Started on	Wednesday, 2 November 2022, 4:19 PM
State	Finished
Completed on	Wednesday, 2 November 2022, 4:54 PM
Time taken	35 mins 45 secs
Marks	18.50/30.00
Grade	**6.17** out of 10.00 (**62**%)

Question **1**

Correct

Mark 1.00 out of 1.00

Consider the following recursive algorithm:

procedure TT(m: positive integer)

if m = 1 **then** TT(m) := 1

else TT(m) := TT(m-1) + 2m -1

What does this algorithm compute?

- a. The sum of the first m positive integers.
- b. The sum of the first m even positive integers.
- c. The sum of the first m odd positive integers. ✔
- d. The sum of the first m even positive integers that are less than m.

The correct answer is:
The sum of the first m odd positive integers.

Question 2

Partially correct

Mark 0.50 out of 1.00

Which propositions are true?

- [ ] a. If a, b, m are integers such that $m \geq 2$ and $a \equiv b \pmod{m}$, then $\gcd(a, m) = \gcd(b, m)$.
- [ ] b. The sum of two primes is always a prime.
- [x] c. The product of any three consecutive integers is divisible by 6.
- [ ] d. There are a finite number of primes.

The correct answers are:
If a, b, m are integers such that $m \geq 2$ and $a \equiv b \pmod{m}$, then $\gcd(a, m) = \gcd(b, m)$.,
The product of any three consecutive integers is divisible by 6.

Question 3

Correct

Mark 1.00 out of 1.00

Using the function f(p)=(3p+1) **mod** 26 to encrypt the message "T". What is a true message?

- A. None of these
- B. "A"
- C. "G" ✔
- D. "B"

The correct answer is:
"G"

Question **4**

Correct

Mark 1.00 out of 1.00

Find f(0) if f(m) = 2f(m-1) - 6 for all positive integers m, and f(1) = 0.

- a. -4
- b. -3
- c. 3 ✔
- d. 4

The correct answer is:
3

Question 5

Correct

Mark 1.00 out of 1.00

Suppose $|A| = 4$ and $|B| = 5$. Find the number of functions from set A to set B.

a. 20
b. 4^5
c. 4!
d. 5^4
e. 5!

The correct answer is:
5^4

Question **6**

Correct

Mark 1.00 out of 1.00

Suppose that f (n) = f(n/3) + 1 when n is a positive integer divisible by 3, and f(1) = 1. Find f(27).

a. 5

b. 4 ✔

c. 3

d. 6

The correct answer is:
4

Question 7

Correct

Mark 1.00 out of 1.00

A **palindrome** is a string whose **reversal** is identical to the string. For example, 1101011 is a palindrome of length 7.
How many bit strings of length 5 are palindromes?

- a. 5
- b. 6
- c. None of these
- d. 6
- e. 8 ✔

The correct answer is: 8

Question 8

Correct

Mark 1.00 out of 1.00

Give a recursive definition of the set of strings S = {1,111,11111,1111111, ...}

(i) $1 \in S; x \in S \rightarrow x11 \in S$
(ii) $1 \in S; x \in S \rightarrow x1 \in S$

a. Both
b. Only (ii)
c. Only (i)
d. Neither

The correct answer is:
Only (i)

Question 9

Correct

Mark 1.00 out of 1.00

Give a recursive definition with initial condition(s) of
$f(n) = (-3)^n$, n = 0, 1, 2, ...

a. None of these

b. f(0) = 1, and f(n) = -f(n+1)/3 for n > 0

c. f(0) = 1, and f(n) = -3f(n-1) for n > 0

d. f(n) = -3f(n-1)

e. f(0) = 1, and f(n) = $-3^{f(n-1)}$ for n > 0

The correct answer is: f(0) = 1, and f(n) = -3f(n-1) for n > 0

Question **10**

Incorrect

Mark 0.00 out of 1.00

Give the function $f(n) = (n^2 + n + n\sqrt{n^2+1}) \log n$ and consider the following statements:

f is O($n^2 \log n$)

f is O(n^3)

f is Ω(n^3)

f is Θ(n^2)

How many correct statements are there?

- A. 1
- B. 0
- C. 3
- D. 2

The correct answer is:
2

Question **11**

Correct

Mark 1.00 out of 1.00

Consider the following procedure

procedure *tinh*(m, *n*: non-negative integer)

if *n* = 0 **then** *return m*

else return {*1* + *tinh*(m, *n* - 1)}

Given m = 13, n= 5, what will be the output of the algorithm?

- a. 12
- b. 18 ✔
- c. 8
- d. 65
- e. None of these

The correct answer is:
18

Question 12

Correct

Mark 1.00 out of 1.00

A multiple-choice test contains 6 questions. There are four possible answers for each question.
In how many ways can a student answer the questions on the test if the student answers every question?

- a. 4^6 ✔
- b. 24
- c. None of these
- d. 6^4
- e. 6!

The correct answer is: 4^6

Question 13

Correct

Mark 1.00 out of 1.00

How many comparisons are needed for a binary search in a set of 64 elements?

- a. 14 ✔
- b. 16
- c. 12
- d. 10

The correct answer is:
14

Question **14**

Incorrect

Mark 0.00 out of 1.00

Let's consider the following procedures :

```
procedure double(n: positive integer)
while n > 0
    n := 2n
return n;
```

```
procedure divide(n: positive integer)
while n ≥ 0 :
       m := 1/n
        n = n – 1
return m;
```

```
procedure sum(n: positive integer)
sum := 0
i := 0
while i < n :
     sum = sum + i
return sum;
```

```
procedure choose(a, b: integers)
x := either a or b
return x;
```

How many procedures are algorithms?

- A. 2
- B. 4
- C. 3
- D. 1

The correct answer is:
1

Question 15

Correct

Mark 1.00 out of 1.00

Give a correct recursive definition for the Factorial of non-negative integer n.

a. None of the other choices is correct ✔

b. n! = n * (n-1), for n≥1
n! = 1 , for n=0

c. n! = (n-2) + (n-1), for n≥1
n! = 1 , for n=0

d. n! = n + (n-1), for n≥1
n! = 1 , for n=0

The correct answer is:
None of the other choices is correct

Question **16**

Incorrect

Mark 0.00 out of 1.00

Suppose A = { x, {x}, y, z} and B = { y, y, z, w}. Find the number of functions from the set A to the set B.

a. 4^3
b. 3^4
c. 3^3
d. 4^4
e. 12

The correct answer is:
3^4

Question 17

Correct

Mark 1.00 out of 1.00

Given a recursive definition of the set of strings S

$1 \in S; x \in S \rightarrow x11 \in S$

Which one is true?

(i) $1111 \in S$
(ii) $11111 \in S$

a. Neither
b. Only (i)
c. Both
d. Only (ii)

The correct answer is:
Only (ii)

Question 18

Correct

Mark 1.00 out of 1.00

Encrypt the message "BUY" using the encryption function f(x) = (5x + 7) mod 26.

- a. MEX
- b. MDX ✔
- c. NDX
- d. NEY
- e. None of the other choices is correct

The correct answer is:
MDX

Question 19

Incorrect

Mark 0.00 out of 1.00

Let a and b be integers such that a|b. Consider the following statements:
(P) If b|a then a=b
(Q) If b|c then a|c
Choose the correct answer.

- A. P is correct and Q is wrong ✖
- B. P is wrong and Q is correct
- C. Both P and Q are wrong
- D. Both P and Q are correct

The correct answer is:
P is wrong and Q is correct

Question 20

Incorrect

Mark 0.00 out of 1.00

Given the pseudorandom number $x_{n+1} = (3x_n + 2)$ **mod** 13 with seed x_0. What is the possible value of x_0 if $x_2 = 1$?

- A. 5
- B. 0
- C. 2
- D. 8

The correct answer is:
5

Question 21

Incorrect

Mark 0.00 out of 1.00

What is the total number of additions and shifts of bits used to multiply a = $(101)_2$ and b = $(100)_2$ by using the Algorithm below?

ALGORITHM 3 Multiplication of Integers.

procedure *multiply*(a, b: positive integers)
{the binary expansions of a and b are $(a_{n-1}a_{n-2} \dots a_1a_0)_2$
and $(b_{n-1}b_{n-2} \dots b_1b_0)_2$, respectively}
for $j := 0$ **to** $n - 1$
if $b_j = 1$ **then** $c_j :=$ a shifted j places
else $c_j := 0$
{$c_0, c_1, \dots, c_{n-1}$ are the partial products}
$p := 0$
for $j := 0$ **to** $n - 1$
$p := add(p, c_j)$
return p {p is the value of ab}

- a. 5
- b. 4
- c. 3
- d. 2

The correct answer is:
4

Question 22

Correct

Mark 1.00 out of 1.00

Suppose you wish to prove that the following is true for all positive integers *n* by using the Principle of Mathematical Induction:

P(n): 2 + 4 + 6 + ... + 2n = n·(n + 1)

Write P(4)

a. 2 + 4 = 6

b. 2 + 4 + 6 + 8 = 4·5 ✔

c. 2 + 4 + 6 + 8

d. None of these

e. 8

The correct answer is:
2 + 4 + 6 + 8 = 4·5

Question 23

Incorrect

Mark 0.00 out of 1.00

Give a recursive definition for the set of odd positive integers.

a. Basis Step: $3 \in S$
Recursive Step: if $x \in S$, then $x - 2 \in S$

b. None of the other choices is correct

c. Basis Step: $1 \in S$
Recursive Step: if $x \in S$, then $x + 2 \in S$ and $x - 2 \in S$

d. Basis Step: $0 \in S$
Recursive Step: if $x \in S$, then $2x + 1 \in S$

The correct answer is:
None of the other choices is correct

Question 24

Incorrect

Mark 0.00 out of 1.00

Prove that P(n) = "for all n ≥ 6 we have n = 3x + 4y with x, y non-negative integers" is true.

In the strong induction proof, assuming that P(k) is true for some k, in order to prove P(k+1) is true we should _____

a. use P(k-1) = "k - 1 = 3a + 4b, (a, b non-negative integers)" is true and k + 1 = (k - 1) + 2.

b. use P(k-4) = "k - 4 = 3a + 4b, (a, b non-negative integers)" is true and k + 1 = (k - 4) + 5.

c. use P(k-2) = "k - 2 = 3a + 4b, (a, b non-negative integers)" is true and k + 1 = (k - 2) + 3.

The correct answer is:
use P(k-2) = "k - 2 = 3a + 4b, (a, b non-negative integers)" is true and k + 1 = (k - 2) + 3.

Question 25

Incorrect

Mark 0.00 out of 1.00

Give a recursive definition of the sequence $\{a_n = 2n + 3\}$, $n = 1, 2, 3, \ldots$.

a. $a_1 = 5$ and $a_n = a_{n-1} + 3$ for $n = 1, 2, 3, \ldots$.

b. $a_n = a_{n-1} + 3$ for $n = 1, 2, 3, \ldots$.

c. $a_1 = 3$ and $a_n = a_{n-2} + 1$ for $n = 1, 2, 3, \ldots$.

d. None of the other choices is correct

The correct answer is:
None of the other choices is correct

Question 26

Correct

Mark 1.00 out of 1.00

What is 3^{2022} mod 100?

- a. 9 ✔
- b. 99
- c. 61
- d. 41

The correct answer is:
9

Question 27

Correct

Mark 1.00 out of 1.00

Find the prime factorization of 11!.

a. $2^7x3^4x5^2x7x11$

b. $2^8x3^5x5^2x7x11$

c. $2^8x3^4x5^2x7x11$ ✔

d. $2^8x3^4x5^2x11$

The correct answer is: $2^8x3^4x5^2x7x11$

Question 28

Correct

Mark 1.00 out of 1.00

How many one-to-one functions are there from a set with five elements to sets with seven elements?

a. 7^5

b. 7!

c. None of these

d. 7.6.5.4.3

e. 5^7

The correct answer is: 7.6.5.4.3

Question 29

Incorrect

Mark 0.00 out of 1.00

Give a recursive definition with initial condition(s) of the set A = {1, 3, 9, 27, 81, ...}.

Which one is true?

(i) $1 \in A; x \in A \rightarrow 3x \in A$.
(ii) $1 \in A; x \in A \rightarrow 3^x \in A$.

a. Only (i)
b. Only (ii)
c. Neither
d. Both

The correct answer is: Only (i)

Question 30

Incorrect

Mark 0.00 out of 1.00

Consider two set A, B such that |A| = 13, |A - B| = 10 and |B - A| = 2. Find |B|.

- a. 7
- b. 6
- c. 5
- d. 8

The correct answer is: 5