Entity Embedding

ACTL3143 & ACTL5111 Deep Learning for Actuaries

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Lecture Outline

- **Entity Embedding**
- Categorical Variables & Entity Embeddings
- Keras' Functional API
- French Motor Dataset with Embeddings
- Scale By Exposure

Continuing on the French motor dataset example

Download the dataset if we don't have it already.

```python
from pathlib import Path
from sklearn.datasets import fetch_openml

if not Path("french-motor.csv").exists():
    freq = fetch_openml(data_id=41214, as_frame=True).frame
    freq.to_csv("french-motor.csv", index=False)
else:
    freq = pd.read_csv("french-motor.csv")

freq
```

Source: Nell et al. (2020), Case Study: French Motor Third-Party Liability Claims, SSRN.

Continuing on the French motor dataset example

	IDpol	ClaimNb	Exposure	Area	VehPower	VehA
0	1.0	1	0.10000	D	5	0
1	3.0	1	0.77000	D	5	0
...	...	...	...	...	...	...
678011	6114329.0	0	0.00274	B	4	0
678012	6114330.0	0	0.00274	B	7	6

678013 rows × 12 columns

Data dictionary

- `IDpol`: policy number (unique identifier)
- `ClaimNb`: number of claims on the given policy
- `Exposure`: total exposure in yearly units
- `Area`: area code (categorical, ordinal)
- `VehPower`: power of the car (categorical, ordinal)
- `VehAge`: age of the car in years
- `DrivAge`: age of the (most common) driver in years
- `BonusMalus`: bonus-malus level between 50 and 230 (with reference level 100)
- `VehBrand`: car brand (categorical, nominal)
- `VehGas`: diesel or regular fuel car (binary)
- `Density`: density of inhabitants per km² in the city of the living place of the driver
- `Region`: regions in France (prior to 2016)

Source: Nell et al. (2020), Case Study: French Motor Third-Party Liability Claims, SSRN.

The model

Have $\{(\mathbf{x}_i, y_i)\}_{i=1,\dots,n}$ for $\mathbf{x}_i \in \mathbb{R}^{47}$ and $y_i \in \mathbb{N}_0$.

Assume the distribution

$$Y_i \sim \mathsf{Poisson}(\lambda(\mathbf{x}_i))$$

We have $\mathbb{E} Y_i = \lambda(\mathbf{x}_i)$. The NN takes $\mathbf{x}_i$ & predicts $\mathbb{E} Y_i$.

Note

For insurance, *this is a bit weird*. The exposures are different for each policy.

$\lambda(\mathbf{x}_i)$ is the expected number of claims for the duration of policy i's contract.

Normally, $\text{Exposure}_i \not\in \mathbf{x}_i$, and $\lambda(\mathbf{x}_i)$ is the expected rate *per year*, then

$$Y_i \sim \mathsf{Poisson}(\text{Exposure}_i \times \lambda(\mathbf{x}_i)).$$

Where are things defined?

In Keras, string options are used for convenience to reference specific functions or settings.

```python
model = Sequential([
    Dense(30, activation="relu"),
    Dense(1, activation="exponential")
])
```

is the same as

```python
from keras.activations import relu, exponential

model = Sequential([
    Dense(30, activation=relu),
    Dense(1, activation=exponential)
])
```

```python
x = [-1.0, 0.0, 1.0]
print(relu(x))
print(exponential(x))
```

```
tf.Tensor([0. 0. 1.], shape=(3,), dtype=float32)
tf.Tensor([0.37 1.   2.72], shape=(3,), dtype=float32)
```

String arguments to `.compile`

When we run

```
model.compile(optimizer="adam", loss="poisson")
```

it is equivalent to

```
from keras.losses import poisson
from keras.optimizers import Adam

model.compile(optimizer=Adam(), loss=poisson)
```

Why do this manually? To adjust the object:

```
optimizer = Adam(learning_rate=0.01)
model.compile(optimizer=optimizer, loss="poisson")
```

or to get help.

Keras' "poisson" loss

```python
help(keras.losses.poisson)
```

```
Help on function poisson in module keras.src.losses.losses:

poisson(y_true, y_pred)
    Computes the Poisson loss between y_true and y_pred.

    Formula:

    ```python
    loss = y_pred - y_true * log(y_pred)
    ```

    Args:
        y_true: Ground truth values. shape = `[batch_size, d0, .. dN]`.
        y_pred: The predicted values. shape = `[batch_size, d0, .. dN]`.

    Returns:
        Poisson loss values with shape = `[batch_size, d0, .. dN-1]`.

    Example:
```

Subsample and split

```python
freq = freq.drop("IDpol", axis=1).head(25_000)

X_train, X_test, y_train, y_test = train_test_split(
  freq.drop("ClaimNb", axis=1), freq["ClaimNb"], random_state=2023)

# Reset each index to start at 0 again.
X_train = X_train.reset_index(drop=True)
X_test = X_test.reset_index(drop=True)
```

What values do we see in the data?

```python
X_train["Area"].value_counts()
X_train["VehBrand"].value_counts()
X_train["VehGas"].value_counts()
X_train["Region"].value_counts()
```

```
Area
C     5507
D     4113
      ...
B     2359
F      475
Name: count, Length: 6, dtype: int64
```

```
VehBrand
B1      5069
B2      4838
        ...
B11      284
B14      136
Name: count, Length: 11, dtype: int64
```

```
VehGas
'Regular'    10773
'Diesel'      7977
Name: count, dtype: int64
```

```
Region
R24     6498
R82     2119
        ...
R42       55
R43       26
Name: count, Length: 22, dtype: int64
```

Preprocess ordinal & continuous

```
from sklearn.compose import make_column_transformer

ct = make_column_transformer(
  (OrdinalEncoder(), ["Area", "VehGas"]),
  ("drop", ["VehBrand", "Region"]),
  remainder=StandardScaler(),
  verbose_feature_names_out=False
)
X_train_ct = ct.fit_transform(X_train)
```

```
X_train.head(3)
```

	Exposure	Area	VehPower
0	1.00	C	6
1	0.36	C	4
2	0.02	E	12

```
X_train_ct.head(3)
```

	Area	VehGas	Exposure
0	2.0	0.0	1.126979
1	2.0	1.0	-0.590896
2	4.0	1.0	-1.503517

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Region column

French Administrative Regions

Source: Nell et al. (2020), Case Study: French Motor Third-Party Liability Claims, SSRN.

One-hot encoding

```python
oe = OneHotEncoder(sparse_output=False)
X_train_oh = oe.fit_transform(X_train[["Region"]])
X_test_oh = oe.transform(X_test[["Region"]])
print(list(X_train["Region"][:5]))
X_train_oh.head()
```

```
['R24', 'R93', 'R11', 'R42', 'R24']
```

	Region_R11	Region_R21	Region_R22	Region_R23	Region
0	0.0	0.0	0.0	0.0	1.0
1	0.0	0.0	0.0	0.0	0.0
...	...	...	...	...	...
3	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	1.0

5 rows × 22 columns

Train on one-hot inputs

```python
num_regions = len(oe.categories_[0])

random.seed(12)
model = Sequential([
  Dense(2, input_dim=num_regions),
  Dense(1, activation="exponential")
])

model.compile(optimizer="adam", loss="poisson")

es = EarlyStopping(verbose=True)
hist = model.fit(X_train_oh, y_train, epochs=100, verbose=0,
    validation_split=0.2, callbacks=[es])
hist.history["val_loss"][-1]
```

```
Epoch 12: early stopping
```

```
0.7526934146881104
```

Consider the first layer

```
every_category = pd.DataFrame(np.eye(num_regions), columns=oe.categories_[0])
every_category.head(3)
```

	R11	R21	R22	R23	R24	R25	R26	R31	R41	R42	...	R53	R54	R72	R73	R74	R8
0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.
1	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.
2	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	...	0.0	0.0	0.0	0.0	0.0	0.

3 rows × 22 columns

```
# Put this through the first layer of the model
X = every_category.to_numpy()
model.layers[0](X)
```

```
<tf.Tensor: shape=(22, 2), dtype=float32, numpy=
array([[-0.21, -0.14],
       [ 0.21, -0.17],
       [-0.22,  0.1 ],
       [-0.83,  0.1 ],
       [-0.01, -0.66],
       [-0.65, -0.13],
       [-0.36, -0.41],
       [ 0.21, -0.03],
       [-0.93, -0.57],
       [ 0.2 , -0.41],
       [-0.43, -0.21],
       [-1.13, -0.33],
       [ 0.17, -0.68],
       [-0.88, -0.55],
       [-0.13,  0.05],
       [ 0.11,  0.  ],
       [-0.46, -0.38],
       [-0.62, -0.37],
       [-0.19, -0.28],
       [-0.22,  0.15],
```

The first layer

```python
layer = model.layers[0]
W, b = layer.get_weights()
X.shape, W.shape, b.shape
```

```
((22, 22), (22, 2), (2,))
```

```python
X @ W + b
```

```
array([[-0.21, -0.14],
       [ 0.21, -0.17],
       [-0.22,  0.1 ],
       [-0.83,  0.1 ],
       [-0.01, -0.66],
       [-0.65, -0.13],
       [-0.36, -0.41],
       [ 0.21, -0.03],
       [-0.93, -0.57],
       [ 0.2 , -0.41],
       [-0.43, -0.21],
       [-1.13, -0.33],
       [ 0.17, -0.68],
       [-0.88, -0.55],
       [-0.13,  0.05],
       [ 0.11,  0.  ],
       [-0.46, -0.38],
       [-0.62, -0.37],
       [-0.19, -0.28],
```

```python
W + b
```

```
array([[-0.21, -0.14],
       [ 0.21, -0.17],
       [-0.22,  0.1 ],
       [-0.83,  0.1 ],
       [-0.01, -0.66],
       [-0.65, -0.13],
       [-0.36, -0.41],
       [ 0.21, -0.03],
       [-0.93, -0.57],
       [ 0.2 , -0.41],
       [-0.43, -0.21],
       [-1.13, -0.33],
       [ 0.17, -0.68],
       [-0.88, -0.55],
       [-0.13,  0.05],
       [ 0.11,  0.  ],
       [-0.46, -0.38],
       [-0.62, -0.37],
       [-0.19, -0.28],
```

Just a look-up operation

```
display(list(oe.categories_[0]))
```

```
['R11',
 'R21',
 'R22',
 'R23',
 'R24',
 'R25',
 'R26',
 'R31',
 'R41',
 'R42',
 'R43',
 'R52',
 'R53',
 'R54',
 'R72',
 'R73',
 'R74',
 'R82',
 'R83',
```

```
W + b
```

```
array([[-0.21, -0.14],
       [ 0.21, -0.17],
       [-0.22,  0.1 ],
       [-0.83,  0.1 ],
       [-0.01, -0.66],
       [-0.65, -0.13],
       [-0.36, -0.41],
       [ 0.21, -0.03],
       [-0.93, -0.57],
       [ 0.2 , -0.41],
       [-0.43, -0.21],
       [-1.13, -0.33],
       [ 0.17, -0.68],
       [-0.88, -0.55],
       [-0.13,  0.05],
       [ 0.11,  0.  ],
       [-0.46, -0.38],
       [-0.62, -0.37],
       [-0.19, -0.28],
```

Turn the region into an index

```python
oe = OrdinalEncoder()
X_train_reg = oe.fit_transform(X_train[["Region"]])
X_test_reg = oe.transform(X_test[["Region"]])

for i, reg in enumerate(oe.categories_[0][:3]):
  print(f"The Region value {reg} gets turned into {i}.")
```

```
The Region value R11 gets turned into 0.
The Region value R21 gets turned into 1.
The Region value R22 gets turned into 2.
```

Embedding

```python
from keras.layers import Embedding
num_regions = len(np.unique(X_train[["Region"]]))

random.seed(12)
model = Sequential([
  Embedding(input_dim=num_regions, output_dim=2),
  Dense(1, activation="exponential")
])

model.compile(optimizer="adam", loss="poisson")
```

Fitting that model

```
es = EarlyStopping(verbose=True)
hist = model.fit(X_train_reg, y_train, epochs=100, verbose=0,
    validation_split=0.2, callbacks=[es])
hist.history["val_loss"][-1]
```

```
Epoch 5: early stopping
```

```
0.7526668906211853
```

```
model.layers
```

```
[<Embedding name=embedding, built=True>, <Dense name=dense_6, built=True>]
```

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Keras’ Embedding Layer

```
model.layers[0].get_weights()[0]
```

```
array([[-0.12, -0.11],
       [ 0.03, -0.  ],
       [-0.02,  0.01],
       [-0.25, -0.14],
       [-0.28, -0.32],
       [-0.3 , -0.22],
       [-0.31, -0.28],
       [ 0.1 ,  0.07],
       [-0.61, -0.51],
       [-0.06, -0.12],
       [-0.17, -0.14],
       [-0.6 , -0.46],
       [-0.22, -0.27],
       [-0.59, -0.5 ],
       [-0.  ,  0.02],
       [ 0.07,  0.06],
       [-0.31, -0.28],
       [-0.4 , -0.34],
       [-0.16, -0.15],
```

```
X_train["Region"].head(4)
```

```
0     R24
1     R93
2     R11
3     R42
Name: Region, dtype: object
```

```
X_sample = X_train_reg[:4].to_numpy()
X_sample
```

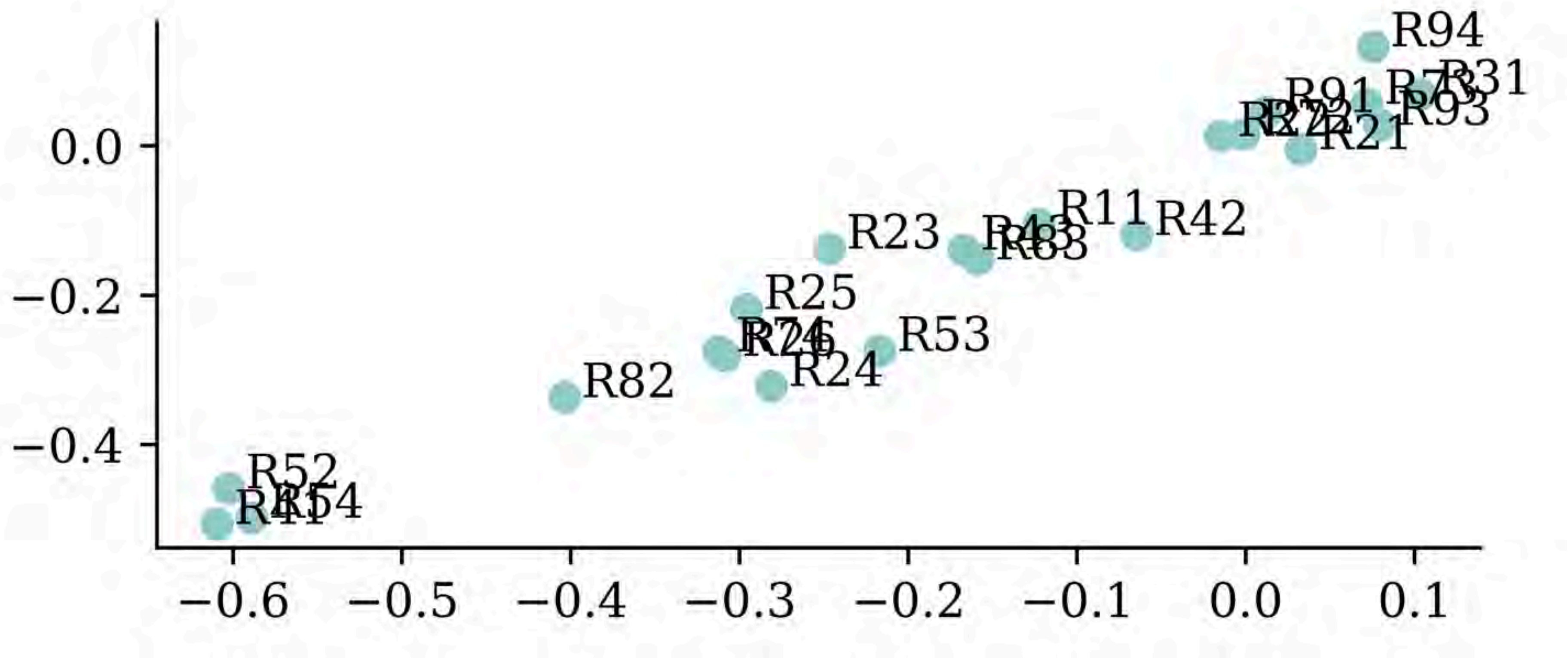
```
array([[ 4.],
       [20.],
       [ 0.],
       [ 9.]])
```

```
enc_tensor = model.layers[0](X_sample)
keras.ops.convert_to_numpy(enc_tensor).
```

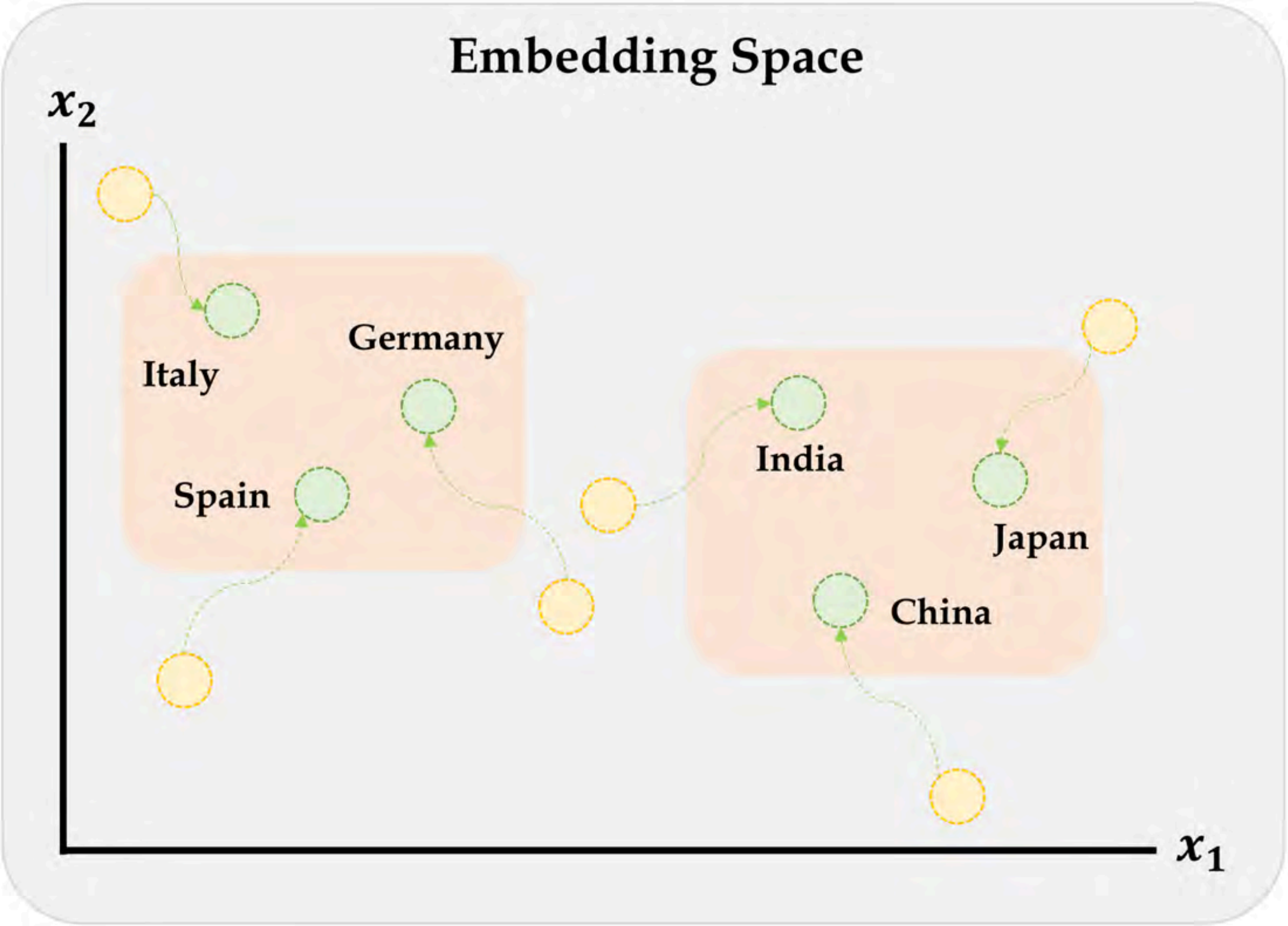
```
array([[-0.28, -0.32],
       [ 0.08,  0.03],
       [-0.12, -0.11],
       [-0.06, -0.12]], dtype=float32)
```

The learned embeddings

```python
points = model.layers[0].get_weights()[0]
plt.scatter(points[:,0], points[:,1])
for i in range(num_regions):
  plt.text(points[i,0]+0.01, points[i,1] , s=oe.categories_[0][i])
```



Entity embeddings



Embeddings will gradually improve during training.

Source: Marcus Lautier (2022).

Embeddings & other inputs

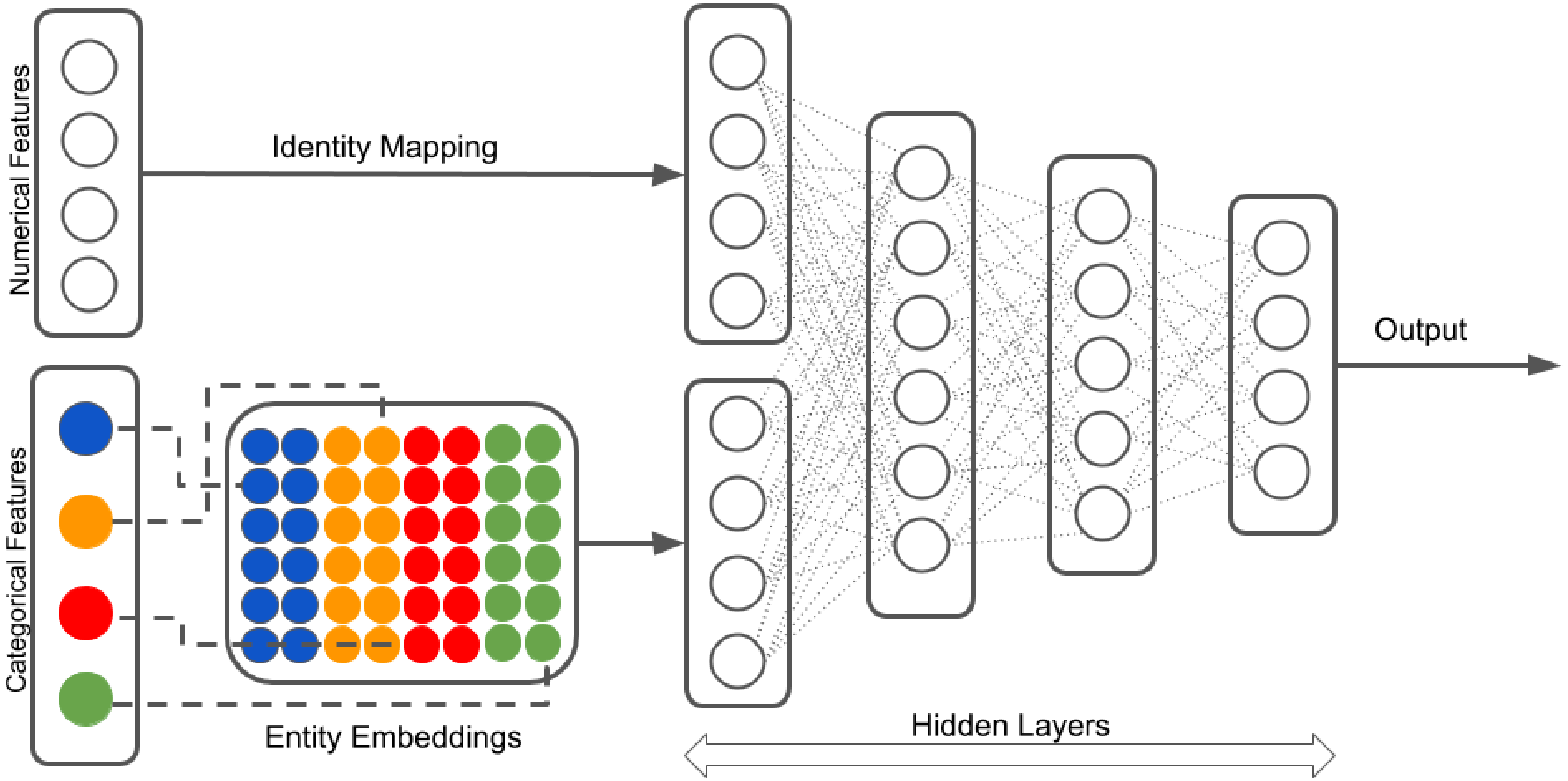


Illustration of a neural network with both continuous and categorical inputs.

We can’t do this with Sequential models…

Source: LotusLabs Blog, Accurate insurance claims prediction with Deep Learning.

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Converting Sequential models

```python
from keras.models import Model
from keras.layers import Input
```

```python
random.seed(12)

model = Sequential([
  Dense(30, "leaky_relu"),
  Dense(1, "exponential")
])

model.compile(
  optimizer="adam",
  loss="poisson")

hist = model.fit(
  X_train_oh, y_train,
  epochs=1, verbose=0,
  validation_split=0.2)
hist.history["val_loss"][-1]
```

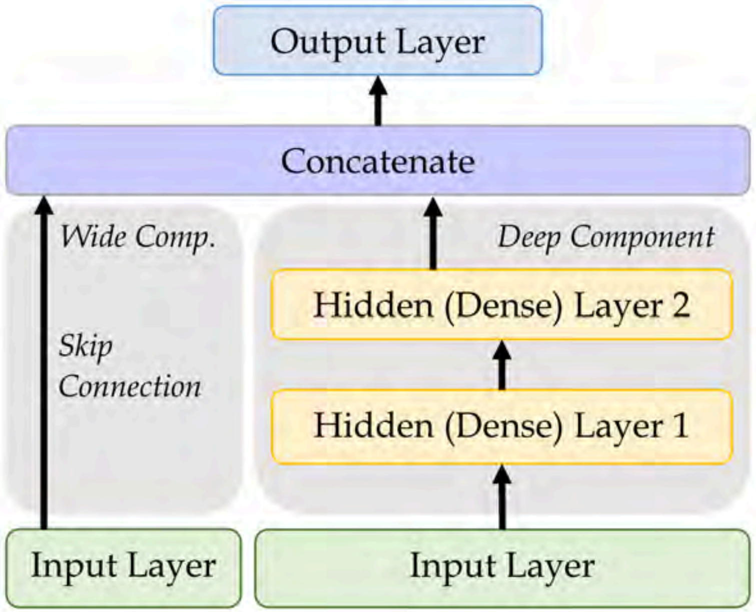
```
0.7535399198532104
```

```python
random.seed(12)

inputs = Input(shape=(X_train_oh.shape[
x = Dense(30, "leaky_relu")(inputs)
out = Dense(1, "exponential")(x)
model = Model(inputs, out)

model.compile(
  optimizer="adam",
  loss="poisson")

hist = model.fit(
  X_train_oh, y_train,
  epochs=1, verbose=0,
  validation_split=0.2)
hist.history["val_loss"][-1]
```

```
0.7535399198532104
```

See one-length tuples.

Wide & Deep network



An illustration of the wide & deep network architecture.

Add a *skip connection* from input to output layers.

```
from keras.layers \
    import Concatenate

inp = Input(shape=X_train.shape[1:])
hidden1 = Dense(30, "leaky_relu")(inp)
hidden2 = Dense(30, "leaky_relu")(hidden1)
concat = Concatenate()(
  [inp, hidden2])
output = Dense(1)(concat)
model = Model(
    inputs=[inp],
    outputs=[output])
```

Naming the layers

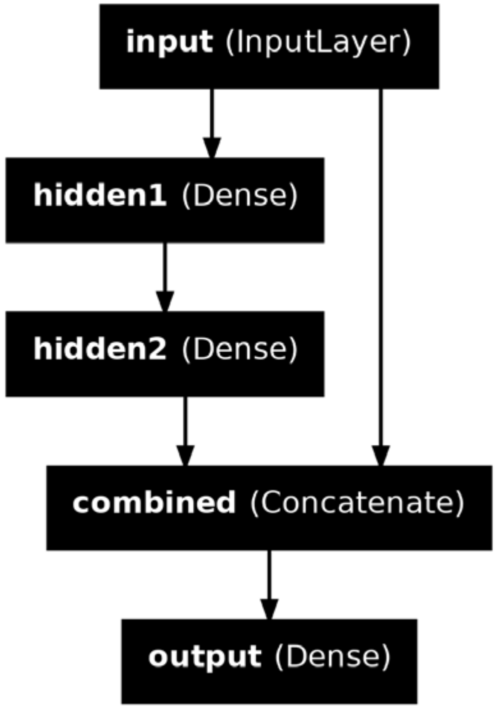
For complex networks, it is often useful to give meaningul names to the layers.

```
input_ = Input(shape=X_train.shape[1:], name="input")
hidden1 = Dense(30, activation="leaky_relu", name="hidden1")(input_)
hidden2 = Dense(30, activation="leaky_relu", name="hidden2")(hidden1)
concat = Concatenate(name="combined")([input_, hidden2])
output = Dense(1, name="output")(concat)
model = Model(inputs=[input_], outputs=[output])
```

Inspecting a complex model

```python
from keras.utils import plot_model
```

```python
plot_model(model, sh
```




```python
model.summary(line_length=75)
```

Model: "functional_8"

Layer (type)	Output Shape	Param #	Connecte
input (InputLayer)	(None, 10)	0	-
hidden1 (Dense)	(None, 30)	330	input[0][
hidden2 (Dense)	(None, 30)	930	hidden1[
combined (Concatenate)	(None, 40)	0	input[0][hidden2[
output (Dense)	(None, 1)	41	combined[

Total params: 1,301 (5.08 KB)
Trainable params: 1,301 (5.08 KB)
Non-trainable params: 0 (0.00 B)

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The desired architecture

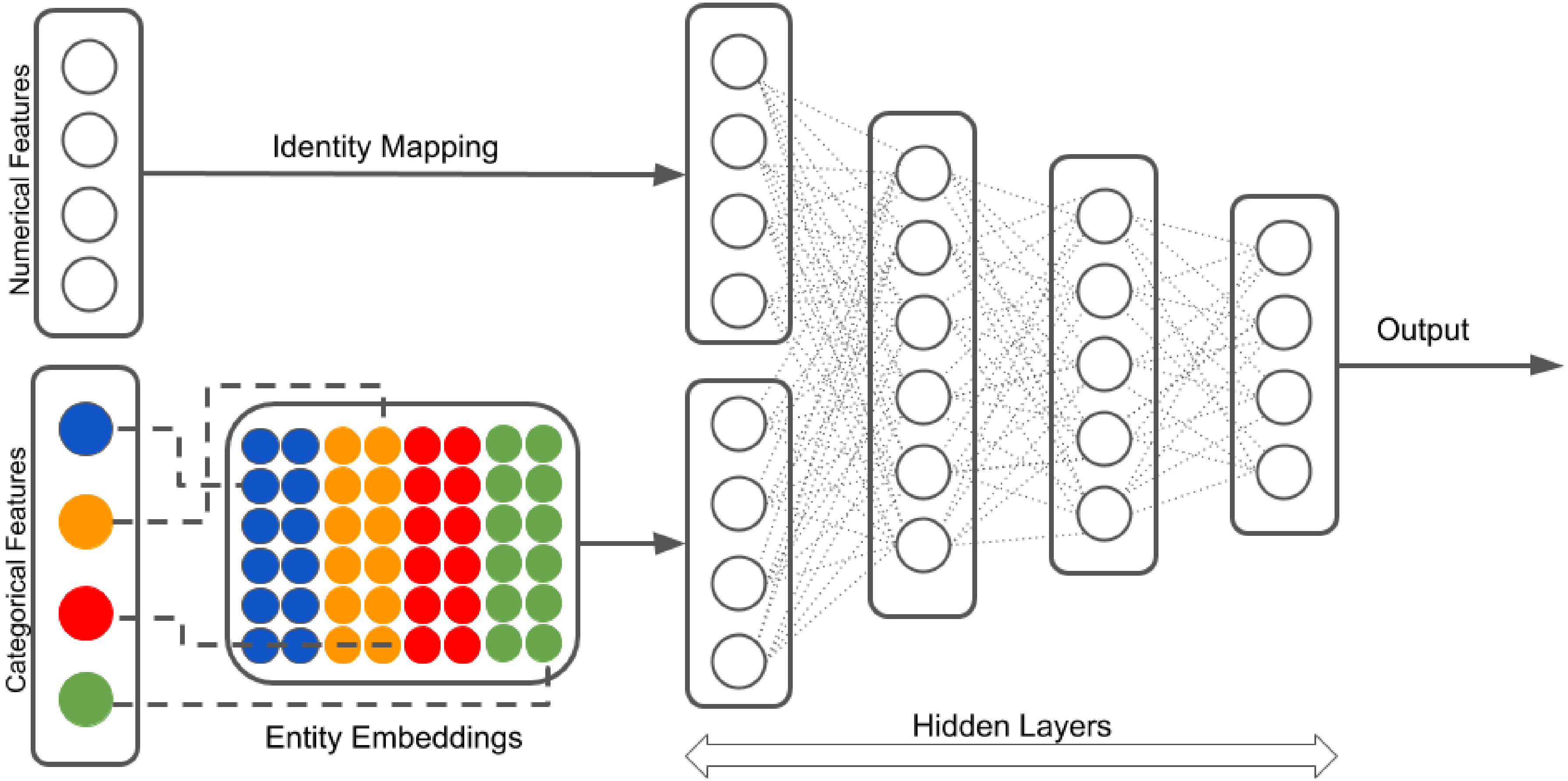


Illustration of a neural network with both continuous and categorical inputs.

Source: LotusLabs Blog, Accurate insurance claims prediction with Deep Learning.

Preprocess all French motor inputs

Transform the categorical variables to integers:

```
num_brands, num_regions = X_train.nunique()[["VehBrand", "Region"]]

ct = make_column_transformer(
  (OrdinalEncoder(), ["VehBrand", "Region", "Area", "VehGas"]),
  remainder=StandardScaler(),
  verbose_feature_names_out=False
)
X_train_ct = ct.fit_transform(X_train)
X_test_ct = ct.transform(X_test)
```

Split the brand and region data apart from the rest:

```
X_train_brand = X_train_ct["VehBrand"]; X_test_brand = X_test_ct["VehBrand"]
X_train_region = X_train_ct["Region"]; X_test_region = X_test_ct["Region"]
X_train_rest = X_train_ct.drop(["VehBrand", "Region"], axis=1)
X_test_rest = X_test_ct.drop(["VehBrand", "Region"], axis=1)
```

Organise the inputs

Make a Keras `Input` for: vehicle brand, region, & others.

```
veh_brand = Input(shape=(1,), name="vehBrand")
region = Input(shape=(1,), name="region")
other_inputs = Input(shape=X_train_rest.shape[1:], name="otherInputs")
```

Create embeddings and join them with the other inputs.

```
from keras.layers import Reshape

random.seed(1337)
veh_brand_ee = Embedding(input_dim=num_brands, output_dim=2,
    name="vehBrandEE")(veh_brand)
veh_brand_ee = Reshape(target_shape=(2,))(veh_brand_ee)

region_ee = Embedding(input_dim=num_regions, output_dim=2,
    name="regionEE")(region)
region_ee = Reshape(target_shape=(2,))(region_ee)

x = Concatenate(name="combined")([veh_brand_ee, region_ee, other_inputs])
```

Complete the model and fit it

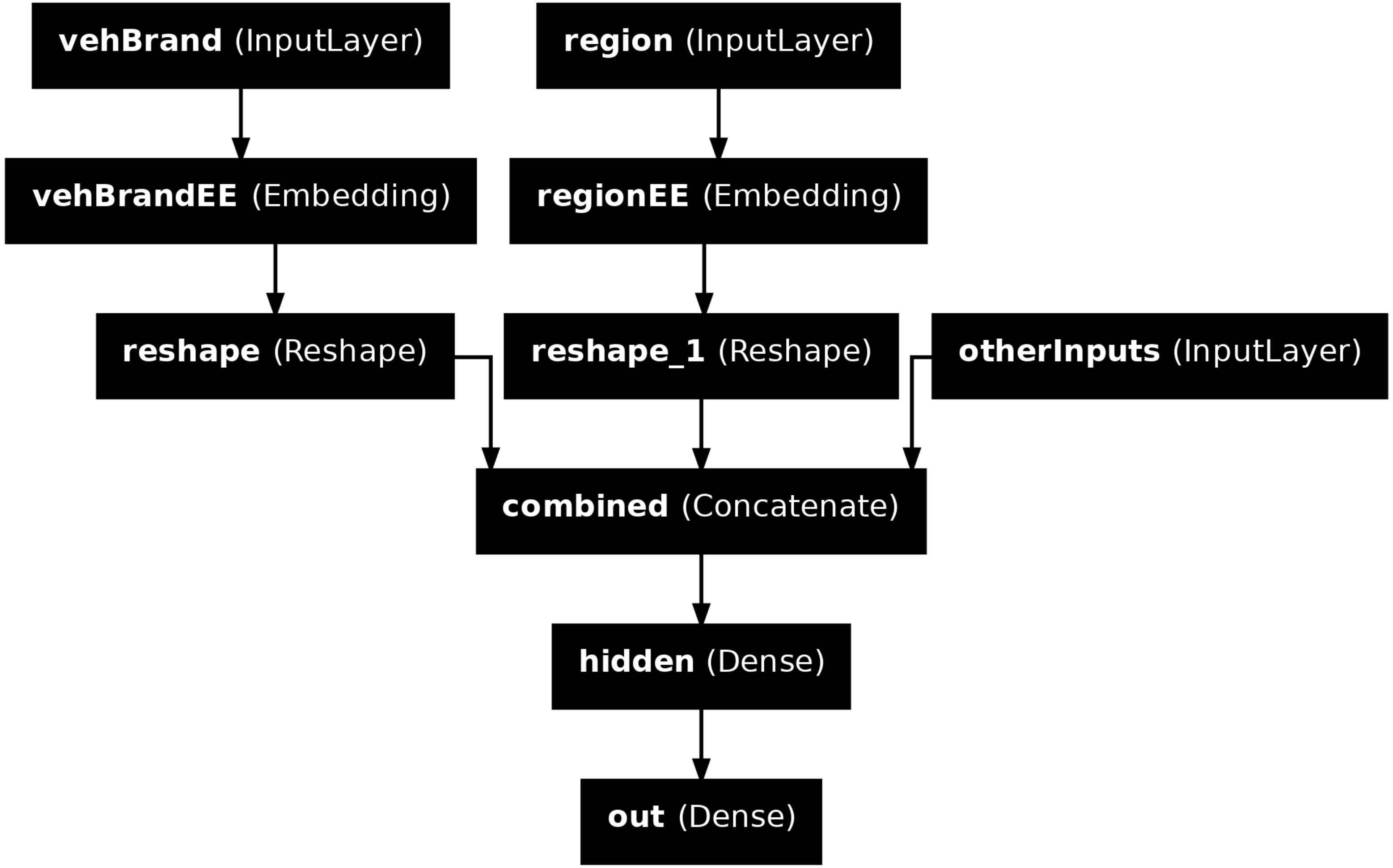
Feed the combined embeddings & continuous inputs to some normal dense layers.

```
x = Dense(30, "relu", name="hidden")(x)
out = Dense(1, "exponential", name="out")(x)

model = Model([veh_brand, region, other_inputs], out)
model.compile(optimizer="adam", loss="poisson")

hist = model.fit((X_train_brand, X_train_region, X_train_rest),
    y_train, epochs=100, verbose=0,
    callbacks=[EarlyStopping(patience=5)], validation_split=0.2)
np.min(hist.history["val_loss"])
```

```
0.6692155599594116
```

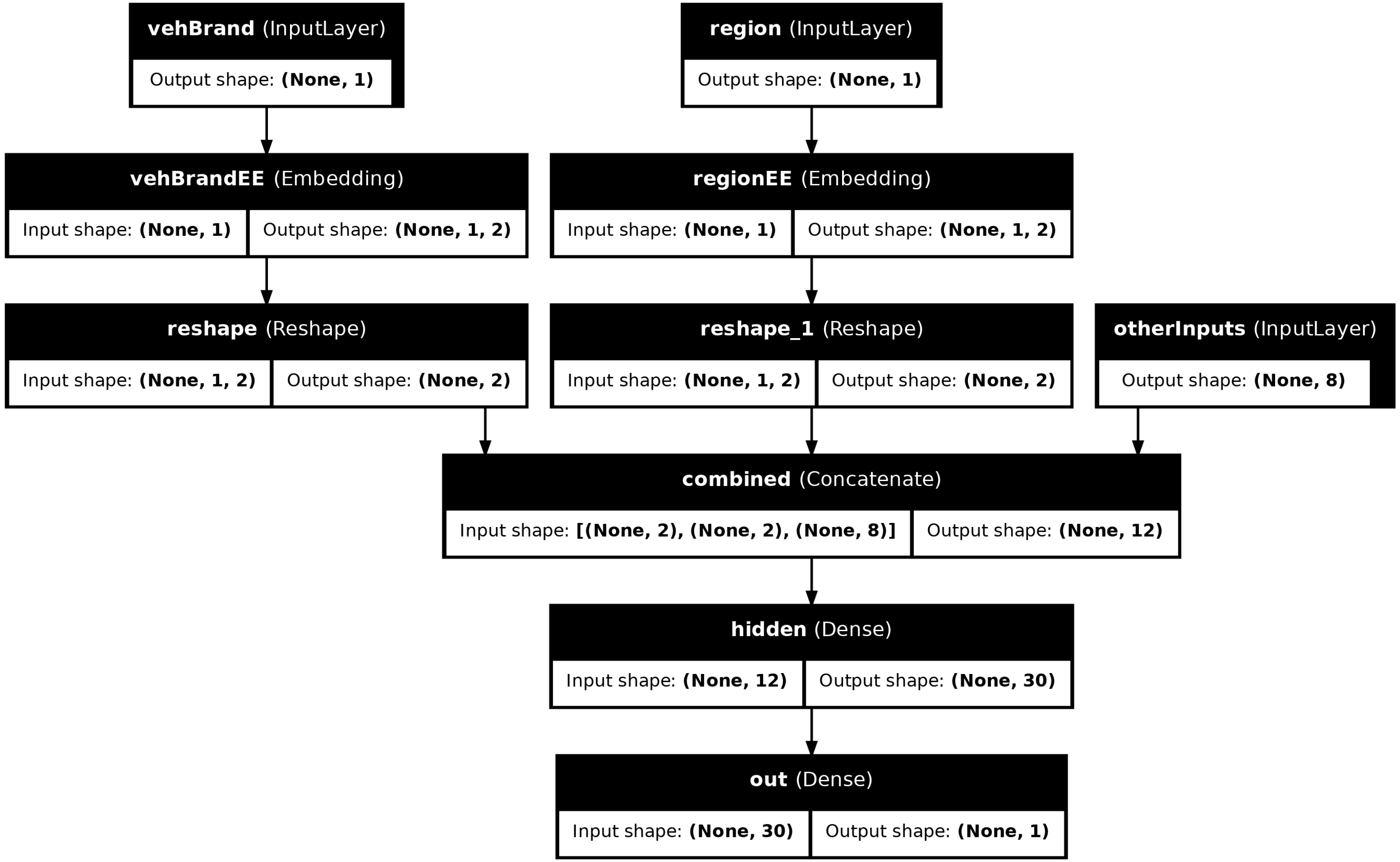
Plotting this model

```
plot_model(model, show_layer_names=True)
```



Why we need to reshape

```python
plot_model(model, show_layer_names=True, show_shapes=True)
```



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- Keras' Functional API
- French Motor Dataset with Embeddings
- **Scale By Exposure**

Two different models

Have $\{(\mathbf{x}_i, y_i)\}_{i=1,\dots,n}$ for $\mathbf{x}_i \in \mathbb{R}^{47}$ and $y_i \in \mathbb{N}_0$.

Model 1: Say $Y_i \sim \mathsf{Poisson}(\lambda(\mathbf{x}_i))$.

But, the exposures are different for each policy. $\lambda(\mathbf{x}_i)$ is the expected number of claims for the duration of policy i's contract.

Model 2: Say $Y_i \sim \mathsf{Poisson}(\text{Exposure}_i \times \lambda(\mathbf{x}_i))$.

Now, $\text{Exposure}_i \notin \mathbf{x}_i$, and $\lambda(\mathbf{x}_i)$ is the rate *per year*.

Just take continuous variables

```
ct = make_column_transformer(
  ("passthrough", ["Exposure"]),
  ("drop", ["VehBrand", "Region", "Area", "VehGas"]),
  remainder=StandardScaler(),
  verbose_feature_names_out=False
)
X_train_ct = ct.fit_transform(X_train)
X_test_ct = ct.transform(X_test)
```

Split exposure apart from the rest:

```
X_train_exp = X_train_ct["Exposure"]; X_test_exp = X_test_ct["Exposure"]
X_train_rest = X_train_ct.drop("Exposure", axis=1)
X_test_rest = X_test_ct.drop("Exposure", axis=1)
```

Organise the inputs:

```
exposure = Input(shape=(1,), name="exposure")
other_inputs = Input(shape=X_train_rest.shape[1:], name="otherInputs")
```

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Make & fit the model

Feed the continuous inputs to some normal dense layers.

```python
random.seed(1337)
x = Dense(30, "relu", name="hidden1")(other_inputs)
x = Dense(30, "relu", name="hidden2")(x)
lambda_ = Dense(1, "exponential", name="lambda")(x)
```

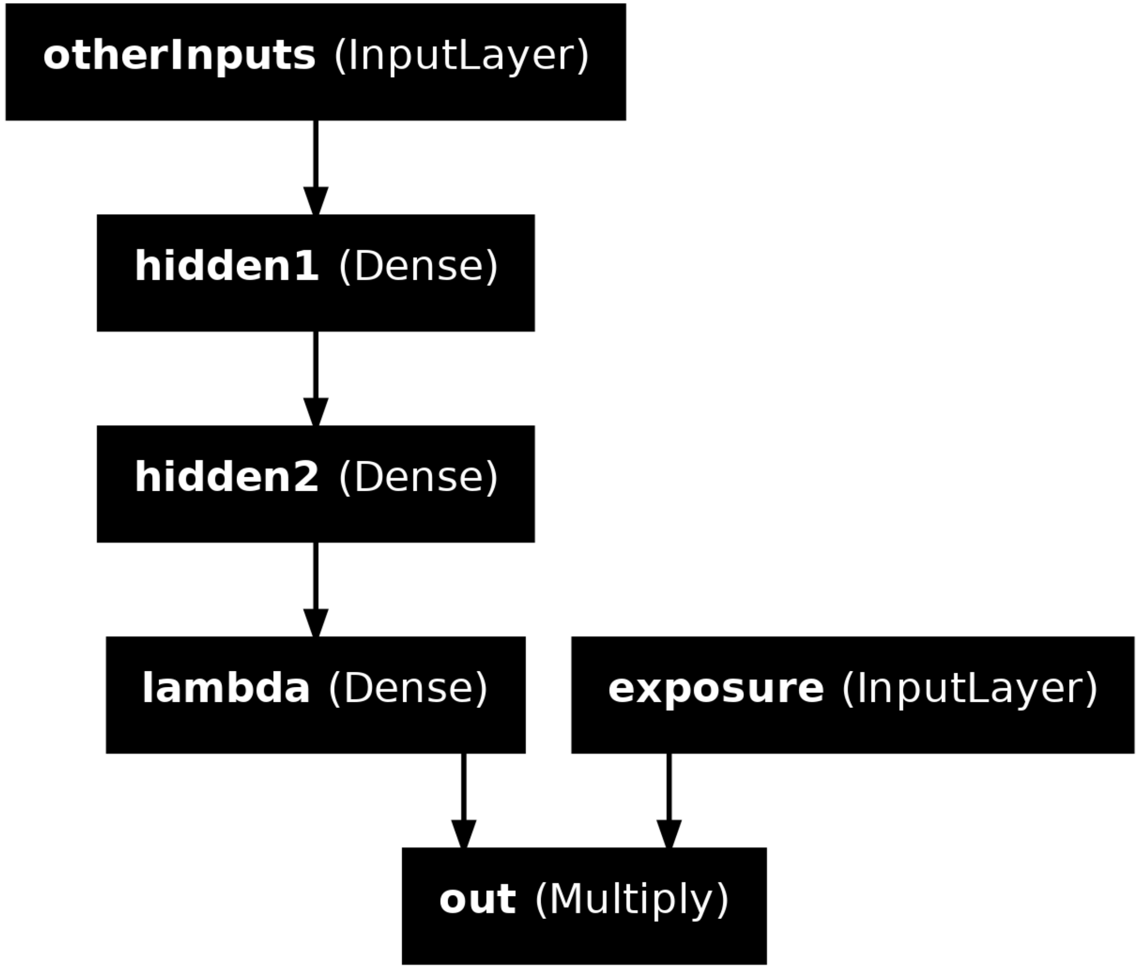
```python
from keras.layers import Multiply

out = Multiply(name="out")([lambda_, exposure])
model = Model([exposure, other_inputs], out)
model.compile(optimizer="adam", loss="poisson")

es = EarlyStopping(patience=10, restore_best_weights=True, verbose=1)
hist = model.fit((X_train_exp, X_train_rest),
    y_train, epochs=100, verbose=0,
    callbacks=[es], validation_split=0.2)
np.min(hist.history["val_loss"])
```

```
Epoch 40: early stopping
Restoring model weights from the end of the best epoch: 30.
```

```
0.8829042911529541
```

Plot the model

```
plot_model(model, show_layer_names=True)
```



Package Versions

```python
from watermark import watermark
print(watermark(python=True, packages="keras,matplotlib,numpy,pandas,seaborn,scipy,torch
```

```
Python implementation: CPython
Python version       : 3.11.9
IPython version      : 8.24.0

keras     : 3.3.3
matplotlib: 3.9.0
numpy     : 1.26.4
pandas    : 2.2.2
seaborn   : 0.13.2
scipy     : 1.11.0
torch     : 2.3.1
tensorflow: 2.16.1
tf_keras  : 2.16.0
```

Glossary

- entity embeddings
- Input layer
- Keras functional API
- Reshape layer
- skip connection
- wide & deep network structure