## Entity Embedding

ACTL3143 & ACTL5111 Deep Learning for Actuaries
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#### **Lecture Outline**

- Entity Embedding
- Categorical Variables & Entity Embeddings
- Keras' Functional API
- French Motor Dataset with Embeddings
- Scale By Exposure





## Continuing on the French motor dataset example

Download the dataset if we don't have it already.

```
from pathlib import Path
from sklearn.datasets import fetch_openml

if not Path("french-motor.csv").exists():
    freq = fetch_openml(data_id=41214, as_frame=True).frame
    freq.to_csv("french-motor.csv", index=False)

else:
    freq = pd.read_csv("french-motor.csv")

freq
```





# Continuing on the French motor dataset example

	IDpol	ClaimNb	<b>Exposure</b>	Area	VehPower	VehAş
O	1.0	1	0.10000	D	5	O
1	3.0	1	0.77000	D	5	О
• • •	•••	•••	•••	•••	•••	•••
678011	6114329.0	О	0.00274	В	4	О
678012	6114330.0	О	0.00274	В	7	6

678013 rows × 12 columns





#### Data dictionary

- IDpol: policy number (unique identifier)
- **ClaimNb**: number of claims on the given policy
- **Exposure**: total exposure in yearly units
- Area: area code (categorical, ordinal)
- VehPower: power of the car (categorical, ordinal)
- VehAge: age of the car in years
- DrivAge: age of the (most common) driver in years

- BonusMalus: bonus-malus level between 50 and 230 (with reference level 100)
- VehBrand: car brand (categorical, nominal)
- VehGas: diesel or regular fuel car (binary)
- **Density**: density of inhabitants per km<sup>2</sup> in the city of the living place of the driver
- Region: regions in France (prior to 2016)





#### The model

Have  $\{(\mathbf{x}_i, y_i)\}_{i=1,...,n}$  for  $\mathbf{x}_i \in \mathbb{R}^{47}$  and  $y_i \in \mathbb{N}_0$ .

Assume the distribution

$$Y_i \sim \mathsf{Poisson}(\lambda(\mathbf{x}_i))$$

We have  $\mathbb{E}Y_i = \lambda(\mathbf{x}_i)$ . The NN takes  $\mathbf{x}_i \ \mathcal{E}$  predicts  $\mathbb{E}Y_i$ .

#### (i) Note

For insurance, this is a bit weird. The exposures are different for each policy.

 $\lambda(\mathbf{x}_i)$  is the expected number of claims for the duration of policy i's contract.

Normally, Exposure<sub>i</sub>  $\notin \mathbf{x}_i$ , and  $\lambda(\mathbf{x}_i)$  is the expected rate *per year*, then

$$Y_i \sim \mathsf{Poisson}(\mathrm{Exposure}_i imes \lambda(\mathbf{x}_i)).$$





#### Where are things defined?

In Keras, string options are used for convenience to reference specific functions or settings.

```
1 model = Sequential([
2     Dense(30, activation="relu"),
3     Dense(1, activation="exponential")
4 ])
```

#### is the same as

```
from keras.activations import relu, exponential

model = Sequential([
    Dense(30, activation=relu),
    Dense(1, activation=exponential)

x = [-1.0, 0.0, 1.0]
print(relu(x))
print(exponential(x))

tf.Tensor([0. 0. 1.], shape=(3,), dtype=float32)
tf.Tensor([0.37 1. 2.72], shape=(3,), dtype=float32)
```





## String arguments to .compile

#### When we run

```
1 model.compile(optimizer="adam", loss="poisson")
```

#### it is equivalent to

```
from keras.losses import poisson
from keras.optimizers import Adam
model.compile(optimizer=Adam(), loss=poisson)
```

#### Why do this manually? To adjust the object:

```
1 optimizer = Adam(learning_rate=0.01)
2 model.compile(optimizer=optimizer, loss="poisson")
```

or to get help.





## Keras' "poisson" loss

```
1 help(keras.losses.poisson)
Help on function poisson in module keras.src.losses.losses:
poisson(y_true, y_pred)
    Computes the Poisson loss between y_true and y_pred.
    Formula:
    ```python
    loss = y_pred - y_true * log(y_pred)
    Args:
        y_true: Ground truth values. shape = `[batch_size, d0, .. dN]`.
        y pred: The predicted values. shape = `[batch size, d0, .. dN]`.
    Returns:
        Poisson loss values with shape = `[batch_size, d0, .. dN-1]`.
    Example:
```





#### Subsample and split

```
freq = freq.drop("IDpol", axis=1).head(25_000)

X_train, X_test, y_train, y_test = train_test_split(
freq.drop("ClaimNb", axis=1), freq["ClaimNb"], random_state=2023)

# Reset each index to start at 0 again.

X_train = X_train.reset_index(drop=True)

X_test = X_test.reset_index(drop=True)
```



#### What values do we see in the data?

```
1 X_train["Area"].value_counts()
2 X_train["VehBrand"].value_counts()
3 X_train["VehGas"].value_counts()
4 X_train["Region"].value_counts()
```

```
VehBrand
Area
     5507
  В1
  5069
     4113
  4838
  B2
  ...
     2359
  B11
   284
      475
  B14
   136
Name: count, Length: 6, dtype: int64
VehGas
  Region
'Regular'
              10773
  6498
  R24
'Diesel'
               7977
  R82
  2119
Name: count, dtype: int64
  • • •
  R42
  55
```

```
B1 5069
B2 4838
...
B11 284
B14 136
Name: count, Length: 11, dtype: int64
Region
R24 6498
R82 2119
...
R42 55
R43 26
Name: count, Length: 22, dtype: int64
```





#### Preprocess ordinal & continuous

```
from sklearn.compose import make_column_transformer

ct = make_column_transformer(
    (OrdinalEncoder(), ["Area", "VehGas"]),
    ("drop", ["VehBrand", "Region"]),
    remainder=StandardScaler(),
    verbose_feature_names_out=False
)

X_train_ct = ct.fit_transform(X_train)
```

1 X\_train.head(3)

1 X\_train\_ct.head(3)

	Exposure	Area	VehPower		Area	VehGas	Exposure
O	1.00	C	6	O	2.0	0.0	1.126979
1	0.36	C	4	1	2.0	1.0	-0.590896
2	0.02	E	12	2	4.0	1.0	-1.503517





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## Region column









## One-hot encoding

```
1  oe = OneHotEncoder(sparse_output=False)
2  X_train_oh = oe.fit_transform(X_train[["Region"]])
3  X_test_oh = oe.transform(X_test[["Region"]])
4  print(list(X_train["Region"][:5]))
5  X_train_oh.head()
```

['R24', 'R93', 'R11', 'R42', 'R24']

	Region_R11	Region_R21	Region_R22	Region_R23	Region
O	0.0	0.0	0.0	0.0	1.0
1	0.0	0.0	0.0	0.0	0.0
•••	•••	•••	•••	•••	•••
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	1.0

5 rows × 22 columns





#### Train on one-hot inputs

```
num_regions = len(oe.categories_[0])

random.seed(12)

model = Sequential([
    Dense(2, input_dim=num_regions),
    Dense(1, activation="exponential")
    ])

model.compile(optimizer="adam", loss="poisson")

es = EarlyStopping(verbose=True)
hist = model.fit(X_train_oh, y_train, epochs=100, verbose=0,
    validation_split=0.2, callbacks=[es])
hist.history["val_loss"][-1]
```

Epoch 12: early stopping 0.7526934146881104



### Consider the first layer

```
1 every_category = pd.DataFrame(np.eye(num_regions), columns=oe.categories_[0])
2 every_category.head(3)
```

	<b>R11</b>	R21	R22	R23	R24	R25	R26	R31	R41	R42	•••	<b>R53</b>	<b>R54</b>	R72	R73	R74	R
0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	•••	0.0	0.0	0.0	0.0	0.0	Ο.
1	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	•••	0.0	0.0	0.0	0.0	0.0	Ο.
2	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	•••	0.0	0.0	0.0	0.0	0.0	0.

#### 3 rows × 22 columns

```
1 # Put this through the first layer of the model
2 X = every_category.to_numpy()
3 model.layers[0](X)
```

```
<tf.Tensor: shape=(22, 2), dtype=float32, numpy=
array([[-0.21, -0.14],
       [0.21, -0.17],
       [-0.22, 0.1],
       [-0.83, 0.1],
       [-0.01, -0.66],
       [-0.65, -0.13],
       [-0.36, -0.41],
       [0.21, -0.03],
       [-0.93, -0.57],
       [0.2, -0.41],
       [-0.43, -0.21],
       [-1.13, -0.33],
       [0.17, -0.68],
       [-0.88, -0.55],
       [-0.13, 0.05],
        [ 0.11, 0. ],
       [-0.46, -0.38],
       [-0.62, -0.37],
       [-0.19, -0.28],
       [-0.22, 0.15],
```



#### The first layer

```
1 layer = model.layers[0]
  2 W, b = layer.get weights()
  3 X.shape, W.shape, b.shape
((22, 22), (22, 2), (2,))
  1 \times 0 \times + b
  1 W + b
array([[-0.21, -0.14],
   array([[-0.21, -0.14],
       [0.21, -0.17],
  [0.21, -0.17],
       [-0.22, 0.1],
  [-0.22, 0.1],
       [-0.83, 0.1],
  [-0.83, 0.1],
       [-0.01, -0.66],
  [-0.01. -0.66].
       [-0.65, -0.13],
  [-0.65, -0.13],
       [-0.36, -0.41],
  [-0.36, -0.41],
      [0.21, -0.03],
  [0.21, -0.03],
       [-0.93, -0.57],
  [-0.93, -0.57]
  [0.2, -0.41],
      [0.2, -0.41],
       [-0.43, -0.21],
  [-0.43, -0.21],
       [-1.13, -0.33],
  [-1.13, -0.33],
      [0.17, -0.68],
  [0.17, -0.68],
       [-0.88, -0.55],
  [-0.88, -0.55],
       [-0.13, 0.05],
  [-0.13, 0.05],
       [ 0.11, 0. ],
  [0.11, 0.],
       [-0.46, -0.38],
  [-0.46, -0.38],
       [-0.62, -0.37],
  [-0.62, -0.37],
       [-0.19, -0.28],
  [-0.19, -0.28],
```





#### Just a look-up operation

```
display(list(oe.categories_[0]))
  W + b
['R11',
   array([[-0.21, -0.14],
'R21',
  [0.21, -0.17],
'R22',
  [-0.22, 0.1],
'R23',
  [-0.83, 0.1],
'R24',
  [-0.01, -0.66],
 'R25',
  [-0.65, -0.13],
'R26',
  [-0.36, -0.41],
'R31',
  [0.21, -0.03],
'R41',
  [-0.93, -0.57],
  [0.2, -0.41],
'R42',
'R43',
  [-0.43, -0.21],
'R52',
  [-1.13, -0.33],
  [0.17, -0.68],
'R53',
  [-0.88, -0.55],
'R54',
'R72',
  [-0.13, 0.05],
'R73',
  [0.11, 0.],
'R74',
  [-0.46, -0.38],
  [-0.62, -0.37],
'R82',
'R83',
  [-0.19, -0.28],
```





## Turn the region into an index

```
1  oe = OrdinalEncoder()
2  X_train_reg = oe.fit_transform(X_train[["Region"]])
3  X_test_reg = oe.transform(X_test[["Region"]])
4  
5  for i, reg in enumerate(oe.categories_[0][:3]):
6  print(f"The Region value {reg} gets turned into {i}.")
```

The Region value R11 gets turned into 0. The Region value R21 gets turned into 1. The Region value R22 gets turned into 2.





## Embedding

```
from keras.layers import Embedding
num_regions = len(np.unique(X_train[["Region"]]))

random.seed(12)
model = Sequential([
Embedding(input_dim=num_regions, output_dim=2),
Dense(1, activation="exponential")
])
model.compile(optimizer="adam", loss="poisson")
```





## Fitting that model

```
1 es = EarlyStopping(verbose=True)
2 hist = model.fit(X_train_reg, y_train, epochs=100, verbose=0,
3 validation_split=0.2, callbacks=[es])
4 hist.history["val_loss"][-1]
```

Epoch 5: early stopping 0.7526668906211853

```
1 model.layers
```

[<Embedding name=embedding, built=True>, <Dense name=dense\_6, built=True>]





#### Keras' Embedding Layer

```
model.layers[0].get_weights()[0]
array([[-0.12, -0.11],
       [0.03, -0.],
       [-0.02. 0.01].
       [-0.25, -0.14],
       [-0.28, -0.32].
       [-0.3, -0.22],
       [-0.31, -0.28].
      [0.1, 0.07],
      [-0.61, -0.51],
      [-0.06, -0.12],
      [-0.17, -0.14],
       [-0.6, -0.46],
       [-0.22, -0.27],
       [-0.59, -0.5],
       [-0., 0.02],
       [0.07, 0.06],
       [-0.31, -0.28],
       [-0.4, -0.34],
       [-0.16, -0.15],
```

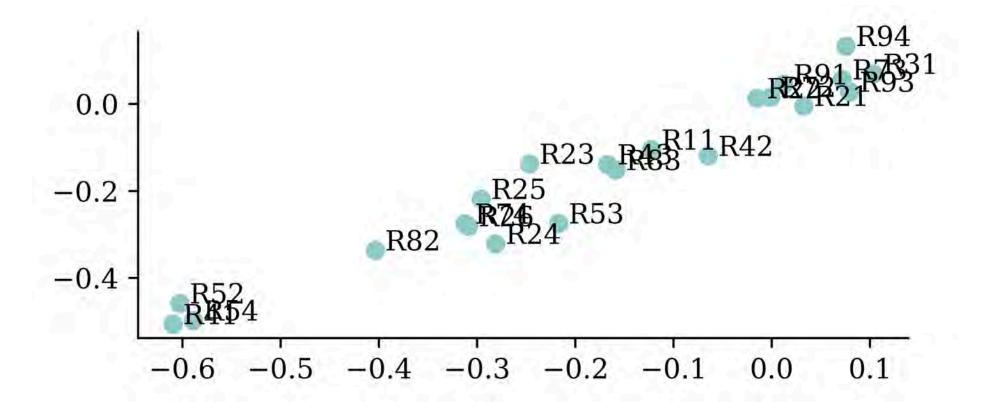
```
1 X_train["Region"].head(4)
     R24
    R93
     R11
     R42
Name: Region, dtype: object
  1 X_sample = X_train_reg[:4].to_numpy()
  2 X_sample
array([[ 4.],
       [20.],
       [ 0.],
       [9.11)
  1 enc_tensor = model.layers[0](X_sample)
  2 keras.ops.convert_to_numpy(enc_tensor).
array([[-0.28, -0.32],
       [0.08, 0.03],
       [-0.12, -0.11],
       [-0.06, -0.12], dtype=float32)
```





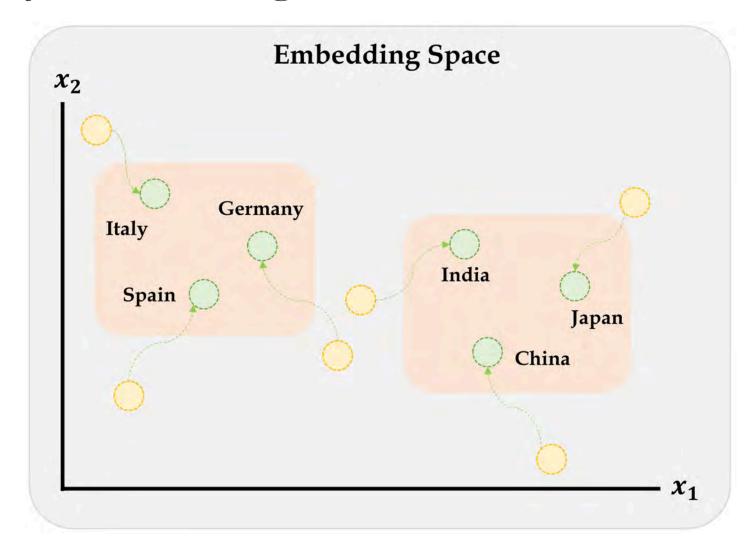
#### The learned embeddings

```
points = model.layers[0].get_weights()[0]
plt.scatter(points[:,0], points[:,1])
for i in range(num_regions):
   plt.text(points[i,0]+0.01, points[i,1] , s=oe.categories_[0][i])
```





## Entity embeddings



Embeddings will gradually improve during training.





#### Embeddings & other inputs

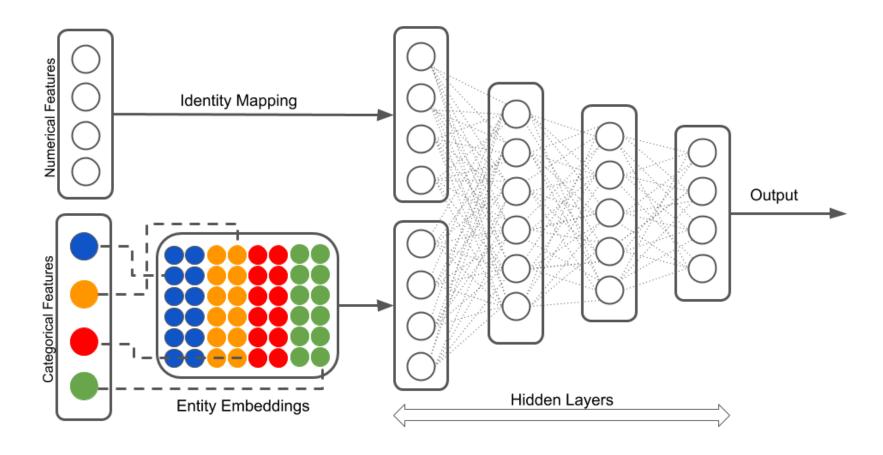


Illustration of a neural network with both continuous and categorical inputs.

We can't do this with Sequential models...





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#### Converting Sequential models

```
1 from keras.models import Model
2 from keras.layers import Input
```

10

11

14

```
1 random.seed(12)
2
3 model = Sequential([
4    Dense(30, "leaky_relu"),
5    Dense(1, "exponential")
6 ])
7
8 model.compile(
9    optimizer="adam",
10    loss="poisson")
11
12 hist = model.fit(
13    X_train_oh, y_train,
14    epochs=1, verbose=0,
15    validation_split=0.2)
16 hist.history["val_loss"][-1]
```

0.7535399198532104

random.seed(12)

model.compile(

12 hist = model.fit(

3 inputs = Input(shape=(X train oh.shape[

4 x = Dense(30, "leaky\_relu")(inputs)

5 out = Dense(1, "exponential")(x)

6 model = Model(inputs, out)

X\_train\_oh, y\_train,

epochs=1, verbose=0,

validation split=0.2)

16 hist.history["val loss"][-1]

optimizer="adam",

loss="poisson")

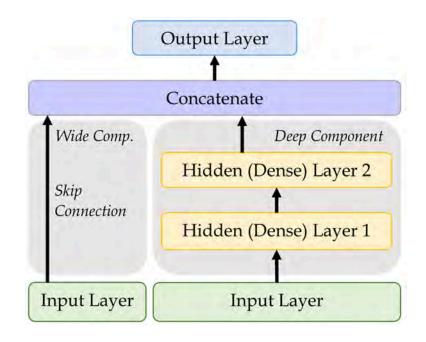
0.7535399198532104

See one-length tuples.





#### Wide & Deep network



An illustration of the wide & deep network architecture.

Add a *skip connection* from input to output layers.

```
from keras.layers \
       import Concatenate
   inp = Input(shape=X_train.shape[1:])
   hidden1 = Dense(30, "leaky_relu")(inp)
   hidden2 = Dense(30, "leaky_relu")(hidden1)
   concat = Concatenate()(
     [inp, hidden2])
   output = Dense(1)(concat)
   model = Model(
       inputs=[inp],
11
       outputs=[output])
12
```

## Naming the layers

For complex networks, it is often useful to give meaningul names to the layers.

```
input_ = Input(shape=X_train.shape[1:], name="input")
hidden1 = Dense(30, activation="leaky_relu", name="hidden1")(input_)
hidden2 = Dense(30, activation="leaky_relu", name="hidden2")(hidden1)
concat = Concatenate(name="combined")([input_, hidden2])
output = Dense(1, name="output")(concat)
model = Model(inputs=[input_], outputs=[output])
```



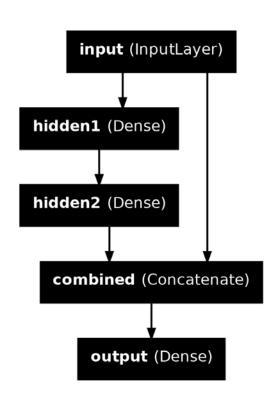


## Inspecting a complex model

1 from keras.utils import plot\_model

plot\_model(model, she

1 model.summary(line\_length=75)



#### Model: "functional\_8"

Layer (type)	Output Shape	Param #	Connected
input (InputLaye	er) (None, 10)	0	-
hidden1 (Dense)	(None, 30)	330	input[0][
hidden2 (Dense)	(None, 30)	930	hidden1[@
combined (Concatenate)	(None, 40)	0	input[0][ hidden2[0
output (Dense)	(None, 1)	41	combined[

Total params: 1,301 (5.08 KB)
Trainable params: 1,301 (5.08 KB)
Non-trainable params: 0 (0.00 B)







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#### The desired architecture

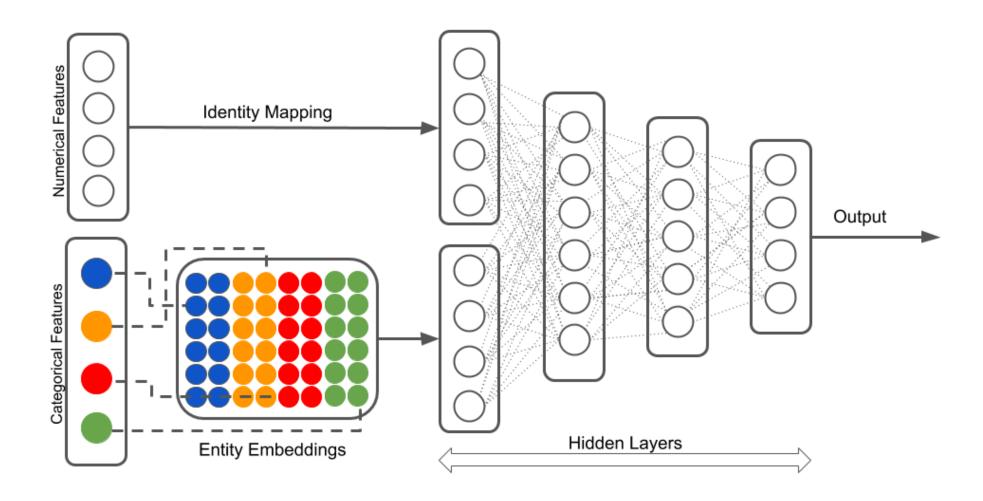


Illustration of a neural network with both continuous and categorical inputs.





## Preprocess all French motor inputs

Transform the categorical variables to integers:

```
num_brands, num_regions = X_train.nunique()[["VehBrand", "Region"]]

ct = make_column_transformer(
    (OrdinalEncoder(), ["VehBrand", "Region", "Area", "VehGas"]),
    remainder=StandardScaler(),
    verbose_feature_names_out=False

)

X_train_ct = ct.fit_transform(X_train)

X_test_ct = ct.transform(X_test)
```

Split the brand and region data apart from the rest:

```
1 X_train_brand = X_train_ct["VehBrand"]; X_test_brand = X_test_ct["VehBrand"]
2 X_train_region = X_train_ct["Region"]; X_test_region = X_test_ct["Region"]
3 X_train_rest = X_train_ct.drop(["VehBrand", "Region"], axis=1)
4 X_test_rest = X_test_ct.drop(["VehBrand", "Region"], axis=1)
```





#### Organise the inputs

Make a Keras Input for: vehicle brand, region, & others.

```
veh_brand = Input(shape=(1,), name="vehBrand")
region = Input(shape=(1,), name="region")
other_inputs = Input(shape=X_train_rest.shape[1:], name="otherInputs")
```

Create embeddings and join them with the other inputs.





#### Complete the model and fit it

Feed the combined embeddings & continuous inputs to some normal dense layers.

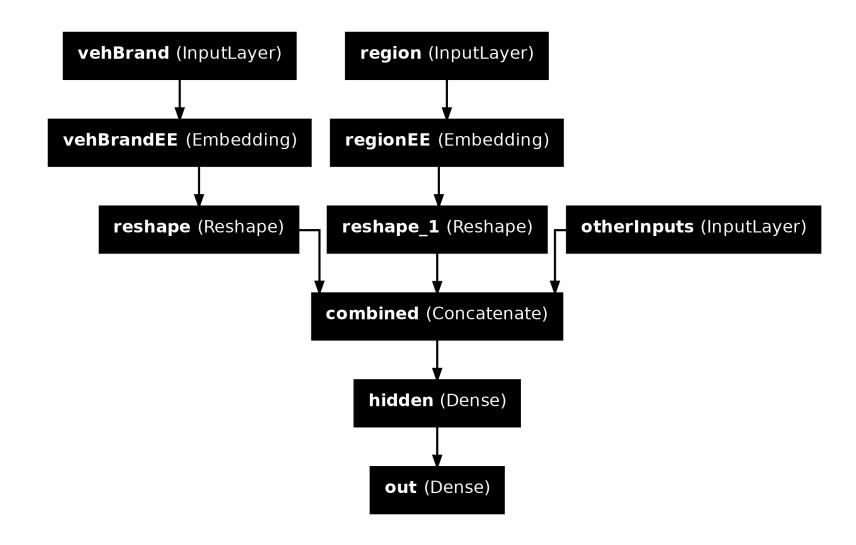
0.6692155599594116





## Plotting this model

1 plot\_model(model, show\_layer\_names=True)



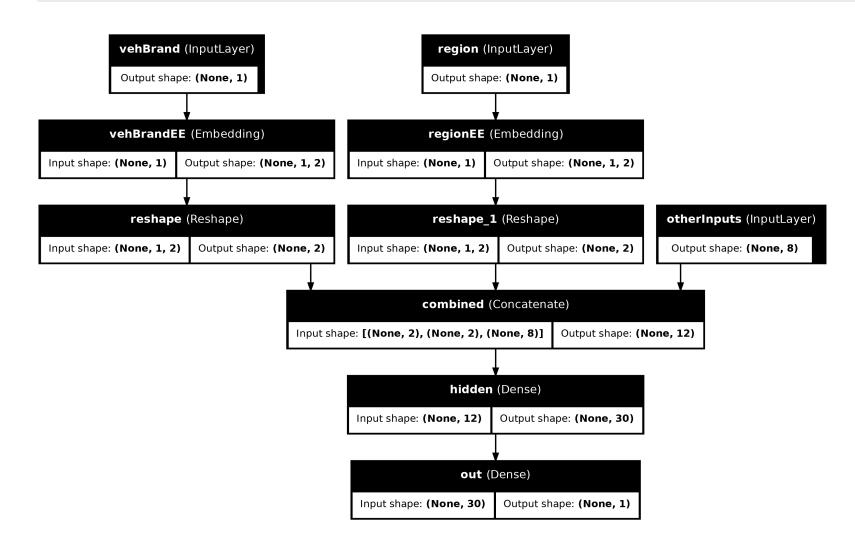






## Why we need to reshape

1 plot\_model(model, show\_layer\_names=True, show\_shapes=True)









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#### Two different models

Have  $\{(\mathbf{x}_i, y_i)\}_{i=1,...,n}$  for  $\mathbf{x}_i \in \mathbb{R}^{47}$  and  $y_i \in \mathbb{N}_0$ .

**Model 1:** Say  $Y_i \sim \mathsf{Poisson}(\lambda(\mathbf{x}_i))$ .

But, the exposures are different for each policy.  $\lambda(\mathbf{x}_i)$  is the expected number of claims for the duration of policy i's contract.

**Model 2:** Say  $Y_i \sim \mathsf{Poisson}(\mathsf{Exposure}_i \times \lambda(\mathbf{x}_i))$ .

Now, Exposure<sub>i</sub>  $\notin \mathbf{x}_i$ , and  $\lambda(\mathbf{x}_i)$  is the rate *per year*.





### Just take continuous variables

```
1 ct = make_column_transformer(
2    ("passthrough", ["Exposure"]),
3    ("drop", ["VehBrand", "Region", "Area", "VehGas"]),
4    remainder=StandardScaler(),
5    verbose_feature_names_out=False
6  )
7    X_train_ct = ct.fit_transform(X_train)
8    X_test_ct = ct.transform(X_test)
```

#### Split exposure apart from the rest:

```
1 X_train_exp = X_train_ct["Exposure"]; X_test_exp = X_test_ct["Exposure"]
2 X_train_rest = X_train_ct.drop("Exposure", axis=1)
3 X_test_rest = X_test_ct.drop("Exposure", axis=1)
```

#### Organise the inputs:

```
1 exposure = Input(shape=(1,), name="exposure")
2 other_inputs = Input(shape=X_train_rest.shape[1:], name="otherInputs")
```





#### Make & fit the model

Feed the continuous inputs to some normal dense layers.

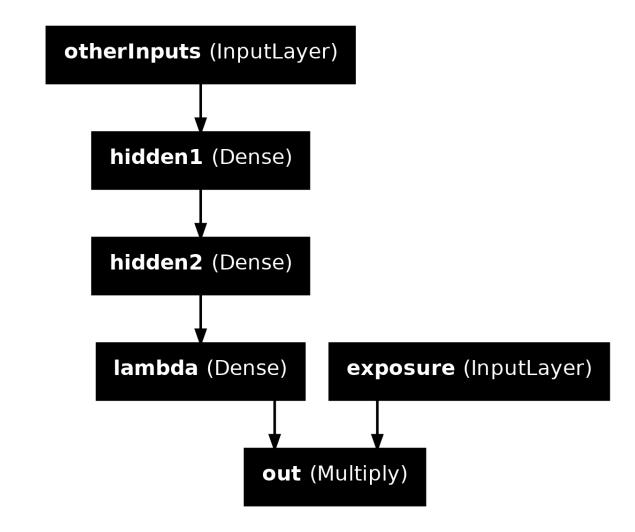
```
1 random.seed(1337)
  2 x = Dense(30, "relu", name="hidden1")(other inputs)
  3 x = Dense(30, "relu", name="hidden2")(x)
  4 lambda = Dense(1, "exponential", name="lambda")(x)
  1 from keras.layers import Multiply
  3 out = Multiply(name="out")([lambda_, exposure])
    model = Model([exposure, other inputs], out)
     model.compile(optimizer="adam", loss="poisson")
     es = EarlyStopping(patience=10, restore best weights=True, verbose=1)
  8 hist = model.fit((X train exp, X train rest),
         y_train, epochs=100, verbose=0,
         callbacks=[es], validation split=0.2)
 11 np.min(hist.history["val loss"])
Epoch 40: early stopping
Restoring model weights from the end of the best epoch: 30.
0.8829042911529541
```





#### Plot the model

1 plot\_model(model, show\_layer\_names=True)







## Package Versions

```
1 from watermark import watermark
2 print(watermark(python=True, packages="keras,matplotlib,numpy,pandas,seaborn,scipy,torch
```

Python implementation: CPython Python version : 3.11.9
IPython version : 8.24.0

keras : 3.3.3
matplotlib: 3.9.0
numpy : 1.26.4
pandas : 2.2.2
seaborn : 0.13.2
scipy : 1.11.0
torch : 2.3.1
tensorflow: 2.16.1
tf\_keras : 2.16.0





## Glossary

- dropout
- entity embeddings
- Input layer
- Keras functional API

- Reshape layer
- skip connection
- wide & deep network structure



