Average Values Associated with Motion

You have been given a set of sixteen cards. Sort them into three groups: one group of 2 cards, one group of 5 cards, and another group of 5 cards. Cards 1, 2, and 3 have verbal statements and must each be in a different group. All the remaining cards in each group must be mathematically equivalent to the given verbal statements on those three cards. There will be 4 extra cards that should be discarded. These four cards represent common mistakes students make in representing average values associated with motion problems.

In this activity, $x(t)$, represents the position of a particle moving along the x-axis at time t, $v(t)$ represents the velocity of the particle at time t, and $a(t)$ represents the acceleration of the particle at time t.

Understanding the Vocabulary of Motion and Definite Integrals

Part 1

Use the words and phrases in the word bank to complete the fill-in-the-blank sentences given. Note: Each word or phrase will be used only once.

Word Bank

absolute value	position	starting position
displacement	speed	velocity

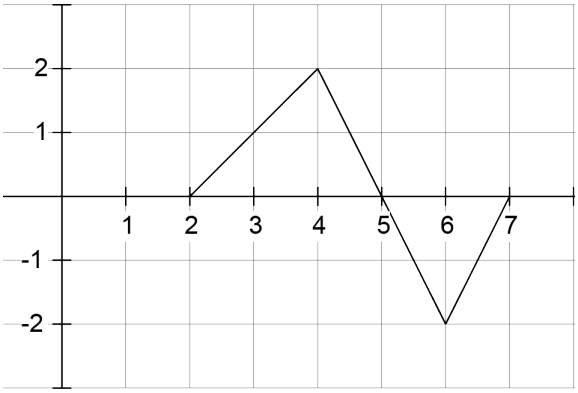
1. To find the new position of a particle moving on the x-axis, add _____ to the integral of _____ from starting time to new time.

2. To find total distance travelled, integrate _____, which is the _____ of velocity.

3. To find the change in _____, integrate velocity over the time interval. This is also called _____.

Part 2

A particle moves along the x-axis so that at time t its position is given by $x(t)$ and velocity given by $v(t)$. At $t = 2$, the particle is located at $x = 5$. The following graph gives the velocity of the particle over the time interval from $t = 2$ to $t = 7$. Match the numerical expressions to their verbal statements.



A. $3+2$ B. $3-2$ C. $5+3-2$

	1. Total distance travelled by the particle over the interval from $t = 2$ to $t = 7$
	2. Position of the particle at $t = 7$
	3. Change in position between $t = 2$ and $t = 7$

Part 3

A particle moves along the x-axis so that at time t its position is given by $x(t)$ and velocity given by $v(t)$. Match the following integral expressions to their verbal statements.

A. $x(2)+\int_2^7 v(t)dt$ B. $\int_2^7 v(t)dt$ C. $\int_2^7 |v(t)|\,dt$

	1. Total distance travelled by the particle over the interval from $t = 2$ to $t = 7$
	2. Position of the particle at $t = 7$
	3. Change in position between $t = 2$ and $t = 7$

Part 4

A particle moves along the x-axis so that, at time t, its position is given by $x(t)$ and velocity given by $v(t)$. At $t = 2$, the particle is located at $x = -1$. The following table gives the velocity of the particle at selected times over the time interval from $t = 2$ to $t = 7$. If integrals are approximated using a left Riemann sum, match the numerical expressions to their verbal statements.

Time	2	5	7
Velocity	-4	6	3

A. $-1+(-4)(3)+(6)(2)$ B. $(4)(3)+(6)(2)$ C. $(-4)(3)+(6)(2)$

	1. Total distance travelled by the particle over the interval from $t = 2$ to $t = 7$
	2. Position of the particle at $t = 7$
	3. Change in position between $t = 2$ and $t = 7$

Check your understanding

Explain in your own words the difference between position, displacement (change in position) and total distance travelled. Which of these can be negative and why?

Using the NUT Strategy to Interpret Motion Integrals

The position of a particle moving on the *x*-axis, in feet from the origin, is given by $x(t)$. The velocity of the particle, in feet per second, is given by $v(t)$, and the acceleration is given by $a(t)$. For each of the integral expressions given in the following table, interpret its meaning by filling in the missing noun, appropriate units, and/or the correct time value or interval.

Integral Expression	Noun	Units	Time
$x(2)+\int_2^4 x'(t)dt$			at t = 4 seconds
$\int_1^4 x'(t)dt$	Change in position of the particle		
$\frac{1}{4-1}\int_1^4 \lvert v(t)\rvert dt$		in feet per second	
$x(4)+\int_4^2 v(t)dt$		in feet from the origin	
$\int_1^4 \lvert v(t)\rvert dt$			from t = 1 to t = 4 seconds
$\frac{1}{4-1}\int_1^4 a(t)dt$	Average acceleration of the particle		

Check your understanding

1. Show how $x(4)+\int_4^2 v(t)dt$ is mathematically equivalent to $x(2)$.

2. What are three different ways a definite integral with an integrand of velocity can be used?

3. What are three different but mathematically equivalent expressions that will find average velocity over a time interval from $t = a$ to $t = b$?