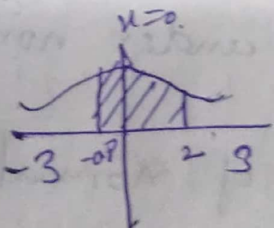
when $x_2 = 40$ then $z_2 = \frac{x_2 - \mu}{\sigma} = \frac{40-30}{5} = 2$

$p(26 \le X \le 40) = P(-0.8 \le z \le 2)$

$= A(2) + A(0.8)$

$= 0.4772 + 0.2881$

$= 0.7653$



ii) $P(X \ge 45)$

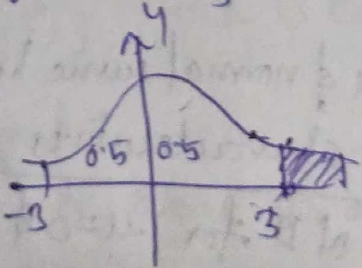
when $x_1 = 45$ then $z_1 = \frac{x_1 - \mu}{\sigma} = \frac{45-30}{5} = 3$

$P(X \ge 45) = P(z \ge 3)$

$= 0.5 - A(3)$

$= 0.5 - 0.4987$

$= 0.0013$



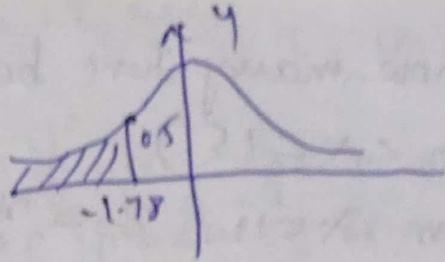
Pr If z is a standard normal variate find the area i) To the left of $z = -1.78$

ii) To the right of $z = -1.45$

iii) Corresponding to $-0.8 \le z \le 1.53$.

(iv) to the left of $z = -2.52$ & to the right of $z = 1.83$

Q) i) area to the left of $-1.78 =$

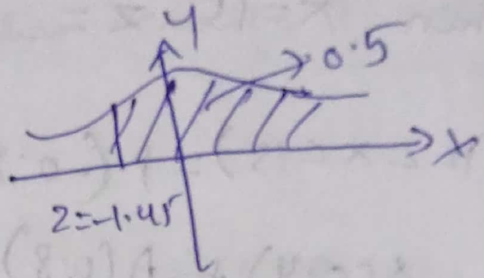


$0.5 - A(1.78)$

$= 0.5 - 0.4625$

$= 0.0375$

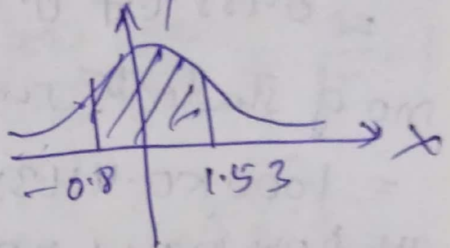
ii) area to the right of -1.45



$= 0.5 + A(1.45)$

$= 0.5 + 0.4265 = 0.9265$

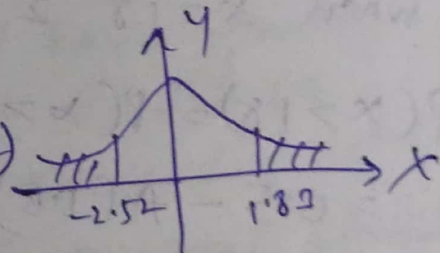
(iii) area corresponding to $-0.8 \le z < 1.53$



$= A(1.53) + A(0.8)$

$= 0.4370 + 0.2881 = 0.7251$

(iv) Area to the left of $z = -2.52$ & to the right of $z = 1.83$



$= 0.5 - A(1.83) + 0.5 - A(2.52)$

$= 0.5 - 0.4664 + 0.5 - 0.4941$

$= 0.0395$

1) In a sample of 1000 cases, the mean of certain test is 14 & SD is 2.5. Assuming that the distribution is normal (i) find how many students score between 12 & 15

(ii) how many score above 18

(iii) how many score below 18

Sol $\mu = 14$ $\sigma = 2.5$ $N = 1000$

(i) how many sone between 12 & 15

$P(12 < x < 15)$

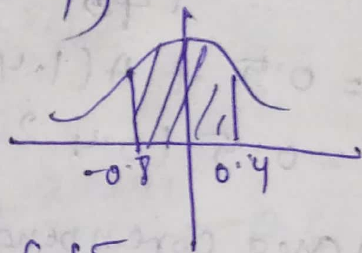
when $x = 12$ $\quad z = \dfrac{x-\mu}{\sigma} = \dfrac{12-14}{2.5} = -0.8$

when $x = 15$ $\quad z = \dfrac{x-\mu}{\sigma} = \dfrac{15-14}{2.5} = 0.4$

$P(12 < x < 15) = P(-0.88 < z < 0.4)$

$A(0.4) + A(0.8)$

$= 0.1554 + 0.2881 = 0.4435$



no. of students score between 12 & 15

$= 1000 \times 0.4435 = 443$

(ii) how many sone above 18

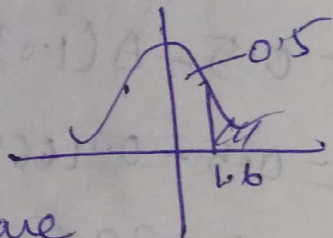
$P(x > 18)$

when $x = 18$ $\quad z = \dfrac{x-\mu}{\sigma} = \dfrac{18-14}{2.5} = 1.6$

$P(x > 18) = P(x > 1.6)$

$= 0.5 - A(1.6)$

$= 0.5 - 0.4452 = 0.0548$



no of stuctuts score chove 18 are

$= 1000 \times 0.0548 = 55$

(iii) how many srore at below 18

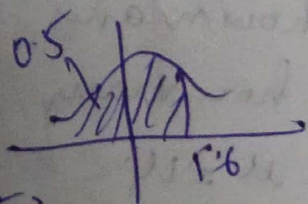
$P(x < 18)$

when $x = 18 \Rightarrow z = \dfrac{x-\mu}{\sigma} = \dfrac{18-14}{2.5} = 1.6$

$P(x < 18) = P(x < 1.6)$

$= A(1.6) + 0.5$

$0.4452 + 0.5 = 0.9452$



=

no. of students score below 17 are
= 1000 × 0.9452 = 945

Q: A manufacturer knows from experience that the resistance of resistors he produces is normal with mean 100 ohms & S.D 2 ohms & what percentage of resistors will have resistance between 98 & 102 ohms.

Sol: $\mu = 100 \quad \sigma = 2$

$P(98 < x < 102)$

when $x = 98$ $\quad z = \frac{x-\mu}{\sigma} = \frac{98-100}{2} = -1$

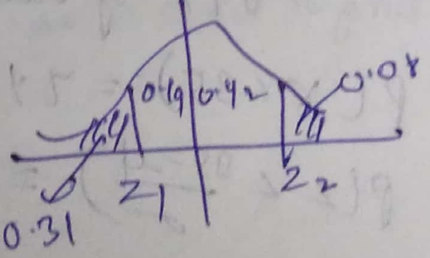
when $x = 102$ $\quad z = \frac{x-\mu}{\sigma} = \frac{102-100}{2} = \frac{2}{2} = 1$

$P(98 < x < 102) = P(-1 < x < 1)$
$= A(+1) + A(1)$
$= 0.3413 + 0.3413$
$= 0.6826$

What percentage of resistors will have resistance between 98 & 102 is $100 \times 0.6826 = 68.26\%$ ~~68~~

Q: In a ND 31% of the items are under 45 & 8% are over 64. Find mean & variance of distribution

Sol: ND 31% of the items under 45 &
8% are ~~are~~ over 64



$P(x < 45) = 0.31$

$P(z < \frac{45-\mu}{\sigma}) = 0.31$

$P(z < z_1) = 0.31$ where $z_1 = \frac{45-\mu}{\sigma}$

$-A(z_1) = 0.5 - 0.31 = 0.19$

$z_1 = -0.5$

$\frac{45-\mu}{\sigma} = -0.5$

$45-\mu = -0.5\sigma \rightarrow (1)$

$P(x>64) = 8\% = 0.08$

$P(z > \frac{64-\mu}{\sigma}) = 0.08$

$P(z>z_2) = 0.08$ where $z_2 = \frac{64-\mu}{\sigma}$

$A(z_2) = 0.5 - 0.08$
$= 0.42$

$z_2 = 1.41$

$\frac{64-\mu}{\sigma} = 1.41$

$64-\mu = 1.41$

$64-\mu = 1.41\sigma \rightarrow (2)$

solving (1) & (2)

$\sigma^2 = (9.9)^2$ $\mu = 49.9$

Pr In a N.D 7% of the items are under 35 and 89% are under 63. find mean & SD of the SD of the distributi

Sol :- 7% of items are under 35
89% of items are under 63

$P(x<35) = 7\% = 0.07$

$P(z < \frac{35-\mu}{\sigma}) = 0.07$

$P(z<z_1) = 0.07$ when $z_1 = \frac{35-\mu}{\sigma}$

$A(z_1) = 0.5 - 0.07 = 0.43$

$z_1 = -1.48$

-9
-49.9

$$\frac{35-\mu}{\sigma} = -1.48$$

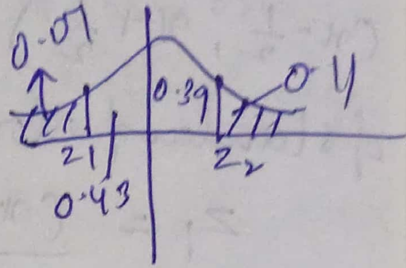
$$35-\mu = -1.48\sigma \rightarrow (1).$$

It is given that 89% of the items are under 63

i.e

$P(X \le 63) = 89\% = 0.89$

$\therefore P(X > 63) = 1 - 0.89 = 0.11$



$P\left(Z > \frac{63-\mu}{\sigma}\right) = 0.11$

$P(Z > z_2) = 0.11$ when $z_2 = \frac{63-\mu}{\sigma}$

$A(z_2) = 0.5 - 0.11 = 0.39$

$z_2 = 1.23$

$$\frac{63-\mu}{\sigma} = 1.23$$

$$63-\mu = 1.23\sigma \rightarrow (2)$$

~~0.09~~ ~~4.86~~.

$\mu = 50.2915$

$\sigma = 10.3324$

Solving (1) & (2)

Normal approximation to Binomial Distribution:-

The ND can be used to approximate the B.D. for large values of n the calculation of binomial probability is difficult, in such cases the binomial curve can be replaced by the normal curve and used to find the required probability

Case-(i):- when $p = q = \frac{1}{2}$

$np = \mu$ & $\sigma = \sqrt{npq}$

Hence the corresponding standard normal variate is

$$Z = \frac{x-\mu}{\sigma} = \frac{X-np}{\sqrt{npq}} \sim N(0,1)$$

Case (ii): when $p \neq q$ for large values of n, we can approximate the binomial Distribution by N.D. For this the real class interval will be $(x-\frac{1}{2}, x+\frac{1}{2})$ Hence the lower & upper limits of standard Normal variate are

$$z_1 = \frac{(x-\frac{1}{2})-np}{\sqrt{npq}} \qquad z_2 = \frac{(x+\frac{1}{2})-np}{\sqrt{npq}}$$

Q: Find the probability of getting an even number on face 3 to 5 times in throwing 10 dice together

Sol $n=10$ p = prob of throwing an even number

$$= \frac{1}{6}+\frac{1}{6}+\frac{1}{6} = \frac{3}{6} = \frac{1}{2}$$

$q = 1-\frac{1}{2} = \frac{1}{2}$

here $p=q=\frac{1}{2}$

$P(3 \le x \le 5)$ when $x=3$ $z = \frac{x-np}{\sqrt{npq}}$

$np = 10 \times \frac{1}{2} = 5$ $\quad npq = 10 \times \frac{1}{2} \times \frac{1}{2} = 2.5$

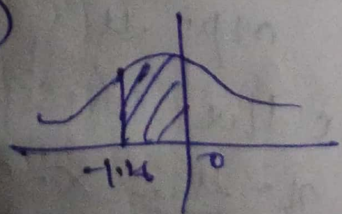
$z = \frac{3-5}{\sqrt{2.5}} = -1.26$

when $x=5$, $z = \frac{x-np}{\sqrt{npq}} = \frac{5-5}{\sqrt{2.5}} = 0.$

$P(3 \le x \le 5) = P(-1.26 \le z \le 0)$

$= A(1.26)$

$= 0.3962$



P) find the probability that by guess work a student can correctly answer 25 to 30 questions in a multiple choice quiz consisting of 80 questions. Assume that in each question with four choices, only one choice is correct! And student has no knowledge on the subject.

Sol/ Given $n = 80$.

p (in each question with four choices) $= \frac{1}{4}$

$q = \frac{3}{4}$

$p \neq q$ (case-ii)

we need to find $P(25 \leq x \leq 30)$

$np = 80 \times \frac{1}{4} = 20$ $\quad npq = 80 \times \frac{1}{4} \times \frac{3}{4} = 15$

$\sqrt{npq} = 3.87$

$P(25 \leq x \leq 30) =$

when $x = 25$, $z_1 = \frac{(x - \frac{1}{2}) - np}{\sqrt{npq}} = \frac{(25 - \frac{1}{2}) - 20}{3.87}$

$= 1.16$

when $x = 30$, $z_2 = \frac{(x + \frac{1}{2}) - np}{\sqrt{npq}} = \frac{(30 + \frac{1}{2}) - 20}{3.87}$

$= \cancel{1.42}\ 2.71$

$P(25 \leq x \leq 30) = P(1.16 \leq z \leq 2.71)$

$= A(2.71) - A(1.16)$

$= 0.4966 - 0.3770 = 0.1196$