8/6/23

Unit - 2

Random variables

Real Random Variable :-

→ Real variable whose value is determined the outcome of a random experiment is called random variable.

→ This is also defined as real valued function defined on the sample space 'S' of a random experiment such that for each point x of the sample space the corresponding probablity occured.

Eg:- when two coins are tossed at a time the sample space consists of

$S = \{HH, HT, TH, TT\}$

Let x be a random variable which takes no. of heads on S. Thus x takes the values ~~x={0, 1, 2}~~ $x = \{0, 1, 2\}$

x	0	1	2
$P(x_i)$	1/4	2/4	1/4
	$P(x=0)$	$P(x=1)$	$P(x=2)$

Types of Random variable (RV)

→ RV are of two types:-

1) Discrete RV

2) Continious RV

Discrete RV :-

→ A RV which takes only finite number of discrete values in any domain is called a discrete RV.

→ In other words If the random variable takes the values only on the set {0, 1, 2, 3...} is called a discrete ~~random~~ RV.

Eg:-

RV denoting the No. of students in a class

$S = \{\text{students in the class}\}$

$X = \{x / x \text{ is a +ve integer}\}$

properties of discrete RV

1) **probablity mass function (pmf)**

for a discrete RV 'x' the real value function $p(x_i) = p(x = x_i)$ is called probablity mass function of a discrete RV

if $p(x_i)$ is a probablity function of a RV 'x' then it possess the following 2 properties :-

1) $P(x_i) \geq 0 \quad \forall i = 1, 2, 3, \ldots$

2) $\Sigma p(x_i) = 1$

2) ~~Probabli~~ **probablity distribution function. (cummulative distribution function) (cdf)**

Let x be a discrete RV then probablity distribution function associated with x is defined as the probablity that the outcome of an Experiment will be one of the outcome for which x belongs to that sample space

→ It is denoted by $F(x)$

$F(x) = P(X \le x_i)$

Properties of cdf :-

1) $P(a \le x \le b) = F(b) - F(a)$

2) $F(-\infty) = 0$

3) $F(\infty) = 1$

Distribution table of descrete RV

If x is a descrete RV with PMF $P(x_i)$ then the probablity distribution table of descrete RV X is given by

X	x_1	x_2	......	x_n
$P(x_i) = P(X = x_i)$	$P(x_1)$	$P(x_2)$	....	$P(x_n)$

i) $P(X < x_i) = P(x = x_1) + P(x = x_2) + \dots + P(x = x_{i-1})$

ii) $P(X \le x_i) = P(x = x_1) + P(x = x_2) + \dots + P(x = x_i)$

Expectations, Mean, Variance of DRV :-

Mean :-

Let x is a DRV with the values $x_1, x_2 \ldots x_n$ along with the probablities $P_1, P_2 \ldots, P_n$ then mathematical Expectation (or) Mean (or) Expected value of x denoted by Expectation of x $E(x)$

and is defined as sum of product of x with their probablities.

$$\text{Mean} = \mu = E(x) = x_1 P_1 + x_2 P_2 + \cdots + x_n P_n$$

$$= \sum_{i=1}^{n} x_i P_i$$

In general the expected value of any function of a RV x is called as population mean and it is denoted by ~~μ~~

$$\mu = E(x) = \sum_{i=1}^{n} x_i \, p(x_i) = \sum_{i=1}^{n} x_i P_i$$

Variance:- Variance characterises the variablity in the distribution since the two distributions with same mean can still have different dispersions of the data with their means.

→ Variance of the probablity distributions of a RV X is the ~~expectation~~ $E(X - E(X))^2$

$\therefore V(X) = E(X - E(X))^2$

$= E(X^2 + (E(X))^2 - 2XE(X))$

$= E(X^2) + E(X))^2 - 2E(X)E(X)$

$= E(X^2) + (E(X))^2 - 2(E(X))^2$

$V(X) = E(X^2) - (E(X))^2$

$\Rightarrow \sum_{i=1}^{n} x_i^2 P_i - \left\{ \sum_{i=1}^{n} x_i P_i \right\}^2$

$\Rightarrow \sum_{i=1}^{n} x_i^2 P_i - \mu^2$

standard Diveation — positive square root of the variance is called the S.D and it is denoted by $\sigma = \sqrt{V(x)}$

Q1) find mean and variance of the following data.

x	0.3	0.2	0.1	0	1	2	3
$P(x=x_i)$	0.05	0.10	0.30	0	0.30	0.15	0.1

$\text{mean} = E(x) = \sum_{i=1}^{n} x_i P_i$

0.015 0.02 0.03 0.3 0.3 0.3

$\Rightarrow 0.9650$

Variance 1.8115

$V(x) = E(x^2) - (E(x))^2$

$\Rightarrow \sum x_i^2 P(x_i) - (E(x))^2$

$E(x^2) = \sum x_i^2 P(x_i)$

$\Rightarrow (0.3)^2 \times 0.05 + (0.2)^2 \times 0.10 +$

$(0.1)^2 \times 0.3 + 1 \times 0.3 + (2)^2 \times 0.15 + (3)^2 \times 0.1$

$\Rightarrow 1.8115$

$$v(x) = 1.8115 - (\cancel{\quad}\, 0.9650)^2$$

$$= 0.8803$$

Q2) The probablity function of x is given by

x	0	1	2	3	4	5	6
$P(x)$	k	$3k$	$5k$	$7k$	$9k$	$11k$	$13k$

find i) k

ii) $P(x < 4)$

iii) $P(x \geq 5)$

iv) $P(3 < x \leq 6)$

i) we ~~th~~ know that

$$\Sigma P(x_i) = 1$$

$$k + 3k + 5k + 7k + 9k + 11k + 13k = 1$$

$$49k = 1$$

$$k = \frac{1}{49}$$

$$k = 0.0204$$

Scanned with OKEN Scanner

i) $P(x<4)$

$\Rightarrow P(x=0)+P(x=1)+P(x=2)+P(x=3)$

$\Rightarrow k+3k+5k+7k$

$\Rightarrow 16k$

$\Rightarrow 16\times\frac{1}{49} \Rightarrow \frac{16}{49}$

$\Rightarrow 0.3265$

ii) $P(x\geq 5)= P(x=5)+P(x=6)$

$11k+13k$

$\Rightarrow 24k$

$\Rightarrow \frac{24}{49}$

$\Rightarrow 0.4898$

iv) $P(3<x\leq 6)= P(x=4)+P(x=5)+P(x=6)$

$\Rightarrow 9k+11k+13k$

$\Rightarrow 33k$

$\Rightarrow 33\times\frac{1}{49}$

$\Rightarrow 0.6735$

Scanned with OKEN Scanner

Q) from the above table find value of a for which $P(x \le a) > 0.3$

$$P(x \le a) > 0.3$$

Let $a = 0$

i) $P(x \le 0) = P(x = 0)$

$= k = \frac{1}{49} = 0.0204$

Let $a = 1$

ii) $P(x \le 1) = P(x=0) + P(x=1)$

$= k + 3k = \frac{4}{49} = 0.081$

iii) Let $a = 2$

$P(x \le 2) = k + 3k + 5k = 9k = \frac{9}{49} = 0.18$

iv) Let $a = 3$

$P(x \le 3) = 0.326$

$P(x \le 3) > 0.3$

$\therefore$ minimum value of a is 3.

12/6/23

Q) find mean & variences of uniform probability distribution. $f(x) = \frac{1}{n}$ for $x = 1, 2, 3 \ldots$

x	1	2	3	....	n
$P(x_i)$	$\frac{1}{n}$	$\frac{1}{n}$	$\frac{1}{n}$	.....	$\frac{1}{n}$

Mean = $E(x) = \Sigma x\, p(x_i)$

$= 1 \times \frac{1}{n} + 2 \times \frac{1}{n} + 3 \times \frac{1}{n} + \ldots\ n \times \frac{1}{n}$

$\Rightarrow \frac{1}{n} [1 + 2 + 3 + \ldots + n]$

$\Rightarrow \frac{1}{\cancel{n}} \times \frac{\cancel{n}(n+1)}{2} \Rightarrow \frac{n+1}{2}$

$E(x^2) = \Sigma x^2 p(x)$

$= 1^2 \times \frac{1}{n} + 2^2 \times \frac{1}{n} + 3^2 \times \frac{1}{n} + \ldots\ n^2 \times \frac{1}{n}$

$\Rightarrow \frac{1}{n} \{1^2 + 2^2 + 3^2 \ldots n^2]$

$\Rightarrow \frac{1}{\cancel{n}} \times \frac{\cancel{n}(n+1)(2n+1)}{6}$

$\Rightarrow \frac{(n+1)(2n+1)}{6}$

$$V(x) = E(x^2) - (E(x))^2$$

$$\Rightarrow \frac{(n+1)(2n+1)}{6} - \left(\frac{n+1}{2}\right)^2$$

$$\Rightarrow \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$\Rightarrow \frac{n+1}{2}\left[\frac{2n+1}{3} - \frac{n+1}{2}\right]$$

$$\Rightarrow \frac{n+1}{2}\left[\frac{4n+2-[3n+3]}{6}\right]$$

$$\Rightarrow \frac{n+1}{2}\left[\frac{n-1}{6}\right]$$

$$\frac{n^2-1}{12}$$

$$\Rightarrow \frac{n^2-1}{12}$$

---

ii) A sample of 4 items is selected at random from a box containing 12 items of which 5 are defective. find the expected number E of the defective items.

$$\begin{array}{ccc} & 12 & \\ \swarrow & & \searrow \\ 5D & & 7N \end{array}$$

$$P(x=0) = \frac{(5_0)(7_4)}{(12_{C_4})} = 0.0707$$

$$P(x=1) = \frac{(5_{C_1})(7_{C_3})}{(12_{C_4})} = 0.3535$$

$$P(x=2) = \frac{(5_2)(7_2)}{(12_4)} = 0.4242$$

$$P(x=3) = \frac{(5_3)(7_1)}{(12_4)} = 0.1414$$

$$P(x=4) = \frac{(5_{C_4})(7_{C_0})}{(12_{C_4})} = 0.0101$$

X	0	1	2	3	4
$P(X=x_i)$	0.0707	0.3535	0.4242	0.1414	0.0101

Expected ~~no~~ $= 0 \times 0.007 + 1 \times 0.3535 + 2 \times 0.424$

$+ 3 \times 0.1414 + 4 \times 0.0101$

$\Rightarrow 1.6665$

8)

x	0	1	2	3	4	5	6	7
$P(x)$	0	k	$2k$	$2k$	$3k$	$3k$	k^2	$7k^2+k$

find i) k

ii) $P(x<6)$

iii) $P(x \geq 6)$

$\Sigma P(x_i) = 1$

i) $k+2k+2k+3k+3k+k^2+7k^2+k=1$

$8k^2+12k-1=0$

$-\frac{3+\sqrt{11}}{4}, \frac{-3-\sqrt{11}}{4}$

$k = 0.0792, -1.5792$

$k = 0.0792$

ii) $P(x<6)$

$0+k+2k+2k+3k+3k$

$\Rightarrow 11k$

Scanned with OKEN Scanner

$\Rightarrow 11 \times 0.0792$

$\Rightarrow 0.8712$

iii) $P(x \geq 6)$

$\Rightarrow k^2 + 7k^2 + k$

$\Rightarrow 8k^2 + k$

$\Rightarrow 0.1294$

---

9)

x	0	1	2	3	4	5	6	7	8
	$\frac{k}{45}$	$\frac{k}{15}$	$\frac{k}{9}$	$\frac{k}{5}$	$\frac{2k}{45}$	$\frac{6k}{45}$	$\frac{7k}{45}$	$\frac{8k}{45}$	$\frac{4k}{45}$

$$\frac{k}{45} + \frac{k}{15} + \frac{k}{9} + \frac{k}{5} + \frac{2k}{45} + \frac{6k}{45} + \frac{7k}{45} + \frac{8k}{45} + \frac{4k}{45} = 1$$

$$\frac{k + 3k + 5k + 9k + 2k + 6k + 7k + 8k + 4k}{45} = 1$$

$$\frac{45k}{45} = 1 \quad \boxed{k = 1}$$

$E(x) = \Sigma x_i p_i$

$\Rightarrow 0 \times \frac{k}{45} + 1 \times \frac{k}{15} + 2 \times \frac{k}{9} + 3 \times \frac{k}{5} + 4 \times \frac{2k}{45} +$

$5 \times \frac{6k}{45} + 6 \times \frac{7k}{45} + 7 \times \frac{8k}{45} + 8 \times \frac{4k}{45}$

$\Rightarrow \frac{k}{15} + \frac{2k}{9} + \frac{3k}{5} + \frac{8k}{45} + \frac{30k}{45} + \frac{42k}{45} + \frac{56k}{45} + \frac{32k}{45}$

$\Rightarrow \frac{3k + 10k + 27k + 8k + 30k + 42k + 56k + 32k}{45}$

$\Rightarrow \frac{\cancel{213}\,208\,k}{45} \Rightarrow \frac{\cancel{213(1)}\,208}{45}$

$\Rightarrow \cancel{4.7333}$ $4.6222 \longrightarrow 4.62$

$E(x^2) = 0^2 \times \frac{k}{45} + 1^2 \times \frac{k}{15} + 2^2 \times \frac{k}{9} + 3^2 \times \frac{k}{5}$

$+ 4^2 \times \frac{2k}{45} + 5^2 \times \frac{6k}{45} + 6^2 \times \frac{7k}{45} + 7^2 \times \frac{8k}{45} + 8^2 \times \frac{4k}{45}$

$\frac{k}{15} + \frac{4k}{9} + \frac{9k}{5} + \cancel{\ } \frac{32k}{45} + \frac{150k}{45} + \frac{252k}{45} + \frac{392k}{45} + \frac{256k}{45}$

$\Rightarrow 30.8$

$\Rightarrow E(x^2) - (E(x))^2$

$\Rightarrow 30.8 - (4.6222)^2$

$\Rightarrow 9.4353 \longrightarrow 4.$

Q) calculate Expected value and S.D.

x	-1	0	1	2	3
f	0.3	0.1	0.1	0.3	0.2

$E(x) = \sum x_i f_i$

$\Rightarrow -1 \times 0.3 + 0 \times 0.1 + 1 \times 0.1 + 2 \times 0.3 + 3 \times 0.2$

$\Rightarrow 1$

$E(x^2) =$

$\Rightarrow (-1)^2 \times 0.3 + 0^2 \times 0.1 + 1^2 \times 0.1 + 2^2 \times 0.3 + 3^2 \times 0.2$

Scanned with OKEN Scanner

$0.3 + 0.1 + 1.2 + 1.8$

~~Mean E(x) 3.4~~ $E(x^2) = 3.4$

Variance

$\Rightarrow E(x^2) - (E(x))^2$

$\Rightarrow 3.4 - (1)^2$

Variance $\Rightarrow 2.4$

S.D $\Rightarrow = \sqrt{\text{Variance}}$

$\Rightarrow \sqrt{2.4}$

$\Rightarrow 1.5492$

Some important results on variences (or) properties of variance

1) Variance of a constant is zero

2) If x is a random variable and k is a constant then variance of $v(x+k) = v($

3) If x is a random variable then variance of $v(x) = E(x^2) - (E(x))^2$

4) If x is a random variable then

$$v(ax+b) = a^2 v(x)$$

Proof :-

Let $y = ax+b$ ——— ①

$E(y) = E(ax+b)$

$E(y) = aE(x)+b$ ——— ②

① - ②

$Y - E(Y) = (ax+b) - (aE(x)+b)$

$= a(x-E(x)) \cancel{+b-b}$

$Y - E(Y) = a(x-E(x))$

taking expectations on B S

$E(Y-E(Y))^2 = E[(a(X-E(X)))^2]$ (by defination of variance)

$V(Y) = a^2 E(X-E(X))^2$

$V(aX+b) = a^2 V(X)$

Q) for the following probablity distributio find variance of X $V(X)$ and variance of $2X-3$ $V(2X-3)$

X	-3	-2	-1	0	1	2	3
P(X)	0.001	0.01	0.1	K	0.1	0.01	0.001

find i) K
ii) $V(X)$
iii) $V(2X-3)$
iv) $E(X^2+2X+3)$

$\Sigma P(x_i) = 1$

$0.001+0.01+0.1+K+0.1+0.01+0.001 = 1$

$K+0.222 = 1$

$K = 1-0.222$

$K = 0.7780$

ii) $v(x)$

$= E(x^2) - (E(x))^2$

$\cdot\ E(x^2) = \Sigma x_i^2 P(x_i)$

$\Rightarrow -3\times 0.001 + (-2)0.01 + (-1)0.1 + 0(k) + 1(0.1)$

$+ 2(0.01) + 3(0.001)$

$\Rightarrow 0$

$E(x^2)$

$9\times 0.001 + 4\times 0.01 + 0.1 + 0.1 + 4\times 0.01 +$

$\cancel{3x}\ 9\times 0.001$

$\Rightarrow 0.2980$

$v(\cancel{x}x) = E(x^2) - (E(x))^2$

$\Rightarrow 0.2980$

iii) $v(2x-3) = 2^2 v(x)$

$\Rightarrow 4\times 0.2980$

$\Rightarrow 1.1920$

Scanned with OKEN Scanner

iv)

$E(x^2+2x+3) = E(x^2) + 2E(x) + 3$

$\Rightarrow 0.2980 + 2(0) + 3$

$\Rightarrow 3.2980$

Q) If $P(x) = 1/4$ $x = 0, 1, 2, 3$ find Cummulative distribution function and also sketch its graph.

$P(x) = 1/4$

$F(x) = 0 \qquad x < 0$

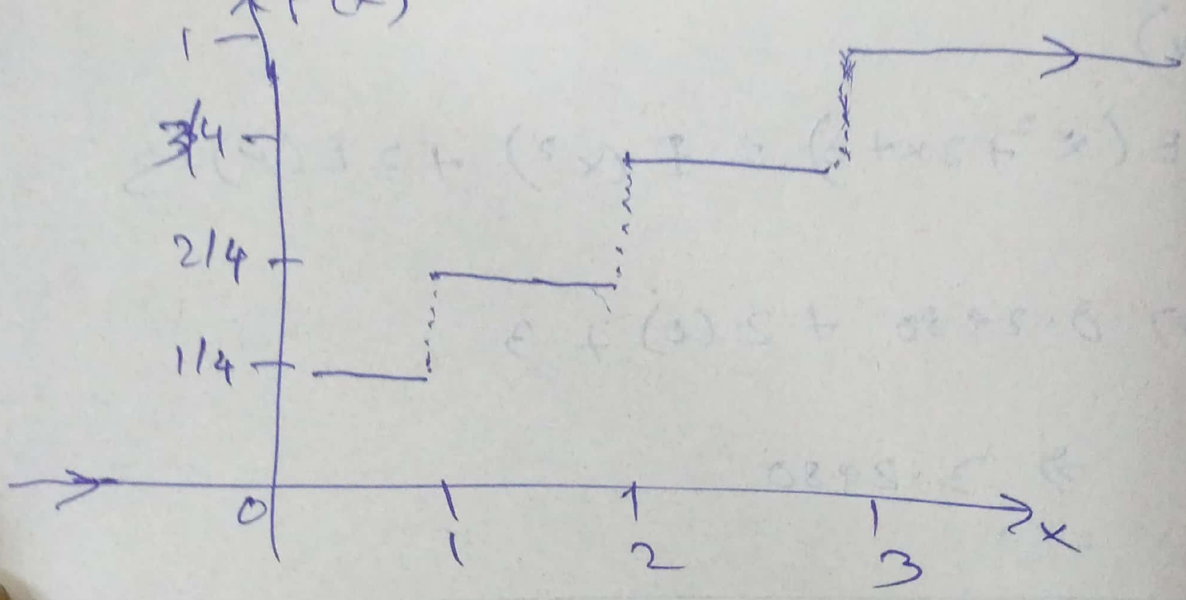
$F(x) = 1/4 \qquad 0 \leq x < 1$

$F(x) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4} \qquad 1 \leq x < 2$

$F(x) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{3}{4} \qquad 2 \leq x < 3$

$F(x) = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} \qquad x \geq 3$

$\Rightarrow 1$



Descrete random variable x has the proba--blity distribution function (cdf)

$$F(x) = \begin{cases} 0 & \text{for } x < 1 \\ 1/3 & 1 \le x < 4 \\ 1/2 & 4 \le x < 6 \\ 5/6 & 6 \le x < 10 \\ 1 & \ge 10 \end{cases}$$

find i) $P(2 \le x \le 6)$

ii) $P(x = 5)$

iii) $P(x \le 6)$

i) $P(2 \le x \le 6) = F(6) - F(2)$

$$= \frac{5}{6} - \frac{1}{3} = \frac{1}{2}$$

$P(x=5) = P(x \le 5) - P(x < 5)$

$\Rightarrow F(5) - P(x<5)$

$\Rightarrow \frac{1}{2} - \frac{1}{2}$

$\Rightarrow 0$

iii) $P(x \le 6) = F(6)$

$= 5/6$

iv) $P(x=6)$

$P(x \le 6) - P(x < 6)$

$\Rightarrow F(6) - P(x < 6)$

$\Rightarrow 5/6 - \frac{1}{2}$

$\Rightarrow 0.3333$

Continuous RV :-

When a RV ~~let~~ x takes every value in an interval is called continious RV.

Eg:- The distribution defined the variates like temperature, hights and weights or continuous distributions.

Probablity Density function (PDF)

For a continuous RV, problity function is called PDF because it is defined for every point in the range and not for the certain value.

Properties of PDf

1) PDf is denoted by $f(x)$ and it has to satisfy the following two properties

i) $f(x) \geq 0$

ii) $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Note:- In CRV's the probablities are calulated as $P(a<x<b) = P(a \leq x \leq b) = \int_a^b f(x)\,dx$

Scanned with OKEN Scanner

CDF (Cumulative distribution function) of a CRV

If x is a continious RV then its CDF is denoted by $F(x)$ and it is defined as $F(x) = \int_{-\infty}^{x} f(x)\,dx$,

where,

$f(x)$ is the PDF of x CRV

properties of CDF :-

i) $F(-\infty) = 0$

ii) $F(\infty) = 1$

iii) $P(a < x < b) = P(a \le x \le b) = \int_{a}^{b} f(x)\,dx$

iv) $F(x) = \int_{-\infty}^{x} f(x)\,dx$

$$f(x) = \frac{d}{dx} F(x)$$

Mean , Variance & S.D of a CRV

Mean:-

If x is a continious RV with PDF $f(x)$, then

$$\text{Mean} = E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$$

Variance:-

If x is a CRV with PDF $f(x)$ then

~~Varian~~ $V(x) = E(x^2) - (E(x))^2$

where,
$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

Standard deviation

Positive square root of the variance

$$S.D \text{ of } x = \sqrt{V(x)}$$

Q) If x is a CRV with PDF

$$f(x) = \begin{cases} k x e^{-\lambda x} & x \geq 0,\ \lambda > 0 \\ 0 & \text{otherwise} \end{cases}$$

find i) k
ii) Mean
iii) Variance

i) we know that $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$$\int_{-\infty}^{\infty} f(x)\, dx = \int_{-\infty}^{0} f(x) dx + \int_{0}^{\infty} f(x)\, dx = 1$$

$\Rightarrow 0 + \int_0^\infty k\, x e^{-\lambda x}\, dx = 1$

$$k\left(x\left(\frac{e^{-\lambda x}}{-\lambda}\right) - 1 \times \frac{e^{-\lambda x}}{\lambda^2} \right)_0^\infty = 1$$

$$k\left(0 + \frac{1}{\lambda^2}\right) = 1$$

$$\frac{k}{\lambda^2} = 1 \Rightarrow k = \lambda^2$$

ii) Mean $= E(x) = \int_{-\infty}^{\infty} x \cdot f(x)$

$$0 + \int_0^\infty x\, k\, x e^{-\lambda x}$$

$$k \left[x^2\left(\frac{e^{-\lambda x}}{-\lambda}\right) - 2x\left(\frac{e^{-\lambda x}}{\lambda^2}\right) + 2\left(\frac{e^{-\lambda x}}{-\lambda^3}\right) \right]_0^\infty$$

$$\Rightarrow k\left[0 - \left(\frac{-2}{\lambda^3}\right)\right]$$

$$\Rightarrow \frac{2k}{\lambda^3} = \frac{2\lambda^2}{\lambda^3} \Rightarrow \boxed{\frac{2}{\lambda}}$$

iii) $v(x) = E(x^2) - (E(x))^2$

$$E(x^2) = \int_0^\infty x^2\, k x e^{-\lambda x}\, dx$$

$$\Rightarrow k\int_0^\infty x^3 e^{-\lambda x}\, dx$$

$$\Rightarrow k\left[x^3\left(\frac{e^{-\lambda x}}{-\lambda}\right) - 3x^2\left(\frac{e^{-\lambda x}}{\lambda^2}\right) + 6x\left(\frac{e^{-\lambda x}}{-\lambda^3}\right) - 6\left(\frac{e^{-\lambda x}}{\lambda^4}\right)\right]_0^\infty$$

$$+6k\left(\frac{e^0}{\lambda^4}\right)$$

$$\Rightarrow \frac{6\lambda^2}{\lambda^4} \Rightarrow \boxed{\frac{6}{\lambda^2}}$$

$$v(x) = \frac{6}{\lambda^2} - \left(\frac{2}{\lambda}\right)^2$$

$$\Rightarrow \frac{6}{\lambda^2} - \frac{4}{\lambda^2}$$

$$\Rightarrow \boxed{\frac{2}{\lambda^2}}$$

Q) If x CRV with PDF ~~$f(x)$~~

$$f(x) = \frac{1}{\pi(1+x^2)} \qquad -\infty \le x \le \infty$$

i) is $f(x)$ is a pdf

ii) ~~fo~~ If so, find its mean

To prove ~~P~~ $f(x)$ is PDF ~~f~~(x) has to satisfy

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\int_{-\infty}^{\infty} \frac{1}{\pi(1+x^2)}\,dx \qquad \Rightarrow \frac{1}{\pi}$$

$$\boxed{\begin{aligned} \because \int_{-a}^{a} f(x)\,dx &= 2\int_{0}^{a} f(x)\,dx \quad \text{if } f(x) \text{ is even fn} \\ &= 0 \quad \text{if } f(x) \text{ is odd fn} \end{aligned}}$$

$$\Rightarrow \frac{1}{\pi} \times 2 \int_{0}^{\infty} \frac{1}{1+x^2}\,dx$$

$$\Rightarrow \frac{1}{\pi} \times 2 \left(\tan^{-1}x\right)_0^{\infty}$$

$$\Rightarrow \frac{2}{\pi}\left\{\tan^{-1}\infty - \tan^{-1}0\right\}$$

$$\Rightarrow \frac{2}{\pi}\left[\frac{\pi}{2} - 0\right]$$

$$\Rightarrow 1$$

Mean $= E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$\Rightarrow \int_{-\infty}^{\infty} x \times \frac{1}{\pi(1+x^2)}\,dx$

$\Rightarrow \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x}{1+x^2}\,dx$

$\Rightarrow \cancel{\frac{1}{\pi} \times \frac{1}{2} \int_{-\infty}^{\infty} \frac{2x}{1+x^2}}$

$\Rightarrow 0 \quad [\because f(x) \text{ is odd}]$

---

8) If x is a continuous RV whose PDF is given by

$$f(x) = \begin{cases} \frac{1}{16}(3+x)^2 & -3 \le x \le -1 \\ \frac{1}{16}(6-2x^2) & -1 \le x \le 1 \\ \frac{1}{16}(3-x)^2 & 1 \le x \le 3 \end{cases}$$

show that area under the curve $f(x)$ above the x-axis is unity. also find mean of x.

to prove $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\left(\int_{-\infty}^{-3} + \int_{-3}^{-1} + \int_{-1}^{1} + \int_{1}^{3} + \int_{3}^{\infty} \right) f(x)\,dx$$

$$\Rightarrow 0 + \int_{-3}^{-1} \frac{1}{16}(3+x)^2\,dx + \int_{-1}^{1} \frac{1}{16}(6-2x^2)\,dx + \int_{1}^{3} \frac{1}{16}(3-x)^2\,dx + 0$$

$$\Rightarrow \frac{1}{16}\left[\frac{(3+x)^3}{3}\Bigg|_{-3}^{-1} + \left(6x + 2\frac{x^3}{3}\right)\Bigg|_{-1}^{1} + \frac{(3-x)^3}{-3}\Bigg|_{1}^{3} \right]$$

$$\Rightarrow \frac{1}{16}\left[\frac{1}{3}(8-0) + 6(1-(-1)) - \frac{2}{3}(1-(-1)) - \frac{1}{3}(0-8) \right]$$

$$\Rightarrow \frac{1}{16}\left[\frac{8}{3} + 12 - \frac{4}{3} + \frac{8}{3} \right]$$

$\Rightarrow 1$

$\therefore$ area under the curve $f(x)$ above x-axis is unity

mean $= E(x) = \int_{-\infty}^{\infty} x f(x)\,dx$

$$\Rightarrow \left(\int_{-\infty}^{-3} + \int_{-3}^{-1} + \int_{-1}^{1} + \int_{1}^{3} + \int_{3}^{\infty} \right) x f(x)\,dx$$

$$\Rightarrow \int_{-3}^{-1} \frac{x}{16}(3+x)^2\,dx + \int_{-1}^{1} \frac{x}{16}(6-2x^2)\,dx + \int_{-1}^{3} \frac{x}{16}(3-x)^2\,dx$$

$$\Rightarrow \frac{1}{16}\left[\int_{-3}^{-1}(9x+6x^2+x^3)\,dx + \int_{-1}^{1}(6x-2x^3)\,dx\right.$$

$$\left. + \int_{1}^{3}(9x-6x^2+x^3)\,dx\right]$$

$$\Rightarrow \frac{1}{16}\left(\frac{9x^2}{2}+\frac{6x^3}{3}+\frac{x^4}{4}\Big|_{-3}^{-1} + \left(\frac{6x^2}{2}-\frac{2x^4}{4}\right)\Big|_{-1}^{1}\right.$$

$$\left. + \frac{9x^2}{2}-6\frac{x^3}{3}+\frac{x^4}{4}\Big|_{1}^{3}\right]$$

$$\Rightarrow \frac{1}{16}\left[\frac{9}{2}[1-(9)] + 2[-1+27] + \frac{1}{4}[1-81]\right.$$

$$+ 3[1-1] - \frac{1}{2}[1-1]$$

$$\left. + \frac{9}{2}[9-1] - 2[27-1] + \frac{1}{4}[81-1]\right]$$

$$\Rightarrow \frac{1}{16}\Big[\cancel{\tfrac{9}{2}(-8)} + \cancel{2[26]} + \cancel{\tfrac{1}{4}[-80]}$$

$$+ \cancel{3[0]} - \cancel{\tfrac{1}{2}[0]} + \cancel{\tfrac{9}{2}[8]}$$

$$- \cancel{2[26]} + \cancel{\tfrac{1}{4}[80]}\Big]$$

$$\Rightarrow \frac{1}{16}[0]$$

$$\Rightarrow 0$$

Q) The PDF of x is given by

$$f(x) = \begin{cases} kx^2 & 0 < x < 3 \\ 0 & \text{otherwise} \end{cases}$$

find

i) k ii) $F(x)$ (cdf) iii) $P(1 \le x \le 2)$

we know that $\int_{-\infty}^{\infty} f(x)\,dx = 1$

$$\left(\int_{-\infty}^{0} + \int_{0}^{3} + \int_{3}^{\infty}\right) f(x)\,dx = 1$$

$$0 + \int_0^3 kx^2\,dx + 0 = 1$$

$$k\left\{\frac{x^3}{3}\right\}_0^3 = 1$$

$$\frac{k}{3}\left[27 - 0\right] = 1$$

$$9k = 1$$

$$k = \frac{1}{9}$$

(ii) $F(x) = cdf$

$$\int_{-\infty}^{x} f(x)\,dx$$

$$\Rightarrow \int_{-\infty}^{0} f(x)\,dx + \int_0^x f(x)\,dx = 1$$

$$\Rightarrow 0 + \int_0^x kx^2\,dx = 1$$

$$\Rightarrow k \frac{x^3}{3}\Big|_0^x = 1$$

$$\Rightarrow \frac{k}{3}\left[x^3 - 0\right] \Rightarrow$$

$\frac{1}{9\times 3} x^3 \Rightarrow \boxed{\frac{x^3}{27}}$

iii) $P(1 \le x \le 2)$

$\int_1^2 f(x)\,dx$

$\int_1^2 k x^2\,dx$

$k \frac{x^3}{3} \Big|_1^2$

$\Rightarrow \frac{k}{3}[8-1] \Rightarrow \frac{k}{3}(7)$

$\Rightarrow \frac{7}{9\times 3}$

$\Rightarrow \boxed{\frac{7}{27}}$

$\Rightarrow 0.2593$

Q) The cdf of x is

$$F(x) = \begin{cases} 1 - e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

find i) Pdf $f(x)$

ii) Mean

iii) Variance

$$F(x) = \int_{-\infty}^{\infty} f(x)\,dx$$

i) $f(x) = \frac{d}{dx} F(x)$

$$= \frac{d}{dx}\left(1 - e^{-2x}\right) \quad x \geq 0$$

$$f(x) = \begin{cases} 0 - e^{-2x}(-2) & x \geq 0 \\ 0 & x < 0 \end{cases}$$

$$f(x) = \begin{cases} 2e^{-2x} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

ii) $E(x) = \text{Mean} = \int_{-\infty}^{\infty} x f(x)\,dx$

$\Rightarrow \left(\int_{-\infty}^{0} + \int_{0}^{\infty}\right) x f(x)\,dx$

$\Rightarrow 0 + \int_{0}^{\infty} x\left(2e^{-2x}\right) dx$

$\Rightarrow 2\left[x\left(\frac{e^{-2x}}{-2}\right) - 1 \times \frac{e^{-2x}}{4}\right]_{0}^{\infty}$

$\Rightarrow 2\left\{0 + \frac{1}{4}\right\} \Rightarrow \boxed{\frac{1}{2}}$

iii) Variance =

$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\,dx$

$\Rightarrow 0 + \int_{0}^{\infty} x^2\left(2e^{-2x}\right) dx$

$\Rightarrow 2\left\{x^2\left[\frac{e^{-2x}}{-2}\right] - 2x\left(\frac{e^{-2x}}{4}\right] + 2\left[\frac{e^{-2x}}{-8}\right]\right\}_{0}^{\infty}$

$\Rightarrow 2\left[\cancel{0} - \left(-\frac{2}{8}\right)\right]$

$\Rightarrow \frac{4}{8} \Rightarrow \frac{1}{2}$

$v(x) = \frac{1}{2} - \left(\frac{1}{2}\right)^2$

$\Rightarrow \frac{1}{4} \Rightarrow 0.25$

$$f(x) = \begin{cases} 2kxe^{-x^2} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

find i) k ii) distribution function of x (cdf)

sol)

w.k.that

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\left(\int_{-\infty}^{0} + \int_{0}^{\infty}\right) f(x)\,dx = 1$$

$0 + \int_0^\infty 2kxe^{-x^2} dx = 1$

$2k \int_0^\infty xe^{-x^2} dx = 1$

$2k \int_0^\infty e^{-k} \frac{dt}{2} = 1$ put $x^2 = t$

$2x\,dx = dt$

$x\,dx = \frac{dt}{2}$

$x=0,\ t=0$

$x \to \infty,\ t \to \infty$

$\frac{\cancel{2}k}{\cancel{2}} \int_0^\infty e^{-t} dt = 1$

$k \left(\frac{e^{-t}}{-1} \right|_0^\infty = 1$

$-k(0-1) = 1$

$k = 1$

---

ii) $cdf = F(x) = \int_{-\infty}^{x} f(x)\,dx$

$\Rightarrow \left(\int_{-\infty}^{0} + \int_0^{x} \right) f(x)\,dx$

$0 + \int_0^x 2kxe^{-x^2} dx$

$\Rightarrow 2k \int_0^x x e^{-x^2} dx$

$\Rightarrow 2k \int_0^x \frac{dt}{2} e^{-t}$

put $x^2 = t$

$\Rightarrow \frac{2k}{2} \int_0^x e^{-t} dt$

$2x\,dx = dt$

$x\,dx = \frac{dt}{2}$

$\Rightarrow k\left[-e^{-t}\right]_0^x$

$x = 0 \quad t = 0$

$x = x$

$\Rightarrow \cancel{k - e^{-x} + 1}$

$\Rightarrow \cancel{-e^{-x} + 1}$

$k\left[-e^{-x^2}\right]_0^x$

$\Rightarrow -e^{-x^2} + 1$

$\Rightarrow$

$\Rightarrow$ If x is a R.V with density function given by

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2-x & 1 \le x < x \\ 0 & \text{otherwise} \end{cases}$$

find

i) mean

$$E(x) = \int_{-\infty}^{\infty} x f(x)\, dx$$

$$\left[\int_{-\infty}^{0} + \int_{0}^{1} + \int_{1}^{2} + \int_{2}^{\infty}\right) x f(x) dx$$

$$0 + \int_{0}^{1} x(x\, dx) + \int_{1}^{2} x(2-x) dx + 0$$

$$\Rightarrow \int_{0}^{1} x^2 dx + \int_{1}^{2} 2x - x^2 dx$$

$$\Rightarrow \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{2x^2}{2}\right]_1^2 - \left\{\frac{x^3}{3}\right\}_1^2$$

$$\Rightarrow \left[\frac{1}{3}\right] + \left[\left(4 - 1\right) - \left(\frac{8}{3} - \frac{1}{3}\right)\right)$$

$$\Rightarrow \frac{1}{3} + 4 - \frac{8}{3} - 1 + \frac{1}{3}$$

$$\Rightarrow \cancel{\frac{1+12-24-3+1}{3}} \Rightarrow \boxed{1}$$

varience

$$v(x) = E(x^2) - (E(x))^2$$

$$E(x^2) = \int_{-\infty}^{\infty} x^2 f(x)\, dx$$

$$\Rightarrow \left[\int_{-\infty}^{0} + \int_{0}^{1} + \int_{1}^{2} + \int_{2}^{\infty}\right] x^2 f(x)\, dx$$

$$0 + \int_{0}^{1} x \cdot x^2 dx + \int_{1}^{2} x^2 (2-x)\, dx + 0$$

$$\Rightarrow \int_{0}^{1} x^3 dx + \int_{1}^{2} 2x^2 dx - \int_{1}^{2} x^3 dx$$

$$\left[\frac{x^4}{4}\right]_0^1 + \left[\frac{2x^3}{3}\right]_1^2 - \left[\frac{x^4}{4}\right]_1^2$$

$$\Rightarrow \frac{1}{4} + \frac{16}{3} - \frac{2}{3} - \left[\frac{16}{4} - \frac{1}{4}\right]$$

$$\Rightarrow \frac{1}{4} + \frac{16}{3} - \frac{2}{3} - 4 + \frac{1}{4}$$

$$\Rightarrow \boxed{7}$$

$\Rightarrow \frac{7}{6} - (1)^2$

$\Rightarrow \frac{7}{6} - 1 \Rightarrow \frac{1}{6}$

iii) $E(25x^2 + 30x - 5)$

$\Rightarrow 25\,E(x^2) + 30\,E(x) - 5$

$\Rightarrow 25 \times \frac{7}{6} + 30 \times 1 - 5$

$\Rightarrow \cancel{29.1667}$ 54.1667

---

2) $f(x) = \begin{cases} 3x^2 & 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$

be a pdf of 'x' find 'a' when $p(x \ge a) = 0.05$

$P(x \ge a) = 0.05$

$\int_a^{\infty} f(x)\,dx = 0.05$

$\left(\int_a^1 + \int_1^{\infty}\right) f(x)\,dx = 0.05$

$$\int_a^1 f(x)\,dx + 0 = 0.05$$

$$\int_a^1 3x^2\,dx = 0.05$$

$$3\left(\frac{x^3}{3}\right)_a^1 = 0.05$$

$$\cancel{\frac{3}{a} = 0 = 0}$$

$$1^3 - a^3 = 0.05$$

$$1 - 0.05 = a^3$$

$$a^3 = 0.95$$

$$a = (0.95)^{\frac{1}{3}} = 0.98$$

Multiple Random variables

Discrete RV

Consider two RV x and y or a 2-D RV (x, y) for the outcome of a trial is a pair of numbers $(x = x_i, y = y_j)$, we call (x, y) and its distribution as descrete if x and y can assume only finite no. of pairs.

Join probablity Law

Two RV x and y are said to be jointly distributed if they are defined on the same probablity space.

Join probablity mass function

Let x and y be the RV defined on a sample space s with $x(s) = \{x_1, x_2, \dots x_i \dots x_n\}$ $y(s) = \{y_1, y_2, \dots y_j \dots y_m\}$ the join probablity of (x, y) is defined as

$$P(x = x_i, y = y_j) = P(x_i, y_j)$$ and

the probablity functis is

$$P_{ij} = P(x = x_i \cap y = y_j) = P(x_i, y_j)$$ is

called the join probablity function of (x, y) and is represented by following table.

Scanned with OKEN Scanner

$X \backslash Y$	y_1	y_2	....	y_j	....	y_m	total
x_1	P_{11}	P_{12}	....	P_{1j}	....	P_{1m}	$P_{1\cdot}$
x_2	P_{21}	P_{22}	....	P_{2j}	....	P_{2m}	$P_{2\cdot}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
x_i	P_{i1}	P_{i2}	----	P_{ij}	---	P_{im}	$P_{i\cdot}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$	$\vdots$
x_n	P_{n1}	P_{n2}	......	P_{nj}	...	P_{nm}	$P_{n\cdot}$
	$P_{\cdot 1}$	$P_{\cdot 2}$	....	$P_{\cdot j}$	---	$P_{\cdot m}$	$\sum_{i=1}^{n}\sum_{j=1}^{m} P_{ij} = 1$

Marginal probablity functions

· suppose join probablity function of two RV x and y is given then the marginal probablity function of X is denoted by $P_X($ and defined as $P_X(x) = P(X = x_i \cap Y = y_1) +$

$P(x = x_i \cap y = y_2) + P(x = x_i \cap y = y_3) + \dots\dots$

$= P_{i1} + P_{i2} + \dots\dots$

$= P_{i\cdot}$

similarly the <u>MPf</u> of Y is denoted by P_Y and defined as

$$P_Y(y) = P(x = x_1 \cap Y = y_j) + P(x = x_2 \cap Y = y_j$$

$$\Rightarrow P_{1j} + P_{2j} + \dots$$

$$\Rightarrow P_{\cdot j}$$

The probablities $P_{i\cdot}$ and $P_{\cdot j}$ are called

MPf of x and Y

Conditional probablity functions

i) $P(X=x_i/Y=y_j) = \dfrac{P(X=x_i \cap Y=y_j)}{P(Y=y_j)}$

2) $\dfrac{P(x_i, y_j)}{P_y(y)} = \dfrac{P_{ij}}{P_{\cdot j}}$

ii) $P(Y=y_j/X=x_i) = \dfrac{P(Y=y_j \cap X=x_i)}{P(X=x_i)}$

$= \dfrac{P(x_i, y_j)}{P_x(x)} = \dfrac{P_{ij}}{P_{i\cdot}}$

Join probablity distribution function (cdf)

Let (x,y) be a 2-D RV then join pdf is denoted by $F_{XY}(x,y)$ and defined as $F_{XY}(x,y) = P(X \le x_i, Y \le y_j)$

$= P(-\infty \le X \le x_i, -\infty \le Y \le y_j)$

Properties

1) $F(\infty, \infty) = 1$

2) $F(-\infty, -\infty) = 0$

3) $0 \le F_{XY}(x,y) \le 1$

4) probablity of $P(x \le a, y \le b) = F(b,a) - F(a,a)$

similarly

$$P(x_1 \le x \le x_2, Y \le y_1) = F(x_2, y_1) - F(x_1, y_1)$$

---

problems

1) for the following bivariate distribution of X and Y find,

i) $P(X \le 1, Y = 2)$

ii) $P(X \le 1)$

iii) $P(Y = 3)$ iv) $P(Y \le 3)$ v) $P(X < 3, Y \le 4)$

X \ Y	1	2	3	4	5	6	
0	0	0	1/32	2/32	2/32	3/32	1/4
1	1/16	1/16	1/8	1/8	1/8	1/8	5/8
2	1/32	1/32	1/64	1/64	0	2/64	1/8
total	3/32	3/32	11/64	13/64	3/16	[illegible]	1

i) $P(X \le 1, Y = 2) = P(X = 0, y = 2) + P(x = 1, y = 2)$

$\Rightarrow 0 + 1/16$

$\Rightarrow \boxed{\frac{1}{16}}$

ii) $P(X \le 1) = P(X = 0) + P(X = 1)$

$\Rightarrow \frac{1}{4} + \frac{5}{8} \Rightarrow \boxed{\frac{7}{8}}$

iii) $P(Y=3) = \frac{11}{64}$

iv) $P(Y \le 3) = P(y=1) + P(y=2) + P(y=$

$\Rightarrow \frac{3}{32} + \frac{3}{32} + \frac{11}{64}$

$\Rightarrow \frac{23}{64}$

v) $P(X \le 3, Y \le 4)$

$\Rightarrow P(x<3, y=1) + P(x<3, y=2) + P(x<3,$
$y=3) + P(x<3, y=4)$

$\Rightarrow (0 + 1/16 + 1/32) + (0 + 1/16 + 1/32) + (1/32 + \frac{1}{8} + \frac{1}{64}) + (2/32 + 1$
$+ 1/16$

$\Rightarrow \frac{3}{32} + \frac{3}{32} + \frac{11}{64} + \frac{13}{64}$

$\Rightarrow 9/16$

---

2) Two balls are drawn at random from a box containing 3 red, 2 Green, 4 white balls. If X and Y are RV represents the No. of Red balls and Green ball respectively including among the two balls drawn from the box.

find, i) Joint prob of X and Y

ii) Marginal Prob of X and Y

Scanned with OKEN Scanner

iii) Conditional probability ... $Y=1$

i) Let X is an event of drawing Red balls
Y is an event of drawing green balls

$$P(\overset{R}{0},\overset{G}{0}) = \frac{(3_0\ 2_0\ 4_2)}{9_2} = \frac{6}{36} \Rightarrow \boxed{\frac{1}{6}}$$

$$P(\overset{R}{0},\overset{G}{1}) = \frac{(3_0)(2_1)(4_1)}{{}^9C_2} = \frac{1\times 2\times 4}{36} = \boxed{\frac{2}{9}}$$

$$P(\overset{R}{0},\overset{G}{2}) = \frac{(3_0)(2_2)(4_0)}{{}^9C_2} = \frac{1}{36}$$

$$P(\overset{R}{1},\overset{G}{0}) = \frac{(3_1)(2_0)(4_1)}{9_2} \Rightarrow \frac{3\times 1\times 4}{36} \Rightarrow \frac{1}{3}$$

$$P(1,1) = \frac{(3_1)(2_1)(4_0)}{9_2} \Rightarrow \frac{3\times 2}{36} \Rightarrow \frac{1}{6}$$

$P(1,2) = 0$

$$P(2,0) = \frac{(3_2)(2_0)(4_0)}{9_2} = \frac{3}{36} \Rightarrow \boxed{\frac{1}{12}}$$

$P(2,1) = 0$
$P(2,2) = 0$
$P(3,0) = 0$

because we are drawing 2 balls only

$P(3,1) = 0$

$P(3,2) = 0$

x \ y	0	1	2
0	$P(0,0)$	$P(0,1)$	$P(0,2)$
1	$P(1,0)$	$P(1,1)$	$P(1,2)$
2	$P(2,0)$	$P(2,1)$	$P(2,2)$
3	$P(3,0)$	$P(3,1)$	$P(3,2)$

x \ y	0	1	2	total
0	1/6	2/9	1/36	5/12
1	1/3	1/6	~~1/12~~ 0	1/2
2	1/12	0	0	1/12
3	0	0	0	0
	7/12	7/18	1/36	1

ii) Marginal probabilty of x and y

x	$P_{i\cdot}$
0	5/12
1	1/2
2	1/12
3	0

y	$P_{\cdot j}$
0	7/12
1	7/18
2	1/36

iii) conditional prob of x given y = 1

$$P(X=x \mid Y=1) = \frac{P(X=x \cap Y=1)}{P(Y=1)}.$$

$$\Rightarrow \frac{P(X=0 \cap Y=1)}{P(Y=1)} + \frac{P(X=1 \cap Y=1)}{P(Y=1)} + \frac{P(X=2 \cap Y=1)}{P(Y=1)}$$

$$+ \frac{P(X=3 \cap Y=1)}{P(Y=1)}$$

$$\Rightarrow \frac{\frac{2}{9}}{\frac{7}{18}} + \frac{\frac{1}{6}}{\frac{7}{18}} + 0 + 0 \Rightarrow 1$$

---

38) Joint probablity mass function of x and Y given by $P(x,y) = K(2x+3y)$ for $x = 0, 1, 2$ $y = 1, 2, 3$

i) find K ii) Marginal prob fun of x and y

iii) conditional prob of x given y = 1

iv) " " of y given x = 2

v) prob distribution of x + y

x \ y	1	2	3	
0	3k	6k	9k	18k
1	5k	8k	11k	24k
2	7k	10k	13k	30k
	15k	24k	33k	72k

i) we know that

$$\sum\sum P_{ij} = 1$$

ie $72k = 1$

$$\boxed{k = \frac{1}{72}}$$

ii) Marginal prob fun of x and y

x	$P_{i\cdot}$
0	1/4
1	1/3
2	5/12

Y	$P_{\cdot j}$
1	5/24
2	1/3
3	11/24

iii) Conditional prob of x given Y=1

$$\Rightarrow \frac{P(x=0 \cap Y=1)}{P(Y=1)} + \frac{P(x=1 \cap y=1)}{P(y=1)} + \frac{P(x=2 \cap Y=1)}{P(Y=1)}$$

$$\Rightarrow \frac{\frac{1}{24}}{5/24} + \frac{5/72}{5/24} + \frac{7/72}{5/24}$$

$$\Rightarrow \frac{1}{5} + \frac{1}{3} + \frac{7}{15} \Rightarrow \boxed{1}$$

iv) y given $x=2$

$$\frac{P(y=1 \wedge x=2)}{P(x=2)} + \frac{P(y=2 \wedge x=2)}{P(x=2)} +$$

$$\frac{P(y=3 \wedge x=2)}{P(x=2)}$$

$$\Rightarrow \frac{\frac{7}{72}}{\frac{5}{12}} + \frac{\frac{5}{36}}{\frac{5}{12}} + \frac{\frac{13}{72}}{\frac{5}{12}}$$

$\Rightarrow$

v) ~~Prob~~ distribution of $x+y$

$z=x+y$	Prob
1	$P_{01} = 3k = \frac{3}{72}$
2	$P_{02} + P_{11} = 6k + 5k = \frac{11}{72}$
3	$P_{03} + P_{12} + P_{21} = 9k + 8k + 7k \Rightarrow \frac{24}{72} = \frac{1}{3}$
4	$P_{13} + P_{22} = 11k + 10k = 21$ $\frac{}{7}$

5	$P_{23} = 13k = \frac{13}{72}$

$x+y$	$P(x+y)$
1	3/72
2	11/72
3	1/3
4	7/24
5	13/72

62416

4Q) If x and y are two R.V having the joint prob mass fun $P(x,y) = \frac{1}{27}(2x+y)$ where x and y assumed to be the values of 0, 1, 2 find the joint prob distribution of x and y and marginal prob of x and y also find the conditional prob of $y = 1$ given $x = x$.

$x \backslash y$	0	1	2	
0	0	1/27	2/27	1/9
1	2/27	3/27	4/27	1/3
2	4/27	5/27	6/27	5/9
	2/9	1/3	4/9	1

Marginal probability function

x	$P_{i\cdot}$
0	~~[illegible]~~ 1/9
1	1/3
2	5/9

y	$P_{\cdot j}$
0	2/9
1	1/3
2	4/9

(iii) Conditional prob $y=1$ & $x = x$

$x = x \mid y = 1$

$$\Rightarrow \frac{P(x=0 \cap y=1)}{P(y=1)} + \frac{P(x=1 \cap y=1)}{P(y=1)}$$

$$+ \frac{P(x=2 \cap y=1)}{P(y=1)}$$

$$\Rightarrow \frac{\frac{1}{27}}{\frac{1}{3}} + \frac{\frac{3}{27}}{\frac{1}{3}} + \frac{\frac{5}{27}}{1/3}$$

$\Rightarrow 1$

Continious joint probablities

joint density

The 2-D RV (x,y) having the probablity density function $f_{xy}(x,y)$ has to satisfy the following properties

i) $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{xy}(x,y)\,dx\,dy = 1$$

ii) $$F_{xy}(x,y) \geq 0$$

Marginal density function :- If $f_{xy}(x,y)$ is the prob density function of the continious RV x and y then its marginal prob density functions of x and y are given by

i) $$f_x(x) = \int_y f_{xy}(x,y)\,dy$$

ii) $$f_y(y) = \int_x f_{xy}(x,y)\,dx$$

Joint prob distribution function (cdf) :-

for 2-D continious RV (x,y) $F_{xy}(x,y)$ is called the cdf it has to satisfy the following :-

i) $$F_{xy}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{xy}(x,y)\,dx\,dy$$

ii) If its ~~prob~~ ~~p~~ density cdf

to find its pdf we use

$$f_{xy}(x,y) = \frac{\partial^2 F_{xy}(x,y)}{\partial x \, \partial y}.$$

Conditional probablities

i) conditional probablities density function of

$$f(x=x/y=y) = \frac{f_{xy}(x,y)}{f_y(y)}$$

ii) $$f(y=y/x=x) = \frac{f_{xy}(x,y)}{f_x(x)}$$

Q) verify that $f(x,y) = \begin{cases} \frac{2}{5}(2x+3y) & 0 \le x \le 1,\ 0 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$

is the pdf or not, if so find $P((x,y) / 0 \le x < 1/2, 1/4 < y < \frac{1}{2})$

sol) to prove $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = 1$

$$\left(\int_{-\infty}^{0}\int_{-\infty}^{0} + \int_{0}^{1}\int_{0}^{1} + \int_{1}^{\infty}\int_{1}^{\infty}\right) f(x,y)\,dx\,dy$$

$\Rightarrow 0 + \int_0^1 \int_0^1 \frac{2}{5}(2x+3y)\,dx\,dy + 0$

$\Rightarrow \frac{2}{5}\int_0^1 \left(\frac{\cancel{2}x^2}{\cancel{2}} + 3y(x)\right)_0^1 dy$

$\Rightarrow \frac{2}{5}\int_0^1 (1-0) + 3y(1-0)\,dy$

$\Rightarrow \frac{2}{5}\left[(y)_0^1 + \left(\frac{3y^2}{2}\right)_0^1\right]$

$\Rightarrow \frac{2}{5}\left[(1-0) + \left(\frac{3}{2}-0\right)\right]$

$\Rightarrow \frac{2}{5}\left(1+\frac{3}{2}\right) \Rightarrow \frac{2}{5} \times \frac{5}{2} = 1$

$\Rightarrow 1$

$\Rightarrow P\left((x,y) \mid 0 < x < 1/2,\ 1/4 < y < 1/2\right)$

$\Rightarrow \int_{x=0}^{1/2}\int_{y=1/4}^{1/2} \frac{2}{5}(2x+3y)\,dy\,dx$

$\Rightarrow \frac{2}{5}\int_{x=0}^{1/2} 2x(y)_{1/4}^{1/2} + \left(\frac{3y^2}{2}\right)_{1/4}^{1/2} dx$

$\Rightarrow \frac{2}{5}\int_{x=0}^{1/2} 2x\left(\frac{1}{2}-\frac{1}{4}\right) + \frac{3}{2}\left(\frac{1}{4}-\frac{1}{16}\right) dx$

Scanned with OKEN Scanner

$$\Rightarrow \frac{2}{5}\left(\frac{1}{4}\, 2\left(\frac{x^2}{2}\right)_0^{1/2} + \frac{3}{2} \times \frac{3}{16}(x)_0^{1/2}\right]$$

$$\Rightarrow \frac{2}{5}\left[\frac{1}{4} \times \frac{1}{4} \times \frac{3}{2} \times \frac{3}{16} \times \frac{1}{2}\right]$$

---

26) 6/23

Q) The cdf of x & y is

$$F(x,y) = \begin{cases} (1-e^{-x})(1-e^{-y}) & x>0, y>0 \\ 0 & \text{otherwise} \end{cases}$$

find i) ~~do~~ joint pdf

ii) $P(1 < x < 3,\ 1 < y < 2)$

joint pdf

i) $$f(x,y) = \frac{\partial^2}{\partial x\, \partial y} F(x,y)$$

$$= \frac{\partial^2}{\partial x\, \partial y}(1-e^{-x})(1-e^{-y})$$

$$\Rightarrow \frac{\partial}{\partial x}(1-e^{-x})\, \frac{\partial}{\partial y}(1-e^{-y})$$

$\Rightarrow \frac{\partial}{\partial x}(1-e^{-x})\frac{\partial}{\partial y}(1-e^{-y})$

$\Rightarrow [0-(-e^{-x})][0-(-e^{-y})]$

$\Rightarrow e^{-x}e^{-y}$

$\Rightarrow e^{-(x+y)}$

i) $P(1<x<3, 1<y<2) = \int_{x=1}^{3}\int_{y=1}^{2} e^{-x} \times e^{-y}\, dy\, dx$

$\Rightarrow \int_{x=1}^{3} e^{-x}\left(\frac{e^{-y}}{-1}\right)_1^2 dx$

$\Rightarrow -\int_{x=1}^{3} e^{-x}(e^{-2}-e^{-1})\, dx$

$\Rightarrow -e(e^{-2}-e^{-1})(-e^{-x})^3$

$\Rightarrow (e^{-2}-e^{-1}) \times (e^{-3}-e^{-1})$

$\Rightarrow \left(\frac{1}{e^2}-\frac{1}{e}\right) \times \left(\frac{1}{e^3}-\frac{1}{e}\right)$

3a) The joint probability of x and y is given by $f(x,y) = \frac{8}{9}xy$ $1 \le x \le 2$, $1 \le y \le 2$ find

i) marginal densities of x & y

ii) conditional densities of x & y.

i) Marginal density of x is

$$f_x(x) = \int_y f(x,y)\,dy$$

$$\Rightarrow \int_1^2 \frac{8}{9}xy\,dy$$

$$\Rightarrow \frac{8}{9}x\left(\frac{y^2}{2}\right)_1^2$$

$$\Rightarrow \frac{8}{9}x\left[\frac{4}{2} - \frac{1}{2}\right]$$

$$\Rightarrow \frac{8}{9}x\left[\frac{3}{2}\right]$$

$$\Rightarrow \frac{24}{18}x$$

$$\Rightarrow \boxed{\frac{4}{3}x}$$

of y is

$$f_y(y) = \int_x f(x,y)\,dx$$

$$\int_1^2 \frac{8}{9} xy \, dx$$

$$\frac{8}{9} y \left[\frac{x^2}{2}\right]_1^2$$

$$\frac{8}{9} y \left[\frac{4}{2} - \frac{1}{2}\right]$$

$$\Rightarrow \boxed{\frac{4}{3} y}$$

ii) conditional densities

$$f(X = x / Y = y) = \frac{f_{xy}(x, y)}{f_y(y)}$$

$$\Rightarrow \frac{\frac{8}{9} x\cancel{y}}{\frac{4}{3} \cancel{y}}$$

$$\Rightarrow \frac{8}{9} \times \frac{3}{4} x \Rightarrow \boxed{\frac{2}{3} x}$$

$$f(Y = y / X = x) = \frac{f_{xy}(x, y)}{f_x(x)}$$

$$\Rightarrow \frac{\frac{8}{9} xy}{\frac{4}{3} x}$$

$$\Rightarrow \boxed{\frac{2}{3} y}$$

Scanned with OKEN Scanner

Statistical independence (statisti.

Two variables ~~x~~ x and y are said to be statistically independent if their joint prob density function is the product of marginal prob densities of x and y.

i.e $f_{xy}(x,y) = f_x(x)\, f_y(y)$

1) Joint prob density function (pdf) is given by $f(x,y) = 4xy\, e^{-(x^2+y^2)}$ $x \geq 0, y \geq 0$

test weather x & y are statistically independent are not.

i) Marginal density of x is

$$f_x(x) = \int_y f(x,y)\, dy$$

$$\int_0^\infty 4xy\, e^{-(x^2+y^2)}\, dy$$

$$4x \int_0^\infty y e^{-(x^2+y^2)}\, dy$$

$$4xe^{-x^2}\int_0 y e^{-y^2} dy$$

$$y^2 = t$$

$$2y\,dy = dt$$

$$y\,dy = \frac{dt}{2}$$

$$4xe^{-x^2}\int_0^\infty e^{-t}\,\frac{dt}{2}$$

$$\frac{4xe^{-x^2}}{2}\int_0^\infty \left[-e^{-t}\right]$$

$$-2x e^{-x^2}\left[-e^{0}-1\right]$$

$$\Rightarrow \cancel{8x^2 e^{-x^2}}\ 2x e^{-x^2}$$

$$f_Y(y) = \int_x f(x,y)\,dx$$

$$\Rightarrow \int_{x=0}^{\infty} 4xy\, e^{-x^2} \times e^{-y^2}\,dx$$

$$\Rightarrow 4ye^{-y^2}\int_{x=0}^{\infty} x e^{-x^2}\,dx$$

$$\Rightarrow 2ye^{-y^2}$$

$$\therefore f_{XY}(x,y) = f_X(x) f_Y(y)$$

$\therefore$ Hence X and Y are statistically independant.

2) Joint prob density fun : x, y

$f(x,y) = 8xy \quad 0 < x < 1 \quad 0 < y < 1$

Marginal density of x is

$f_x(x) = \int_y f(x,y)\, dy$

$\Rightarrow \int_0^1 8xy\, dy$

$\Rightarrow 8x \left(\frac{y^2}{2}\right)_0^1$

$\Rightarrow 8x \left[\frac{1}{2}\right]$

$\Rightarrow \boxed{4x}$

of y is

$f_y(y) = \int_x f(x,y)\, dx$

$\Rightarrow \int_0^1 8xy\, dx$

$\Rightarrow 8y \left(\frac{x^2}{2}\right)_0^1$

$\Rightarrow 8y \left(\frac{1}{2}\right)$

$\Rightarrow \boxed{4y}$

$f(x,y) = f_x(x) \cdot f_y(y)$

$\therefore$ Hence x and y are statestically independent.

ii) conditional

$$f(X=x, Y=y) = \frac{f_{xy}(x,y)}{f_y(y)}$$

$$\Rightarrow \frac{8x^2y}{4xy}$$

$$\Rightarrow \boxed{2x}$$

$$f(Y=y \mid X=x) = \frac{f_{xy}(x,y)}{f_x(x)}$$

$$\Rightarrow \frac{\cancel{8x^2y}}{\cancel{4x}}$$

$$\Rightarrow \boxed{2y}$$

Q) If $f(x,y) = \begin{cases} e^{-(x+y)} & x \geq 0, y \geq 0 \\ 0 & \text{otherwise} \end{cases}$

find i) $P(x, y<1)$

ii) $P(x>y)$

iii) $P(x+y<1)$

i) $P(x<1, y<1) = \left(\int_{-\infty}^{1}\int_{-\infty}^{1} + \int_{1}^{\infty}\int_{1}^{\infty}\right) f(x,$

$\Rightarrow \int_{-\infty}^{0}\int_{-\infty}^{0} + \int_{0}^{1}\int_{0}^{1} + \int_{1}^{\infty}\int_{1}^{\infty} f(x,y)\,dx\,d$

$0 + \int_{0}^{1}\int_{0}^{1} e^{-(x+y)}\,dx\,dy + 0$

$\Rightarrow \int_{0}^{1}\int_{0}^{1} e^{-x} e^{-y}\,dx\,dy$

$\Rightarrow \int_{0}^{1} e^{-x} \left(-e^{-y}\right)_{0}^{1} dx$

$\Rightarrow \int_{0}^{1} e^{-x} \left(-(e^{-1}-1)\right) dx$

$\Rightarrow 1-e^{-1} \left[-e^{-x}\right]_{0}^{1}$

$\Rightarrow 1-e^{-1}\left[-(e^{-1}-1)\right]$

$\Rightarrow 1-e^{-1}\left[1-e^{-1}\right]$

$\Rightarrow \left(1-\frac{1}{e}\right)^2$

ii) $P(x>y)$

$$\int_{x=y}^{\infty} f(x,y)\,dx$$

$$\int_{x=y}^{\infty} e^{-x}\cdot e^{-y}\,dx$$

$$e^{-y}\left(-e^{-x}\right)_y^{\infty}$$

$$\Rightarrow e^{-y}\left(-\left(0-e^{-y}\right)\right)$$

$$\Rightarrow e^{-y}\cdot e^{-y} \Rightarrow e^{-2y}$$

iii) $P(x+y<1) = P(x<1-y)$

$$= \int_{-\infty}^{1-y} f(x,y)\,dx$$

$$\Rightarrow \left(\int_{-\infty}^{0} + \int_{0}^{1-y}\right) f(x,y)\,dx$$

$$\Rightarrow \int_{0}^{1-y} e^{-x}\cdot e^{-y}\,dx$$

$$\Rightarrow e^{-y}\left[-e^{-x}\right]_0^{1-y}$$

$\Rightarrow e^{-y}\left[-\left(e^{-\{1-y\}} - 1\right)\right]$

$\Rightarrow \cancel{e^{-y}\{-e^{-1-y}}\}$

$\Rightarrow e^{-y}\left\{1 - e^{-[1-y]}\right\}$

$\Rightarrow e^{-y}\left[1 - \dfrac{1}{e^{1-y}}\right]$