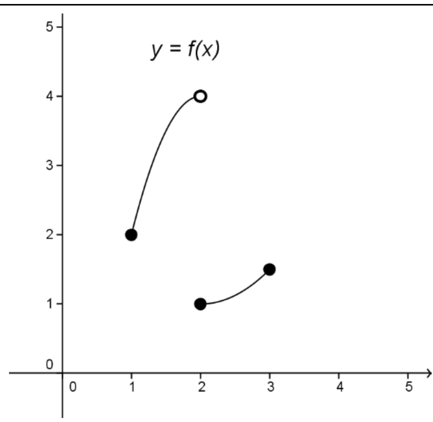
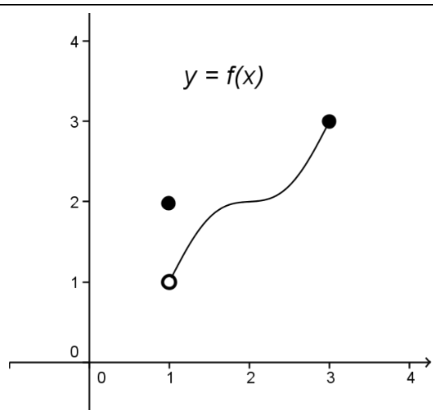
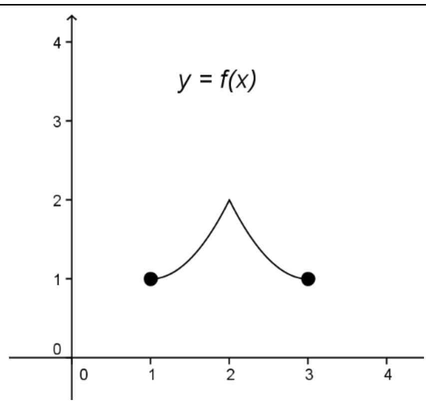
Practice with the Mean Value Theorem

Recall that the **Mean Value Theorem (MVT)** states that if $f(x)$ is continuous on the interval $[a, b]$ and differentiable on the interval (a, b), then for at least one value of c in (a, b), $f'(c) = \frac{f(b) - f(a)}{b - a}$. In words, at some point in the interval the instantaneous rate of change is equal to the average rate of change over that interval. Graphs of three functions with domain $[1, 3]$ are shown in the following table. Explain why the Mean Value Theorem does not apply for any of these functions on the interval $[a, b]$.

	$y = f(x)$	Why the MVT does not apply to f on $[1, 3]$
A.		
B.		
C.		

Does the Mean Value Theorem apply?

For each of the following functions described below, determine whether the Mean Value Theorem can be applied on the interval $[-3,3]$. If it can be applied, explain how you know. If it cannot be applied, explain why not.

Example: $f(x)$ is a function differentiable for all real numbers.

Answer: The MVT can be applied. Since the function is differentiable for all real numbers, it is also continuous for all real numbers. So it is certainly continuous on $[-3,3]$ and differentiable on $(-3,3)$.

1. $f(x)$ is continuous for all real numbers.

2. $f(x)$ is differentiable on $(-3,3)$.

3. $f(x)=x^{2/3}$

4. $f(x)=|x|+4$

5. $f(x)=\sin\left(\frac{\pi x}{3}\right)$

6. $f(x) = \tan\left(\frac{\pi x}{3}\right)$

7. $f(x) = \begin{cases} x^2 + 3, & x \leq 1 \\ 2x + 2, & x > 1 \end{cases}$