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1	07:00	08:30	Mathematics for Engineering	SE1640	209	FUHCM	Closing		
2	08:45	10:15	Mathematics for Engineering	AI1702	215	FUHCM	Closing		
3	10:30	12:00	Mathematics for Engineering	SE1703	214	FUHCM	Closing		
5	14:15	15:45	Mathematics for Engineering	AI1608	220	FUHCM	Closing		
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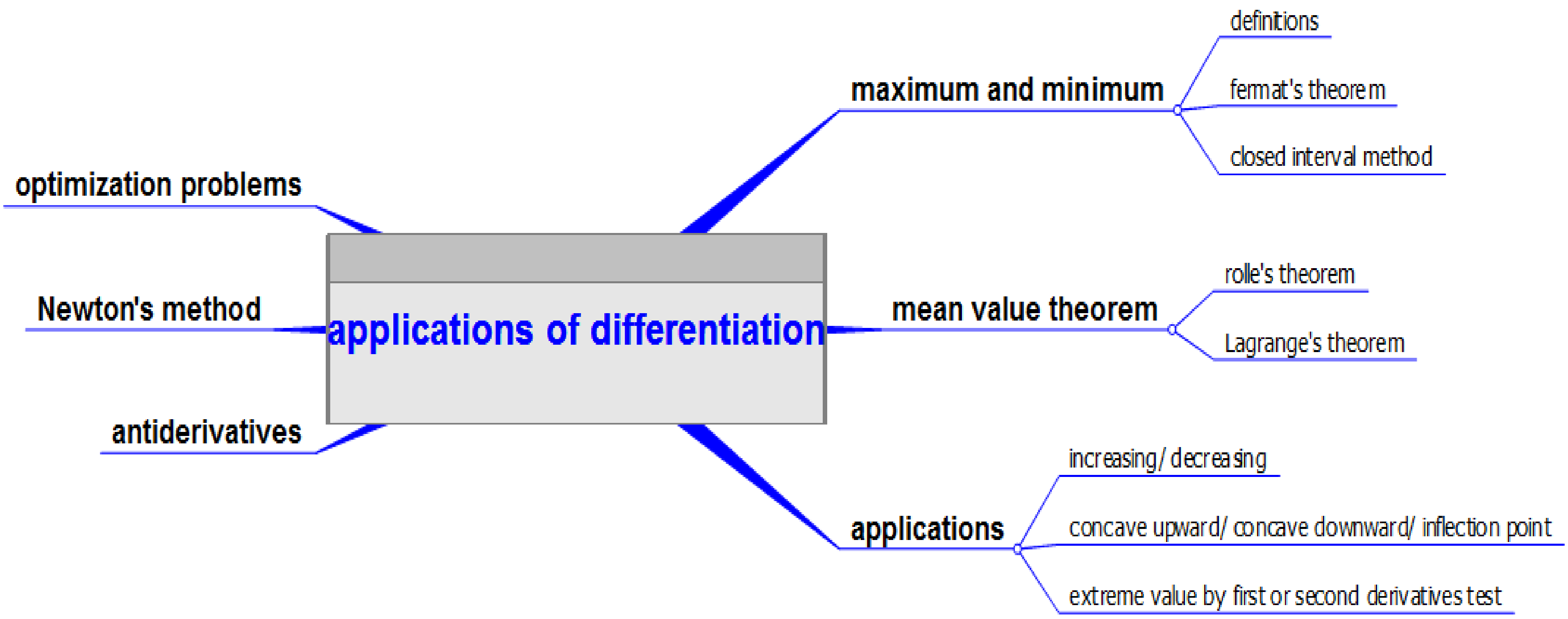

applications of differentiation
maximum and minimum
definitions
fermat's theorem
closed interval method
mean value theorem
rolle's theorem
Lagrange's theorem
applications
increasing/ decreasing
concave upward/ concave downward/ inflection point
extreme value by first or second derivatives test
optimization problems
Newton's method
antiderivatives

3.1

Maximum and Minimum Values

In this section, we will learn:

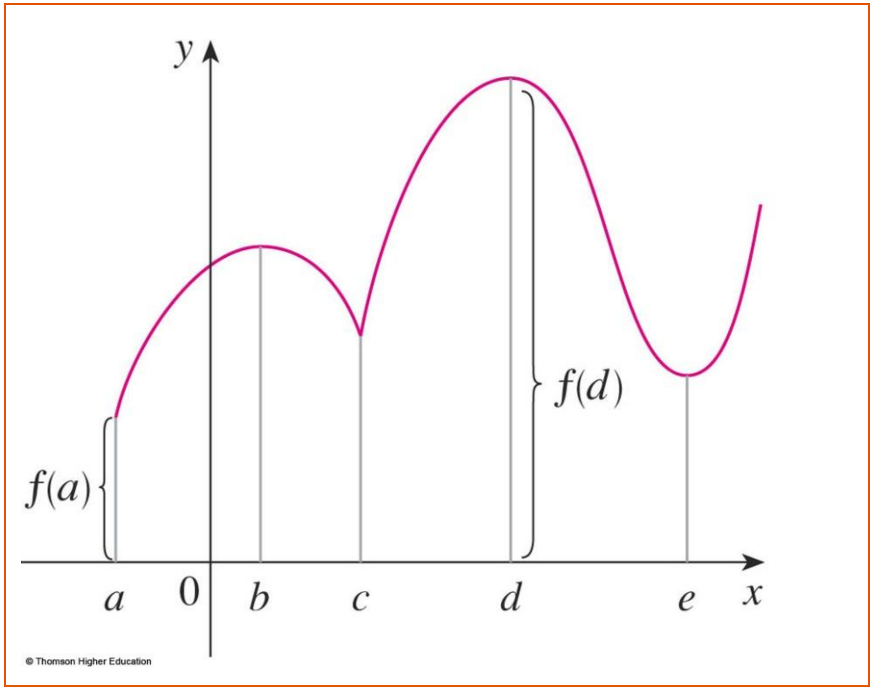
How to find the maximum

and minimum values of a function.

MAXIMUM & MINIMUM VALUES **Definition 1**

- A function f has an **absolute maximum** (or global maximum) at c if $\boldsymbol{f(c) \geq f(x)}$ **for all $\boldsymbol{x}$ in $\boldsymbol{D}$**, where D is the domain of f.

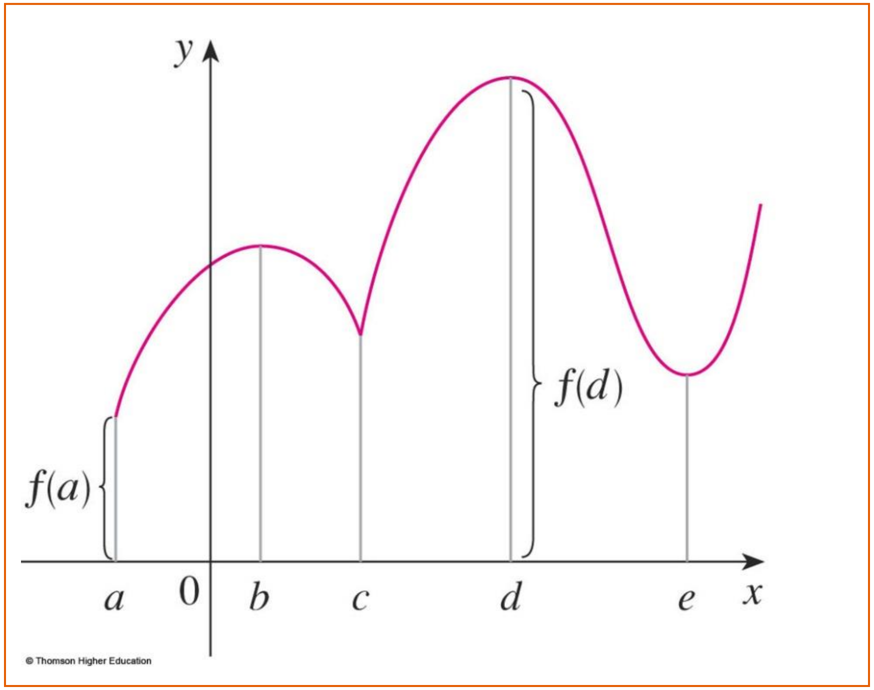
- The number $f(c)$ is called the **maximum value of $\boldsymbol{f}$ on $\boldsymbol{D}$**.



MAXIMUM & MINIMUM VALUES **Definition 1**

- Similarly, f has an **absolute minimum** at c if $f(c) \leq f(x)$ for all x in D and the number $f(c)$ is called the minimum value of f on D.

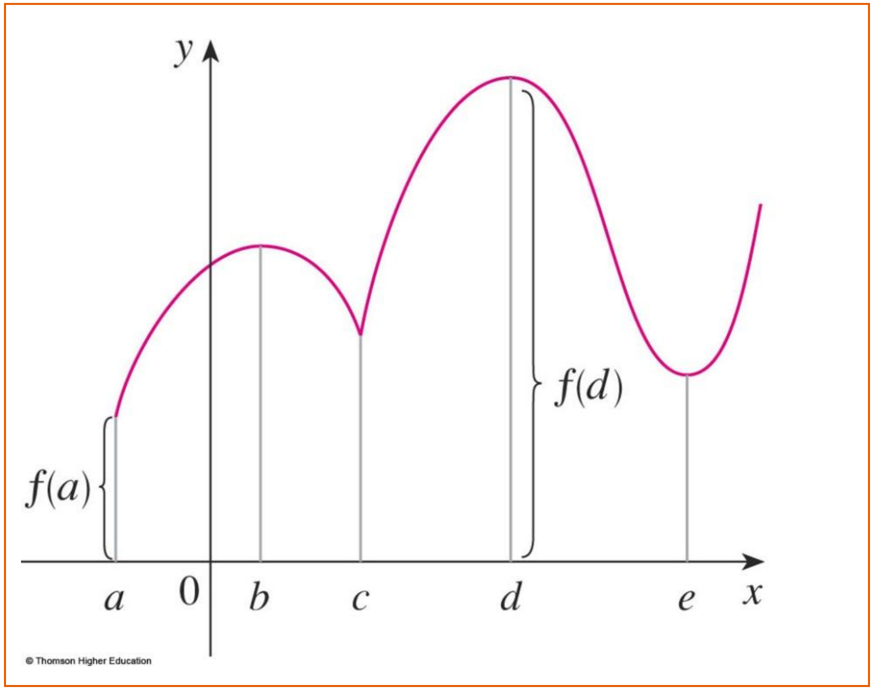
- The maximum and minimum values of f are called the **extreme values** of f.



MAXIMUM & MINIMUM VALUES **Definition 2**

– A function f has a **local maximum** (or relative maximum) at c if $f(c) \geq f(x)$ when ***x* is near *c***.

– Similarly, f has a **local minimum** at c if $f(c) \leq f(x)$ when x is near c.

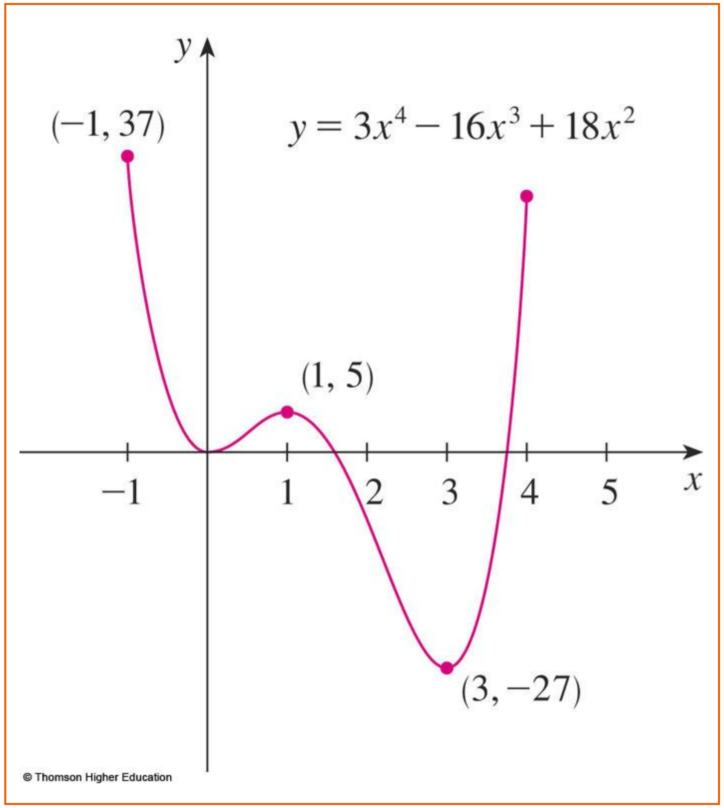


MAXIMUM & MINIMUM VALUES Example 4

- The graph of the function
- $$f(x) = 3x^4 - 16x^3 + 18x^2, \ -1 \leq x \leq 4$$
- is shown here.

- $f(1) = 5$ is a local maximum
- the absolute maximum is $f(-1) = 37$
- $f(0) = 0$ is a local minimum and
- $f(3) = -27$ is both a local

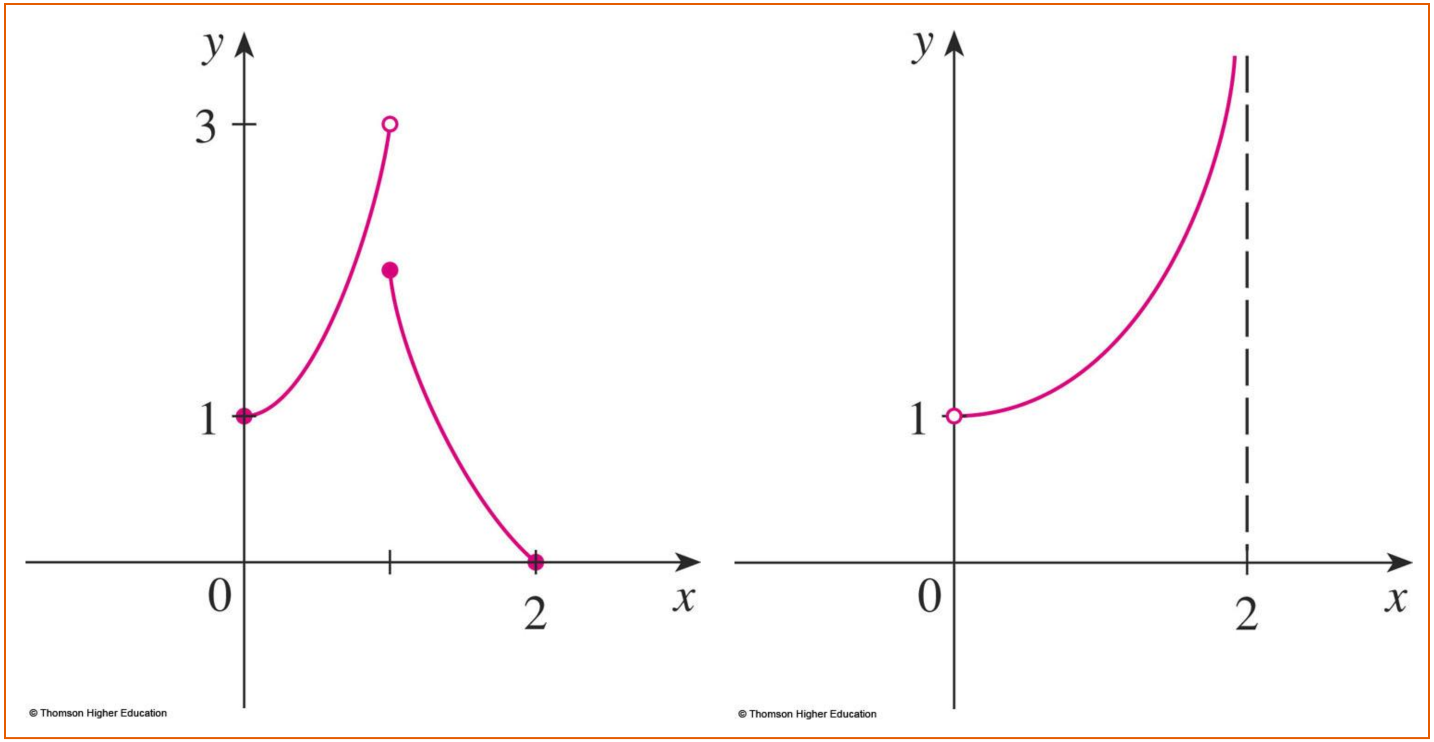
  and an absolute minimum.



EXTREME VALUE THEOREM

In the first figure, why isn't 3 the absolute maximum value?

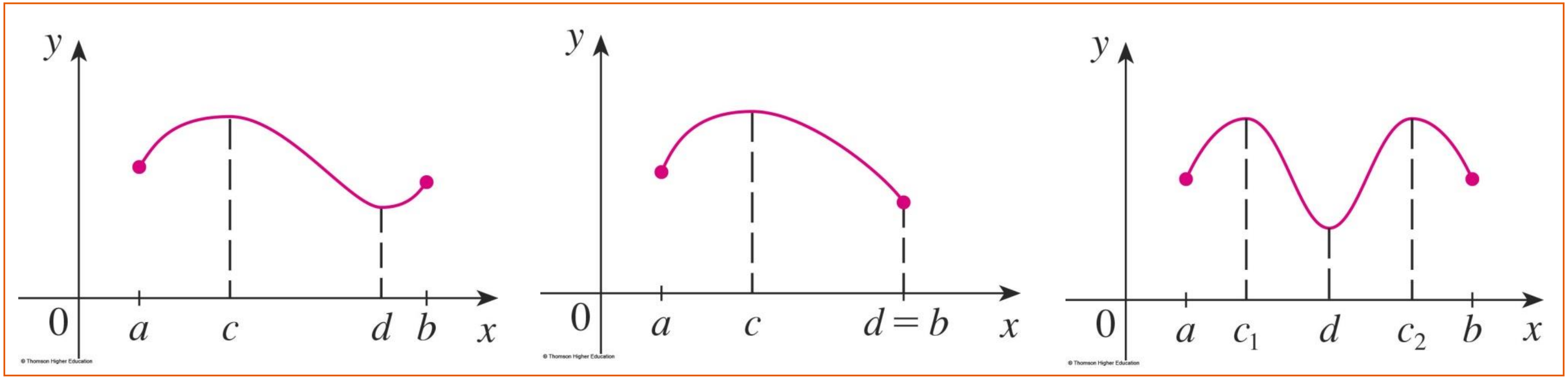
In the second, does it have the absolute maximum and minimum value?



EXTREME VALUE THEOREM

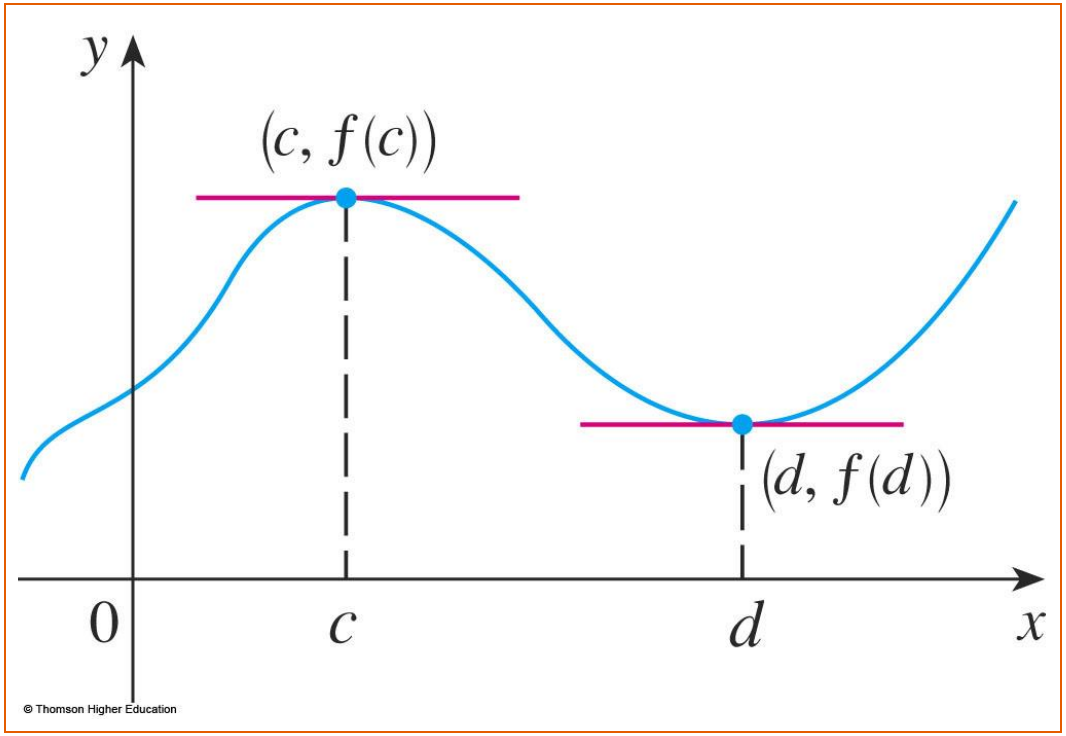
Theorem 3

- If f is continuous on a closed interval $[a, b]$,
- then f attains an absolute maximum value $f(c)$
- and an absolute minimum value $f(d)$
- at some numbers c and d in $[a, b]$.



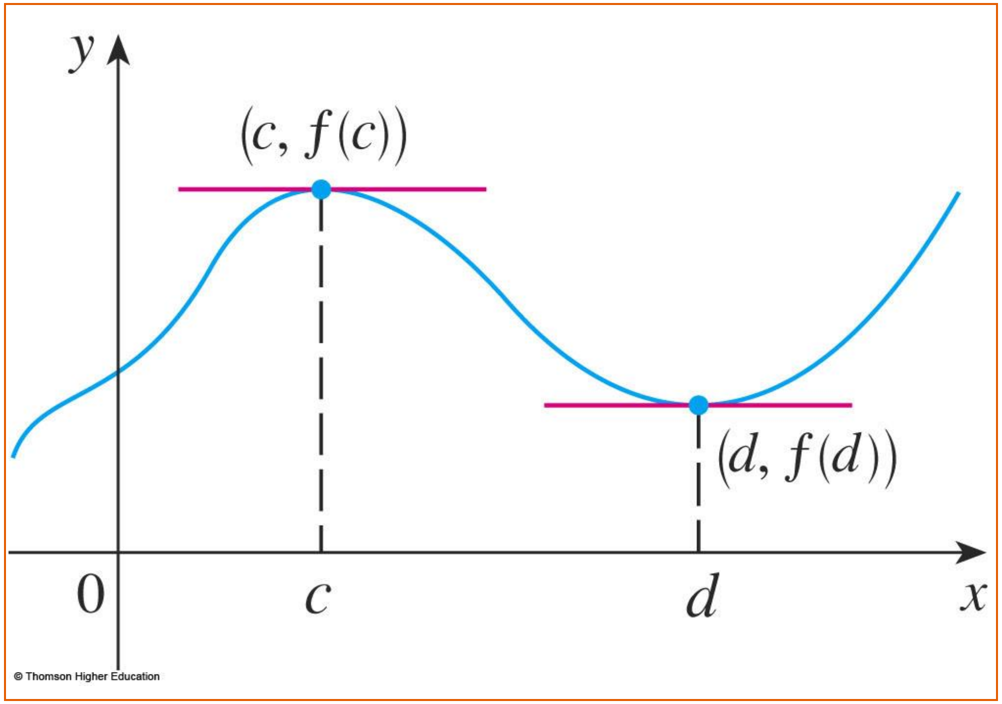
EXTREME VALUE THEOREM

- The theorem does not tell us how to find these extreme values.
  - We start by looking for local extreme values.



FERMAT'S THEOREM **Theorem 4**

- If f ***has*** a local maximum or minimum at c, and
- if $\boldsymbol{f\,'(c)}$ exists,
- then $f\,'(c) = 0$.



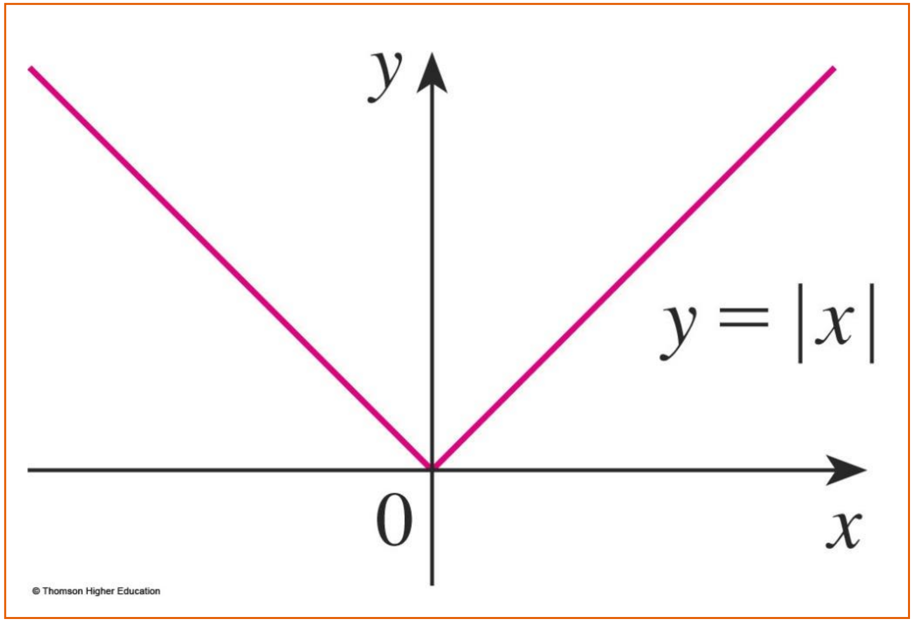
Is it true if say that

” f’(c)=0 if f has local extrem value at c?”

Answer: It is false, see next

CRITICAL NUMBERS **Example**

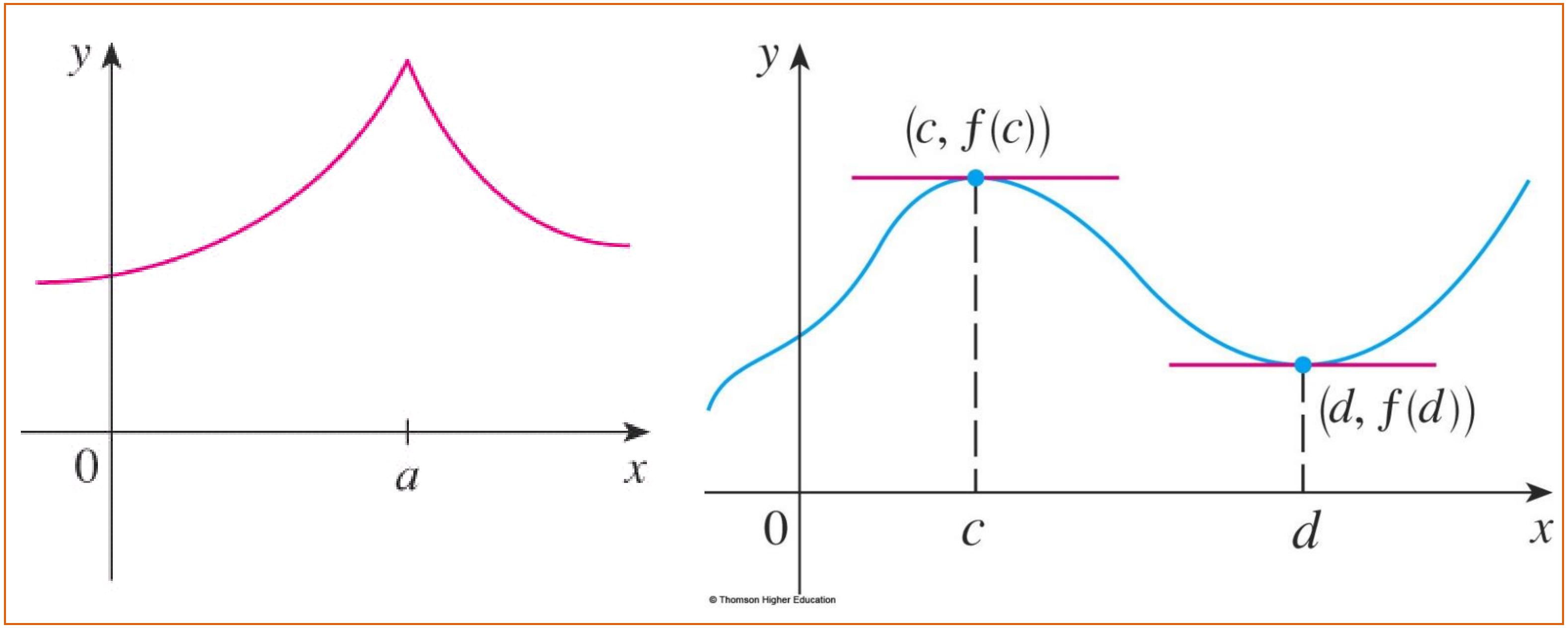
- The function $f(x) = |x|$ has its (local and absolute) minimum value at 0.
- $f'(0)$ does not exist.



CRITICAL NUMBERS

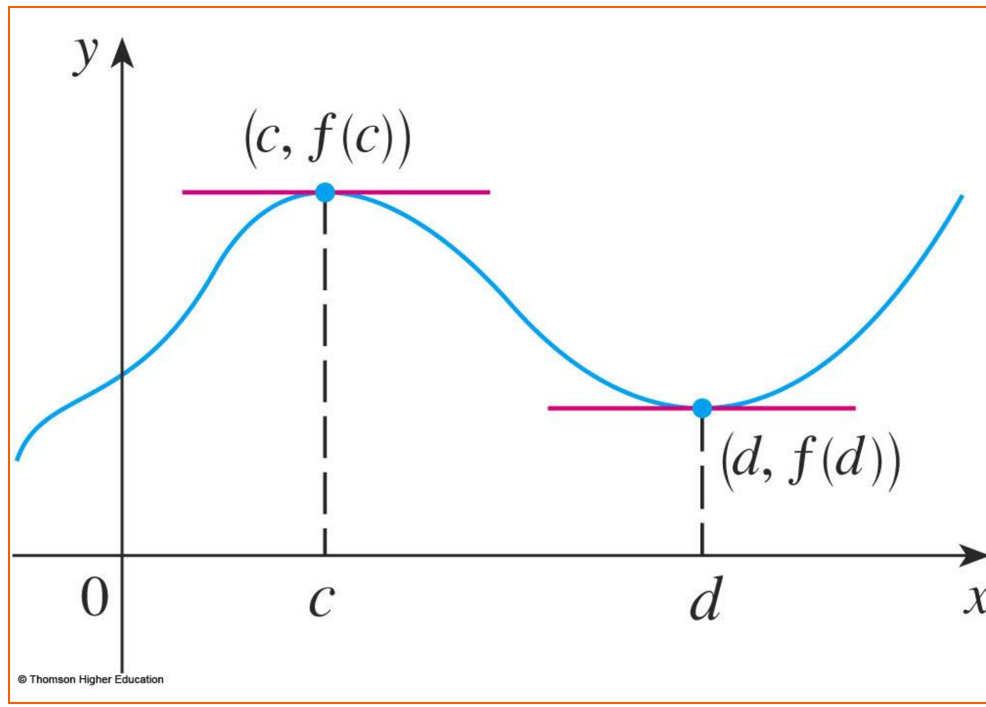
Definition 6

- A **critical number (điểm tới hạn)**
- of a function *f* is a number *c* in the domain of *f*
- such that either $f'(c) = 0$ or $f'(c)$ does not exist.



CRITICAL NUMBERS **Theorem 7**

- If f has a local maximum or minimum at c, then c is a critical number of f.



CLOSED INTERVAL METHOD

- To find the **absolute** maximum and minimum values of a continuous function f on a closed interval $[a, b]$:

1. Find the values of f at the critical numbers of f in (a, b).
2. Find the values of f at the endpoints of the interval.
3. The largest value from 1 and 2 is the absolute maximum value. The smallest is the absolute minimum value.

3.2

The Mean Value Theorem

In this section, we will learn about:

The significance of the mean value theorem.

ROLLE'S THEOREM

- Let f be a function that satisfies the following three hypotheses:

  1. f is continuous on the closed interval $[a, b]$

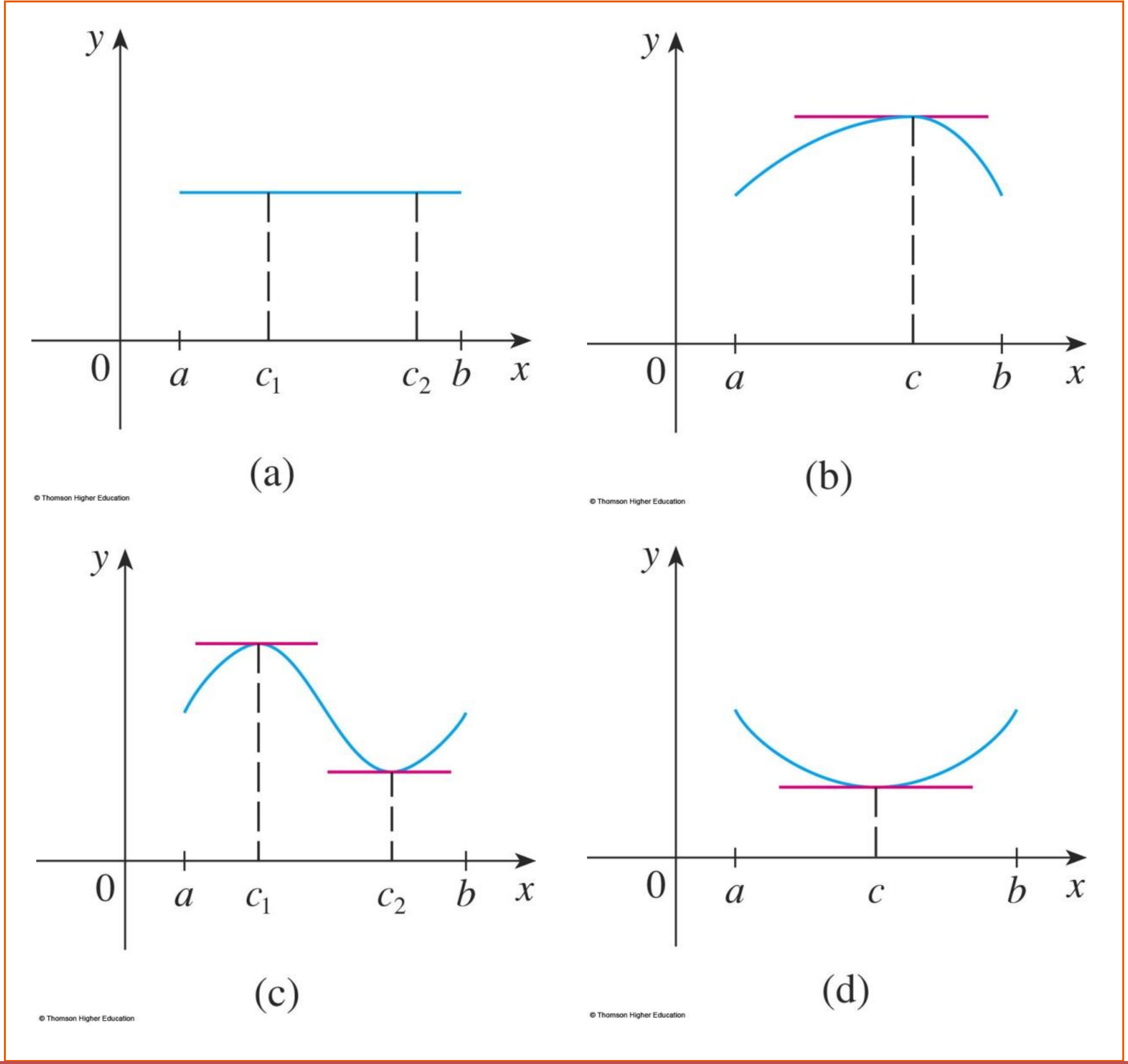
  2. f is differentiable on the open interval (a, b)

  3. $f(a) = f(b)$

- Then, there is a number c in (a, b) such that $f'(c) = 0$.

ROLLE'S THEOREM

- The figures show the graphs of four such functions.



Let $f(x)=x^3-2x^2+x-5$. Find the numbers c in the Rolle's theorem?

MEAN VALUE THEOREM

Equations 1 and 2

- Let *f* be a function that fulfills two hypotheses:

  1. *f* is continuous on the closed interval [*a*, *b*].

  2. *f* is differentiable on the open interval (*a*, *b*).

Then, there is a number *c* in (*a*, *b*) such that

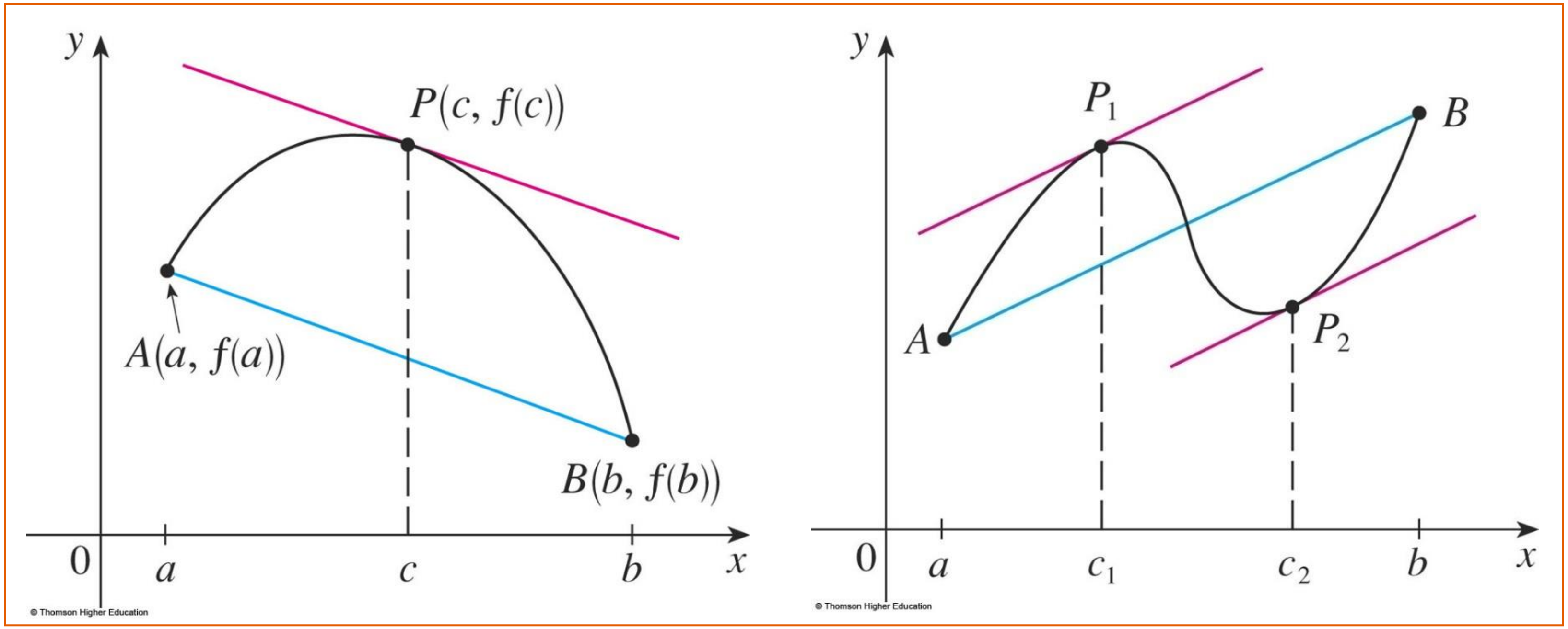
$$\boxed{f'(c)=\frac{f(b)-f(a)}{b-a}}$$

or, equivalently,

$$\boxed{f(b)-f(a)=f'(c)(b-a)}$$

MEAN VALUE THEOREM

- $f'(c)$ is the slope of the tangent line at $(c, f(c))$.
  - There is at least one point $P(c, f(c))$ on the graph where the slope of the tangent line is the same as the slope of the secant line AB.



Example 5

- Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x.
  - How large can $f(2)$ possibly be?

$\Rightarrow$ We are given that f is differentiable - and therefore continuous - everywhere.

$\Rightarrow$ In particular, we can apply the Mean Value Theorem on the interval [0, 2].

- There exists a number c such that

$$f(2) - f(0) = f'(c)(2 - 0)$$

- So, $f(2) = -3 + 2f'(c)$

Example 5

– We are given that $f'(x) \leq 5 \ \forall x$

$\Rightarrow f'(c) \leq 5.$

$\Rightarrow 2f'(c) \leq 10.$

$\Rightarrow f(2) = -3 + 2f'(c) \leq -3 + 10 = 7$

– The largest possible value for $f(2)$ is 7.

MEAN VALUE THEOREM **Theorem 5**

- If $f'(x) = 0$ for all x in an interval (a, b),
- then f is constant on (a, b).

MEAN VALUE THEOREM **Corollary 7**

- If $f'(x) = g'(x)$ for all x in an interval (a, b),
- then $f - g$ is constant on (a, b).
- That is, $f(x) = g(x) + c$ where c is a constant.

3.3

Derivatives and the Shapes of Graphs

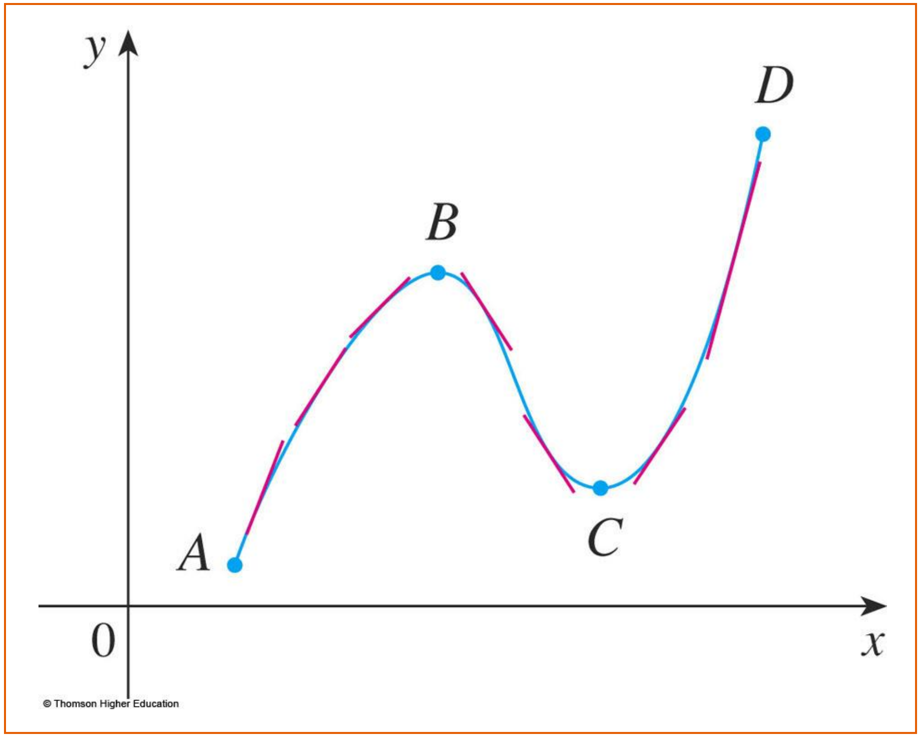
In this section, we will learn:

How the derivative of a function gives us the direction in which the curve proceeds at each point.

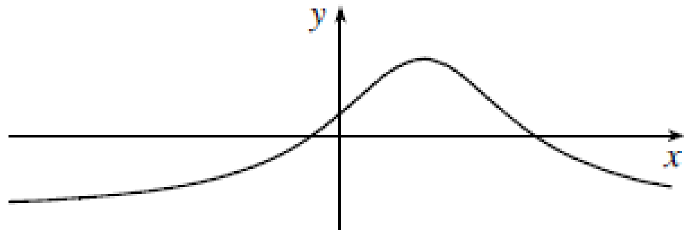
INCREASING/DECREASING TEST (I/D TEST)

a. If $f'(x) > 0$ on an interval, then f is increasing on that interval.

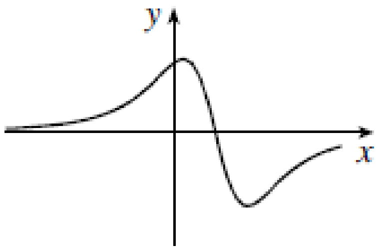
b. If $f'(x) < 0$ on an interval, then f is decreasing on that interval.



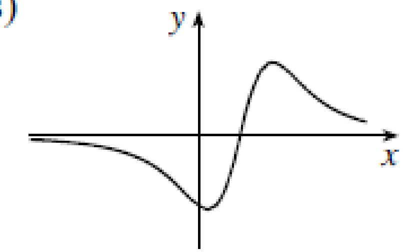
- **Drill Question:** The graph of f is shown below. Which of the following could be the graph of f'?



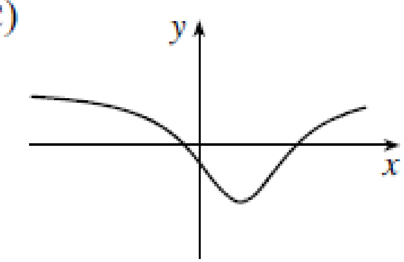
(A)



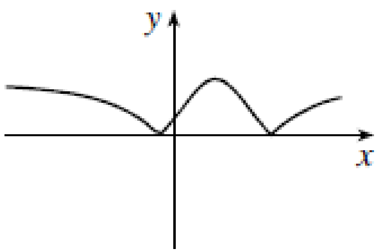
(B)



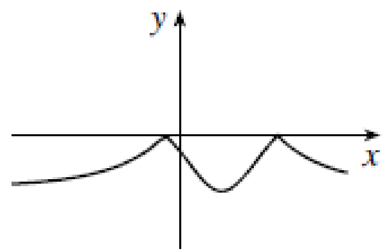
(C)



(D)



(E)

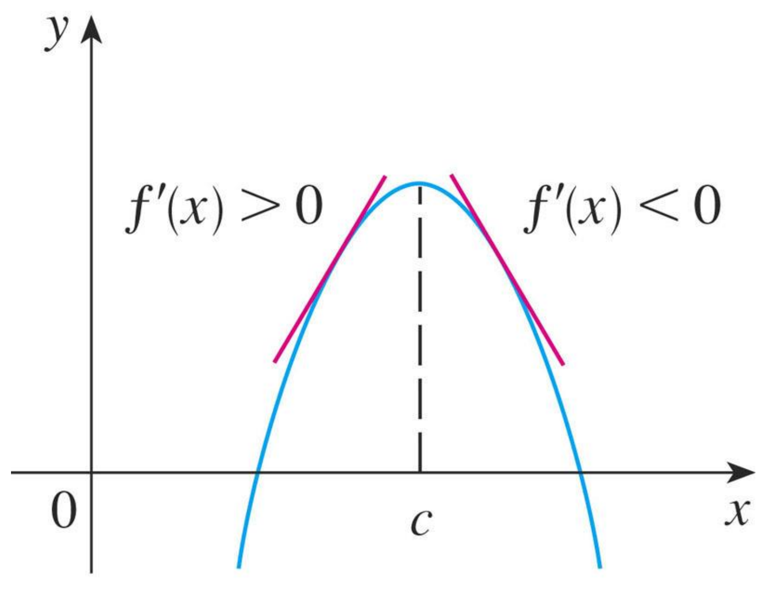


Answer: (A)

FIRST DERIVATIVE TEST

- Suppose that *c* is a critical number of a continuous function *f*.

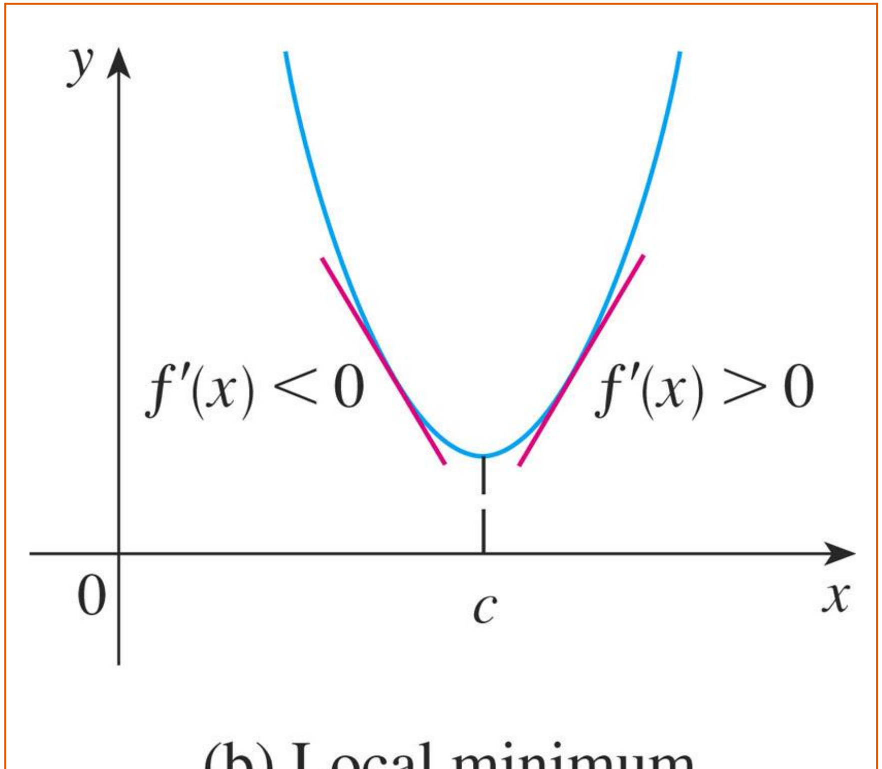
a. If *f'* changes from positive to negative at *c*, then *f* has a local maximum at *c*.



(a) Local maximum

FIRST DERIVATIVE TEST

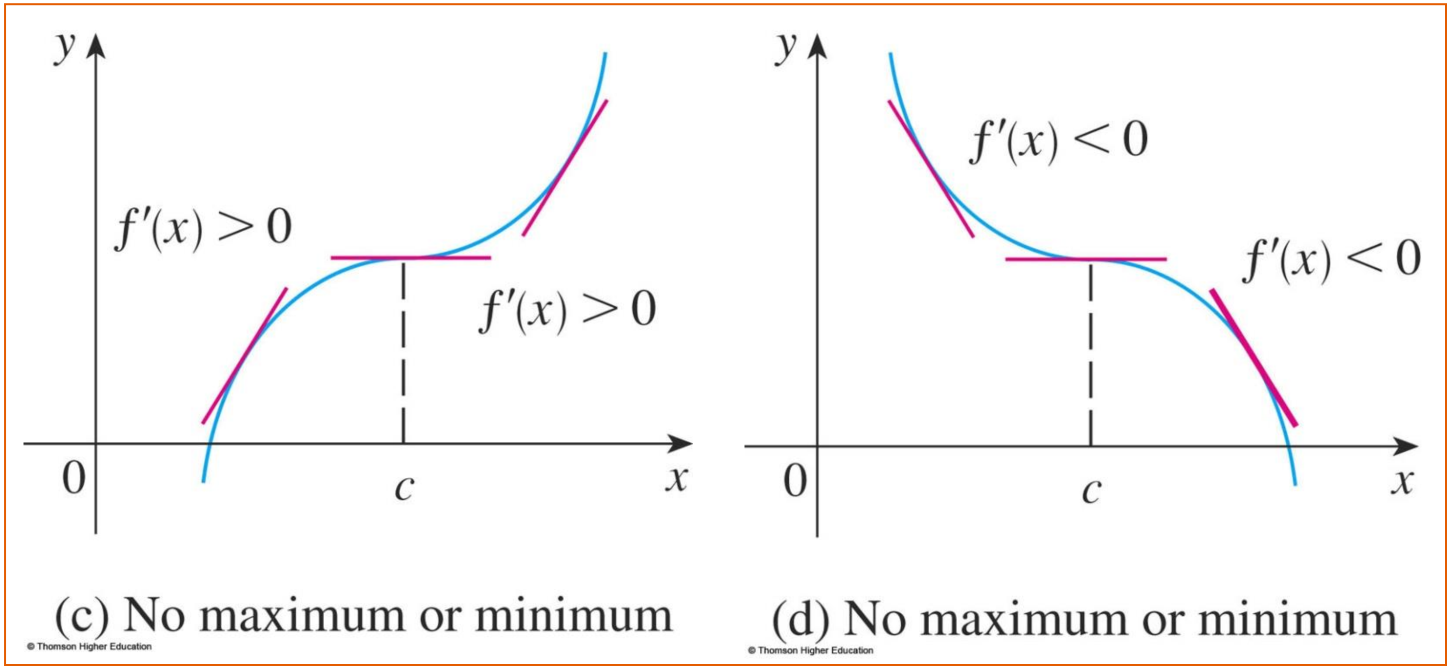
- b. If *f'* changes from negative to positive at *c*, then *f* has a local minimum at *c*.



(b) Local minimum

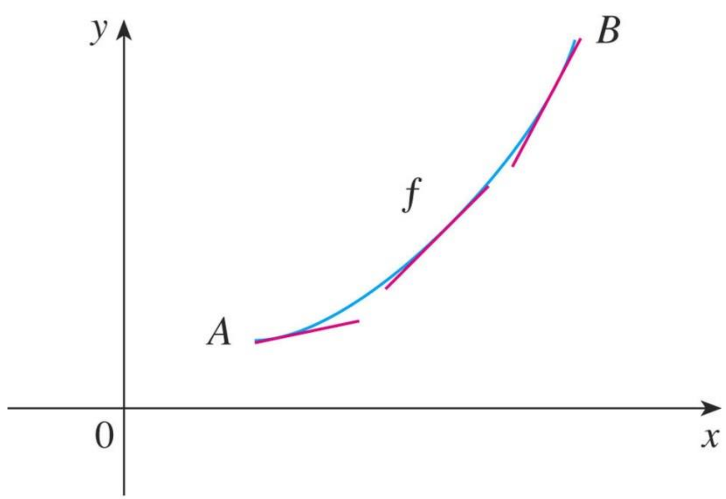
FIRST DERIVATIVE TEST

- c. If *f'* does not change sign at *c*
  then *f* has no local maximum or minimum at *c*.

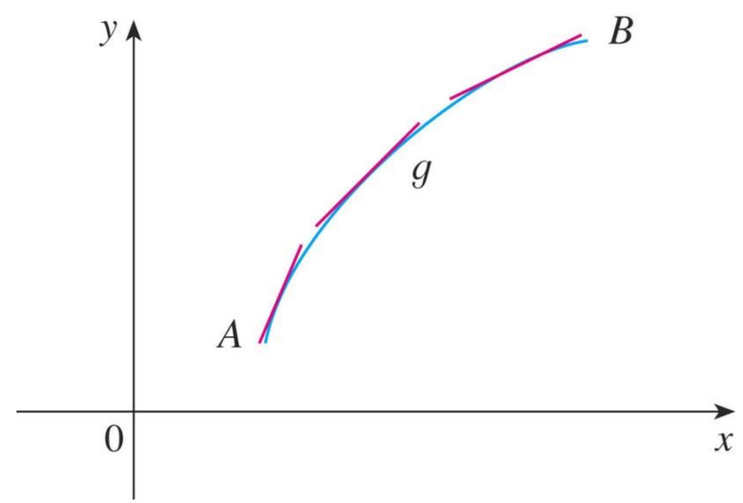


CONCAVE UPWARD/DOWNDWARD

- The curve lies above the tangents and *f* is called **concave upward** (lõm lên) on (*a*, *b*).
- The curve lies below the tangents and *g* is called **concave downward** (lõm xuống) on (*a*, *b*).



(a) Concave upward

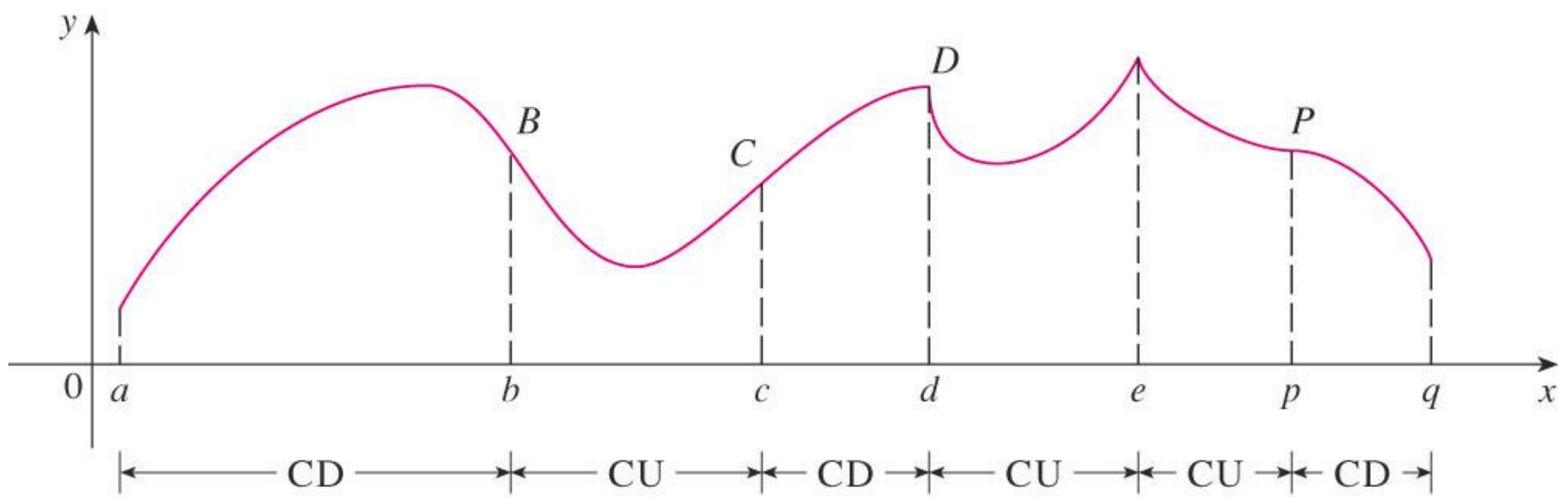


(b) Concave downward

CONCAVITY TEST

a. If $f''(x) > 0$ for all x in I, then the graph of f is concave upward on I.

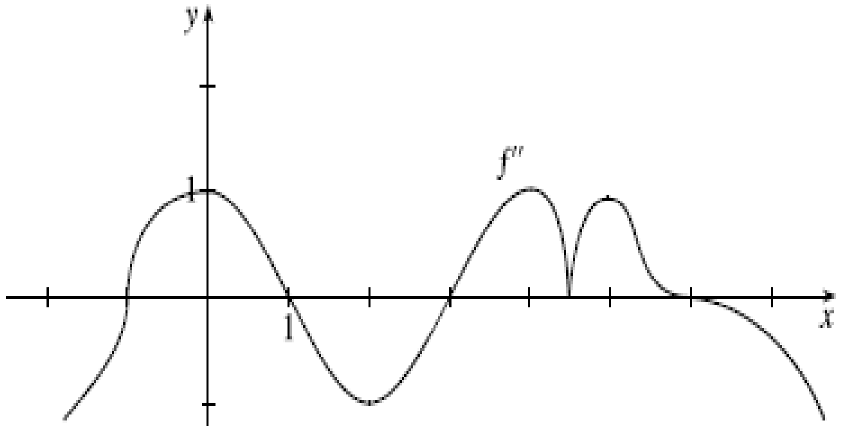
b. If $f''(x) < 0$ for all x in I, then the graph of f is concave downward on I.



INFLECTION POINT—DEFINITION

- A point P on a curve $y = f(x)$ is called an **inflection point (điểm uốn)**
- if f is continuous there and the curve changes
  from concave upward to concave downward
- (or from concave downward to concave upward at P).

- Given a graph of f'' as below, have the students indicate the points of inflection of f, and explain their reasoning.



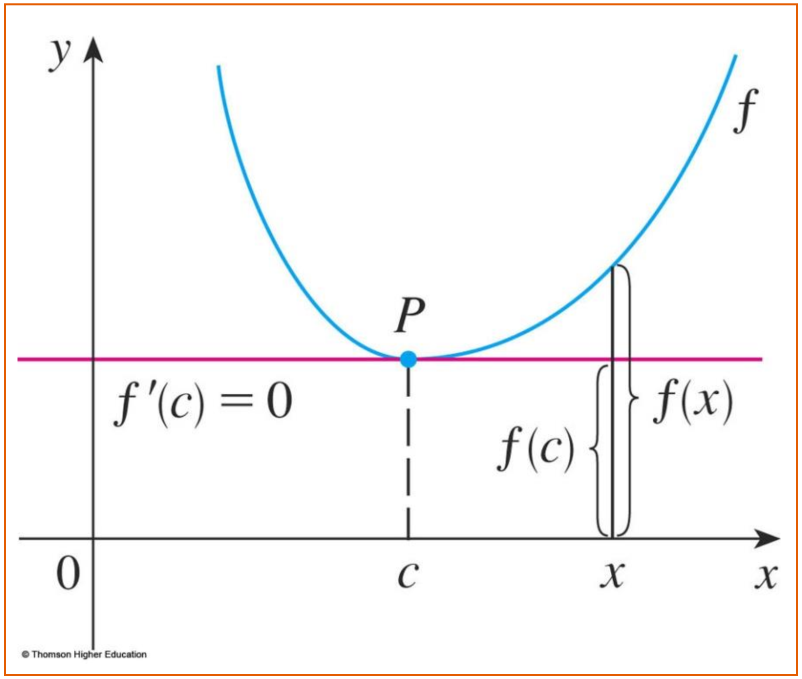
Answer: (-1,f(-1)),(1,f(1)),(3,f(3)),(6,f(6))

SECOND DERIVATIVE TEST

- Suppose f'' is continuous near c.

a. If $f'(c) = 0$ and $\boldsymbol{f''(c) > 0}$,
   then f has a **local minimum** at c.

b. If $f'(c) = 0$ and $\boldsymbol{f''(c) < 0}$,
   then f has a **local maximum** at c.



Choose the correct one.	
A	If f has local extreme value at c then f'(c)=0.
B	If f'(c)=0 then f has local extreme value at c.
C	If f''(3)=0 then (3,f(3)) is an inflection point of f.
D	There exists a function such that f'(x) is nonzero for all x and f(1)=f(0).
E	None of the above

Answer: e

3.5

Optimization Problems

In this section, we will learn:

How to solve problems involving

maximization and minimization of factors.

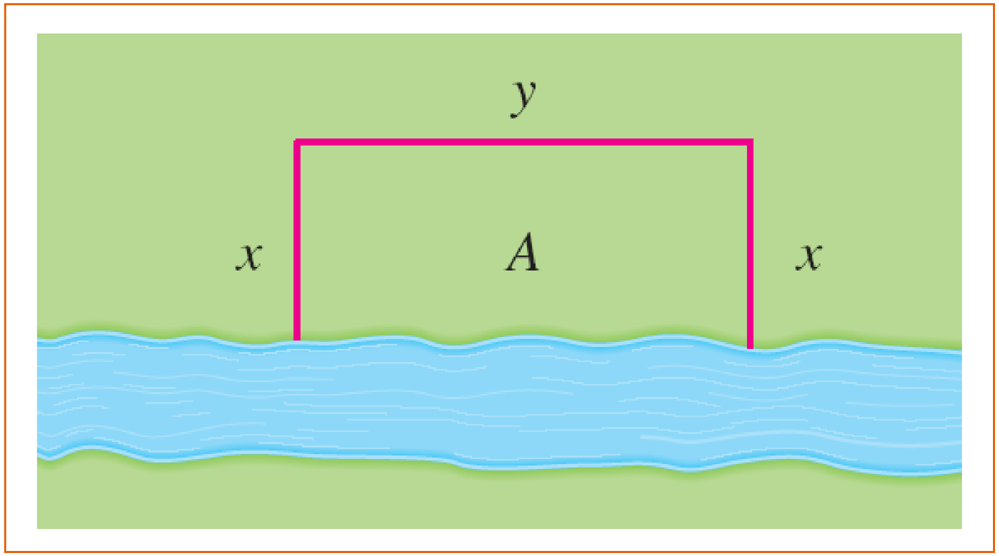
- Read the problem carefully until it is clearly understood.
  - What is the **unknown**?
  - What are the **given quantities**?
  - What are the given **conditions**?

6. FIND THE ABSOLUTE MAX./MIN. VALUE OF *f*

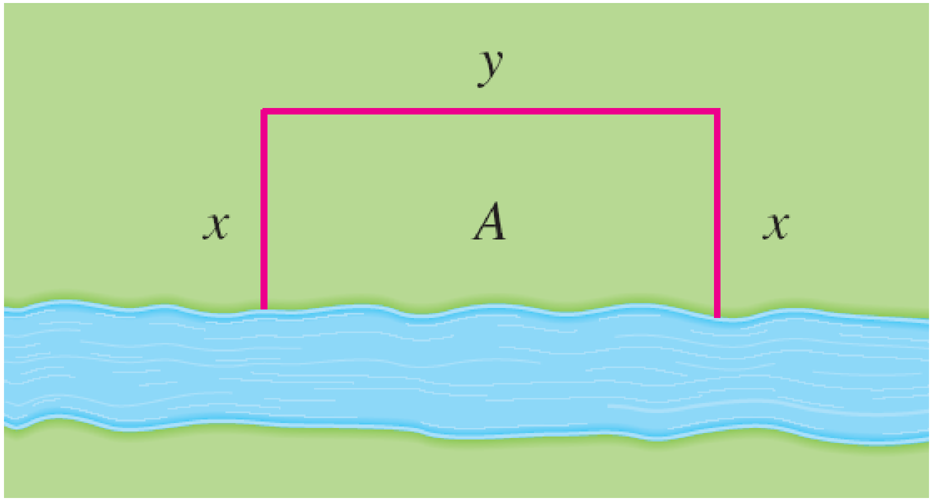
- Use the methods of Sections 4.1 and 4.3 to find the absolute maximum or minimum value of *f*.
  - In particular, if the domain of *f* is a closed interval, then the Closed Interval Method in Section 4.1 can be used.

OPTIMIZATION PROBLEMS **Example 1**

- A farmer has 2400 ft of fencing and wants to fence off a rectangular field that borders a straight river. He needs no fence along the river.
  - What are the dimensions of the field that has the **largest area** ?



OPTIMIZATION PROBLEMS **Example 1**



- This figure illustrates the general case.

- We wish to maximize the area A of the rectangle.
  - Then, we express A in terms of x and y: $A = xy$
  - $2x + y = 2400$
  - So, $A(x) = 2400x - 2x^2$, $0 \leq x \leq 1200$
  - …

3.6
Newton's Method

In this section, we will learn:

How to solve high-degree equations

using Newton's method.

- How do those numerical rootfinders work?
  - They use a variety of methods.
  - Most, though, make some use of Newton’s method, also called the Newton-Raphson method.

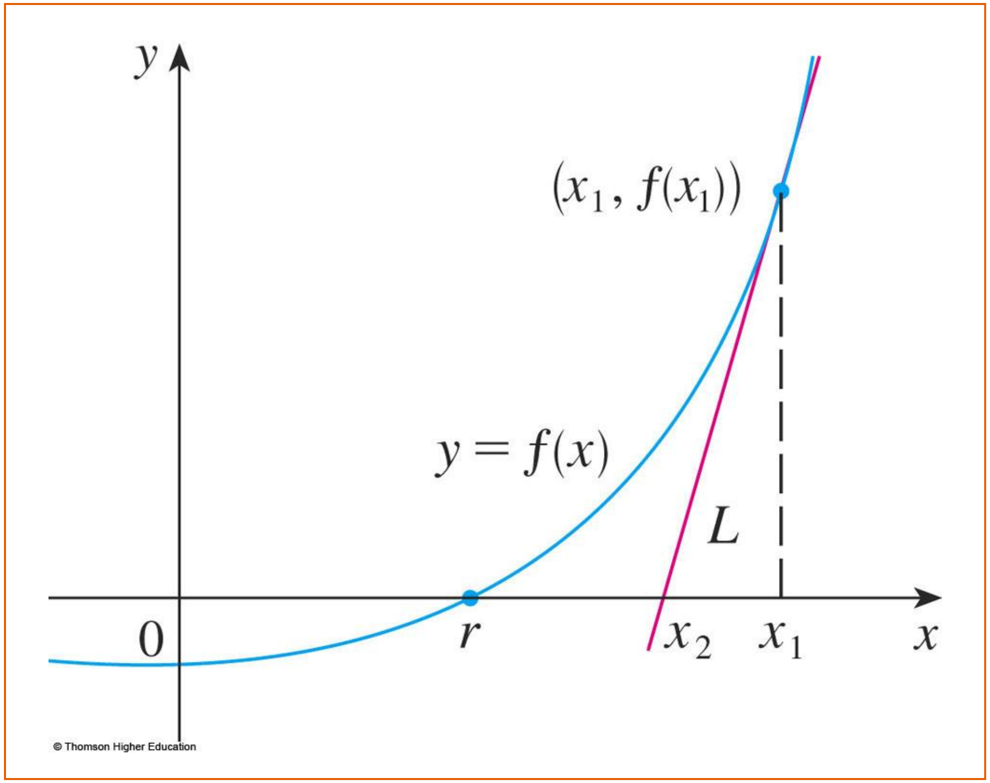
NEWTON'S METHOD

- We start with a first approximation x_1, which is obtained by one of the following methods:

  – Guessing

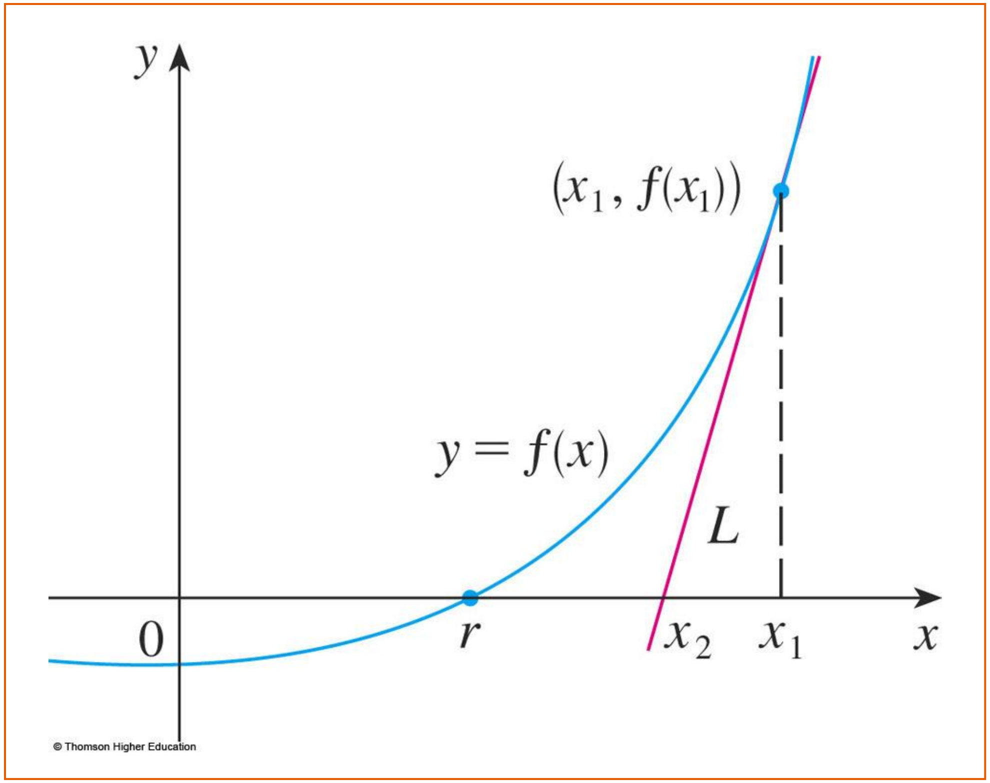
  – A rough sketch of the graph of *f*

  – A computer-generated graph of *f*



NEWTON'S METHOD

- Consider the tangent line L to the curve $y = f(x)$ at the point $(x_1, f(x_1))$ and look at the x-intercept of L, labeled x_2.



- As the x-intercept of L is x_2, we set $y = 0$ and obtain:
    - $0 - f(x_1) = f'(x_1)(x_2 - x_1)$
- If $f'(x_1) \neq 0$, we can solve this equation for x_2:

$$\boxed{x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}}$$

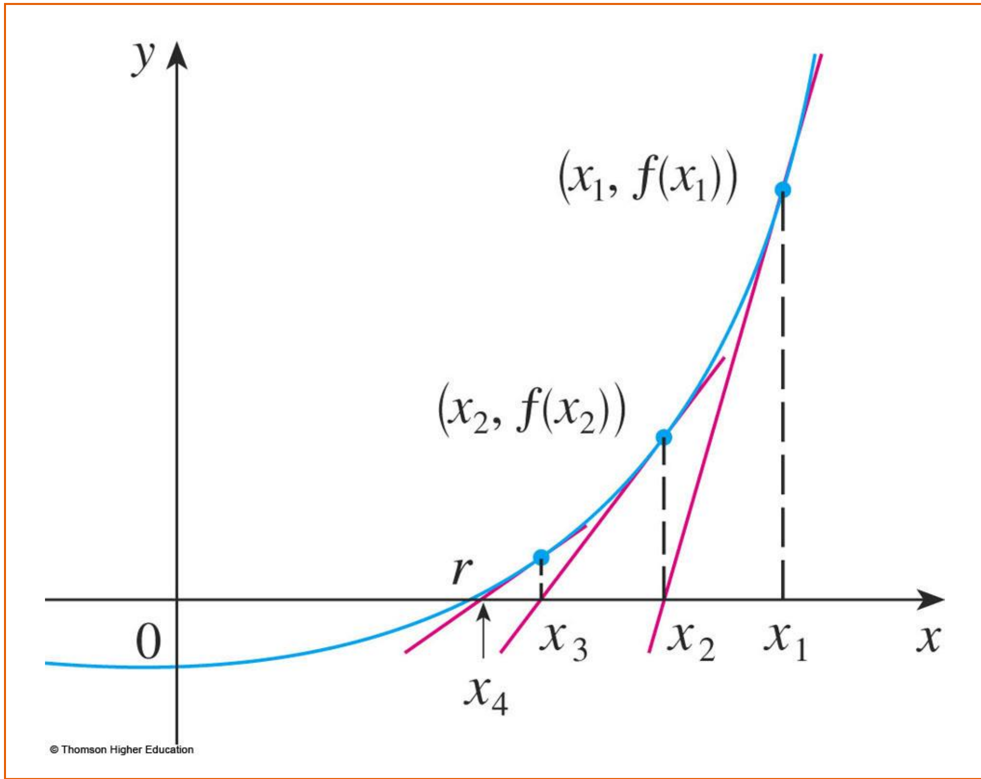
  – We use x_2 as a second approximation to r.

SUBSEQUENT APPROXIMATION

Equation/Formula 2

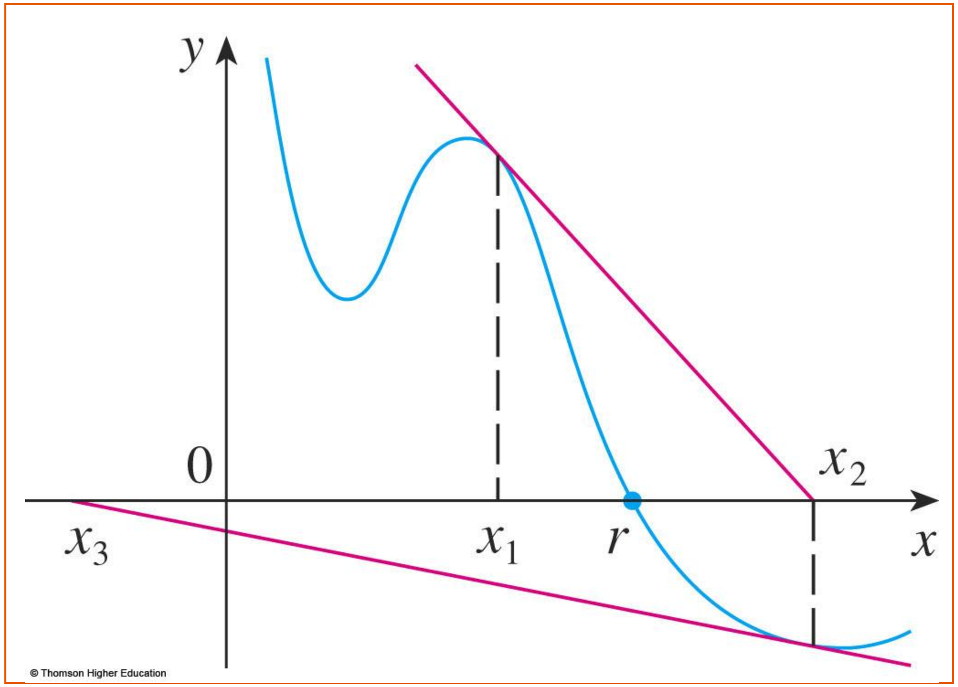
- In general, if the *n*th approximation is x_n and $f'(x_n) \neq 0$, then the next approximation is given by:

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$



- If the numbers x_n become closer and closer to r as n becomes large, then we say that the sequence converges to r and we write:

$$\lim_{n \to \infty} x_n = r$$



NEWTON'S METHOD **Example 2**

- Use Newton's method to find $\sqrt[6]{2}$ correct to eight decimal places.
  - First, we observe that finding $\sqrt[6]{2}$ is equivalent to finding the positive root of the equation $x^6 - 2 = 0$
  - So, we take $f(x) = x^6 - 2$
  - Then, $f'(x) = 6x^5$

NEWTON'S METHOD **Example 2**

- So, Formula 2 (Newton's method) becomes:

$$x_{n+1} = x_n - \frac{x_n^6 - 2}{6x_n^5}$$

NEWTON'S METHOD **Example 2**

- Choosing $x_1 = 1$ as the initial approximation, we obtain:

$$x_2 \approx 1.16666667$$
$$x_3 \approx 1.12644368$$
$$x_4 \approx 1.12249707$$
$$x_5 \approx 1.12246205$$
$$x_6 \approx 1.12246205$$

  - As x_5 and x_6 agree to eight decimal places, we conclude that $\sqrt[6]{2} \approx 1.12246205$ to eight decimal places.

3.7

Antiderivatives

In this section, we will learn about:

Antiderivatives and how they are useful in solving certain scientific problems.

DEFINITION

- A function F is called an **antiderivative** of f on an interval I if $F'(x) = f(x)$ for all x in I.

ANTIDERIVATIVES

Theorem 1

- If F is an antiderivative of f on an interval I, the most general antiderivative of f on I is

$$\boxed{F(x) + C}$$

- where C is an arbitrary constant.

ANTIDERIVATIVE FORMULA **Table 2**

- Here, we list some particular antiderivatives.

Function	Particular antiderivative	Function	Particular antiderivative
$cf(x)$	$cF(x)$	$\sin x$	$-\cos x$
$f(x) + g(x)$	$F(x) + G(x)$	$\sec^2 x$	$\tan x$
$x^n \quad (n \neq -1)$	$\dfrac{x^{n+1}}{n+1}$	$\sec x \tan x$	$\sec x$
$1/x$	$\ln \lvert x \rvert$	$\dfrac{1}{\sqrt{1-x^2}}$	$\sin^{-1} x$
e^x	e^x	$\dfrac{1}{1+x^2}$	$\tan^{-1} x$
$\cos x$	$\sin x$		

RECTILINEAR MOTION

Example 6

- A particle moves in a straight line and has acceleration given by $a(t) = 6t + 4$.
- Its initial velocity is $v(0) = -6$ cm/s and its initial displacement is $s(0) = 9$ cm.
  - Find its position function $s(t)$.

Thanks