Using Integrals to Find the New Value of a Function

One version of the Fundamental Theorem of Calculus looks like this:

$$\int_a^b f'(x)dx = f(b) - f(a)$$

For contextual situations, you can interpret this as saying, "if you integrate the rate of change of a function over an interval from *a* to *b*, you will determine the amount of change in that function between *a* and *b*."

By transposing the terms of the above equation, another version looks like this:

$$f(a) + \int_a^b f'(x)dx = f(b)$$

For contextual situations, you can interpret this as saying, "if you start with an initial value at *a* and add the amount of change that has taken place over the interval from *a* to *b*, you will determine the value of the function at *b*.

For example, if the initial population of a town is 5500 people and the population is changing at the rate of $P'(t)$ people per year, then $5500 + \int_0^{10} P'(t)dt$ will determine the population of the town after 10 years.

Part I: For each of the following situations, write the definite integral that will determine the requested quantity. Note: Do not evaluate the definite integrals yet.

1. There are 30 gallons of pollutant in a lake at *t* = 0. If the number of gallons, $P(t)$, of the pollutant changes at the rate $P'(t) = 1 - 3e^{-0.3\sqrt{t}}$ gallons per day, where *t* is measured in days. Write a definite integral expression to determine the amount of pollutant in the lake after 12 days.

2. Water is leaking from a tank at a rate of $\sqrt{t+1}$ gallons per minute. Ten minutes after the leak is discovered, there are 60 gallons of water in the tank. How much water was in the tank when the leak was initially discovered?

3. A pie is removed from an oven and, after sitting for 10 minutes, is at a temperature of 220 degrees Fahrenheit. The pie cools at a rate of $3\ln(x^2)$ degrees Fahrenheit per minute. What is the expected temperature of the pie 15 minutes after it was removed from the oven?

4. Throughout the day, people have entered a craft fair at the rate of $E(t) = 30t^2 + 16t$ people per hour and have exited at a rate given by $L(t) = 12t - 3$ people per hour. If *t* is measured in hours since the fair opened, and there were 200 people at the fair at the end of the first hour, how many people should be at the fair 3 hours after it opened?

Part II: Some definite integrals can be evaluated by hand and others should be evaluated using the definite integral feature of your calculator. Two of the problems in Part I can be worked by hand, and two require the use of the calculator. Determine which methods will work for each of the four problems and then work each problem to determine the numerical answer.

Problem from Part I	Method		Solution
	Calculator	By Hand Using FTC	
#1			
#2			
#3			
#4			

Matching Definite Integrals to Their Interpretations

Read the problem situation and then match the integral expressions in the left column to their interpretations in the right column.

Situation A

A pipeline company manufactures pipe that sells for $100 per meter. The cost of manufacturing a portion of the pipe varies with its distance from the beginning of the pipe. The company reports that the cost to produce a portion of the pipe that is x meters from the beginning of the pipe is $C(x)$ dollars per meter. (Note: Profit is defined as the difference between the amount of money received by the company for selling the pipe and the amount it costs to manufacture the pipe.)

_______ **1.** $\int_0^{125} C(x)dx$

_______ **2.** $\frac{1}{125}\int_0^{125} 100 - C(x)dx$

_______ **3.** $\frac{1}{125}\int_0^{125} 100 - C'(x)dx$

_______ **4.** $\int_{50}^{100} C(x)dx$

_______ **5.** $\frac{1}{50}\int_{50}^{100} C(x)dx$

_______ **6.** $\int_{50}^{100} 100dx$

_______ **7.** $\int_{50}^{100} 100 - C(x)dx$

A. The difference in sales price, in dollars, between 100 meters of pipe and 50 meters of pipe

B. The cost, in dollars, of manufacturing 125 meters of pipe

C. The average profit, in dollars per meter, made on the sale of 125 meters of pipe

D. The difference in the cost, in dollars, of manufacturing 100 meters of pipe and 50 meters of pipe

E. The average rate of change, in dollars per meter per meter, in the cost per meter of pipe for 125 meters of pipe

F. The difference in profit, in dollars, between selling 100 meters of pipe and 50 meters of pipe

G. The average difference in the cost per meter, in dollars per meter, between manufacturing 100 meters of pipe and 50 meters of pipe

Situation B

A cup of boiling water is removed from a microwave and set on the counter to cool. The initial temperature of the water is 100 degrees Celsius. The temperature of the water *T*, measured in degrees Celsius, as a function of time *t*, measured in minutes, is given by $T(t)$.

______	**1.**	$T'(4)$
______	**2.**	$100+\int_0^k T'(t)dt$
______	**3.**	$\frac{1}{4}\int_0^4 T(t)dt$
______	**4.**	$\frac{1}{4}\int_0^4 T'(t)dt$
______	**5.**	$\int_0^4 T'(t)dt$
______	**6.**	$T(4)$
______	**7.**	$100+\int_0^k T'(t)dt=85$
______	**8.**	$\frac{1}{k}\int_0^k T(t)dt=85$

A. An equation that can be solved to find *k*, the time, in minutes, when the average temperature of the water is 85 degrees Celsius

B. The rate of change in the temperature of the water, in degrees Celsius per minute, at *t* = 4 minutes

C. The average temperature of the water, in degrees Celsius, between *t* = 0 and *t* = 4 minutes

D. The difference in the temperature of the water, in degrees Celsius, between *t* = 0 and *t* = 4 minutes

E. The temperature of the water, in degrees Celsius, at *t* = 4 minutes

F. The average rate of change in the temperature of the water, in degrees Celsius

G. The temperature of the water, in degrees Celsius, at *t* = *k* minutes

H. An equation that can be solved to determine the time *k*, in minutes, when the average temperature of the water, in degrees Celsius, was 85, over the time interval from *t* = 0 to *t* = *k* minutes

Apply Your Understanding: Using Definite Integrals in Contextual Situations

At 3 pm, when the box office opens, there are 75 people waiting in line to buy tickets for a new movie. The rate that the box office sells tickets and moves people out of the line is given by $S(t)$, measured in people per hour. While tickets are being sold, more people are arriving and joining the line at a rate given by $L(t)$, also measured in people per hour.

Match each of the integral expressions given in the table to the correct interpretation.

__________	**1.** $75+\int_3^5 L(t)\,dt$	**A.**	The number of people to whom tickets have been sold between 3 pm and 5 pm
__________	**2.** $\int_3^5 S(t)\,dt$	**B.**	The number of people standing in line at 5 pm
__________	**3.** $75+\int_3^5 L(t)-S(t)\,dt$	**C.**	The total number of people who have stood in line beginning at 3 pm and ending at 5 pm
__________	**4.** $\int_3^5 L(t)\,dt$	**D.**	The change in the number of people in the line between 3 pm and 5 pm
__________	**5.** $\int_3^5 L(t)-S(t)\,dt$	**E.**	The number of people who have arrived to join the line between 3 pm and 5 pm

Check your understanding

At the beginning of an eight-hour shift, there are 55 widgets waiting to be shipped. New widgets are produced at the rate of $W(t)$, measured in widgets per hour. Widgets are shipped out of the factory at a rate of $S(t)$, also measured in widgets per hour. Write a definite integral to find each of the following:

a. The total number of widgets produced during the eight-hour shift

b. The change in the number of widgets waiting to be shipped between the end of the second hour and the beginning of the fourth hour of the shift

c. The number of widgets that are waiting to be shipped at the end of the eight-hour shift

d. The total number of widgets shipped during the second half of the shift