Chapter 2: Matrix Algebra

1. Let $A=\begin{pmatrix}1 & 4\\ 2 & 3\end{pmatrix}, B=\begin{pmatrix}-1 & 1\\ 3 & 2\end{pmatrix}$ and $C=\begin{pmatrix}1 & 3 & -4\\ -1 & 2 & 1\end{pmatrix}$. Compute the matrix

a. $2A-B^T$ b. AB c. BA d. AC

e. CC^T f. C^TC g. A^3 h. B^2A^T

2. Suppose that A and B are nxn matrices. Simplify the expression

a. $(A+B)^2-(A-B)^2$ b. $A(BC-CD)+A(C-B)D-AB(C-D)$

3. Let $A=\begin{pmatrix}3 & 1 & 2\\ 4 & 8 & 0\\ 0 & 1 & 2\end{pmatrix}$ and $B=\begin{pmatrix}0 & 5 & 2 & 1\\ 1 & 8 & 0 & -6\\ 1 & 4 & 3 & 7\end{pmatrix}$.

a. Compute AB

b. Compute $f(A)$ if $f(x)=x^2-3x+2$

4. Find the inverse of each of the following matrices.

a. $\begin{pmatrix}1 & 5\\ 2 & -1\end{pmatrix}$ b. $\begin{pmatrix}2 & 1\\ 2 & -4\end{pmatrix}$ c. $\begin{pmatrix}1 & -1 & 2\\ -5 & 7 & -11\\ -2 & 3 & -5\end{pmatrix}$ d. $\begin{pmatrix}1 & -1 & 3\\ 2 & 0 & 5\\ -1 & 1 & 0\end{pmatrix}$

5. Given $A^{-1}=\begin{pmatrix}1 & -1 & 3\\ 2 & 0 & 5\\ -1 & 1 & 0\end{pmatrix}$. Find a matrix X such that

a. $AX=\begin{pmatrix}1\\ -1\\ 3\end{pmatrix}$ b. $AX=\begin{pmatrix}1 & -1 & 2\\ 0 & 1 & 1\\ 1 & 0 & 0\end{pmatrix}$ c. $XA=\begin{pmatrix}1 & 2 & -1\\ 3 & 1 & 1\end{pmatrix}$

6. Find A when

a. $(3A)^{-1} = \begin{pmatrix} 1 & 2 \\ 0 & -2 \end{pmatrix}$ b. $(I+2A)^{-1} = \begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix}$ c. $\left(A^{-1} - 2I\right)^{T} = -2\begin{pmatrix} 1 & 4 \\ 3 & 11 \end{pmatrix}$

7. Write the system of linear equations in matrix form and then solve them.

a. $\begin{cases} 2x - y = 4 \\ 3x + 2y = -4 \end{cases}$ b. $\begin{cases} 2x + 3y + z = 10 \\ 2x - 3y - 3z = 22 \\ 4x - 2y + 3z = -2 \end{cases}$ c. $\begin{cases} x + y = a \\ 2x + 3y = 1 - 2a \end{cases} (a \in R)$

8. Find A^{-1} if

a. $A^2 - 6A + 5I = 0$ b. $A^2 + 3A - I = 0$ c. $A^4 = I$

9. Solve for X

a. $\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix} X = \begin{pmatrix} 1 & -1 \\ 3 & 3 \end{pmatrix}$ b. $ABXC = B^T$ c. $AX^T BC = B$

(where A, B and C are nxn invertible matrices)

10. Compute $\begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}^{101}$

11. Let $T : R^2 \to R^2$ be a linear transformation, and assume that $T(1,2) = (-1,1)$ and $T(0,3) = (-3,3)$

a. Compute $T(11,-5)$ b. Compute $T(1,11)$

c. Find the matrix of T d. Compute $T^{-1}(2,3)$

12. Let $T : R^2 \to R^2$ be a linear transformation such that the matrix of T is $\begin{pmatrix} 1 & 2 \\ -1 & 3 \end{pmatrix}$.

Find $T(3,-2)$

13. The (2;1)-entry of the product $\begin{pmatrix} 1 & 2 & 0 & 1 \\ 0 & 2 & 5 & 1 \\ 4 & -1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 4 & 2 & 1 \\ 2 & 3 & 2 \\ 5 & 1 & 0 \\ 0 & 4 & 3 \end{pmatrix}$