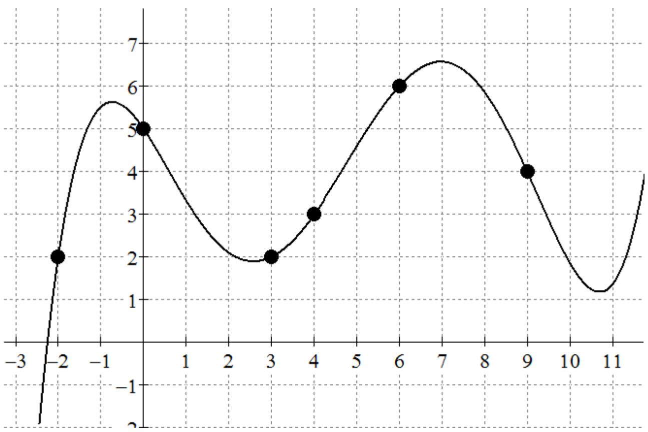
Determining the Type of Approximation

There are four methods by which you should be able to approximate a definite integral:

- a right Riemann sum,
- a left Riemann sum,
- a midpoint Riemann sum,
- or a trapezoidal sum.

This activity will focus on the right Riemann sum and the left Riemann sum methods. Work with your partner to determine which method has been used to solve each of the following problems. (*n* represents the number of sub-intervals for each sum.) Be prepared to defend your choices.

Scenario 1:



Definite Integral	Approximation	Method of Approximation
$\int_{-2}^{3} f(x)dx$	$n = 2 \quad 5(2) + 2(3)$	
$\int_{0}^{9} f(x)dx$	$n = 4 \quad 5(3) + 2(1) + 3(2) + 6(3)$	

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Scenario 2: The table below gives velocity values measured in ft/sec every 3 seconds.

Time (seconds)	**0**	**3**	**6**	**9**	**12**
Velocity (ft/sec)	**10**	**-15**	**20**	**15**	**10**

Definite Integral	**Approximation**	**Method of Approximation**
$\int_0^6 \lvert v(t) \rvert \, dt$	$n = 2$ $\quad 10(3) + 15(3)$	
$\int_0^6 f(x) dx$	$n = 3$ $f(0) \cdot 2 + f(2) \cdot 2 + f(4) \cdot 2$	

Check your understanding

Fill in the blank with the method that is described.

For any subinterval, a ____ involves adding areas of rectangles where the height is the *y*-value on the left side of the interval which is multiplied by the width (change in *x*-values) of that subinterval.

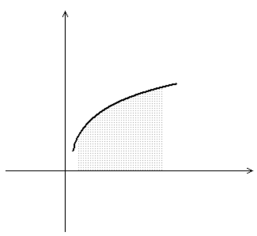
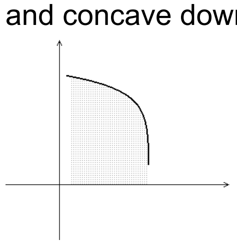
A ____ involves adding areas of rectangles where the height is the *y*-value on the right side of the interval which is multiplied by the width (change in *x*-values) of that subinterval.

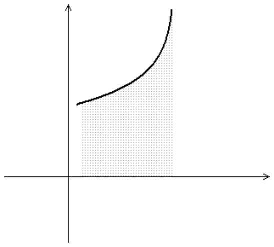
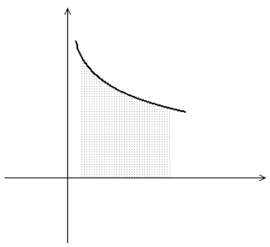
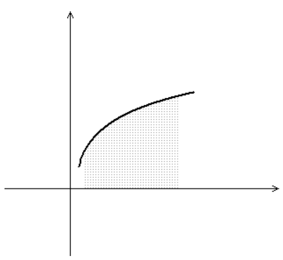
Greater Than or Less Than the Definite Integral?

Notes for this Activity

- On an increasing function, a left Riemann sum underestimates the definite integral and a right Riemann sum overestimates the definite integral
- On a decreasing function, a left Riemann sum overestimates the definite integral and a right Riemann sum underestimates the definite integral
- If a function is concave down, a trapezoidal sum will underestimate the definite integral
- If a function is concave up, a trapezoidal sum will overestimate the definite integral

Directions: For each of the graphs below, determine whether Column A or Column B would be the greater value and explain why in the final column. If not stated, assume equal numbers of sub-intervals.

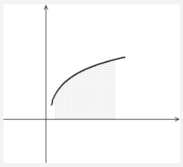
Graph	Column A	Column B	Answer and Explanation
$f(x)$ is increasing and concave down 	Right Riemann Sum	Trapezoidal Sum	Column _____ is greater since a right Riemann sum ____________________ a definite integral on an increasing function and a trapezoid sum ____________________ on a concave down function.
$f(x)$ is decreasing and concave down 	Right Riemann sum	$\int_a^b f(x)dx$	

Graph	Column A	Column B	Answer and Explanation
$f(x)$ is increasing and concave up 	Trapezoidal sum with 3 equal sub-intervals	Trapezoidal sum with 6 equal sub-intervals	
$f(x)$ is decreasing and concave up 	Trapezoidal sum	Left Riemann sum	
$f(x)$ is increasing and concave down 	Right Riemann sum with 3 equal sub-intervals	Right Riemann sum with 6 equal sub-intervals	

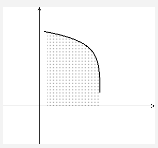
Check your understanding

Match each of the following graphs to the correct description in the blanks below.

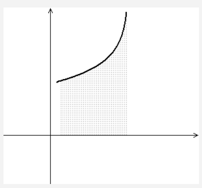
a. $f(x)$ is increasing and concave down:



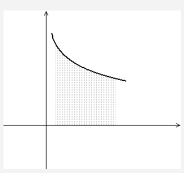
b. $f(x)$ is decreasing and concave down:



c. $f(x)$ is increasing and concave up:



d. $f(x)$ is decreasing and concave up:



_____ 1. Trapezoidal approximation overestimates, and left Riemann underestimates.

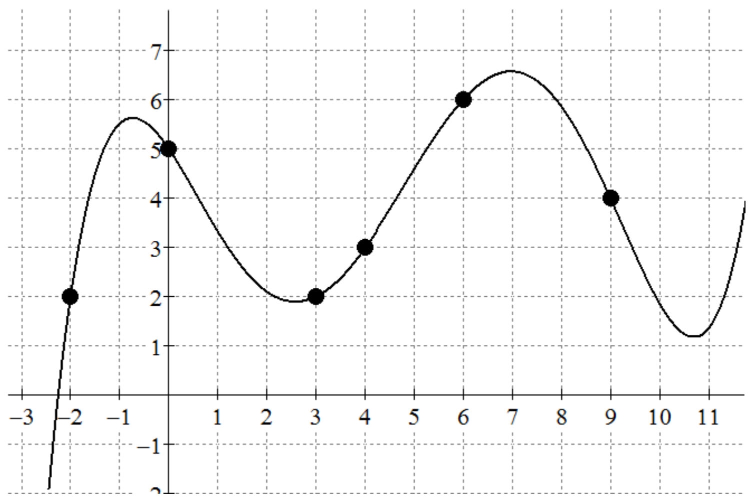
_____ 2. Left Riemann sum overestimates, and trapezoidal approximation underestimates.

_____ 3. Trapezoidal approximation overestimates, and left Riemann overestimates.

_____ 4. Trapezoidal approximation underestimates, and right Riemann overestimates.

Apply Your Understanding of Left and Right Rectangle Riemann Sums

Approximate each of the given definite integrals using the specified method. You may leave your answers in expanded form (as a sum of products). Note that *n* represents the number of sub-intervals for each sum.



Definite Integral	Method	Approximation
$\int_3^9 f(x)dx$	$n = 3$ Left Riemann Sum	
$\int_0^9 f(x)dx$	$n = 4$ Right Riemann Sum	

Given: The table below gives velocity values measured in ft/sec every 4 seconds.

Time (seconds)	0	4	8	12	16
Velocity (ft/sec)	12	20	-8	-12	10

Definite Integral	Method	Approximation
$\int_0^{16} \lvert v(t) \rvert \, dt$	$n = 4$ Left Riemann sum	
$\int_0^{24} f(x) dx$	$n = 3$ Right Riemann sum	

Check your understanding

After comparing papers with your partner, discuss what you think are the three most common errors that students make in approximating definite integrals.