Derivatives and Composite functions

The following table gives selected values for the functions f and g and their first derivatives. Use these values to compute each of the indicated quantities.

x	1	2	3	4	5
$f(x)$	3	5	4	2	1
$f'(x)$	-2	3	2	1	-4
$g(x)$	5	2	6	3	4
$g'(x)$	3	4	4	1	-1

Examples: $f\left(g(3)\right) = f(1) = 3$

$g'\left(g(5)\right) = g'(4) = 1$

1. $g\left(f(1)\right)$

2. $f\left(f(4)\right)$

3. $f'\left(g(1)\right)$

4. $g'\left[\left(f(4)\right)^2\right]$

5. $\left[g'\left(f(4)\right)\right]^2$

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The Chain Rule

Each of the functions $h(x)$ given in the first column below is a composition of functions $f(g(x))$. What is the outer function $f(x)$? What is the inner function $g(x)$? Complete the table below, simplifying the expression for $h'(x)$ in the last column.

	OUTER FUNCTION $f(x)$	INNER FUNCTION $g(x)$	APPLYING THE CHAIN RULE
Example: $h(x)=\left(x^2+3\right)^4$	$f(x)=x^4$	$g(x)=x^2+3$	$h'(x)=f'(g(x))\cdot g'(x)$ $=4\left(x^2+3\right)^3\cdot 2x$ $=8x\left(x^2+3\right)^3$
	$f'(x)=4x^3$	$g'(x)=2x$	
	$f'(g(x))=4\left(x^2+3\right)^3$		
1. $h(x)=\sqrt{3x^5+2x-4}$	$f(x)=$	$g(x)=$	$h'(x)=f'(g(x))\cdot g'(x)$
	$f'(x)=$	$g'(x)=$	
	$f'(g(x))=$		
2. $h(x)=(3x-1)^6$	$f(x)=$	$g(x)=$	$h'(x)=f'(g(x))\cdot g'(x)$
	$f'(x)=$	$g'(x)=$	
	$f'(g(x))=$		
3. $h(x)=\dfrac{1}{2x^3+1}$	$f(x)=$	$g(x)=$	$h'(x)=f'(g(x))\cdot g'(x)$
	$f'(x)=$	$g'(x)=$	
	$f'(g(x))=$		

	OUTER FUNCTION $f(x)$	INNER FUNCTION $g(x)$	APPLYING THE CHAIN RULE
4. $h(x)=\left(3+6\sqrt{x}\right)^{\frac{2}{3}}$	$f(x)=$	$g(x)=$	$h'(x)=$
	$f'(x)=$	$g'(x)=$	
	$f'(g(x))=$		
5. $h(x)=\dfrac{4}{\left(2x^2-x-1\right)^3}$	$f(x)=$	$g(x)=$	$h'(x)=$
	$f'(x)=$	$g'(x)=$	
	$f'(g(x))=$		

Differentiating Combined Algebraic Functions

A very common mistake when applying the chain rule is forgetting the second factor – multiplying by the derivative of the inside function. This is especially common in implicit differentiation and in related rates, primarily because students don't always recognize these as chain rule situations.

For example, if the height h of an object depends on the time t, then the rate of change of $y = h^3$ requires the chain rule: $\frac{d}{dt}\left(h^3\right) = 3h^2 \cdot \frac{dh}{dt}$. This is clear if we instead write h^3 as $\left(h(t)\right)^3$ – then we see the composition of function. Or perhaps it is clear if we use the alternative form of the chain rule: If y is a function of h, and h is a function of t, then $\frac{dy}{dt} = \frac{dy}{dh} \cdot \frac{dh}{dx}$. The error we are focusing on in this activity is that second factor. Since we are taking the derivative with respect to t, we must think of h as "*stuff*" and multiply by its derivative.

In each of the following, decide whether or not the chain rule is needed to find the indicated derivative, and then find the derivative.

1. $y = x^3$; find $\frac{dy}{dx}$

☐ Chain rule needed ☐ No chain rule needed

$\frac{dy}{dx} =$

2. $y = \sqrt{x^4 + 3x + 1}$; find $\frac{dy}{dx}$

☐ Chain rule needed ☐ No chain rule needed

$\frac{dy}{dx} =$

3. $y = x^5 + 2x$ and $x = t^2 - 1$; find $\frac{dy}{dt}$

☐ Chain rule needed ☐ No chain rule needed

$\frac{dy}{dt} =$

4. $y = \sqrt[3]{x} + 4$; find $\frac{d}{dx}\left(y^5\right)$

☐ Chain rule needed ☐ No chain rule needed

$\frac{d}{dx}\left(y^5\right) =$