SUBJECT : PROBABILITY & ALGEBRA B.Tech II-II Sem CSD & CSM

UNIT –I

2 MARKS

1) Define mutually exclusive events with example
2) Prove that $P(A^C) = 1 - P(A)$
3) Prove that $P\left(A \cap B'\right) = P(A) - P(A \cap B)$
4) Define i) conditional probability ii) independent events
5) Define i) Equally likely events ii) Exaustive events
6) If A & B are any two events and $A \subset B\ then\ prove\ that\ P(A) \leq P(B)$
7) Write three axioms of probability
8) State Baye's theorem
9) Define independent events and conditional probability
10) State multiplication theorem on probabilities
11) If P(A) = $\frac{1}{3}$, P(B) = $\frac{1}{4}$, $P(A \cup B) = \frac{1}{2}, then\ find\ P(A/B^C)$

ESSAY QUESTIONS:

1) State and prove addition theorem for two events
2) The probability that a man will be alive in 25 years is 3/5. And probability that his wife will be alive is 2/3. Find the probability that i) both will be alive ii) only man will be alive iii) at least one will be alive
3) In a factory, machine A produces 40% of the total output and machine B produces 60%. On the average 9 items in 1000 produced by A are defective and 1 item in 250 produced by B is defective. An item is drawn at random from a day's output is defective, what is the probability that it was produced by A or B?
4) A, B, C are aiming to shoot a balloon. A will succeed 4 times out of 5 attempts. The chance of B to shoot the balloon is 3 out of 4 and that of C is 2 out of 3. If the three aim the balloon simultaneously then find the probability that at least two of them hit the balloon.
5) Two aero planes bomb a target in succession. The probability of each correctly scoring a hit is 0.3 and 0.2 respectively. The second will bomb only if the first misses the target. Find the probability that i) target is hit ii) both fails to score hits
6) A product is assembled from 3 components X, Y, Z. the probability of these components being defective is 0.01, 0.02, 0.05 respectively. What is the probability that the assembled product will not be defective?
7) State and prove Addition theorem for 'n' events
8) A, B and C in order toss a coin. The first one to toss head wins the game. What are their probabilities of winning, assuming that the game may continue indefinitely?
9) Suppose 5 men out of 100 and 25 women out of 10,000 are colour blind. A colour blind person is chosen at random what is the probability of the person being a male assuming that male and female are equally populated
10) State and prove Baye's theorem
11) The machines A, B, C produce 40%, 30%, 30% of the total number of items of factory. The percentage of defective items of these machines are 4%, 2%, 3%. If an item is selected at random and is found to be defective find the probabilities that it is produced from i) machine A ii) machine C

UNIT-II

2 MARKS

1) 1) If $f(x) = \begin{cases} \frac{1}{2}(x+1) & -1 < x < 1 \\ 0 & \text{elsewhere} \end{cases}$
2) Then find mean of ‘x’
3) If X_1, X_2 are two random variables and a ,b are constants then prove $E(aX_1 + bX_2) = a\,E(X_1)+b\,E(X_2)$
4) Define Marginal probability functions of random variables X and Y
5) If $f(x) = k(2x + 3)$ in $0<x<2$, then find k?
6) Define joint probability distribution functions of two dimensional random variables
7) A random variable X has the following probability function

X	4	5	6	8
P(X)	0.1	0.3	0.4	0.2

8) Find the mean of distribution
9) Define conditional probability functions of two dimensional random variables X and Y
10) Write any two properties of distribution function
11) Prove that $E(ax + b) = a\,E(x) + b$
12) Prove that $V(ax + b) = a^2\,V(x) + b$

ESSAY QUESTIONS

1) The joint probability density function of two dimensional random variable (X,Y) is given by

$f(x,y) = \frac{8}{9}xy, 1 \le x \le 2, 1 \le y \le 2$

find i) marginal densities of X and Y

ii) conditional density functions of X and Y

2) Two –dimensional random variable (X,Y) have the joint density

$f(x,y) = 8xy$, $0<x<1$, $0<y<1$, find i) marginal distributions

ii) conditional distributions

3) The cumulative distribution function for a continuous random variable ‘x’ is

$F(x) = \begin{cases} 1 - e^{-2x} & , x \ge 0 \\ 0 & , x < 0 \end{cases}$ Evaluate (i) density function (ii) Mean (iii) Variance

4) A random variable ‘x’ has the following probability function

x	0	1	2	3	4	5	6	7
P(x)	0	k	2k	2k	3k	K^2	$2k^2$	$7k^2+k$

Find the value of ‘k’

i) Mean

ii) Variance

5) Given joint density function $f(x,y) = \begin{cases} 2 & 0 < x < 1, 0 < y < 1 \\ 0 & other\ wise \end{cases}$

Find i) Marginal density function of x & y

ii) conditional density function 'y' given X = x and 'x given Y = y

6) Probability density function of random variable 'x' is

$$f(x) = \begin{cases} \frac{1}{2}\sin x, & \text{for } 0 \le x \le \pi \\ 0 & \text{elsewhere} \end{cases}$$

then Evaluate i) mean ii) variance

7) A continuous random variable has the probability function

$$f(x) = \begin{cases} K\ x\, e^{-\lambda x} & \text{for } x \ge 0,\ \lambda > 0 \\ 0 & \text{otherwise} \end{cases}$$

8) Determine (i) k (ii) Mean (iii) Variance

9) Two balls are drawn at random from a box containg 3 red,2 green and 4 white balls ,if X and Y are the random variables represents the number of red balls and green balls respectively including among the two balls drawn from the box, find i) joint probability of X & Y

ii) marginal probability of X&Y

iii)conditional distribution of X given Y=1

UNIT-III

2 MARKS

1) The mean and variance of a binomial distribution are 4 & 3 respectively find the mode of the binomial distribution

2) If X is a normal variate, z is a standard normal variate then find the area to the left of z = 1.78

3) If the mean and variance of the binomial variate are 12 and 4 then write binomial distribution

4) If 'z' is a standard normal variate then find the area corresponding to $0.72 \le z \le 2.48$

5) If X is a passion variate such that P(X = 0) = P(X = 2)+ 3 P(X = 4) find the mean

6) The mean and standard deviation of a normal variate are 8 and 4 respectively, then find $P(5 \le X \le 10)$

7) The mean and variance of a binomial distribution are 6 & 3 respectively find the mode of the binomial distribution

8) If X is a passion variate such that P(X = 0) = P(X = 2)+ 3 P(X = 4) then find $P(X \le 2)$

9) Suppose 2% of the people on the average are left handed. Find the probability that none or one left handed

10) If X is normal variate then find the area corresponding to $-0.72 \le z \le 1.48$

ESSAY QIESTIONS

1) 1000 students has written an examination the mean is 35 and standard deviation is 5. Assuming the distribution to be normal find
   (i) How many students marks lie between 25 and 40.
   (ii) How many get more than 40
   (iii) How many students get below 20
   (iv) How many get more than 50
   (v) How many students marks lie between 30 and 40

2) Show that mean and variance of Poission distribution are same

3) Assume that 50% of all engineering students are in good in mathematics .Determine that among 18 engineering students

i) Exactly 10 ii) at least 10 are good in mathematics

4) Derive the mean of the normal distribution

5) Find the mean and S.D of the normal distribution in which 7% of the items are under 35 and 89% are under 63

6) If 10% of the rivets produced by a machine are defective, find the probability that out of 5 rivets chosen of random (i)Non will be defective (ii) one will be defective

7) If X is a passion variate such that P(X = 0) = P(X = 2)+ 3 P(X = 4) then find

$i) P(X \leq 2)$ $ii) P(X > 2)$

8) It has been found that 2% of the tools produced by a certain machine are defective what is the probability that in a shipment of 400 such tools

(iii) i) 3% of more (ii) 2% or less will prove defective

9) In a binomial distribution consisting of 5 independent trails, probabilities of 1and 2 success are 0.4096 and 0.2048 respectively. Find the parameter 'p' of the distribution

10) Show that mean, median, mode of Normal distribution is μ

11) Fit a Binomial distribution to the following data

x	0	1	2	3	4
f	28	62	46	10	4

12) In normal distribution 31% of the items are under 45 and 8% are over 64. Find mean and variance of the distribution

13) Write the chief characteristics of Normal distribution

UNIT-IV

2 MARKS

1) Write the rules to draw the Hasse diagram of a Poset

2) Consider the function f:R to R defined by f(x)= 2x+5, another function g(x)=(x-5)/2.Prove that g is inverse of f.

3) If R is a relation on the set A={ 1,2,3,4} defined by x R y if x exactly divides y .Prove that (A,R) is a Poset.

4) Let f and g be functions from R to R defined by f(x)=ax+b ,g(x)= 1-x+ x^2 .
If (gof) =9x^2-9x+3,determine a,b

5) Let f(X)=x+2 g(x) = x-2, h(x) =3x find i) fog ii) fogoh.

6) If A = {1,2,3,4,5,6,7,8,9,10,11,12} and R= { (x,y)/x-y is multiple of 5} find the partition of A

7) Let R= { [1,1] [2,2] [3,3] [4,4] [5,5] [1,2] [2,1] [5,4] [4,5]} be the equivalence relation on A = {1,2,3,4,5} Find equivalence classes and A/R

8) Find the inverse of the function f(x) = e^x defined from R to R^+

9) Define one to one and onto functions

ESSAY QUESTIONS:

1) Draw the Hasse diagram representing the positive divisors of 36.
2) Show that the relation ‘R’ defined by (a,b) R (c,d) iff a+d=b+c is an equivalence relation
3) If X={1,2,3,4} and R= {(x,y)/x<y} Draw the graph of ‘R’ and also give its matrix
4) Write the procedure to find the maximal compatibility blocks to a compatibility relation.
5) Draw the Hasse diagram representing the positive divisors of 45
6) If R denote a relation on the set of all ordered pairs of positive integers by (a,b)R(c,d) iff ad=bc show that ‘R’ is an equivalence relation.
7) Let X={1,2,3,4,5,} and relation R={(x,y)/x>y}.Draw the graph of ‘R’ and also give its matrix
8) What is Compatibility relation and Write the procedure to find compatibility blocks.
9) Let A be any finite set and P(A) be the power set of A.⊆ be the inclusion relation on the elements of P(A). Draw the Hasse diagrams of (P(A),⊆) for i) A = {a} ii) A = {a,b} iii) A = {a,b,c} iv) A ={a,b,c,d}
10) Let A = B = {x/ -1$\leq x \leq 1$} for each of the following functions state whether it is injective, surjective or bijective

    a) f(x) = IxI b) g(x) = sinπx c) h(x) = $\frac{2x+3}{5}$
11) Show that the relation R={ (a,a),(a,b),(b,a),(b,b)(c,c)} on A={a,b,c} is an equivalence relation and find A/R also find partitions of A
12) Let f:R→ R, $g: R \to R$, where R is the set of real numbers be given by f(x) = $x^2 - 2$ and
13) g(x) = x+4 find f og and gof. state whether these functions are bijective or not
14) Prove that the relation R defined by “a is congruent to b modulo m” on the set of integers is an equivalence relation.
15) Draw the Hasse diagram representing the positive divisors of 45.

UNIT-V

2 MARKS

1) Show that the cube roots of unity forms a group with respect to multiplication.
2) Define a) Left coset of subgroup H in G. b) Right coset of subgroup H in G.
3) If a, b ∈ group G, then prove that $(ab)^{-1}$ =b^{-1} a^{-1}
4) Find the order of each element in the group $\{1, -1, i, -i\}$.
5) If aob=a+b+ab∀ a,b∈Z S.T (Z,0) is a semi group.
5) Define cyclic group with an example.
6) Show that binary operation * defined on (R,*) where x*y=x^y is not associative
7) Define a) Normal subgroup of a group b) Quotient group
8) Define Monoid with example
9) State Lagranges theorem on Cosets

ESSAY QUESTIONS

1) Construct composition table for the roots of equation x^4= 1 and Show that it is a group with respect to operation multiplication.
2) If ‘G’ is a group then prove that $(a^{-1})^{-1}$=a
3) Prove that G ={0,1,2,3,4} is an abelian group of order 5 with respect to addition modulo 5.

4) Show that Q_1 (rational numbers other than1) is an infinite abelian group with respect to * defined by a*b=a+b-ab, where a,b are rational number
5) Prove that the identity element of a group “G’ is same as identity element of its subgroup H
6) Prove that in a group its identity element , inverse element are unique.
7) Show that the fourth roots of unity form a group with respect to multiplication
8) Prove that the identity element of a group “G’ is same as identity elements of its subgroup H
9) State and prove Lagrange’s theorem on cosets
10) Prove that set of non singular matrices of order 2 x 2 is a group but not an abelian group under multiplication.
11) Prove that G ={0,1,2,3,4,5,6} is an abelian group of order 7 with respect to addition modulo 7.
12) Define subgroup, normal subgroup, Quotient group, left and right cosets with an example for each.