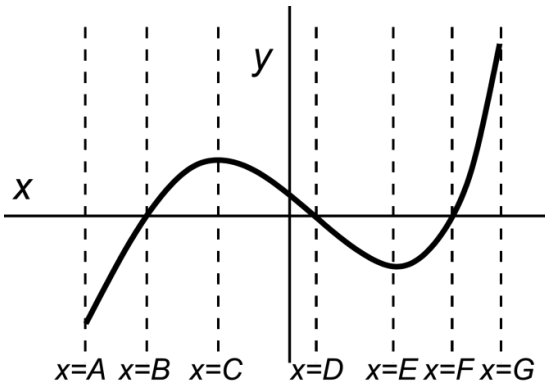
Increasing and Decreasing Function Behavior

You're given three graphs. Each of them is labeled and accompanied by a set of questions. First, answer each question as well as you can individually – use complete sentences. Then compare your answers with those of your group members and discuss any differences.

1. To the right is a graph of the function $y = f(x)$, a function whose domain is the interval [A,G].

a. Write any intervals in which the function $f(x)$ is increasing.

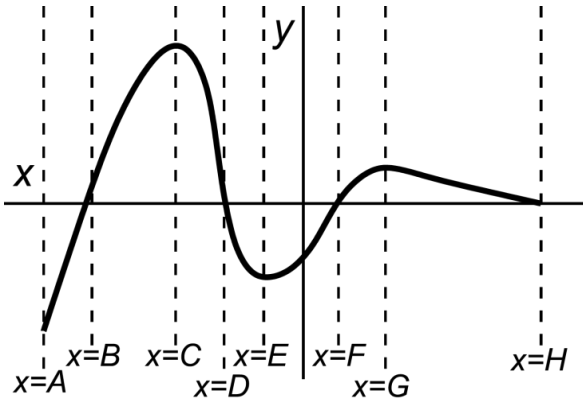
b. Write any intervals in which the function $f(x)$ is decreasing.



A graph of the function $y = f(x)$

2. To the right is a graph of the function $y = g'(x)$, the *derivative* of the function $g(x)$. The domain of this derivative is the interval [*A,H*].

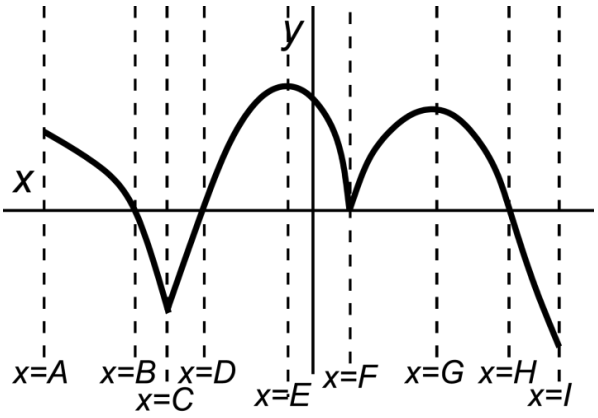
a. State the intervals in which the function $g(x)$ would be increasing. Give reasoning supporting each of your answers.



A graph of the function $y = g'(x)$

b. State the intervals in which the function $g(x)$ would be decreasing. Give reasoning supporting each of your answers.

3. To the right is a graph of the function $y = h'(x)$, the *derivative* of the function $h(x)$. The domain of this derivative is the interval [*A,I*].



A graph of the function $y = h'(x)$

a. State the intervals in which the function $h(x)$ would be increasing. Give reasoning supporting each of your answers.

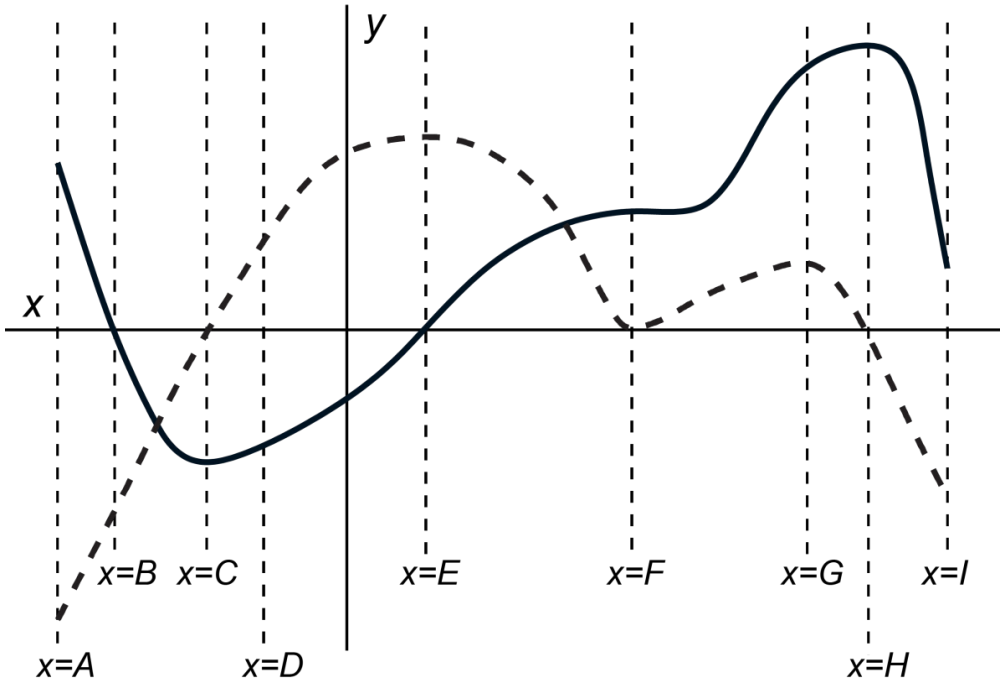
b. State the intervals in which the function $h(x)$ would be decreasing. Give reasoning supporting each of your answers.

Check your understanding

- ☐ When looking at the graph of a function, how can you tell whether the value of that function is *positive* or *negative?*
- ☐ What condition on the derivative of a function would guarantee that the original function is *increasing* in a given interval?
- ☐ What condition on the derivative of a function would guarantee that the original function is *decreasing* in an interval?

Apply Your Understanding: Justifying Behaviors of Functions

Part I: Following is the graph of two functions defined on the interval $[A, I]$. One function is $g(x)$ whose graph is the **solid curve**. The other function is $h(x)$ whose graph is the **dashed curve**. One of these functions is the derivative of the other. That is, either $g(x) = h'(x)$ or $h(x) = g'(x)$. Decide which of these alternatives is correct and support your assertion with as many specific facts regarding features of the graphs as you can.



Part II: Suppose $f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x$. Where is *f*(*x*) increasing and where is it decreasing? Explain your answer.

Check your understanding

What connections do you know between the derivative of a function and the original function?

☐ Suppose you are given a graphical representation of the derivative (a *graph*). How can you use this graph to determine where the original function is increasing or where it is decreasing?

☐ Suppose you are given an analytical representation of a function (a *formula*). How can you use the formula to determine where the function is increasing or where it is decreasing?

Analytical Representations and Critical Points

You're given analytical information (formulas) about three functions. In each case, you're asked to find and classify the critical values of the function. First, answer each question as well as you can individually. You may consult with others in your group for hints about solutions if needed, but your written solutions should be individual efforts. *Any verbal answers should be written using complete sentences.*

1. Suppose $f'(x) = (x-1)(x-2)^2(x-3)^3$. (Note: this is a formula for the *derivative* of the original function). Use the following table to find and classify the critical points for the original function $f(x)$.

Steps	Notes and Solutions
a. Consider the domain of $f'(x)$. Are there any values that x cannot be?	
b. Find the critical numbers for $f(x)$. (**Note:** For this problem, the function $f(x)$ will have <u>three</u> critical numbers.)	
c. Classify each critical number as a *local maximum*, a *local minimum*, or *neither*, and explain your reasoning.	Critical number:
	Critical number:

Steps	Notes and Solutions
continued: Classify each critical number as a *local maximum*, a *local minimum*, or *neither*, and explain your reasoning.	Critical number:

2. Suppose that $g(x) = \dfrac{(x+1)^2}{x^2+8}$.

a. What is the domain of $g(x)$?

b. Compute $g'(x)$ and find the critical numbers of $g(x)$. (**Note:** For this problem, the function $g(x)$ will have <u>two</u> critical numbers.)

c. Classify each of the critical numbers found in part b. Is each a local minimum, local maximum, or neither?

Check your understanding

Suppose you are given an analytical representation of a function (a *formula* for the function). How would you identify *all* the critical values of the function?