

5. Unit

1) Problem on composition table

2(a) Roots of the equation $x^2 = 1$ show that it is a group with respect to multiplication.

2(b) Show that cube roots of unity form a group with respect to multiplication.

3(a) State and prove Lagrange's theorem on cosets
Statement
Proof:

3(b) Prove that identity element in group is unique in group

3(c) Prove that inverse element in group is unique }
}

3(d) $(a^{-1})^{-1} = a$

3(e) $(ab)^{-1} = b^{-1}a^{-1}$

for compulsory

$\Rightarrow Q_1$ is set of natural numbers except (1).

3(f) Q_1 is the set of rational numbers except 1.

$a * b = a + b - ab$, $a, b \in Q_1$, Q_1 is an infinite abelian group.

100) Prove that the set of non singular matrices of order $2/2$ is a group but not an abelian group with respect to multiplication.

Soln $G_1 = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid \begin{array}{l} \text{non singular} \\ \text{det} \neq 0 \end{array} \right. \text{all } 2 \times 2 \text{ matrix in set} \text{ operation is multiplication}$

* closure: $A = \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix}, B = \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \in G_1$

$A \cdot B = \begin{bmatrix} a_1a_2 + b_1c_2 & a_1b_2 + b_1d_2 \\ c_1a_2 + d_1c_2 & c_1b_2 + d_1d_2 \end{bmatrix}$ belong

real numbers add and mult we get real numbers

$|A| \neq 0, |B| \neq 0$

$|AB| = |A| \cdot |B|$

$\neq 0$

Exercises ①

$$AB \in G$$

Associative: 3 elements

$$A, B, C \in G$$

We know that matrix multiplication is associative

$$(AB)C = A(BC)$$

AB property is true

Identity: $\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ is the identity element

$$|I| = 1 - 0 - 1 + 0 = 1 \in G$$

Inverse: let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \in G$

$$ad - bc \neq 0$$

$$A^{-1} = \frac{adj A}{|A|} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ cofactor $= A^{-1}$ is exist $A A^{-1} = I$

$$A^{-1} \in G$$

A^{-1} is two 2×2 matrix

$$|A^{-1}| = \frac{1}{ad - bc} \{da - bc\}$$

for true we have
to prove all $A^{-1} \in G$, $= 1 \neq 0$

In case of ~~all~~ false
we have to do one

1) commutative property $AB \neq BA$

$$A = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$$

$$3+2=5$$

$$AB = \begin{bmatrix} -3 & -2 \\ -2 & 2 \end{bmatrix}$$

$$BA = \begin{bmatrix} -3 & 4 \\ 1 & 2 \end{bmatrix}$$

$$AB \neq BA$$

commutative property is not true so it is a group but not an abelian group.

* + unit:

1) Hasse Diagrams

a) $(D_{36}; |)$ positive divisors of 36
 $\{1, 2, 3, 4, 6, 9, 12, 18, 36\}$

b) $(D_5; |)$

2) Subset relation, $A = \{a\}$, $C = \{1\}$
 $A \subset \{a, b, c\}$, $C \subset \{1, 2, 3\}$
 $A = \{ab, c\}$
 $A = \{abc, cd\}$

3) what is compatibility relation write the procedure to find maximal compatibility blocks

* A reflexive and symmetric is known as compatibility.

* maximal compatibility finding procedure

4) Prove that it is equivalence relation

a) $(a, b) R (c, d)$ iff $ad = bc$

reflex, symm, transitive

b) $(a, b) R (c, d)$ iff $a+b = b+c$

c) $f(x) = \underline{\hspace{2cm}}$ $g(x) = \underline{\hspace{2cm}}$ function

1) $g \circ f$ 2) $f \circ g$

3) $g \circ f = \frac{ac}{bc}$
find a, b, c values for chayale

d) functions to find inverse of function $f(x)$

(Domain) (Codomain)

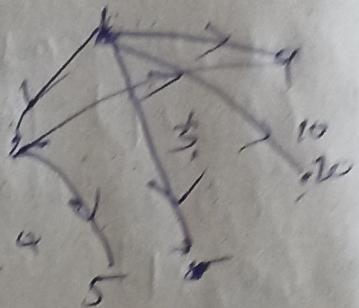
One one, onto exist then it is

Graph the relation

$$X = \{1, 2, 5, 9, 10\}, R = \{(x, y) | x < y\}$$

$$R = \{(1, 2), (1, 5), (1, 9), (1, 10), (2, 5), (2, 9), (2, 10), (5, 9), (5, 10), (9, 10)\}$$

digraph representation,
matrix form



3 unit z values are below,

1) Normal distribution

2) Assuming that 50% of all engineering students are good in sciences. Determine among 18 students

- Exactly 12
- At least 12 are good in science.

so : 50% are good in science
Using Normal distribution

$$N = 18$$

$$P = \frac{50}{100} = 0.5$$

$$q = 1 - P = 0.5$$

$$P = q \quad ? \text{ too far}$$

$$P \neq q$$

$$P(X=12)$$

$$P = q \Rightarrow z = \frac{12-11}{\sigma}$$

$$\mu = np = 18 \times 0.5 = 9$$

$$\sigma = \sqrt{npq} = \sqrt{18 \times 0.5 \times 0.5}$$

$$z = \frac{12-9}{\sigma} \quad \text{find}$$

$$\approx 1.2 - 9$$

$$P(z \approx)$$

