Algebra

Unit – IV

Relations

cartician product :- A & B are two non empty sets the cartisian product of A & B is defined as $A \times B = \{(x,y) \mid x \in A, y \in B\}$

Eg:- Let $A = \{-1, 0, 1\}$ $B = \{2, 4\}$

$A \times B = \{(x,y) \mid x \in A, y \in B\}$

$= \{(-1,2)(-1,4)(0,2)(0,4)(1,2)(1,4)\}$

$B \times A = \{(y,x) \mid x \in A, y \in B\}$

$= \{(2,-1)(2,0)(2,1)(4,-1)(4,0)(4,1)\}$

$\boxed{A \times B \neq B \times A}$

Relations :- Let A & B be two non Empty sets a relation from A to B is a subset of the cartisian Product $A \times B$ and it is defined as

$R = \{(x,y) \mid x \in A, y \in B \; xRy\}$

R is called a binary relation from A to B.

Eg:- Let $A = \{1, 2, 3, 4\}$

$R = \{(x, y) / x < y, \forall x, y \in A\}$

$\Rightarrow \{(1, 2) (1, 3) (1, 4) (2, 3) (2, 4) (3, 4)\}$

Domain of a relation:- ~~It~~ Domain of relation

$D(R) = \{x / xRy, \forall x \in A, y \in B\}$

Range of a relation:- Range of relation R defined as

$Range = \{y / (x, y) \in R, \forall x \in A, y \in B\}$

Inverse of a relation Let R be a relation from set A to B the inverse of R is denoted by R^{-1} and it is defined as

$R^{-1} = \{(y, x) / x \in A\ y \in B, xRy\}$

Ex. Let $A = \{2, 3, 5\}$ $B = \{6, 8, 10\}$

$R = \{(x, y) / x | y, x \in A, y \in B\}$, find domain, range R^{-1}!

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2 divides 4 means 2 exactly divides 4. which means 2 is a multiple of 4

$R = \{(2, 6) (2, 8) (2, 10) (3, 6) (5, 10)\}$

i) $D(R) = \{2, 3, 5\}$

$\forall \rightarrow$ for all

ii) Range of $R = \{6, 8, 10\}$

iii) $R^{-1} = \{(6,2) (8,2) (10,2) (8,3) (10,5)\}$

Eg:- Let $A = \{2,3,4,8\}$ $B = \{2,4,6,10\}$

R is defined from A to B as $R = \{(x,y) / x | y$ & $x \in A, y \in B\}$ find Domain, Range, inverse.

$R = \{(2,2) (2,4) (2,6) (2,10) (3,6) (4,4)$

i) $D(R) = \{2, 3, 4\}$

ii) $Range = \{2, 4, 6, 10\}$

iii) $R^{-1} = \{(2,2) (4,2) (6,2) (10,2) (6,3) (4,4)$

Properties of Relation

1) Reflexive relation :- A relation R on a set is reflexive if $aRa, \forall a \in R \rightarrow A$

i.e, $(a,a) \in R$

$R = \{(a,b) / a = b\}$

Eg:-

$A = \{1,2,3\}$

$R = \{(1,1) (2,2) (3,3) (1,2) (1,3)\}$

2) Irreflexive Relation

→ A relation R on a set is irreflexive if $a \not R a \ \forall a \in A$

i.e,

$(a, a) \notin R$

Eg:-

Let, $A = \{1, 2, 3\}$

$R = \{(1,2) (2,3) (1,3)\}$ Roster

$R = \{(x,y) / x > y, \forall x, y \in N\}$ set builder

3) Non Reflexive Relation

A relation R on a set A is non reflexive if R is neither reflexive nor irreflexive

Eg:-

$A = \{1, 2, 3\}$

$R = \{(1,1) (2,2) (1,2) (2,3)\}$

Symmetric relation A relation R on a set A is symmetric if aRb then bRa

i.e, if $(a, b) \in R$ then $(b, a) \in R$

Eg:- $A = \{1, 2, 3\}$

$R = \{(1,2) (2,1) (2,3) (3,2) (1,1) (2,2) (3,3)\}$

$R = \{(x, y) / x^2 + y^2 = 1 \ \forall x, y \in R\}$

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$R = \{ (1,0)\ (0,1) \}$

Asymmetric relation A relation R on a set A is asymmetric if when ever

$(a,b) \in R$ then $(b,a) \notin R$

i.e, if aRb then $b \not R a$ $\forall a,b \in A$

eg:- $A = \{1, 2, 3\}$

$R = \{ (1,2)\ (2,3)\ (3,1)\ (1,3) \}$

Anti symmetric relation A relation R on a set A is anti symmetric $\forall a,b \in A$ ~~aRb, bRa~~

$aRb, bRa \Rightarrow a=b$

if $(a,b) \in R$, $(b,a) \in R \Rightarrow a=b$

Eg:- $A = \{1, 2, 3\}$

$R = \{(1,1)\ (2,2)\}$

$R = \{(x,y) \mid x \leq y\ y \leq x \Rightarrow x = y \}$

Transidive Relation

A relation R on a set A is said to be transitive

if $aRb, bRc \Rightarrow aRc$

i.e, if $(a,b) \in R$, $(b,c) \in R$

then $(a,c) \in R$

Eg:- if $A = \{1,2,3\}$

$R = \{(1,2)\ (2,1)\ (1,1)\ (1,3)\ (3,1)\ (2,3)\ (3,2)\ (2,2)\}$

Equilance Relation

A relation R on a set A is said to be Equilace relation if it is reflexive, symmetric & transitive

That is R is Equilance relation on A if it has the following 3 properties

i) $(a,a) \in R \ \forall a \in A$ (Reflexive)

ii) if $(a,b) \in R$ then $(b,a) \in R \ \forall a,b \in A$ (symmetric)

iii) if $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R \ \forall a,b,c \in A$ (transitive)

Compatibility Relation

A Relation on the set A is said to compatibility if it is reflexive and symmetric

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partial ordering relation

A binary relation R on the set A is called partial ordering relation if it ~~is~~ satisfies the following 3 properties :-

i) $(a,a) \in R \ \forall a \in R$ (Reflexive)

ii) if $(a,b) \in R$, $(b,a) \in R$ then $a = b$ (anti-symmetric)

iii) if $(a,b) \in R$, $(b,c) \in R$ then $(a,c) \in R \ \forall a,b,c \in A$ (transitive)

10/8/23

Representation of Relations

A relation R from finite set A to itself can be represented by 2 ways :-

1) matrix representation

2) Di graph representation

$a_{[1,2]} = a_1 R a_2$

Matrix representation

elements of a matrix of the relation can be defined as

$$a_{ij} = \begin{cases} 1 & a_i R a_j \\ 0 & a_i \not R a_j \end{cases}$$

Properties

1) A relation matrix is reflexive if all its diagonal elements are ~~unity~~ unity

2) A relation is symmatric if its relation matrix is symmatric matrix. $(A^T = A)$

Digraph representation

i) A relation can be represented by drawing its graph

i) consider the elements of given non empty sets as vertices or nodes ~~of~~

ii) if $a_i R a_j$ ~~then~~ there exist an edge b/w the two vertices.

iii) If $a_i R a_i$ then tere exist an self loop at that vertex.

iv) Graphical representation is not unique.

Properti

i) If a relation is reflexive then there is a self loop at each vertex

ii) If a relation is symmetric then there exist ~~two~~ parallel edges b/w the same vertices

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Eg:- Let $A = \{1, 2, 3, 4\}$

R is defined on A as

$R = \{(x, y) \mid x \leq y, \forall x, y \in A\}$

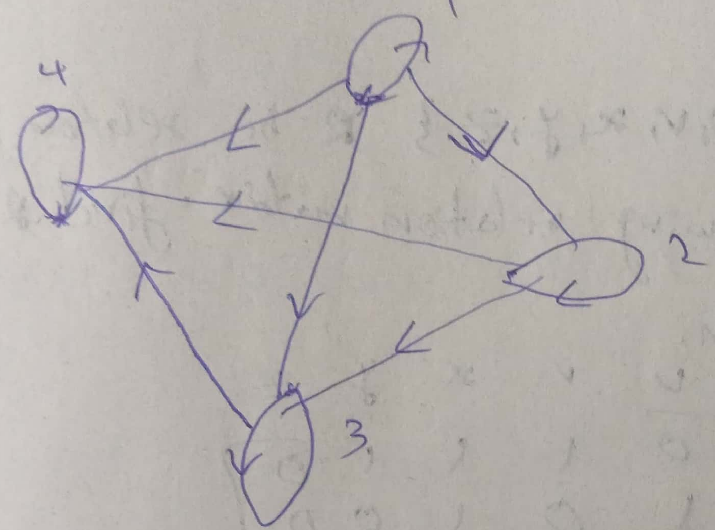
find relational matrix and digraph.

sol) $R = \{(1,1), (1,2), (1,3), (1,4), (2,2), (2,3), (2,4), (3,3), (3,4), (4,4)\}$

relation matrix

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 0 & 1 & 1 & 1 \\ 3 & 0 & 0 & 1 & 1 \\ 4 & 0 & 0 & 0 & 1 \end{array}$$

digraph



2) Let $A = \{1,2,3\}$ and $R = \{(x,y) \mid x > y$ & $x, y \in A\}$ find relation matrix & digraph.

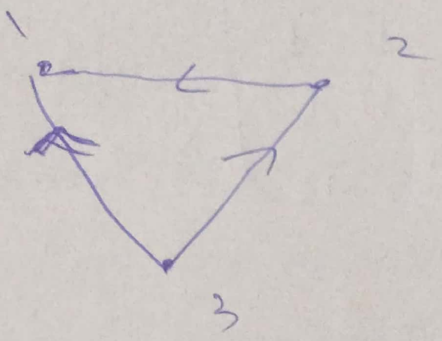
$R = \{(2,1)\ (3,2)\ \cancel{\}}\ \cancel{\&}\ (3,1)\}$

matrix

$$\begin{array}{c|ccc} & 1 & 2 & 3 \\ \hline 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 \end{array}$$

1

digraph

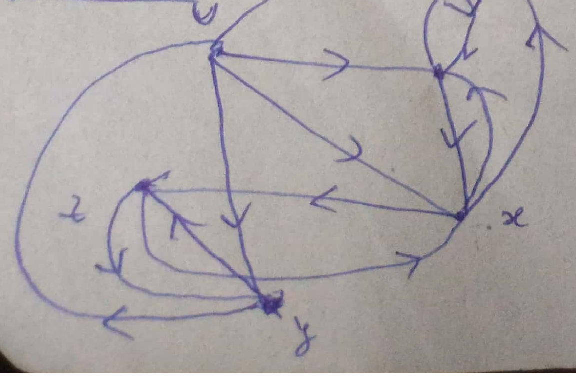


8) Let A = {u, v, x, y, z} R be relation on A with the following relation matrix. find R and draw its graph.

$$M_R = \begin{array}{c} \\ u \\ v \\ x \\ y \\ z \end{array} \begin{array}{c} \begin{array}{ccccc} u & v & x & y & z \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

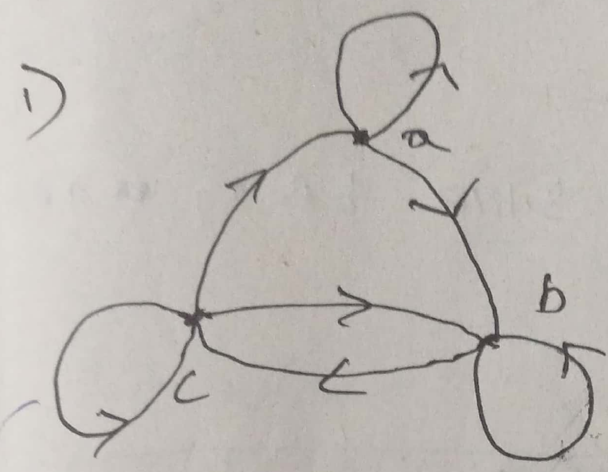
R = {(u, v) (u, x) (u, y) (v, u) (v, x) (x, u) (x, v) (x, z) (y, u) (y, z) (z, x) (z, y)}

graph



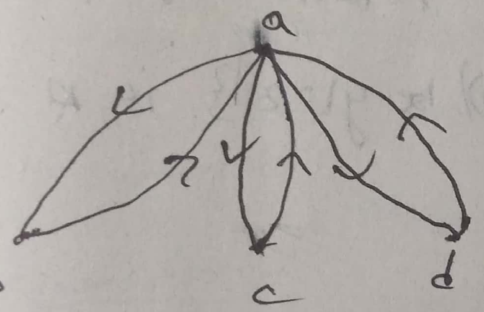
Q) find ~~the~~ weather a relation for digraph shown in the following figures are reflexive, symmetric, transitive

1)



R = {(a,a) (a,b) (b,b) (b,c) (c,c) (c,b) (c,a)

2)



R = {(a,b) (a,c) (a,d) (b,a) (c,a) (d,a)}

→ i) R is reflexive. Since there exist an self loop at each vertex

ii) R is not symmetric. Since there is an edge b/w a to b but there is no return edge from b to a.

iii) R is not transitive. Since there is an edge b/w a to b also from b to c but no edge from a to c. i.e, aRb, bRc but a$\not R$c

2) i) R is not reflexive. Since ~~all~~ all the vertices does not have self loop.

ii) R is symmatric since every edge is having its return edge

iii) R is not transitive. Since bRa, aRb but b R̸b.

Equivalence relation problems

Q) Consider the following relation on $A = \{1,2,3,4,5,6\}$ and $R = \{(x,y) \mid |x-y| = 2\}$ is R is an equivalance relation?

$R = \{(1,3)\ (3,1)\ (2,4)\ (4,2)\ (3,5)\ (5,3)\ (4,6)\ (6,4)\}$

i) R is not Reflexive. Since $(1,1)\ (2,2)\ (3,3)\ (4,4)\ (5,5)\ (6,6) \notin R$

ii) R is symmetric. since $\forall\ (a,b) \in R$ then (b,a) is also $\in R$

iii) R is not transitive since $(1,3)\ (3,1) \in R$ then $(1,1) \notin R$.

8) R be a relation defined on $N \times N$ given by $(a,b) R (c,d)$ if and only if $ad = bc$ show that A is an equivilance relation.

Sol) i) Reflexive :-

$(a,b) R (a,b) \Leftrightarrow ab = ba$

$\Leftrightarrow ba = ab$

$\Leftrightarrow (a,b) R (a,b)$

R is reflexive

ii) $(a,b) R (c,d) \Leftrightarrow ad = bc$

$\Leftrightarrow bc = ad$

$\Leftrightarrow cb = ad$

$\Leftrightarrow (c,d) R (a,b)$

R is symmetric

iii) Transitive :-

Let (a,b) (c,d) $(e,f) \in N \times N$

$(a,b) R (c,d)$

$\Leftrightarrow ad = bc$

$(c,d) R (e,f)$

$\Leftrightarrow cf = de$

Consider $a\cancel{d} \times \cancel{c}f = b\cancel{c} \times \cancel{d}e$

$\Leftrightarrow af = be$

$\Leftrightarrow (a,b) R (e,f)$

R is transitive.

$\therefore$ R is Equivalance relation

8) If R is a relation defined by set of integers given by $R = \{(x,y) / x-y$ is divisible by $6\}$ prove the R is equivalance relation.

Reflexive:-

$xRx \Rightarrow x-x=0$

0 is divisible by 6

Hence R is reflexive

Symmatric:-

$xRy \Rightarrow x-y$ is divisible by 6

$\Rightarrow y-x$ is also divisible by

$\Rightarrow yRx$

$\therefore$ 'R' is symmatric

Transitive

Let $x,y,z \in A$

$xRy, yRz \Rightarrow x-y$ is divisible by 6, $y-z$ is divisible by 6

$\Rightarrow (x-y+y-z)$ is divisible by 6

$\Rightarrow (x-z)$ is divisible by 6

$\Rightarrow xRz$

$\therefore$ 'R' is transitive

$\therefore$ R is Equivalance

Q) ~~Let R be a~~ If I be the set of non 0 integers and the relation R on I is defined as xRy iff $x^y = y^x$ $\forall\, x, y \in I$ show that R is Equivalence

$I = \{-\infty, \ldots\ldots -3, -2, -1, 1, 2, 3, \ldots\ldots \infty\}$

i) Reflexive:-

$xRx \Leftrightarrow x^x = x^x$

$\Leftrightarrow xRx$

R is reflexive

ii) Symmetric:-

$xRy \Leftrightarrow x^y = y^x$

$\Leftrightarrow y^x = x^y$

$\Leftrightarrow yRx$

R is symmetric

iii) Transitive:-

$xRy \Leftrightarrow x^y = y^x \Rightarrow x^{y/x} = y$

$yRz \Leftrightarrow y^z = z^y$

xRz

$x^z = z^x$

consider

$y^z = z^y$

$\left(x^{y/x}\right)^z = z^y$

$\left(x^z\right)^{y/x} = z^y$

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$x^{e^z} = (z^y)^{\frac{x}{y}} = z^x$

R is transitive

∴ R is equivalence

Q5) Give an example of a relation which is

i) Reflex & tansitive but not symmetric

ii) S & t but not R

iii) R & S but not t

Let $A = \{1, 2, 3\}$

i) $R = \{(1,1)\ (2,2)\ (3,3)\ (3,1)\}$

ii) $R = \{(1,1)\ (1,2)\ (2,1)\}$

iii) $R = \{(1,1)\ (2,2)\ (3,3)\}$

Reflexive, irreflexive, non-reflexive, sym, asym, anti sym, transitive $\forall (x,y) \in R$

i) $R_1 = \{(x,y) \mid x \neq y\}$

ii) $R_2 = \{(x,y) \mid x \cdot y \geq 1\}$

iii) $R_3 = \{(x,y) \mid x = y - 1\}$

iv) $R_4 = \{(x,y) \mid x$ is a multiple of $y\}$

v) $R_5 = \{(x,y) \mid x = y^2\}$

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i) $R_1 = \{(x,y) / x \neq y\}$

$R_1 = \{(-1,-2)\ (1,2)\ (2,3)\ (1,3 \ldots\}$

R_1 is irreflexive, symmetric, non-transitive

ii) $R_2 = \{(x,y) / xy \geq 1 \ \forall x,y \in R\}$

$R_2 = \{(-1,-2)\ (1,1)\ (2,2)\ (-1,-1)\ (2,3)\ (3,2)\}$

R_2 is reflexive, symmetric & transitive

$(-1,-2)\ (-2,-3)$

$\therefore R_2$ is equivalance

$(-1,-3)$

iii) $R_3 = \{(x,y) / x = y-1\}$ $1 = 0-1$ $1=-1$

$R_3 = \{(0,1)\ (1,2)\ (-2,-1)\ (-1,0)\ (-3,-2) \ldots\}$

R_3 is ~~not~~ ~~reflexive~~ irreflexive, asymatric

non transitive relation.

iv) $R_4 = \{(x,y) / x$ is a multiple of $y\}$

$R_4 = \{(1,2)\ (1,3)\ (2,4)\ (3,6)\ (2,6)$

$(1,-2)\ (1,-3)\ (2,-4)\ (3,-6) \ldots\}$

$(1,1)\ (2,2)\ (3,3) \ldots$

R_4 is reflexive, asymmetric, transitive

$R_4 = \{(x,y) / x = y^2\}$

$R_4 = \{(1,1)\ (4,2)\ (1,-1)\ (4,-2)$
$(9,3)\ (9,-3) \ldots$

R_4 is non reflexive, asymmetric non transitive

Transitive Closure ~~It is T~~ of a relation R is the smallest transitive Relation which is denoted by R^+ and is given by

$$R^+ = \{R \cup R^2 \cup R^3 \cup \ldots\}$$

Let A ~~by~~ be any non Empty set containing n elements and R be a relation defined on the non empty set A then transitive closure is

$$R \cup R^2 \cup R^3 \ldots$$

Q) find the transitive closure of

$R = \{(1,2)\ (2,3)\ (3,4)\}$

$R^2 = R \circ R$

$= \{(1,2)\ (2,3)\ (3,4)\} \circ \{(1,2)\ (2,3)\ (3,4)\}$

$\Rightarrow \{(1,3)\ (2,4)\}$

* If the given relation itself is a transitive then we get R^+ also same so we should have non transitive R to find R^+.

$R^3 = R^2 \circ R$

$\Rightarrow \{(1,3)\ (2,4)\} \circ \{(1,2)\ (2,3)(3,4)\}$

$\Rightarrow \{(1,4)\}$

$R^4 = R^3 \circ R$

$\rightarrow \{(1,4)\} \circ \{(1,2)\ (2,3)(3,4)\}$

$\Rightarrow \phi$

$R^+ = \{R \cup R^2 \cup R^3 \cup R^4\}$

$\Rightarrow \{(1,2)(2,3)\ (3,4)\ (1,3)\ (2,4)$

$(1,4)\}$

Q) find the transitive closure of the relation R defined on A = {1, 2, 3} where

$R = \{(1,1)\ (2,2)\ (2,3)(1,3)\}$

$R^2 = \{(1,1)\ (2,2)\ (2,3)\ (1,3)\} \circ \{(1,1)(2,2)(2,3)(1,3)\}$

$\Rightarrow \{(1,1)\ (2,2)\ (2,3)\ (1,3)\}$

Since given relation R itself is the transitive relation

$\therefore R^+ = R$

Partition and Blocks

Let S be a given set non empty set $A = \{A_1, A_2, \dots A_n\}$ where each A_i is a subset of S and $S = \bigcup_{i=1}^{n} A_i$ then the set A is called the partion of S and the sets $A_1, A_2 \dots A_n$ said to cover S

$\therefore$ S is called covering in addition to this the elements of A which are the subsets of S ~~in addition~~ are mutually disjoint then $A_1, A_2, \dots A_n$ are called blocks of partition of A of S.

Equalance class Let R be an equalance relation on a set A & $a \in A$ then equalance class of A is denoted by $[a]$ and is defined as $[a] = \{x / xRa \ \forall a \in$

Inducted class / Quotient class The collection of all equalance classes which are defined of set A is called induced class & it is denoted by A/R

$$A/R = \{[a] [b] [c] \dots\}$$

where $a, b, c \in A$

~~If A = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11,~~

Q) ~~x y is~~ find equalance closses and induced closs of $R = \{(1,1)(2,2)(3,3)(4,4)(5,5)(1,2)(2,1)(5,4)(4,5)\}$ defined on $A = \{1,2,3,4,5\}$

Sol) Equilance closs

$[1] = \{1,2\}$ $[4] = \{4,5\}$

$[2] = \{1,2\}$ $[5] = \{4,5\}$

$[3] = \{3\}$

$A/R = \{[1][3][4]\}$

$A/R = \{[2][3][5]\}$

$A/R = \{[1,2][3][4,5]\}$

Q) $A = \{1,2,3,4,5,6,7,8,9,10,11,12\}$ and $R = \{(x,y) \mid x-y$ is multiple of $5\}$ find partitions of A & A/R.

$R = \{(1,1)(2,2)(3,3)\ldots(12,12)\ (1,6)\ (6,1)$
$(2,7)\ (7,2)\ (3,8)(8,3)\ (4,9)\ (9,4)(5,10)$

(10,5) (6,11) (11,6) (7,12) (12,7) (1,11)
(11,1) (2,12) (12,2)}

[1] = {1,6,11}
[2] = {2,7,12
[3] = {3,8}
[4] = {4,9}
[5] = {5,10}
[6] = {1,6,11}
[7] = {2,7,12}
[8] = {3,8}
[9] = {4,9}
[10] = {5,10}
[11] = {1,6,11}
[12] = {2,7,12}

A|R = {[1] [2] [3] [4] [5]}

π_1 = {{1,6,11} {2,7,12} {3,8} {4,9}
{5,10} }

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8) Let A = {a,b,c,d}
R = {(a,a) (b,b) (c,c) (d,d) (a,b) (b,a)
~~find~~ (c,d) (d,c)} find partition of A
and A/R.

$[a] = \{a, b\}$

$[b] = \{a, b\}$

$\{c\} = \{c, d\}$

$\{d\} = \{c, d\}$

$A|R = \{a, c\}$ → This should be disjoint

$\pi_1 = \{\{a, b\} \{c, d\}\}$ → and union of this should be A

$A|R = \{b, d\}$

$\pi_2 = \{\{a, b\} \{c, d\}\}$

Q) If $A = \{1, 2, 3, 4\}$ $\pi = \{\{1, 2, 3\} \{4\}\}$ find equivalence relation determined by π

$R = \{(1,1) (1,2) (1,3) (2,1) (2,2) (2,3)$
$(3,1) (3,2) (3,3) (4,4)\}$

8) for the partition $\pi = \{\{1,3\}\{2,4\}\}$ defined on $A = \{1,2,3,4\}$ find its equilance relation.

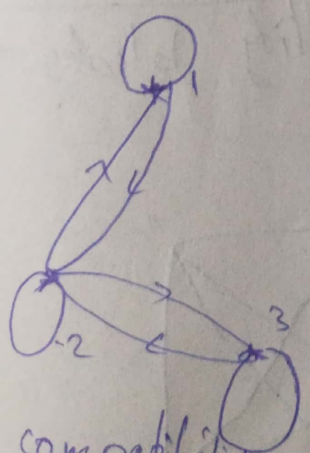
$R = \{(1,1)(1,3)(3,1)(3,3)(2,2)(2,4)(4,2)(4,4)\}$

Compatibility relation

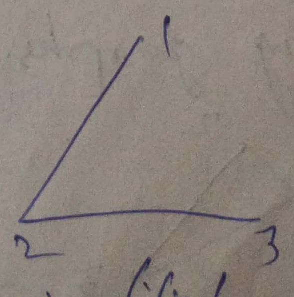
A relation on set A is said to be compatibility relation If it is reflexive & symmatric

→ All Equilance relations are compatibility relation.

→ If a relation is compatibility relation then it is not necessary to draw loops at vertices and parallel edges.



compatibility graph



simplified graph.

maximal compatibility blocks

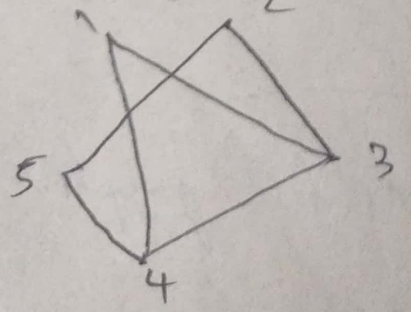
1) Draw simplified graph of compatibility relation

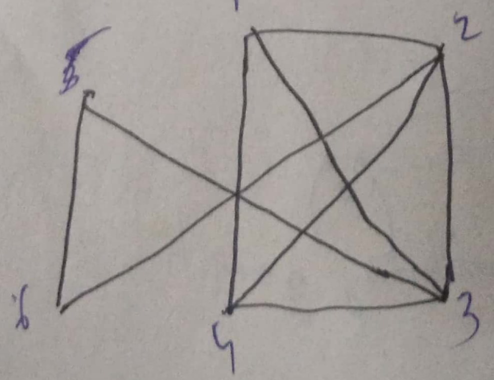
2) Pick from this graph the largest complete polygons (complete polygon is a complete graph in which any vertex is ~~comp~~ connected to every other vertex)

3) any two ~~vertex~~ elements which are compatible to ~~each~~ one another also forms a maximal compatibility blocks.

4) The maximal compatibility blocks need not be disjoint.

Q) find the maximal compatibility blocks for the following graphs.

1) 

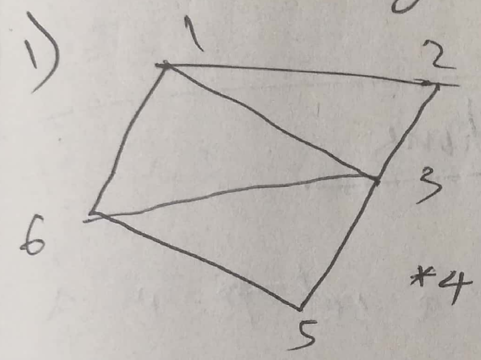
2) 

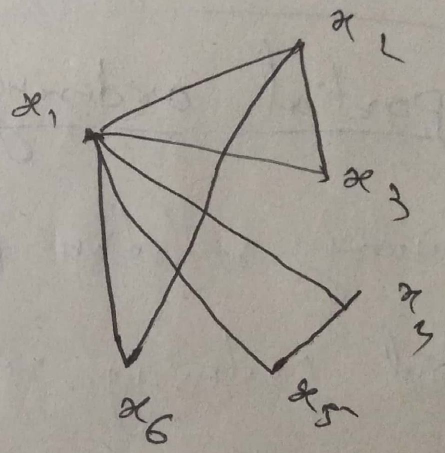
1) {1, 3, 4} {2, 3} {4, 5} {2, 5}

2) {1, 2, 3, 4} {5, 6} {2, 6} {3, 5}

Q) find the relation matrix and also blocks to the following graphs.

1)





1) {1, 2, 3} {1, 3, 6} {3, 5, 6} {4}

2	1				
3	1	1			
4	0	0	0		
5	0	0	1	0	
6	1	0	1	0	1
	1	2	3	4	5

2) $\{x_1, x_2, x_3\}$ $\{x_1, x_4, x_5\}$ $\{x_1, x_2, x_6\}$

	x_1	x_2	x_3	x_4	x_5
x_2	1				
x_3	1	1			
x_4	1	0	0		
x_5	1	0	0	1	
x_6	1	1	0	0	0

partial ordering relations

A binary relation R on a set P is a partial ordering relation if it is reflexive anti symmetric and transitive

Poset (Partial ordered set) :- A relation which satisfies partial ordering relation def on a non Empty set A, then A is called partial Ordered set.

Eg:- Let $\leq$ is a relation defined on non empty set R, show ~~to~~ that $\leq$, R is a poset

Sol)

Let $x, y, z \in R$ where R is real No. syst

i) reflexive :- $x \leq x \; \forall x \in R$

ii) anti symmetric :- $x \le y, y \le x \Rightarrow x = y$

iii) Transitive :- $x \le y, y \le z \Rightarrow x \le z \ \forall x, y, z$

eg:- Let P be a power set, show that $(\subseteq, P)$ is a partial ordered set.

Sol) Let $A, B, C \in P$ where 'P' is power set

i) $A \subseteq A \Rightarrow \subseteq$ is reflexive

ii) $A \subseteq B, B \subseteq A \Rightarrow A = B$ $\subseteq$ is anti symmetric

iii) $A \subseteq B, B \subseteq C \Rightarrow A \subseteq C$ $\subseteq$ is transitive

$\therefore (\subseteq, P)$ is a poset.

Hasse diagram * * *

The digramatic representation of poset is called ~~has~~ hasse diagram.

Rules to construct hasse diagram

1) The elements of poset are represented by a small circle or dot.

2) The elements $x \in P$ ~~wrote~~ write below the $y \in P$ if $x < y$

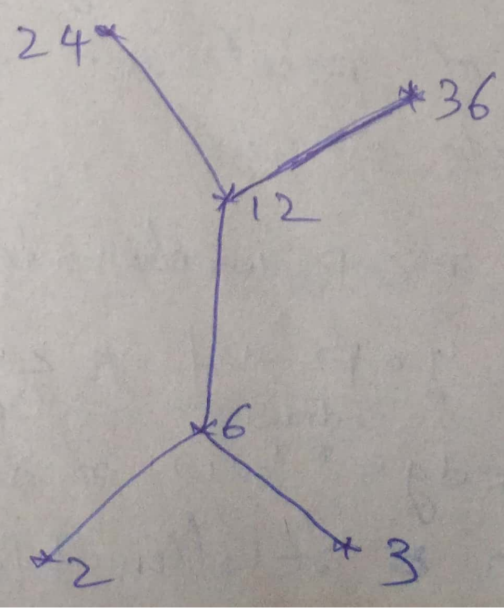
3) An ~~If there~~ edge is drawn b/w x and y if there exist an ~~x~~ relation b/w x & y.

4) If there is no relation b/w the elements we consider those elements $\parallel^{el}$ to each others

---

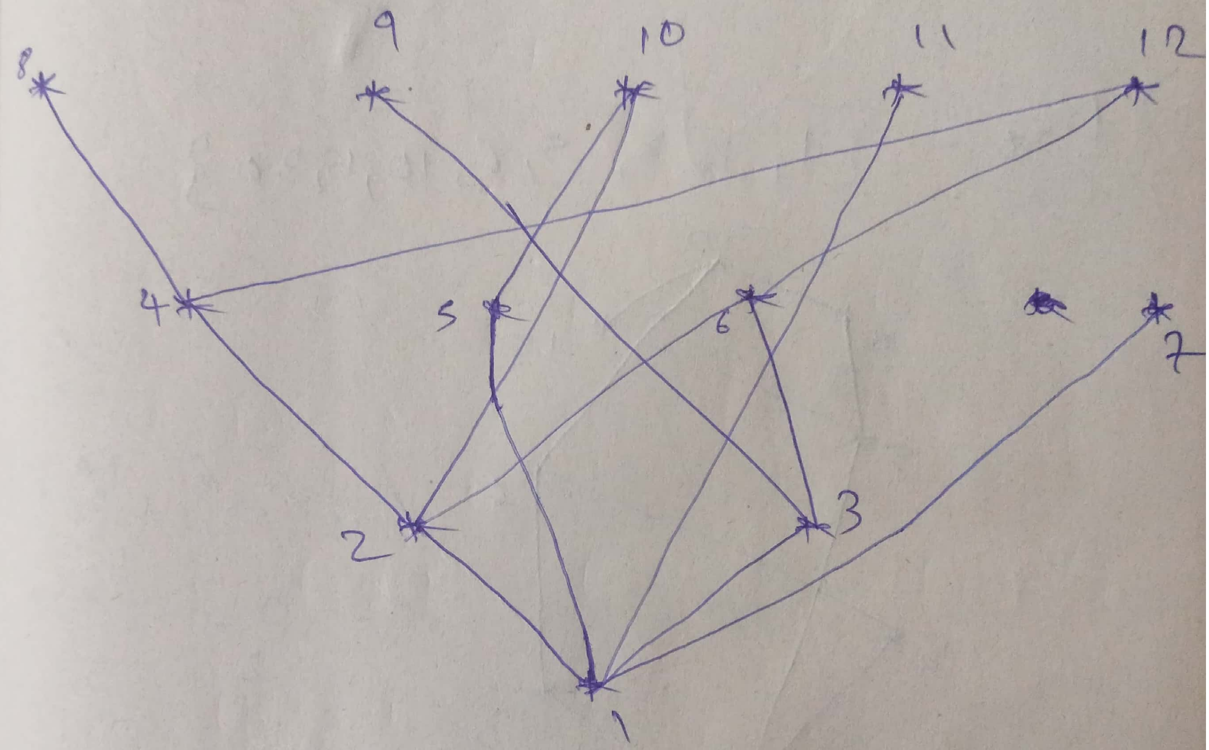
1) Let $A = \{2, 3, 6, 12, 24, 36\}$ ***
and $R = \{(x, y) / x \text{ divides } y\}$
Construct Hasse diagram and also partial ordering relation.

$R = \{(2,2)\ (3,3)\ (6,6)\ (12,12)\ (24,24)$
$\{3, 6, 36\}\ (2,6)\ (2,12)\ (2,24)\ (2,36)$
$(3,6)\ (3,12)\ (3,24)\ (3,36)$
$(6,12)\ (6,24)\ (6,36)$
$(12,24)\ (12,36)\}$



2) Let I_{12} is a set of positive integer which are ≤ 12 draw the Hasse diagram of (I_{12}, ~~divisibility~~ /) → divisibility

$I_{12} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$



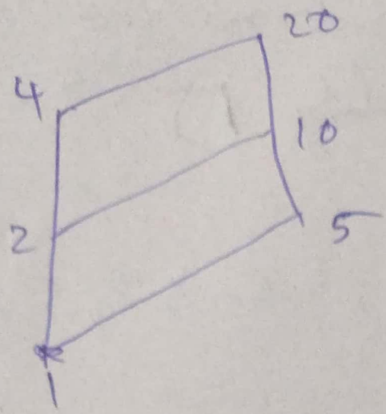
9) Draw the hasse diagram to following

1) $(D_{20}, /)$

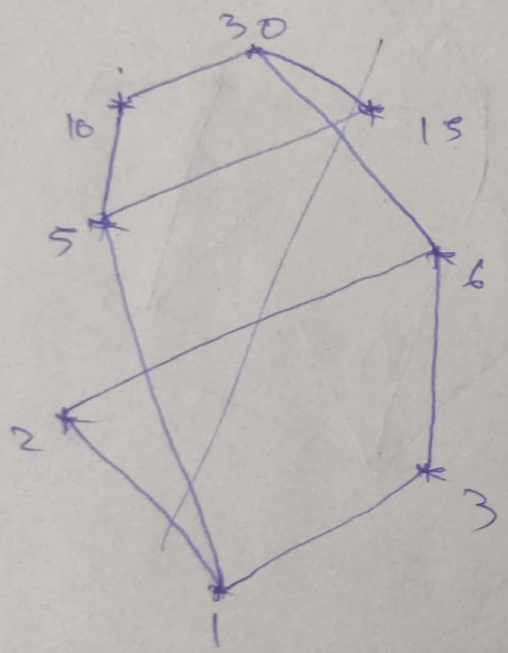
2) $(D_{30}, /)$

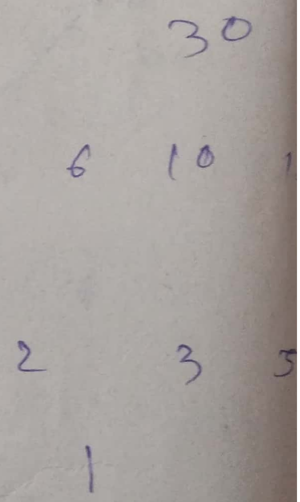
3) $(D_{45}, /)$

i) $D_{20} = \{1, 2, 4, 5, 10, 20\}$

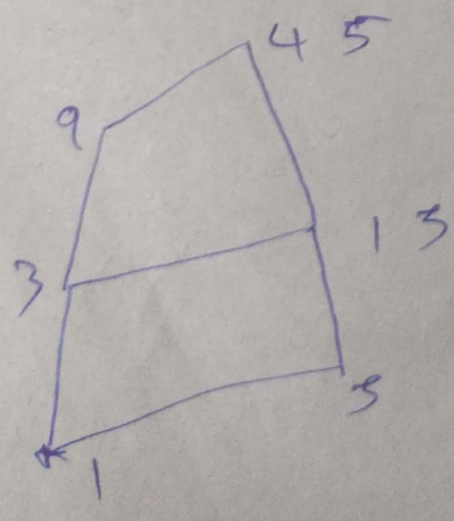


ii) $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$





iii) $D_{45} = \{1, 3, 5, 9, 15, 45\}$



Q) Let A be any finite set and $P(A)$ be the power set of A draw the Hasse diagram of $(P(A), \subseteq)$ for

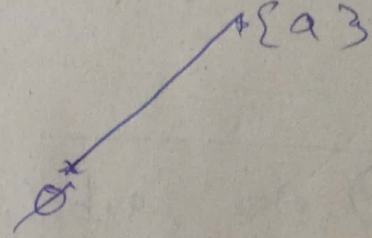
i) $A = \{a\}$

ii) $A = \{a, b\}$

iii) $A = \{a, b, c\}$

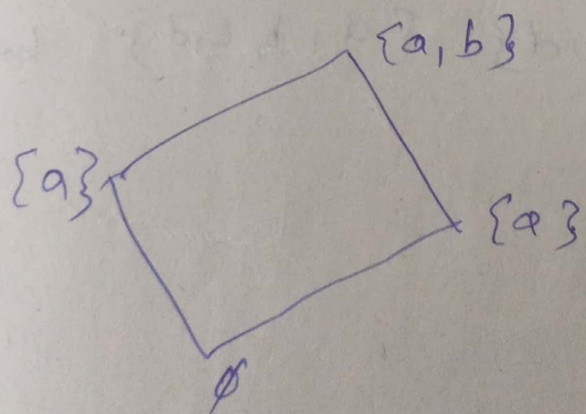
iv) $A = \{a, b, c, d\}$

i) $P(A) = \{\phi, \{a\}\}$



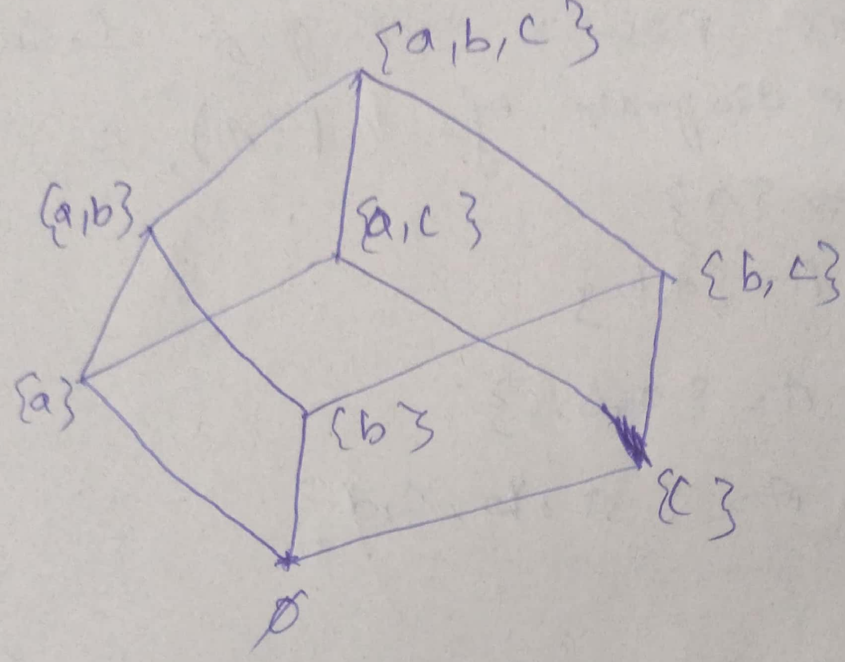
ii) $A = \{a, b\}$

$P(A) = \{\phi, \{a\} \{b\} \{a, b\}\}$



iii) if $A = \{a, b, c\}$

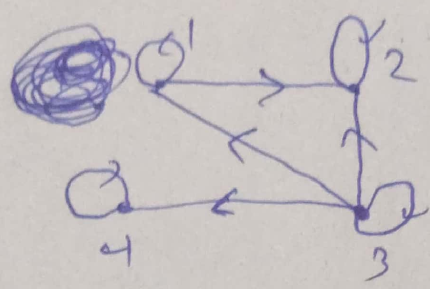
$P(A) = \{\phi, \{a\}, \{b\} \{c\} \{a, b\} \{b, c\}$
$\{c, a\} \{a, b, c\}\}$



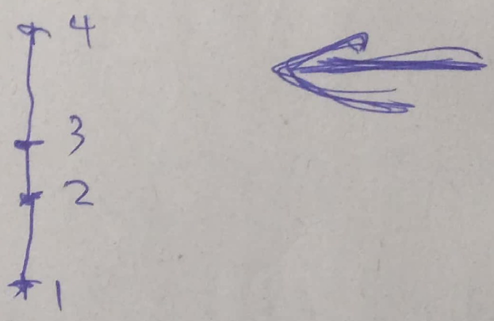
iv) A = {a, b, c, d}

P(A) = { ϕ {a} {b} {c} {d} {a,b} {a,c}
{a,d} {b,c} {b,d} {c,d} {a,b,c} {a,b,d}
{a,c,d} {b,c,d} {a,b,c,d} hasse diagram

5.) Draw the hasse diagram of a graph hence write the corresponding relation.



$R = \{(1,1)\ (2,2)\ (3,3)\ (4,4)\ (1,2)\ (3,2)$
$(3,4)\ (3,1)\}$



Functions:- A function is a special case of relation. Let A & B be 2 non empty sets of a relation R from A to B may or may not be related an element of A to more than 1 element of B but a fn relates each element of A to unique element of B, hence every fn is a relation but a relation need not to be a fn.

Defination of fn Let A, B are any 2 non-empty sets a relation from A to B is said to be a fn if for every element $a \in A$ there is a unique image in B.

Note :- 1) $\forall\ a \in A$ there exist a unique image in B such that ordered pair A, B belongs to f i.e $f(a) = b$

2) b is called image of a

3) a is called pre-image of b.

4) if f maps from a to b is a fn then A is called domain & B is called co-domain.

Q) state whether each of the following relations are functions or not. defined on the sets $A = \{a, b, c\}$ $B = \{1, 2, 3\}$

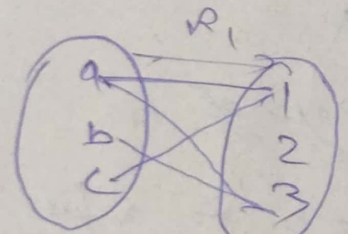
i) $R_1 = \{(a,1)\ (a,3)\ (b,3)\ (c,1)\}$

ii) $R_2 = \{(a,2)\ (b,3)\}$

iii) $R_3 = \{(a,1)\ (b,3)\ (c,1)\}$

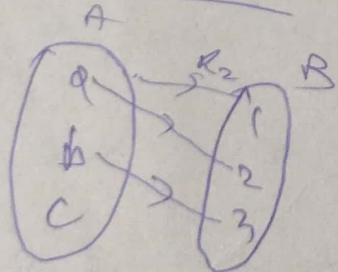
iv) $R_4 = \{(a,1)\ (b,1)\ (c,1)\}$

v) $R_5 = \{(a,2)\ (a,3)\ (b,3)\ (c,3)\}$

1) 

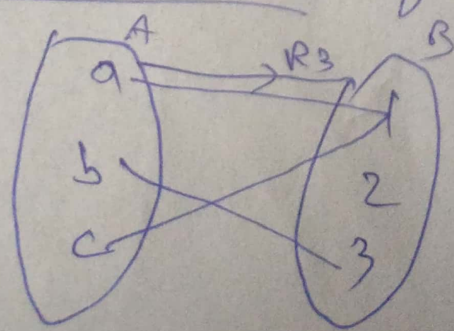
R_1 is not function
since for $a \in A$ $\exists$ two images in B

---

2) $R_2 =$ 

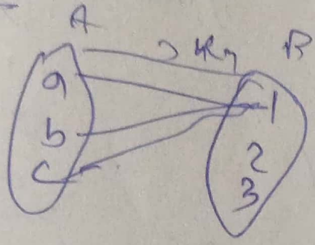
R_2 is not a function
since $c \in A$ no image in B

---

3) 

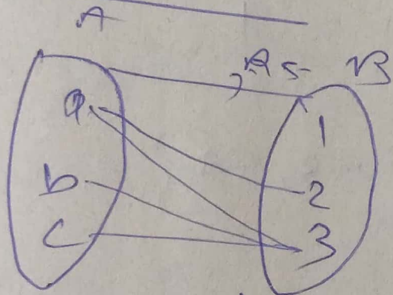
R_3 is a function
since $\forall\, a \in A\ \exists$ image in B

4)



R_4 is a function
since $\forall\, a \in A\ \exists$ image in B

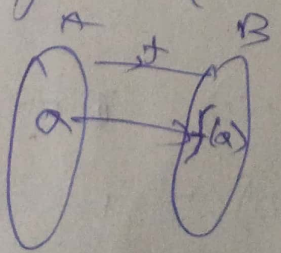
5)



R_5 is not a function
$\because\ \exists\, a \in A$ having two images in B.

Range of a function

If $f : A$ to B is a function defined over 2 non empty sets A & B then
range of $f = \{ f(a) / \forall\, a \in A \}$



Q) if $f: R \to R$ $A = \{0, \pm 1, \pm 2, 3\}$

$f(x) = x^3 - 2x^2 - 3x + 1$, find range of f

$-1 - 2 + 1 +$
$-8 + 8 + 6$
$22 \quad -18 - 9$

$f(x) = x^3 - 2x^2 - 3x + 1$

$f(0) = 1$ $\qquad f(2) = -5$

$f(1) = -3$ $\qquad f(-2) = 7$

$f(-1) = -1$ $\qquad f(3) = 1$

Range of $f = \{1, -3, -5, -9\}$

Let $f: R \to R$ $f(x) = x^2 + 1$ determine images of following subsets

i) $A_1 = \{2, 3\}$ ii) $A_2 = \{-2, 0, 3\}$ iii) $A_3 = (0,$

iv) $A_4 = [-6, 3]$

$A_1 = \{2, 3\}$ $f(2) = 4 + 1 = 5$

$f(3) = 10$

$f(A_1) = \{5, 10\}$

$A_2 = \{-2, 0, 3\}$ $f(-2) = 5$

$f(0) = 1$

$f(3) = 10$

$f(A_2) = \{1, 5, 10\}$

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$A_3 = (0,1)$

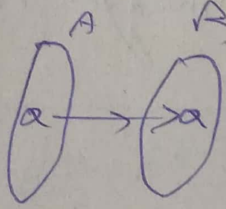
$f(A_3) = \{f(x) \mid 0 < x < 1 \ \forall\, x \in A_3\}$

$A_4 = [-6, 3]$

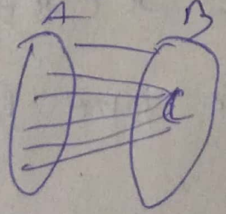
$f(A_4) = \{f(x) \mid -6 \leq x \leq 3 \ \forall\, x \in A_4\}$

Types of functions

1) Identify function :- A function $f: A \to A$ i.e $f(a) = a \ \forall\, a \in A$ is called identify function on A.



2) constant function :- A function $f: A \to B$ i.e, and $f(a) = c \ \forall\, a \in A$ is mapped to fixed image in B is called constant function

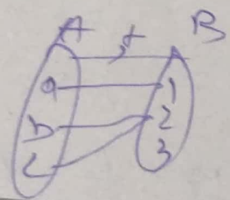


3) One to One (injective) :- A function $f: A \to B$ is said to be one to one fn if ~~every~~ different elements of A having distinct images in B (or) when ever $x_1, x_2 \in A$ if $f(x_1) = f(x_2)$ then $x_1 = x_2$

4) <u>Onto fn</u> :- A fn $f: A \to B$ is said to be onto fn if every of B having pre image in A (or) if f is said to be onto fn when Range of f = co-domain of f

i.e, $f(a) = b$

5) <u>into fn</u> (many to one) :- Let $f: A \to B$ be a fn if there exist any element in B having no pre image in A then f is said to be into fn.



Q :- a) if $f: R \to R$ $g: R \to A$ and

$f(x) = 3x+7$ $g(x) = x(x^3-1)$ $\forall x \in R$

verify that f is one to one but not g.

$f(x) = 3x+7$

Let $x_1, x_2 \in R$

$f(x_1) = f(x_2) = 3x_1 + 7 = 3x_2 + 7$

$x_1 = x_2$

$\therefore$ f is one to one

Let $x_1, x_2 \in R$

Let $g(x_1) = g(x_2)$

$$x_1(x_1^3 - 1) = x_2(x_2^3 - 1)$$

$$x_1^4 - x_1 = x_2^4 - x_2$$

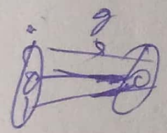
$$x_1^4 - x_2^4 - (x_1 - x_2) = 0$$

$g(0) = 0$

$g(1) = 0$

for 0, 1 having same image 0

$\therefore$ g is not 1-1



Inverse fn :- A fn is said to be invertible fn if it is both 1-1 and onto

i.e $f : A \to B$ is any fn then its invertible fn $f^{-1} : B \to A$

i.e if $f(x) = y$

then $x = f^{-1}(y)$

Bijective fn :- A fn is both 1-1 and onto then it is called bijective fn.

Q) find inverse of fn $f(x) = \frac{x+1}{x}$

Let $f(x) = \frac{x+1}{x} = y$

$x+1 = xy$

$x = xy - 1$

$x - xy = -1$

$x(1-y) = -1$

$x = \frac{-1}{1-y}$

$x = \frac{1}{y-1}$

$f^{-1}(y) = \frac{1}{y-1}$

$f^{-1}(x) = \frac{1}{x-1}$

Q)

$f(x) = 4e^{3x+1}$

Let $f(x) = y$

$4e^{3x+1} = y$

$\log(4e^{3x+1}) = \log y$

$\log 4 + \log e^{3x+1} = \log y$

$\log 4 + (3x+1) = \log y$

$3x+1 = \log y - \log 4$

$3x+1 = \log \frac{y}{4}$

$3x = \log \frac{y}{4} - 1$

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$x = \frac{1}{3}\left(\log\frac{y}{4} - 1\right)$

$f^{-1}(y) = \frac{1}{3}\left(\log\frac{y}{4} - 1\right)$

$f^{-1}(x) = \frac{1}{3}\left(\log\frac{x}{4} - 1\right)$

---

8) $f(x) = \frac{10}{\sqrt[5]{7-3x}}$

Let $f(x) = y$

$$\frac{10}{\sqrt[5]{7-3x}} = y$$

$$10 = y\sqrt[5]{7-3x}$$

$$\frac{10}{y} = \sqrt[5]{7-3x}$$

$$\frac{10^5}{y^5} = 7-3x$$

$$\frac{1}{3}\left(\frac{\frac{10^5}{y^5} - 7}{\cancel{\ }}\right) = x$$

$$f(x) = \begin{cases} 3x-5 & x>0 \\ -3x+1 & x \le 0 \end{cases}$$

find $f^{-1}(0)$ & $f^{-1}(1)$

Let

$f(x) = y \Rightarrow x = f^{-1}(y)$

$$y = \begin{cases} 3x-5 & x>0 \\ -3x+1 & x \le 0 \end{cases}$$

$f^{-1}[-5,5] \Rightarrow 3x-5 = y$

$x > 0$

$3x = y+5$

$x = \frac{y+5}{3}$

$f^{-1}(y) = \frac{y+5}{3}$

$f^{-1}(x) = \frac{x+5}{3}$

$y = -3x+1$

$y-1 = -3x$ $\quad x \le 0$

$\frac{1-y}{3} = x$

$f^{-1}(y) = \frac{1-y}{3}$

$f^{-1}(x) = \frac{1-x}{3}$

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$\cancel{f^{-1}(x) =}$ $f^{-1}(x) = \begin{cases} \frac{x+5}{3} & x > 0 \\ \frac{1-x}{3} & x \leq 0 \end{cases}$

$f^{-1}(0) = \frac{1-0}{3} = \frac{1}{3}$

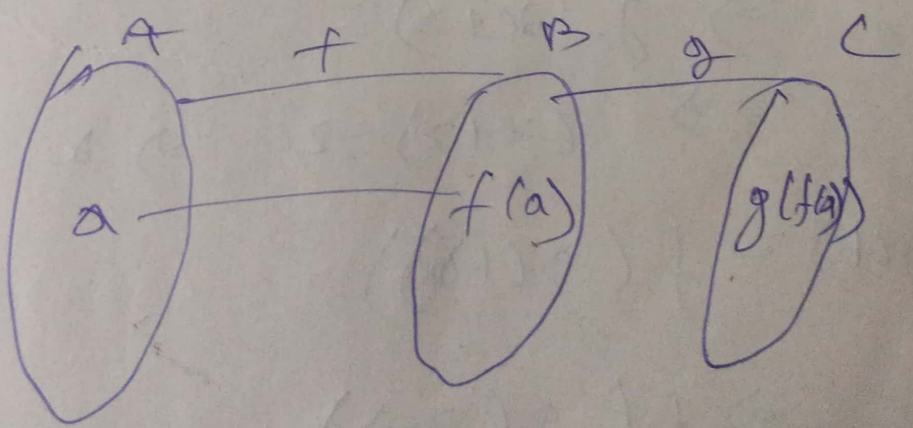
$f^{-1}(1) = \frac{x+5}{3} = \frac{1+5}{3} = \frac{6}{3} = 2$

$f^{-1}\{-5, 5\} = f^{-1}\{-5\} = \frac{1-(-5)}{3} = 2$

$f^{-1}\{5\} = \frac{5+5}{3} = \frac{10}{3} = 3.33$

$f^{-1}\{-5, 5\} = \{2, 3.33\}$

Composition of functions



Let $f : A \to B$, $g : B \to C$ are any two mappings, then the composition mapping from $f \to g$ is denoted by $g \circ f$ and it is defined by $g \circ f(a) = g(f(a))$

Q) Let $f(x) = x+2$ $g(x) = x-2$ $h(x) = 3x$

then find i) fog ii) gof iii) $fo(goh)$

iv) $fohog$ v) fof

$fog = fog(x)$

$= f(g(x))$

$= f(x-2)$

$\Rightarrow (x-2)+2 = x$

$gof = gof(x)$

$\Rightarrow g(f(x))$

$\Rightarrow g(x+2)$

$\Rightarrow (x+2)-2 \Rightarrow x$

$fo(goh) = f(g(h(x)))$

$\Rightarrow f(g(3x))$

$\Rightarrow f(3x-2)$

$\Rightarrow 3x-2+2$

$\Rightarrow 3x$

$\therefore f \circ h \circ g = f(h(g(x)))$

$\Rightarrow f(h(x-2))$

$\Rightarrow f(3x-6)$

$\Rightarrow 3x-6+2$

$\Rightarrow 3x-4$

$f \circ f = f(f(x))$

$\Rightarrow f(x+2)$

$\Rightarrow x+2+2$

$\Rightarrow x+4$