# Matching Integrands to Outcomes

Match each expression in column A to its corresponding outcome in column B. Writer the letter from column B on the blank line next to the expression in column A.

| | Column A<br>**If you integrate:** | Column B<br>**You will find:** |
|---|---|---|
| ______ | 1. The rate snow is falling on a driveway | A. The amount of oil removed |
| ______ | 2. The rate that water is flowing into a tank | B. The amount of weight gained |
| ______ | 3. The rate oil is being removed from an oil spill | C. The amount of a product that has been sold |
| ______ | 4. The rate a pizza is cooling | D. The amount of water added to the tank |
| ______ | 5. The rate a product is selling | E. The amount of new vocabulary acquired |
| ______ | 6. The rate a drug is entering a patient's bloodstream | F. The amount of a drug that has entered the bloodstream |
| ______ | 7. The rate at which a puppy is gaining weight | G. The amount of snow that has fallen on the driveway |
| ______ | 8. The rate at which a student acquires new vocabulary | H. The amount by which the temperature of the pizza has changed |

**Check your understanding**

Complete each of the following sentences.

- If you integrate the rate that a house is being painted, you will find:
  ______________________________________________.
- If you integrate the rate that grass clippings are decomposing in a bin, you will find:
  ______________________________________________.
- If you integrate the rate that sand is eroding off of a beach, you will find:
  ______________________________________________.

# Using the NUT Strategy to Identify Errors in Interpretation

A correct interpretation for a definite integral includes an appropriate noun (N), the correct units (U), and a time element (T) – or, the "NUT" strategy. For each of the following situations, a student has attempted to interpret a definite integral using the NUT strategy, but has made a mistake in one of the three parts. First identify the student's error (was it the noun, the units, or the time?), then provide a correction to the statement. The first one has been done as an example.

**Example:**

A tank initially contains 50 gallons of honey. Honey is added to the tank at a rate of $H(t)$ gallons per hour and removed from the tank at a rate of $R(t)$ gallons per hour.

**Student's Response:** $50+\int_0^3 H(t)-R(t)dt$ represents <u>the amount of honey in the tank</u> [NOUN], in <u>gallons</u> [UNITS], <u>over the time interval from $t$ = 0 to $t$ = 3 hours</u> [TIME].

| Which part contains the error? | Correction: |
|---|---|
| ☐ NOUN<br>☐ UNITS<br>☑ TIME | It should be the amount <u>at $t$ = 3 hours</u>, rather than over an interval of three hours of time. |

**Situation One**

The number of mosquitoes on a tropical island is given by $M(t)$, where M is the number of mosquitoes in thousands, and $t$ is the number of days since the beginning of the year.

**Student's Response:** $\frac{1}{30}\int_0^{30} M'(t)dt$ represents the average number of mosquitoes on an island, in thousands per day, between the beginning of the year and the thirtieth day of the year.

| Which part contains the error? | Correction: |
|---|---|
| ☐ NOUN<br>☐ UNITS<br>☐ TIME | |

**Situation Two**

The temperature in a house is modeled by a function $H(t)$, where $H$ is measured in degrees Fahrenheit, and $t$ is measured in hours since midnight.

**Student's Response:** $H(0)+\int_0^8 H'(t)dt$ represents temperature of the house, in degrees Fahrenheit, between midnight and 8 am.

| Which part contains the error? | Correction: |
|---|---|
| ☐ NOUN<br>☐ UNITS<br>☐ TIME | |

**Situation Three**

A cup of tea is cooling on a counter. The temperature is a differentiable function $T(t)$, where $T$ is measured in degrees Celsius, and $t$ is measured in minutes.

**Student's Response:** $\int_2^5 T'(t)dt$ represents the average temperature of the tea, in degrees Celsius, between $t$ = 2 minutes and $t$ = 5 minutes.

| Which part contains the error? | Correction: |
|---|---|
| ☐ NOUN<br>☐ UNITS<br>☐ TIME | |

**Situation Four**

The rate that sand is removed by the tide is modeled by the differentiable function $S(t)$, where $S$ is measured in cubic yards per hour, and $t$ is measured in hours since midnight. While sand is removed by the tide, a pumping station nearby returns sand to the beach at a rate of $R(t)$, where $R$ is measured in cubic yards per hour.

**Student's Response:** $\int_8^2 R(t) - S(t)dt$ represents the change in the amount of sand on the beach, in cubic yards per hour, between 2 am and 8 am.

| Which part contains the error? | Correction: |
|---|---|
| ☐ NOUN<br>☐ UNITS<br>☐ TIME | |

# Apply Your Understanding: Definite Integrals in Context

In each of the following problems, a student has produced an answer that is **INCORRECT**. Describe the student's mistake or misunderstanding, and then provide a correct answer.

**Situation A:** The rate that cars pass through an intersection is given by $C(t)$ measured in cars/hour for $t$ = 6 hours until $t$ = 12 hours.

| Problem | Incorrect Answer | Describe the student's mistake or misunderstanding | Correct answer |
|---|---|---|---|
| Write an equation involving an integral to determine the time $t$ = k at which 500 cars would have passed through the intersection starting from $t$ = 6 hours. | $\int_k^6 C(t)\,dt = 500$ | | |
| Interpret the meaning of the $\frac{1}{4}\int_6^{10} C(t)\,dt$ in the context of this problem. | This represents the average number of cars that pass through the intersection between $t$ = 6 hours and $t$ = 10 hours. | | |
| Write an integral to determine the number of cars that pass through the intersection between $t$ = 6 hours and $t$ = 10 hours. | $\int_6^{10} C'(t)\,dt$ | | |

**Situation B:** Tribbles, a particularly annoying pest, are born at the rate of $B(t)$ tribbles per hour, and are exterminated at the rate of $D(t)$ tribbles per hour. When the infestation of tribbles was discovered at $t = 0$ hours, there was already a population of 2000 tribbles.

| Problem | Incorrect Answer | Describe the student's mistake or misunderstanding | Correct answer |
|---|---|---|---|
| Write an expression, involving at least one integral, to determine the total number of tribbles alive at any time $t = \text{k}$. | $\int_0^k B(t) - D(t)\,dt$ | | |
| Write an integral to represent the number of tribbles born between $t = 2$ hours and $t = 6$ hours. | $\int_2^6 B'(t)\,dt$ | | |
| Interpret the meaning of the integral $\frac{1}{4}\int_7^{11} D(t)\,dt$. | This represents the average rate that tribbles were dying, in tribbles per hour, over a 4-hour time period. | | |