

In the second section of Chapter 3, different slope triangles are explored for a given line or segment. For each line, the slope remains constant no matter where the slope triangle is drawn on that line or how large or small each slope triangle is. All the slope triangles for a given line are similar. These similar slope triangles can be used to write proportions to calculate side lengths and angle measures. This constant slope ratio is known as the **tangent** (trigonometric) relationship. Later in the section, the tangent button on a calculator is used to determine measurements in application problems.

See the Math Notes boxes in Lessons 3.2.2, 3.2.4, and 3.2.5 for more information about slope angles and the tangent ratio.

### Example 1

The line graphed at right passes through the origin. Draw in three different slope triangles for the line. For each triangle, what is the slope ratio,  $\frac{\Delta y}{\Delta x}$ ? What is true about all three ratios?

Note:  $\Delta x$  (delta  $x$ ) and  $\Delta y$  (delta  $y$ ) are read “change in  $x$ ” and “change in  $y$ .”

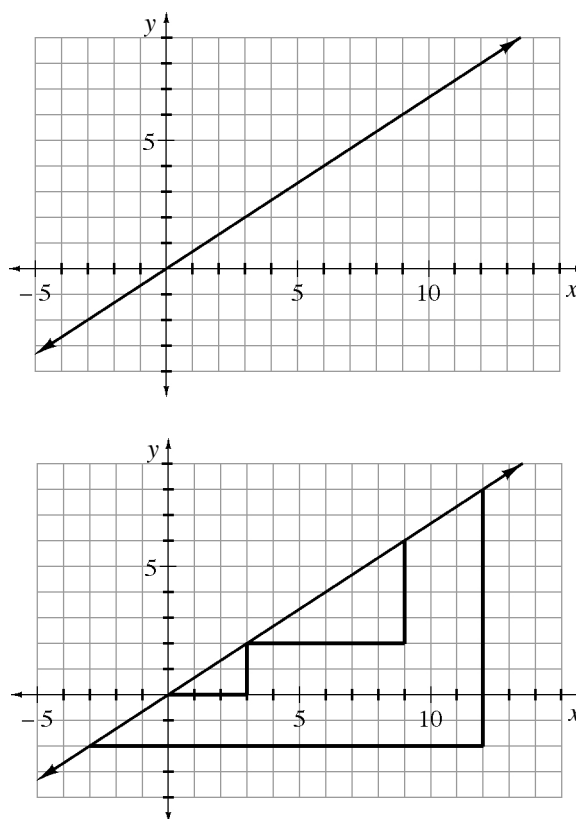
A slope triangle is a right triangle that has its hypotenuse on the line that contains it. This means that the two legs of the right triangle are parallel to the axes: one leg runs vertically, the other horizontally. There are infinitely many slope triangles that can be drawn, but it is easiest if the triangles that have their vertices on lattice points (that is, their vertices have integer coordinates). The length of the horizontal leg is  $\Delta x$  and the length of the vertical leg is  $\Delta y$ . In the diagram at right there are three possible slope triangles. For the smallest triangle,  $\Delta x = 3$  (the length of the horizontal leg), and  $\Delta y = 2$  (the length of the vertical leg). Therefore  $\frac{\Delta y}{\Delta x} = \frac{2}{3}$ .

In the medium-sized triangle,  $\Delta x = 6$  and  $\Delta y = 4$ , which means  $\frac{\Delta y}{\Delta x} = \frac{4}{6}$ .

Lastly, the leg lengths of the largest triangle are  $\Delta x = 15$  and  $\Delta y = 10$ , so  $\frac{\Delta y}{\Delta x} = \frac{10}{15}$ .

These slope ratios are all equivalent, so no matter where the slope triangles for this line is drawn, the slope ratio remains constant.

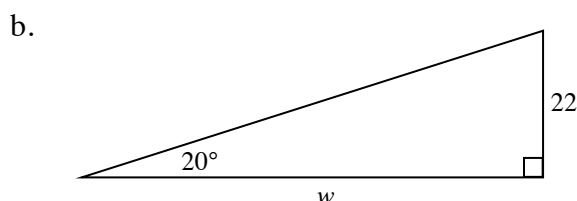
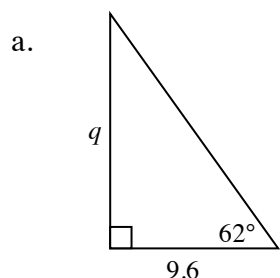
$$\frac{\Delta y}{\Delta x} = \frac{2}{3} = \frac{4}{6} = \frac{10}{15}$$



In Lesson 3.2.2 students connect specific slope ratios of slope triangles to their related angles and record these values in a Trig Table Graphic Organizer (Lesson 3.2.2 Resource Page). They use this information to calculate missing side lengths and angle measures of right triangles. In Lesson 3.2.4, students learn to use the tangent button on their calculator to calculate these ratios and solve for missing information.

## Example 2

Write an equation and use the tangent function on your calculator to calculate the missing side length in each triangle.



When using the tangent button on a calculator to solve these problems, be sure that the calculator is in degree mode and not radian mode. Since we know that the slope ratio depends on the angle, we can use the angle measure and the tangent function on a calculator to determine unknown lengths of the triangles.

The tangent of an angle in a right triangle is the ratio  $\frac{\text{opposite leg}}{\text{adjacent leg}}$ . This allows us to write and solve the equations as shown below.

Part (a):  $\tan 20^\circ = \frac{22}{w}$

$$w \tan 20^\circ = 22$$

$$w = \frac{22}{\tan 20^\circ}$$

$$w \approx 60.44$$

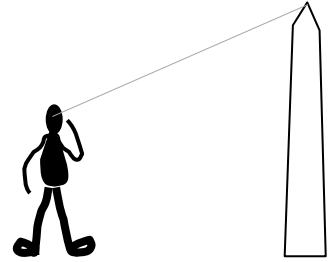
Part (b):  $\tan 62^\circ = \frac{q}{9.6} \quad \left( \frac{\text{opposite leg}}{\text{adjacent leg}} \right)$

$$9.6(\tan 62^\circ) = q$$

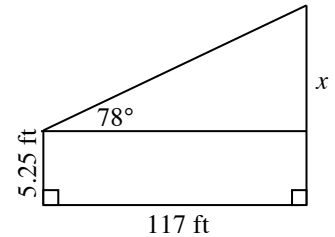
$$q \approx (9.6)(1.88) \approx 18.05$$

### Example 3

Talula is standing 117 feet from the base of the Washington Monument in Washington, D.C. She uses her clinometer to measure the angle of elevation to the top of the monument to be  $78^\circ$ . If Talula's eye height is 5 feet, 3 inches, what is the height of the Washington Monument?



With all problems representing an everyday situation, it is helpful to draw a diagram of what the problem is describing. In this situation, Talula looking up at the top of a monument. We know how far away Talula is standing from the monument, we know her eye height, and we know the angle of elevation of her line of sight.



We use this information to draw a diagram, as shown at right. Then we write an equation using the tangent ratio and solve for  $x$ .

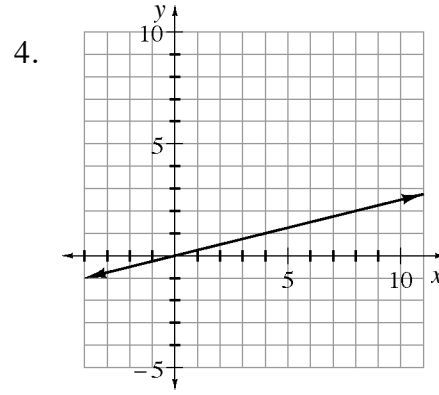
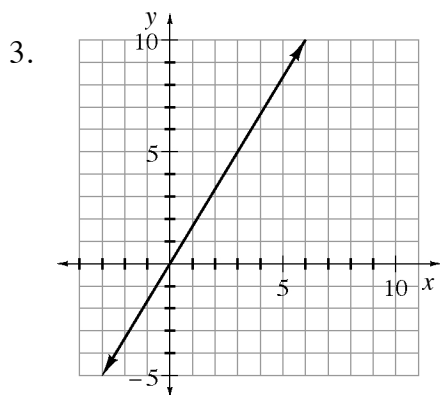
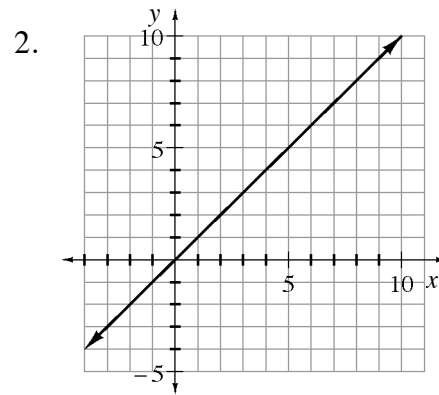
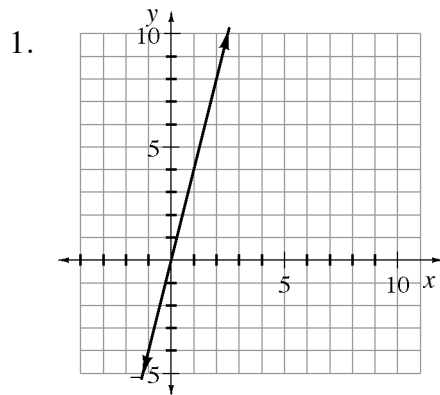
We add the “eye height” to the value of  $x$  to determine the height of the Washington Monument. Write the answer to the nearest foot.

$$\begin{aligned}\tan 78^\circ &= \frac{x}{117} \\ 117(\tan 78^\circ) &= x \\ x &\approx 549.9 \text{ feet}\end{aligned}$$

$$549.9 + 5.25 \approx 555.15, \text{ or about } 555 \text{ feet}$$

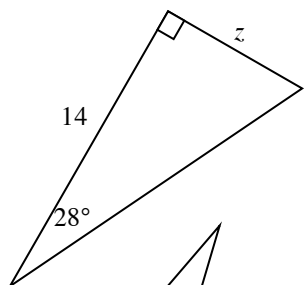
### Problems

For each line, draw several slope triangles. Then calculate the slope ratios.

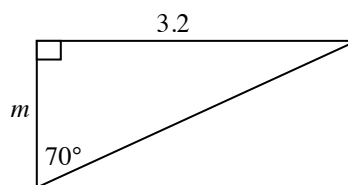


Calculate the measures indicated by the variables. It may be helpful to rotate the triangle so that it resembles a slope triangle. If you write a tangent equation, use the tangent button on your calculator not your Trig Table Graphic Organizer to solve the equation. Note: Some calculations require the Pythagorean Theorem. For lengths, write your answer to the nearest hundredth. For angles, write your answer to the nearest degree.

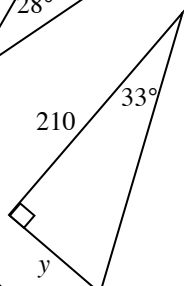
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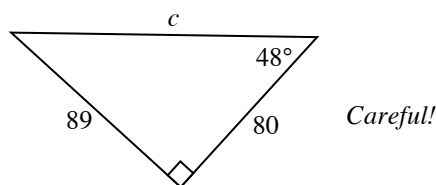
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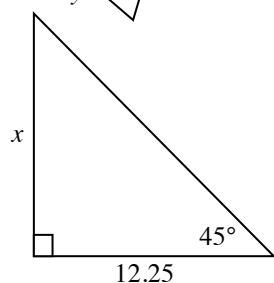
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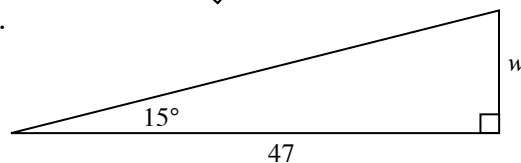
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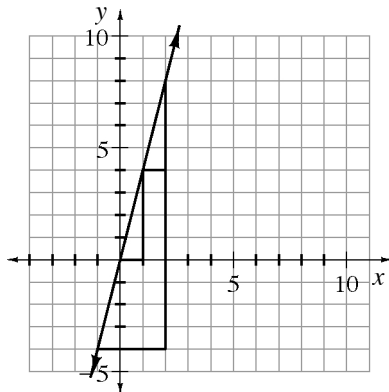
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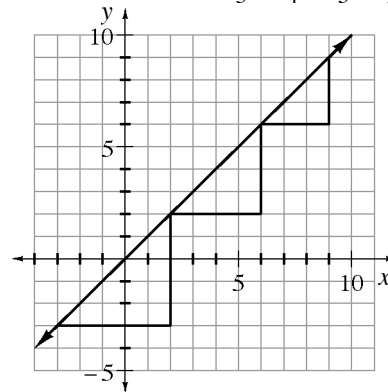
11. A ladder leaning against a wall makes a  $75^\circ$  angle with the ground. The base of the ladder is 5.0 feet from the wall. How high up the wall does the ladder reach? Write your answer with the appropriate precision.
12. Davis and Tess are 30 feet apart when Tess lets go of her helium-filled balloon, which rises straight up into the air. (It is a windless day.) After 4 seconds, Davis uses his clinometer to sight the angle of elevation to the balloon at  $35^\circ$ . If Davis' eye height is 4 feet, 6 inches, what is the height of the balloon after 4 seconds? Write your answer with the appropriate precision.

## Answers

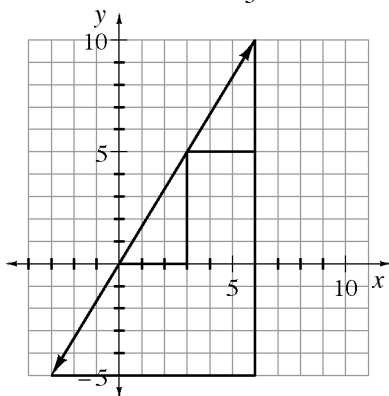
1. In each case the slope ratio is  $\frac{4}{1} = 4$ .



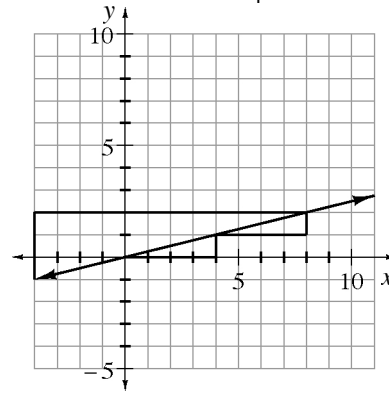
2. The slope ratio is  $\frac{5}{5} = \frac{4}{4} = \frac{3}{3} = 1$ .



3. The slope ratio is  $\frac{5}{3}$ .



4. The slope ratio is  $\frac{1}{4}$ .



5.  $\tan(28^\circ) = \frac{z}{14}$ ,  $z \approx 7.44$

6.  $\tan(70^\circ) = \frac{3.2}{m}$ ,  $m \approx 1.16$

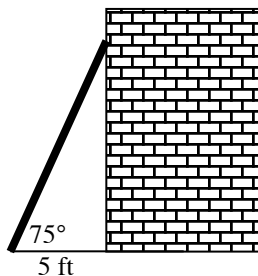
7.  $\tan(33^\circ) = \frac{y}{210}$ ,  $y \approx 136.38$

8.  $c \approx 119.67$  (Pythagorean Theorem)

9.  $x = 12.25$

10.  $\tan(15^\circ) = \frac{w}{47}$ ,  $w \approx 12.59$

11.  $\tan(75^\circ) = \frac{h}{5}$ ; The ladder reaches about 18.7 feet up the wall.



12.  $\tan(35^\circ) = \frac{h}{30}$ ,  $h \approx 21 + 4.5 \approx 25.5$ ; After 4 seconds the balloon is about 25.5 feet above the ground.

