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Question 1
Not answered
Marked out of 0.50

The difference in age between two brothers is 5. 8 years ago, the older of the two was twice as old as the younger. Find the present age of the younger brother.

a) 10 b) 11 c) 12 d) 13 e) 14

$$\begin{cases} x - y = 5 \\ (x-8) = (y-8)\cdot 2 \end{cases} \Leftrightarrow \begin{cases} x = 18 \\ y = 13 \end{cases}$$

a. E
b. A
c. F
d. C
e. D ✓
f. B

The correct answer is:
D

Question 2
Not answered
Marked out of 0.50

Which of the statements below is correct for the system of equations S where

$$S = \begin{cases} x - 2y + z + w = 2 \\ 3x + 2z - 2w = -8 \\ 4y - z - w = 2 \\ 2x + y + z - w = k \end{cases} ?$$

a) S has a unique solution $(0, 2, 1, 5)$ if $k = -2$.
b) S is inconsistent if $k = -2$.
c) S has an infinite number of solutions if $k = -2$.
d) S has the trivial solution of $k = -2$.

a. B
b. D
c. C
d. A

The correct answer is:
C

Question 3
Not answered
Marked out of 0.50

Solve the system of equations $\begin{cases} x + 2y - z - w = 0 \\ z + 2w = 4 \\ -x - 2y + 2z + 4w = 5 \end{cases}$ for y.

a) 1 b) 2 c) 3 d) y is arbitrary e) -1 f) -2

$(y \in \mathbb{R})$

a. E
b. B
c. F
d. D
e. C
f. A

The correct answer is:
D

Question 4
Not answered
Marked out of 0.50

$7 > 6$

A homogeneous linear system of 6 equations in 7 unknowns must have:

a) exactly the same number of solutions as unknowns,

b) many solutions or none,

c) one solution or none,

d) many solutions or exactly one solution,

e) many solutions.

f) The system has either the trivial solution only or infinitely many solutions.

a. E
b. A
c. F
d. C
e. D
f. B

The correct answer is:
E

Question 5
Not answered
Marked out of 0.50

For a homogeneous system of 10 equations in 12 unknowns, state which combination of answers to the following questions is correct.

- Can the system be inconsistent? (no solution) ✗
- Can the system have infinitely many solutions? ✓
- Can the system have only one solution? ✗

a) Yes, Yes, No	b) No, No, Yes	c) Yes, No, Yes
d) No, Yes, No	e) Yes, Yes, Yes	f) No, No, No

a. A
b. B
c. F
d. E
e. C
f. D

The correct answer is:
D

Question 6
Not answered
Marked out of 0.50

Solve the system of equations
$$\begin{cases} x_1 + x_2 + 5x_4 = 6 \\ x_1 + 2x_2 + x_3 = 4 \\ 2x_2 + x_3 + x_4 = 6 \\ 3x_1 - 4x_4 = 2 \end{cases}$$
for x_4.

a) -10	b) 56	c) -98	d) -8	e) -40	f) 24

a. D
b. B
c. F
d. A
e. C
f. E

The correct answer is:
D

Question 7

Not answered

Marked out of 0.50

For a system of four equations in three unknowns, which statements are true?

a) There is always at least one solution.

b) There may be exactly 3 solutions.

c) There may be exactly 4 solutions.

d) There may be exactly 1 solutions.

e) There may be no solution. ✓

f) There may be infinitely many solutions. ✓

g) If the system is homogeneous, then there are always infinitely many solutions.

h) If the system is homogeneous, then there is always at least one solution. ✓

$$\begin{cases} x_1 + x_2 + x_3 = 4 \\ x_1 - x_3 = -1 \\ 2x_1 + x_2 - x_3 = 0 \\ x_1 + x_2 + 2x_3 = 0 \end{cases}$$

trivial solution $(0, 0, 0)$

$$\begin{cases} x_1 + x_2 + x_3 = 4 \\ x_1 - x_3 = -1 \\ 2x_1 + 2x_2 + 2x_3 = 8 \\ 3x_1 + 3x_3 = 3 \end{cases}$$

- a. H
- b. A
- c. B
- d. E
- e. F
- f. G
- g. C
- h. D

The correct answers are:
E,
F,
H

Question 8

Not answered

Marked out of 0.50

Given that rank $\begin{bmatrix} -1 & 4 & 5 & 3 \\ 2 & 3 & -2 & 6 \\ 3 & 10 & a & 15 \end{bmatrix}$ is 2, find a :

$\xrightarrow[r_2 - 2r_1,\ r_3 - 3r_1]{-r_1} \begin{bmatrix} 1 & -4 & -5 & -3 \\ 0 & 11 & 8 & 12 \\ 0 & 22 & a+15 & 24 \end{bmatrix} \xrightarrow[r_3 - 2r_2]{\frac{1}{11} r_2} \begin{bmatrix} 1 & -4 & -5 & -3 \\ 0 & 1 & 8/11 & 12/11 \\ 0 & 0 & a-1 & 0 \end{bmatrix}$

$a - 1 = 0 \Rightarrow a = 1$

a) 0 b) −1 c) 4 d) $\frac{1}{2}$ e) 1 f) −2

- a. E
- b. D
- c. B
- d. C
- e. A
- f. F

The correct answer is:
E

Question 9
Not answered
Marked out of 0.50

Solve the system of equations $\begin{cases} & & - & 2x_3 & + & & + & 7x_5 & = & 12 \\ 2x_1 & + & 4x_2 & - & 10x_3 & + & 6x_4 & + & 12x_5 & = & 28 \\ 2x_1 & + & 4x_2 & - & 5x_3 & + & 6x_4 & - & 5x_5 & = & -1 \end{cases}$ for x_3.

a) -5 b) 3 c) -2 d) 2 e) -1 f) 1

$l_2 - l_3 = -5x_3 + 17x_5 = 29$
$l_1 = -2x_3 + 7x_5 = 12$
$\Rightarrow \begin{cases} x_3 = 1 \\ x_5 = 2 \end{cases}$

- a. E
- b. C
- c. A
- d. F
- e. B
- f. D

The correct answer is:
F

Question 10
Not answered
Marked out of 0.50

$5 > 4$

For a homogeneous system S of 4 equations in 5 unknowns, which of the following statements is (are) true?

(i) S can be inconsistent.

(ii) S can have a unique solution.

(iii) S can have infinitely many solutions.

a) (i) b) (ii) c) (iii) d) (i) and (iii) e) (ii) and (iii) f) (i) and (ii)

- a. D
- b. C
- c. F
- d. B
- e. E
- f. A

The correct answer is:
C

Question 11

Not answered

Marked out of 0.50

For what value(s) of λ will the system of equations $\begin{cases} (1+\lambda)x + y + z = 1 \\ x + (1+\lambda)y + z = 1 \\ x + y + (1+\lambda)z = 1 \end{cases}$ have infinitely many solutions?

a) -1 b) -3 c) 1 d) 0 e) 0 and 1 f) $-1, 0$ and -3

a. F
b. D
c. B
d. C
e. E
f. A

The correct answer is:
D

Question 12

Not answered

Marked out of 0.50

Find all values of x and y so that the matrix $\begin{bmatrix} 1 & 2 \\ x & y \end{bmatrix}$ is in reduced row-echelon form.

a) $x = 0, y$ is arbitrary b) $x = 1, y$ is arbitrary c) $x = 0, y = 0$
d) $x = 0, y = 1$ e) $x = 1, y = 1$ f) $x = 1, y = 0$

a. A
b. C
c. F
d. B
e. E
f. D

The correct answer is:
C

Question 13
Not answered
Marked out of 0.50

Which of the following matrices are in reduced row-echelon form?

(1) $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 1 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(4) $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \end{bmatrix}$ (5) $\begin{bmatrix} 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

a) (1) and (2) b) (2) and (5) c) (1), (3) and (4)
d) (1), (2) and (5) e) (3), (4) and (5) f) only (2)

- a. E
- b. C
- c. D
- d. A
- e. B
- f. F

The correct answer is:
B

Question 14
Not answered
Marked out of 0.50

For a non-homogeneous system of 17 equations in 9 unknowns, state which combination of answers to the following questions is correct.

- Can the system have no solutions at all?
- Can the system have one – and only one – solution?
- Can the system have an infinite number of solutions?

a) Yes, No, No b) Yes, Yes, No c) Yes, No, Yes
d) Yes, Yes, Yes e) No, Yes, Yes f) No, No, No

- a. A
- b. C
- c. E
- d. F
- e. D
- f. B

The correct answer is:
D

Question 15
Not answered
Marked out of 0.50

A linear system of 212 equations in 312 unknowns:

a) is always consistent and has exactly 100 parameters in the general solution.

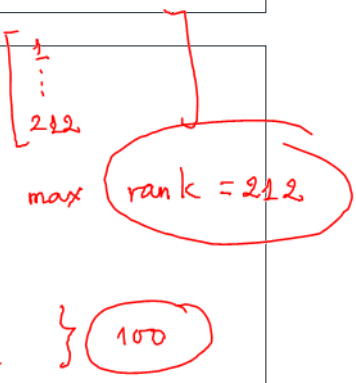
b) is always consistent and has at most 100 parameters in the general solution.

c) is always consistent and has at least 100 parameters in the general solution.

d) which is consistent has exactly 100 parameters in the general solution.

e) which is consistent has at most 100 parameters in the general solution.

f) which is consistent has at least 100 parameters in the solution.



- a. D
- b. B
- c. C
- d. F
- e. A
- f. E

The correct answer is:
F

Question 16
Not answered
Marked out of 0.50

The system of equations $\begin{cases} 3x_1 + 2x_2 + x_3 = 0 \\ x_1 + x_2 + x_3 = 1 \\ x_1 - x_3 = -1 \\ x_2 - x_3 = 0 \end{cases}$

a) has infinitely many solutions.

b) has a unique solution and $x_1 = \frac{2}{3}$.

c) has a unique solution and $x_1 = -\frac{1}{3}$.

d) has infinitely many solutions and $x_2 = \frac{2}{3}$.

e) has no solution.

f) has the unique solution $(0, 0, 0)$.

- a. A
- b. F
- c. B
- d. E
- e. D
- f. C

The correct answer is:
E

Question 17
Not answered
Marked out of 0.50

The coefficient matrix A in a homogeneous system of 12 equations in 16 unknowns is known to have rank 6. How many free parameters are there in the solution?

a) 10 b) 6 c) 4 d) none e) 12 f) 16

16 - 6 = 10

- a. D
- b. F
- c. A
- d. E
- e. B
- f. C

The correct answer is:
A

Question 18
Not answered
Marked out of 0.50

The system of equations $\begin{cases} x + y - z = 3 \\ x - y + z = 0 \\ 2x + y + 2z = 3 \end{cases}$:

$\Rightarrow \begin{cases} x = 3/2 \\ y = 1 \\ z = -1/2 \end{cases}$

$(x, y, z) = (3/2, 1, -1/2)$

a) is inconsistent.
b) has exactly 2 solutions.
c) has exactly 1 non-trivial solution.
d) has an infinite number of solutions.
e) has exactly 3 solutions.
f) has the trivial solution only.

- a. E
- b. C
- c. F
- d. D
- e. A
- f. B

The correct answer is:
C

Question 19
Not answered
Marked out of 0.50

The rank of $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 3 \\ 5 & 3 & 4 \end{bmatrix}$ is:

a) 0 b) 2 c) 3 d) $\frac{1}{2}$ e) -1 f) -2

a. A
b. C
c. B
d. E
e. D
f. F

The correct answer is:
B

Question 20
Not answered
Marked out of 0.50

Find all (x, y) so that the matrix $\begin{bmatrix} 1 & 0 & 1 \\ x & y & 0 \end{bmatrix}$ is in reduced row-echelon form.

a) $(0,0)$ b) $(0,0)$ and $(1,0)$ c) $(0,1)$
d) $(0,0)$ and $(0,1)$ e) $(1,0)$ and $(0,1)$ f) $(1,1)$

a. D
b. A
c. C
d. B
e. E
f. F

The correct answer is:
D