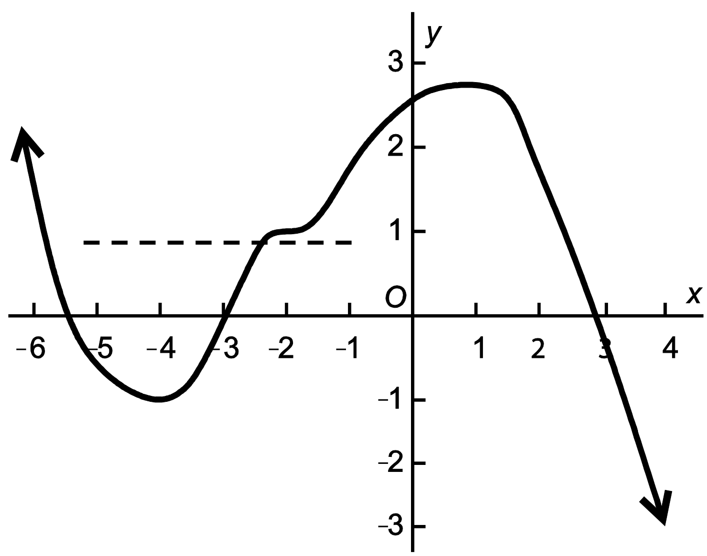
Concavity and Inflection

Question 1

Suppose that f is differentiable and that $f''(x) = (x - 1)^2 (x - 2)^3$. Which x-values have $f''(x) = 0$? What are the first coordinates of any inflection points of f?

Question 2

Suppose that g is differentiable. A graph of the *derivative* of g, that is, $y = g'(x)$, is displayed below. Use that graph to answer these questions: which x-values have $g''(x) = 0$, and what are the first coordinates of any inflection points of $g(x)$?



Graph of $y = g'(x)$, the derivative of $g(x)$
(This graph has a horizontal tangent at $x = -2$.)

Finding Inflection Points

Answer the questions in the scenarios below, then compare your answer with those of your group members and discuss any differences.

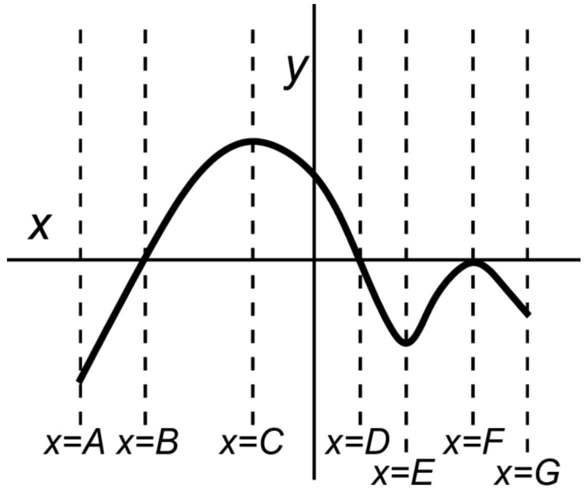
Scenario 1: Suppose that $f(x) = x^4 + x^3 - 3x^2$.

a. Find the first and second derivatives of $f(x)$.

b. Does $f(x)$ have any inflection points? If it does, find their coordinates and explain why they are inflection points.

Scenario 2: Suppose that $g(x) = \dfrac{x^3 + 2}{x^2 + x + 1}$. Then the first derivative, $g'(x)$, is $\dfrac{x^4 + 2x^3 + 3x^2 - 4x - 2}{(x^2 + x + 1)^2}$, and the second derivative, $g''(x)$, is $\dfrac{18x(x+1)}{(x^2 + x + 1)^3}$. Does $g(x)$ have any inflection points? If it does, find their coordinates and explain why they are inflection points.

Scenario 3: This is a graph of the *derivative* of $h(x)$, which is a function defined and continuously differentiable on the interval $[A,G]$. Use this graph of $y = h'(x)$ to answer the following questions.



The graph of *y* = *h'*(*x*)

a. What are the *x*-coordinates of the inflection points of $h(x)$?

b. Justify why those *x*-values are inflection points.

Justifying Inflection Points

Some values of a twice differentiable function, $f(x)$, and its first and second derivatives, $f'(x)$ and $f''(x)$ respectively, are given in the table below. For example, $f'(3) = -2$. Use the table to answer the questions that follow.

x	1	2	3	4
$f'(x)$	4	5	1	2
$f''(x)$	0	0	–2	0
$f(x)$	2	0	0	–3

1. Does $A(x) = (f(x))^2$ have a critical point at $x = 4$? If $A(x)$ does have a critical point, can you determine whether it is a local maximum, local minimum, or neither? Explain your answer.

2. Does $B(x) = f(x^2)$ have a critical point at $x = 2$? If $B(x)$ does have a critical point, can you determine whether it is a local maximum, local minimum, or neither? Explain your answer.

3. Does $C(x) = f(f(x))$ have a critical point at $x = 3$? If $C(x)$ does have a critical point, can you determine whether it is a local maximum, local minimum, or neither? Explain your answer.

4. Does $D(x) = f(4x - 2)$ have a critical point at $x = 1$? If $D(x)$ does have a critical point, can you determine whether it is a local maximum, local minimum, or neither? Explain your answer.

5. Does $E(x) = f(x + 3)$ have a critical point at $x = 0$? If $E(x)$ does have a critical point, can you determine whether it is a local maximum, local minimum, or neither? Explain your answer.

Check your understanding

- [ ] Suppose you are given an analytical representation (a formula) for a function $f(x)$. What steps could you use to identify the inflection points of the function?

- [ ] Suppose you are given a graphical representation for $f'(x)$, the derivative of a function $f(x)$. How could you identify the inflection points of the function?