Inner and Outer Functions

In each of the following indefinite integrals, some part of the integrand involves a composition of functions of the form $f(g(x))$. In each case, identify the "outside function", $f(x)$, and the "inside function", $g(x)$, by completing the second and third columns below.

	Integral	Outside function $f(x)$	Inside function $g(x)$
Example:	$\int x^2\left(x^3+1\right)^4 dx$	x^4	x^3+1
1.	$\int \frac{2x+5}{\sqrt{x^2+5x+1}}dx$		
2.	$\int \frac{dx}{\sqrt{x}\left(1+\sqrt{x}\right)^3}$		
3.	$\int \left(x^3+x\right)\left(x^4+2x^2+7\right)^{3/4} dx$		
4.	$\int \frac{x\,dx}{\sqrt{x+4}}$		
5.	$\int x^3\left(x^2+1\right)^9 dx$		

Applying Procedures for *U*-Substitution

The table below gives the same indefinite integrals from the Introductory Activity. This time, let u equal the inside function identified before, determine its differential du, and then use this substitution to construct an equivalent integral in terms of u and du only.

Integral	Inside function u	du	Integral in terms of u
Example: $\int x^2\left(x^3+1\right)^4 dx$	$u = x^3 + 1$	$du = 3x^2 dx$	
1. $\int \frac{2x+5}{\sqrt{x^2+5x+1}} dx$			
2. $\int \frac{dx}{\sqrt{x}\left(1+\sqrt{x}\right)^3}$			
3. $\int \left(x^3+x\right)\left(x^4+2x^2+7\right)^{3/4} dx$			
4. $\int \frac{x\,dx}{\sqrt{x+4}}$			
5. $\int x^3\left(x^2+1\right)^9 dx$			

Finding Antiderivatives and Definite Integrals

Part I: Evaluate each integral in the left-hand column. Then check your answer by using the space in the right-hand column to differentiate it and check that you obtain the integrand.

Evaluate each integral:	Differentiate and check your answer:
1. $\int \sqrt{x+1}\,dx$	☐ Matches $\int \sqrt{x+1}\,dx$
2. $\int 2x\sqrt{x^2+1}\,dx$	☐ Matches $\int 2x\sqrt{x^2+1}\,dx$
3. $\int x^2(x^3-1)^7\,dx$	☐ Matches $\int x^2(x^3-1)^7\,dx$

Evaluate each integral:	Differentiate and check your answer:
4. $\int \frac{x^2+2}{(x^3+6x+1)^3}dx$	☐ Matches $\int \frac{x^2+2}{(x^3+6x+1)^3}dx$
5. $\int x^3\sqrt[3]{x^2+4}dx$	☐ Matches $\int x^3\sqrt[3]{x^2+4}dx$

Part II: Explain which error the student made in the work shown below:

Example : $\int_1^9 \sqrt[3]{7x+1}\,dx = \frac{1}{7}\int_1^9 \sqrt[3]{u}\,du = \frac{1}{7}\cdot\frac{3}{4}u^{4/3}\Big|_1^9 = \frac{3}{28}\left(9^{4/3}-1\right)$

Answer: *The student correctly substituted* $u = 7x$ *, but did not change the limits of integration to reflect the substitution.*

Identify the error made, if any, in the solution of the following two definite integrals.

1. $\int_1^4 (2x+3)^2\,dx$ $\qquad \int_5^{11} u^2 du = \frac{u^3}{3}\Big|_5^{11} = \frac{11^3}{3} - \frac{5^3}{3} = 402$

2. $\int_4^{12} \frac{1}{\sqrt{1+2x}}\,dx$ $\qquad \int_4^{25} \frac{u^{\frac{-1}{2}}}{2}\,du = u^{1/2}\Big|_4^{25} = 3$

Find the following definite integrals by using substitution, including changing the limits of integration appropriately.

3. $\int_1^6 \sqrt{x+3}\,dx$

4. $\int_0^2 \frac{x\,dx}{(x^2+1)^3}$

5. $\int_{-2}^4 \left(x^2+5x\right)^4 (6x+15)\,dx$