Unit-1 - Basic Probability

Probability & Algebra

Random experiment: An experiment conducted repeatedly under the same conditions is called Random experiment

eg: Tossing a coin 15 times
Tossing a dice 30 times

Trial: Each performance of the random experiment is called trial.

→ Trials under the same conditions is called random experiment

Outcome: Result of a trial is called outcome

Sample space: The set of all possible outcomes of a random experiment is called sample space.

eg: when we toss a single coin sample space consists of $S = \{H, T\}$

when we toss two coins at a time sample space consists of $S = \{HH, HT, TH, TT\}$

Event: Subset of sample space is called event

eg: let $S = \{HH, HT, TH, TT\}$

E_1 = all heads = $\{H, H\}$

E_2 = atleast one Head = $\{HH, HT, TH\}$

Mutually exclusive events: Two events are said to be mutually exclusive if the happening of anyone event prevents the happening of any other events. i.e no two or more events can happen simultaneously in the same trial.

Equally likely events: If the preference of one event cannot expect the preference of other event than the two events are said to be equally likely / when there is no reason to expect any one of them happens rather than any one of the others can happen.

eg: when a card is drawn from a pack any card may obtained in this case all the 52 cards are happened equally likely.

Exhaustive events: Possible events in any random experiment are known as exhaustive events. i.e total no. of elements in a sample space are called exhaustive events.

eg: In tossing a coin there are two exhaustive events can be head & tail eg: In throwing a dice there are 6 exhaustive events namely $\{1, 2, 3, 4, 5, 6\}$

Probability: In a random experiment if there are n mutually exclusive and eqully likely events let E be any event of the experiment & if m elementary events from the event E then the Probability of E can be defined as:

$$P(E) = \frac{m}{n} = \frac{\text{no. of elementary events}}{\text{Total no. of events in the random exp}}$$

$$= \frac{\text{no. of favorable chances}}{\text{Total no. of chances}}$$

Prob

1. If three coins are tossed find the Probability of getting ① three heads ② two heads ③ no heads

Sol: Sample space $S = \{HHH, HHT, HTH, THH, HTT, THT, TTH, TTT\} = 2^3$

① Probability of 3 heads

$P(3\text{ heads}) = \frac{1}{8}$

② $P(2\text{ heads}) = \frac{4}{8} = \frac{1}{2}$

③ $P(\text{no heads}) = \frac{1}{8}$

2. Two cards are drawn at random from 10 cards numbered 1 to 10. find the Probability that the sum is even if

(1) two cards are drawn together

(2) Two cards are drawn one after the other with replacement

(1) No. of exhaustive cases $= 10C_2$

i.e selecting two cards together out of 10 cards. The sum is even if both are odd or both are even

→ Their are 5 odd and 5 even cards

Both are even cases $= 5C_2$

Both are odd cases $= 5C_2$

required Probability of getting sum is even if two cards are drawn together $= \frac{5C_2 + 5C_2}{10C_2} = \frac{10+10}{45} = 0.44$

(2) Two cards are drawn one after other with replacement

Total no. of chances $= (10C_1)(10C_1)$

no. of possible cases for both are even $= 5C_1 \times 5C_1$

no. of possible cases for both are odd $= 5C_1 \times 5C_2$

required Probability $= \frac{5C_1 \times 5C_1 + 5C_1 + 5C_1}{(10C_1)(10C_1)}$

$= \frac{50}{100} = 0.5$

3. Out of 15 item 4 are not in good condition 4 are selected at random find the probability that

(1) all are not good

(2) 2 are not good

(1) no of ways of selecting 4 items out of 15 items $= 15C_4$

suppose 4 items of selected all are not good. No of ways of such selection $= 4C_4$

$\therefore$ Probability that all are not good $= \frac{4C_4}{15C_4} = 0.0007$

(2) no. of selecting two items are good and remaining are not good $= 4C_2 \times 11C_2$

$$\text{Probability} = \frac{4C_2 \times 11C_2}{15C_4} = 0.2418$$

4. A class consists of 6 girls and 10 boys, if a commity of 3 are choosen at random in a class. find the Probability that

(1) three boys are selected

(2) exactly two girls are selected

Sol

No. of ways of selecting 3 students out of 16 students $= 16C_3 = 560$

(1) 3 boys are selected out of 10 boys $= 10C_3 = 120$

Probability of selecting 3 boys $= \frac{10C_3}{16C_3} = 0.2143$

(2) exactly two girls are selected out of 6 girls and remaining one boy is selected out of 10 boys $= 6C_2 \times 10C_1 = 150$

Probability of getting Exactly 2 girls: $\frac{6C_2 \times 10C_1}{16C_3} = 0.2679$

5. A, B, C in order toss a coin the first one to toss a head wins a game what are their probabilities of winning assuming that the game continues indefinitly

Sol A wins the game if he gets head first & he may win the game in the first round with Probability 1/2

He may win in the 2nd round after he failed in the first round and also B&C with the Probability

$\left(\frac{1}{2}\right)^3 \times \frac{1}{2}$

In the third round he may win with Probability $\left(\frac{1}{2}\right)^6 \times \frac{1}{2}$

A	B	C
$\frac{1}{2}$	$\frac{1}{2} \times \frac{1}{2}$	$\left(\frac{1}{2}\right) \times \frac{1}{2} \times \frac{1}{2}$
$\left(\frac{1}{2}\right)^3 \times \frac{1}{2}$	$\left(\frac{1}{2}\right)^4 \times \frac{1}{2}$	$\left(\frac{1}{2}\right)^5 \times \frac{1}{2}$
$\left(\frac{1}{2}\right)^6 \times \frac{1}{2}$		

$P(H) = \frac{1}{2}$

$P(T) = \frac{1}{2}$

$P(A) = \left(\frac{1}{2} + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^7 \ldots\ldots\right]$

$= \frac{1}{2}\left[1 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^6 + \ldots\right]$

G.S with ratio $= \frac{1}{8}$

$= \frac{1}{2}\left[\frac{1}{1-\frac{1}{8}}\right]$

$= \frac{1}{\cancel{2}} \times \frac{\cancel{8}}{7} = \frac{4}{7}$

$P(B) = \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^5 + \left(\frac{1}{2}\right)^8$

$= \left(\frac{1}{2}\right)^2\left[1 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^6 \ldots\right]$

$=$ G.S with ratio $= \frac{1}{8}$

$= \frac{1}{4}\left[\frac{1}{1-\frac{1}{8}}\right] = \frac{1}{\cancel{4}} \times \frac{\cancel{8}^2}{7} = \frac{2}{7}$

$P(C) = \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^9$

$= \left(\frac{1}{2}\right)^3\left[1 + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^3 \ldots\right]$

G.S with ratio $= \frac{1}{8}$

$= \frac{1}{8}\left[\frac{1}{1-1/10}\right]$

$= \frac{1}{8} \times \frac{8}{7} = \frac{1}{7}$

$P(E) = \frac{1}{7}$

6. A & B throws a pair of dice A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6 if A begins the game and the game continuous indefinetly find the chances of winning the game by A & B.

Sol:

If we throw a pair of fair dice total chances $= 6^2 = 36$

sum 6 = (1,5) (2,4) (3,3) (4,2) (5,1)

$P(\text{sum } 6) = \frac{5}{36}$

$P(\text{not getting sum } 6) = 1 - \frac{5}{36} = \frac{31}{36}$

sum 7 = (1,6) (2,5) (3,4) (4,3) (5,2) (6,1)

$P(\text{sum } 7) = \frac{6}{36}$

$P(\text{not getting sum } 7) = 1 - \frac{6}{36} = \frac{30}{36}$

A

B

$\frac{5}{36}$

$\frac{31}{36}\times\frac{6}{36}$

$\frac{31}{36}\times\frac{30}{36}\times\frac{5}{36}$

$\left(\frac{31}{36}\right)^2\times\frac{30}{36}\times\frac{6}{36}$

$\left(\frac{31}{36}\right)^2\times\left(\frac{30}{36}\right)^2\times\frac{5}{36}$

$\left(\frac{31}{36}\right)^3\times\left(\frac{30}{36}\right)^2\times\frac{6}{36}$

$$P(A) = \frac{5}{36}+\frac{31}{36}\times\frac{30}{36}\times\frac{5}{36}+\cdots$$

$$= \frac{5}{36}\left[1+\frac{31}{36}\times\frac{30}{36}+\left(\frac{31}{36}\right)^2\times\left(\frac{30}{36}\right)^2+\cdots\right]$$

$$= \frac{5}{36}\times\left[\frac{1}{1-\left(\frac{31\times30}{(36)^2}\right)}\right] = 0.4918$$

$$P(B) = \left(\frac{31}{36}\right)^2\times\frac{30}{36}\times\frac{6}{36}+\frac{31}{36}\times\frac{6}{36}\cdots$$

$$= \frac{36}{36}\times\frac{6}{36}\left[1+\left(\left(\frac{31}{36}\times\frac{30}{36}+\left(\frac{31}{36}\right)^2\left(\frac{30}{36}\right)^2\cdots\right)\right]\right]$$

$$= \frac{31}{36}\times\frac{6}{36}\left[\frac{1}{1-\frac{31}{36}\times\frac{30}{36}}\right] = 0.5082$$

Axioms of Probability.

Let S be a finite sample space & E is any arbitrary event defined on sample space S the following 3 axioms are hold by any subset defined on S

① Axioms of Positivity – $0 \le P(E) \le 1$

② Axiom of certainity – $P(S) = 1$

③ " " union – If A, B are mutually exclusive M.E events then Probability of $P(A \cup B) = P(A) + P(B)$

Theorm (1) – $P(\phi) = 0$

Let S is the sample space ~~with~~ which is the set of all outcomes of a random experiment also $S' = \phi$

W.K.T $P(S') = 1 - P(S)$

i.e $P(S') = 0$

$P(\phi) = 0$

Theorm (2) – $P(A) = 1 - P(A')$

Proof Let A be any event defined on a sample space S. W.K.T $A \cap A' = \phi$

$A \cup A' = S$

$P(A \cup A') = P(A) + P(A')$

$P(S) = P(A) + P(A')$

$1 = P(A) + P(A')$

$P(A) = 1 - P(A')$

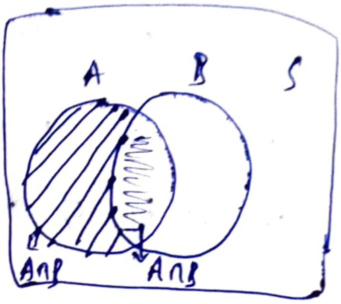
Theorm(3) - $P(A \cap B') = P(A) - P(A \cap B)$

$\phi = (A \cap B') \cap (A \cap B)$

$(A \cap B') \cup (A \cap B) = A$

$P(A) = P(A \cap B') + P(A \cap B)$

$P(A \cap B') = P(A) - P(A \cap B$



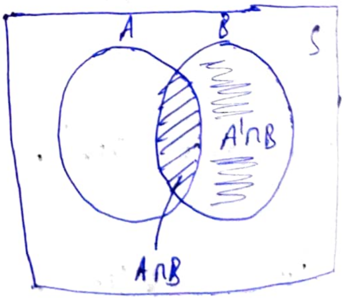
let A, B two events defined on a sample space S

A can be divided into two mutually exclusive events

$(A \cap B')$ & $(A \cap B)$

Theorm(4) - $P(A' \cap B) = P(B) - P(A \cap B)$

B can be divided into two disjoint subsets or two mutually exclusive events



$A \cap B$ and $A' \cap B$

$B = (A \cap B) \cup (A' \cap B)$ — ①

$(A \cap B) \cap (A' \cap B) = \phi$

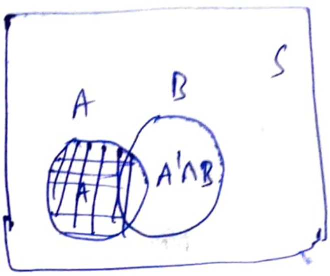
Taking Probabilities on b.s to ①

$P(B) = P(A \cap B) + P(A' \cap B)$ [From A_3]

$P(A' \cap B) = P(B) - P(A \cap B)$

Theorm(5):- Addition theorm for two events

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$



~~(AUB can be divided into)~~

Proof Let A & B are any two events define on a sample space S

AUB can be divided into two mutually exclusive event

One is A & other is $A' \cap B$

$\therefore A \cup (A' \cap B) = A \cup B$ ←①

$A \cap (A' \cap B) = \phi$

Taking Probability on b.s to ①

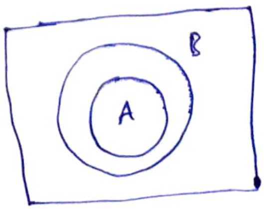
$P(A \cup (A' \cap B)) = P(A \cup B)$

$P(A \cup B) = P(A) + P(A' \cap B)$ [$\because A \cap (A' \cap B) = \phi$ from A_3]

$= P(A) + P(B) - P(A \cap B)$ [from Th:4]

Theorm (6):- If A is containing B then A is subset of B

then $P(A) \leq P(B)$ ~~if A is subset~~



A & B are any two events of sample space S since A is subset of B

$A \cap B = A$

from the above theorm $P(A' \cap B) = P(B) - P(A \cap B)$

$$P(A' \cap B) = P(B) - P(A)$$

since $A \subseteq B$ also from A,

$\boxed{\subseteq \rightarrow \text{Subset}}$

$P(B) \geq P(A)$

$P(A) \leq P(B)$

[Let $P(A \cap B) = 1/2$

$P(B) = 3/4$

$P(A) = 1/4$]

$P(B) \geq P(A)$

Probm

1. 3 students A, B, C are in a running race A & B have same Probability of winning and each is twice as likely too win of C. find the Probability that B or C wins the race

sol Three students A, B, C are participating in a running race & S is the sample space containing A, B, C

$\therefore$ $A \cup B \cup C = S$

given $P(A) = P(B)$

$P(A) = P(B) = 2P(C)$

$P(S) = P(A \cup B \cup C)$

$1 = P(A) + P(B) + P(C)$ [$\because$ Total Prob = 1]

$P(A) + P(A) + \frac{1}{2} P(A) = 1$

$P(A)\left(1 + 1 + \frac{1}{2}\right) = 1$

$P(A)\left(\frac{5}{2}\right) = 1$, $\Rightarrow P(A) = \frac{2}{5}$

$P(B) = \frac{2}{5}$

$P(C) = \frac{1}{5}$

Prob of B or C wins the race

$= P(B \cup C)$

$= P(B) + P(C)$

$= \frac{2}{5} + \frac{1}{5} = \frac{3}{5}$

2. A card is drawn from a well shuffled deck of cards what is the Probability that it is either spade or an ace only one card

A be a event of spade cards

Sol Probability of getting a spade card =

$P(A) = \frac{13}{52}$

B be an event of Ace cards

Probability of getting a ace card

$P(B) = \frac{4}{52}$

One ace is a spade Ace $= \frac{1}{52} = P(A \cap B)$

$\therefore$ By addition theorm

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Probability of getting spade or ace card

$$= \frac{13}{52} + \frac{4}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

Note: ~~Adding~~ Addition theorm for 3 events:

→ Let A B C are any 3 events defined on a sample space S then $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(A \cap C) + P(A \cap B \cap C)$

Conditional event: If A, B are events of a sample space S and if B happened after happening of an event A then the event of happening of B after A is called C.E. It is denoted by A/B, B/A (given)

Conditional Probability: If A & B are two events in a sample space S & $P(A) \neq 0$ then the conditional probability of B after the event A is happened is given by $P(B|A) = \frac{P(B \cap A)}{P(A)}$ $[\because P(A) \neq 0]$

Similarly the conditional probability of A given B is

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

(or)

C.P is defined as finding the probability of any event if the C.P is given

Multiplication theorm of Probability: In a random experiment if A, B are two events such that $P(A) \neq 0$, $P(B) \neq 0$

then $P(A \cap B) = P(A/B)\,P(B)$

$P(B/A)\,P(A)$

Proof: Let S be the sample space Associated with the random experiment let A, B be two events of S such that $P(A) \neq 0$, $P(B) \neq 0$ then by definition of conditional Probability $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

also $P(B/A) = \dfrac{P(B \cap A)}{P(A)}$

Similarly $P(A \cap B) = P(B \cap A)$

$P(A \cap B) = P(A/B)\,P(B)$

$= P(B/A)\,P(A)$

Independent events: Two events are said to be independent if happening of one event does not depending on happening of other event the events are said to be dependent.

Prob

1. three machines 1, 2, 3 Produces 40%, 30%, 30% of the total no. of items of factory. The Percentage of defective items are 4%, 2%, 3% respectively. If an item is selected at random find the Probability that the selected item is defective

Sol Let A, B, C are three events that the machines 1, 2, 3 be chosen for Production

Let D be the event which denotes the defective item.

Given that $P(A) = 40\% = 0.4$

$P(B) = 30\% = 0.3$

$P(C) = 30\% = 0.3$

The % of defective items of the machines given by

$P(D|A) = 4\% = 0.04$

$P(D|B) = 2\% = 0.02$

$P(D|C) = 3\% = 0.03$

$P(D) = P(A \cap D) + P(B \cap D) + P(C \cap D)$

$= P(A)\,P(D|A) + P(B)\,P(D|B) + P(C)\,P(D|C)$

$= 0.4 \times 0.04 + 0.3 \times 0.02 \times 0.3 \times 0.03$

$= 0.0310$

2. In a class 2% of boys & 3% of girls having blue eyes there are 30% girls in class if a student is selected in random what is the probability that the student having blue eyes

Sol Let A ~~be~~ denote the event of selected student is a girl

Let B " " " " is a boy

Let X be an event that student having blue eyes

$P(A) = 30\% = 0.3$

$P(B) = 70\% = 0.7$

also given

$P(X|A) = 0.03$

$P(X|B) = 0.02$

The selected student is a blue eyes

$P(X) = P(A)P(X|A) + P(B)P(X|B)$

$= 0.3 \times 0.03 + 0.7 \times 0.02$

$= 0.0230$

3. A Problem in a subject is given to three students whose chances of solving it are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively what is the probability that the prob is solved

Sol A B C are solving independently

$P(A) = \frac{1}{2}$ $P(A') = 1 - \frac{1}{2} = \frac{1}{2}$

$P(B) = \frac{1}{3}$ $P(B') = 1 - \frac{1}{3} = \frac{2}{3}$

$P(C) = \frac{1}{4}$ $P(C') = 1 - \frac{1}{4} = \frac{3}{4}$

Probability of solving Problem

$P(A \cup B \cup C) = 1 - P(A \cup B \cup C)'$ (Th : 2)

$= 1 - [P(A' \cap B' \cap C')]$ [demorgan law]

$= 1 - P(A')P(B')P(C')$ [independent law)

$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4}$

$= 0.75$

4. The probability that india wins a cricket match against W.I is $\frac{2}{3}$. In IN & W.I play 3 test matches what is the Prob that

① IN will lose all the three matches

② IN will will atleast 1 test match

③ IN will win atmost 1 match

Sol ~~(Let A, B, C be the three events of India and W.I)~~
~~(matches)~~

~~(Probability that India winning match)~~

Let A be the event of India wins against West Indies

$P(A) = \frac{2}{5}$ $\qquad P(A') = 1 - \frac{2}{5} = \frac{3}{5}$

① P(lose all matches) = $P(A')P(A')P(A')$

$= \frac{3}{5} \times \frac{3}{5} \times \frac{3}{5}$

$= 0.2160$

② P(atleast 1 match)

~~$(P(X \geq 1) = 1 - P(X=0))$~~ 1 − Probability (loose all matches)

$= 1 - \frac{27}{125} = 0.7840$

③ P(win at most one match) =

Probability that India will win + Probability that India will win 1 match = $\frac{27}{125} + P(A \cap A' \cap A') +$

$P(A' \cap A \cap A') + P(A' \cap A' \cap A)$

$= \frac{27}{125} + P(A) \times P(A') \times P(A') + P(A') \times P(A) \times P(A') + P(A') \times P(A') \times P(A)$

$= \frac{27}{125} + 3 \times \frac{2}{5} \times \left(\frac{3}{5}\right)^2 = 0.6480$

Note: ① If the happening of an event A is not affected by the happening of event B then A & B are said to be not affected

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A)P(B)}{P(B)} = P(A)$$

$$P(B/A) = P(B)$$

② Three events A, B, C are said to be pairwise independent events if $P(A \cap B) = P(A)P(B)$

$P(B \cap C) = P(B)P(C)$

$P(A \cap C) = P(A)P(C)$

Prob:

1. Among the workers in a factory only 50% received the bonus. Among those who receiving the bonus 30% are skilled what is the probability of randomly selected worker who is skilled and receiving bonus

Sol Let A denotes the event of receiving bonus

B denotes the event of considering skilled workers

$P(A) = 50\% = \frac{1}{2}$

Probability of skilled persons receiving the bonus

$P(B/A) = 30\% = \frac{30}{100} = 0.3$

Prob of selected worker skilled & receiving the bonus

$$P(B \cap A) = P(B/A)\,P(A)$$

$$= 0.33 \times 0.5 = 0.1650$$

2. (1) Find Probability of $P(B|A)$

(2) $P(A|B')$

$P(A) = \frac{1}{3}$ $\quad P(B) = \frac{1}{4}$ $\quad P(A \cup B) = \frac{1}{2}$

(1) $$P(B/A) = \frac{P(B \cap A)}{P(A)}$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{1}{2} = \frac{1}{3} + \frac{1}{4} - P(A \cap B)$$

$$P(A \cap B) = \frac{1}{3} + \frac{1}{4} - \frac{1}{2} = 0.08333 = \frac{1}{12}$$

(2) $$P(A/B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A) - P(A \cap B)}{P(B')}$$

$$= \frac{\frac{1}{3} - \frac{1}{12}}{\frac{3}{4}} = 0.3 = \frac{1}{3}$$

3. Two aeroplanes bomb a target in succession the probability of each correctly hit the target is 0.3 & 0.2 respectively. The 2nd will bomb only if the first misses the target. Find the probability that

(1) Target is Hit

② Both fails to Hit

Sol

Let A is an event of first plane hits the target

B is the event of 2nd plane hits the target

$P(A) = 0.3$ $\quad$ $P(A') = 0.7$

$P(B) = 0.2$ $\quad$ $P(B') = 0.8$

P(A hits or (A fails and B hits))

① $P(\text{target hits}) = P(A \cup (A' \cap B))$

$= P(A) + P(A' \cap B)$

$= P(A) + P(A')P(B)$

$= 0.3 + 0.7 \times 0.2$

$= 0.44$

② $P(\text{both fails to hit}) = P(A' \cap B')$

$= P(A') \times P(B')$

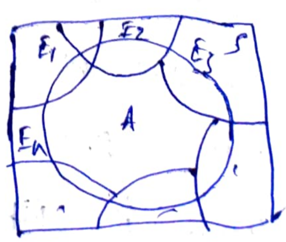
$= 0.7 \times 0.8 = 0.56$

Baye's theorm:

Statement: Suppose $E_1, E_2, \ldots E_n$ are n mutually exclusive events of a sample space S $P(E_i) > 0$ for $i = 1, 2 \ldots n$ and A is any arbitrary event of S such that $P(A) > 0$ & $A \subseteq \bigcup_{i=1}^{n} E_i$ then
$$P(E_i|A) = \frac{P(A|E_i)P(E_i)}{\sum_{i=1}^{n} P(A|E_i)P(E_i)}$$

Proof:

Let $E_1, E_2, \ldots E_n$ are n mutually exclusively events defined on a sample space S



also given A is any arbitrary event defined on S

$\therefore$ A is subset of S $(A \subseteq S)$

$$A \subseteq \bigcup_{i=1}^{n} E_i$$

$$A \subseteq (E_1 \cup E_2 \cup \cdots \cup E_n)$$

$$A \cap \left(\bigcup_{i=1}^{n} E_i\right) = A$$

$$A \cap (E_1 \cup E_2 \cup \cdots \cup E_n) = A$$

$$(A \cap E_1) \cup (A \cap E_2) \cup \cdots \cup (A \cap E_n) = A$$

since $A \cap E_i$ are mutually exclusive

$$(A \cap E_1) \cap (A \cap E_2) \cap \cdots \cap (A \cap E_n) = \phi$$

also $(A \cap E_1) \cup (A \cap E_2) \cup \cdots \cup (A \cap E_n) = A$

applying probabilities on both sides

$$P((A \cap E_1) \cup (A \cap E_2) \cup \cdots \cup (A \cap E_n)) = P(A)$$

$$P(A \cap E_1) + P(A \cap E_2) + \cdots + P(A \cap E_n) = P(A) \quad [\text{from } A_3]$$

$$P(A) = \sum_{i=1}^{n} P(A \cap E_i)$$

$$P(A) = \sum_{i=1}^{n} P(A \mid E_i) P(E_i) \quad [\text{from multiplication Th}]$$

But def of conditional probability

$$P(E_i \mid A) = \frac{P(E_i \cap A)}{P(A)}$$

$$= \frac{P(E_i) P(A \mid E_i)}{P(A)}$$

$$= \frac{P(E_i) P(A \mid E_i)}{\sum_{i=1}^{n} P(A \mid E_i) P(E_i)} \quad [\text{from } ①]$$

Prob

1. In a bolt factory ~~missions~~ Machines A, B, C manufactures 20%, 30%, 50% of their total O/P and 6%, 3%, 2% are defective. A bolt is drawn at random and is found to be defective what is the Probability that it was manufactured by machine B

Sol: Let X be the defective bolt manufactured by machines A, B, C

Probability of Producing bolts from machine A

$P(A) = 20\% = \frac{20}{100} = 0.2$

Prob of Bolts Produced by machine B

$P(B) = 30\% = \frac{30}{100} = 0.3$

Prob of bolts Produced by machine C

$P(C) = 50\% = \frac{50}{100} = 0.5$

$P(X|A) = 6\% = 0.06$

Prob of defective bolts from machine B, A, C

$P(X|B) = 3\% = 0.03$

$P(X|C) = 2\% = 0.02$

From Baye's theorm Prob of randomly chosen bolt is defective & it was manufactured by machine B is

$$P(B/X) = \frac{P(B)\,P(X/B)}{P(A)P(X|A) + P(B)P(X|B) + P(C)\,P(X|C)}$$

$$= \frac{0.3 \times 0.03}{0.2\times 0.06 + 0.3\times 0.03 + 0.5\times 0.02} = 0.2903$$

2. Companies B_1, B_2, B_3 Produces 30%, 45%, 25% of the cars respectively it is known that 2%, 3%, 2% of these Cars are defective

① What is the Probability that a car purchased is defective

② If a car purchased and is defective what is the Probability that this car produced by the company B_1

sol let X be the ~~defective~~ event that the items produced by the company B_1, B_2, B_3 is defective

$P(B_1) = 30\% = 0.3$

$P(B_2) = 45\% = 0.45$

$P(B_3) = 25\% = 0.25$

Prob that defective car produced by company B_1, B_2, B_3

$P(X|B_1) = 2\% = 0.02$

$P(X|B_2) = 3\% = 0.03$

$P(X|B_3) = 2\% = 0.02$

① Prob that purchased car is defective

$P(X) = P(B_1 \cap X) + P(B_2 \cap X) + P(B_3 \cap X)$

$= P(B_1)P(X|B_1) + P(B_2)P(X|B_2) + P(B_3)P(X|B_3)$

$= 0.3 \times 0.02 + 0.45 \times 0.03 + 0.25 \times 0.02$

$= 0.0245$

(2) Prob that randomly selected car is defective and it is produced by

$$P(B_1/X) = \frac{P(B_1)P(X|B_1)}{P(B_1)P(X|B_1) + P(B_2)P(X|B_2) + P(B_3)P(X|B_3)}$$

$$= \frac{0.3 \times 0.02}{0.0245} = 0.2449$$