

Started on	Tuesday, 18 October 2022, 5:05 PM
State	Finished
Completed on	Tuesday, 18 October 2022, 5:13 PM
Time taken	7 mins 58 secs
Marks	23.00/29.00
Grade	7.93 out of 10.00 (79%)

Question 1

Correct

Mark 1.00 out of 1.00

If a , b and c are integers, which of the following statements is/are TRUE?

Note that $a \mid b$ means b is multiple of a , that is, there is an integer k such that $b = ka$

(i) If $a \mid bc$, then $a \mid b$.

(ii) If $ab \mid c$, then $b \mid c$.

- ☐ a. Both (i) and (ii)
- ☐ b. (i) only
- ☒ c. (ii) only
- ☐ d. Neither (i) nor (ii)



Question 2

Correct

Mark 1.00 out of 1.00

Find the least integer k such that $f(n)$ is $O(n^k)$

if $f(n) = 7n^3 + n^3 \log n$

- ☐ a. 2
- ☐ b. 3
- ☐ c. 1
- ☒ d. 4



Your answer is correct.

$f(n) = 7n^3 + n^3 \log n$ is $O(n^3 \log n)$

$\implies k = 4$

Question 3

Incorrect

Mark 0.00 out of 1.00

Suppose that a computer has only the memory locations 0, 1, 2, ..., 29. Use the hashing function h where $h(x) = (x + 5) \bmod 30$ to determine the memory locations in which 97, 32, and 16 are stored.

- ☐ a. 12, 2, 16
- ☒ b. 12, 7, 16
- ☐ c. None of the others
- ☐ d. 7, 2, 16
- ☐ e. 12, 7, 21



Question 4

Correct

Mark 1.00 out of 1.00

Which of the following statements is/are TRUE?

- (i) The bubble sort has $O(n^2)$ time complexity.
- (ii) The merge sort has $O(n \log n)$ time complexity.

- ☐ a. (ii) only
- ☐ b. (i) only
- ☐ c. Neither (i) nor (ii)
- ☒ d. Both (i) and (ii)



Question 5

Correct

Mark 1.00 out of 1.00

Determine the big-oh time complexity of the following algorithm:

```
procedure  $tt(a_1, a_2, \dots, a_n: \text{integers})$   
    count := 0  
     $v := a_1$   
    for  $i := 2$  to  $n$  do  
        if  $a_i = v$  then  
            count := count + 1  
             $a_i := a_i + i$   
    return (count)
```

- ☒ a. $O(n)$
- ☐ b. $O(1)$
- ☐ c. $O(\log n)$



Question 6

Correct

Mark 1.00 out of 1.00

Give as good a big-O estimate as possible for $(n \log n + n^2)(n^3 + 1)$.

- ☐ a. $o(n^2 \log n)$
- ☒ b. $O(n^5)$
- ☐ c. $O(n^4 \log n)$
- ☐ d. None of these
- ☐ e. $O(n^3)$



Your answer is correct.

Question 7

Correct

Mark 1.00 out of 1.00

Give the “best” big-oh notation to describe the complexity of the algorithm that prints all bit strings of length n .

- ☒ a. $O(2^n)$
- ☐ b. $O(n)$
- ☐ c. $O(n^2)$
- ☐ d. $O(n \log n)$



Question 8

Correct

Mark 1.00 out of 1.00

Convert $(43)_5$ to its decimal expansion.

- ☐ a. 35
- ☐ b. 130
- ☐ c. 103
- ☒ d. 23
- ☐ e. None of the others



$$4 \cdot 5 + 3 = 23$$

Question 9

Incorrect

Mark 0.00 out of 1.00

Which of the following statements is/are TRUE?

(i) 2^n is $O(n^2)$

(ii) 2^n is $O(n^3)$

- ☒ a. Both (i) and (ii)
- ☐ b. (i) only
- ☐ c. (ii) only
- ☐ d. Neither (i) nor (ii)



Question **10**

Correct

Mark 1.00 out of 1.00

Find $17 \bmod 5$ and $-17 \bmod 5$.

- ☐ a. None of the others
- ☐ b. 3 and -3
- ☒ c. 2 and 3
- ☐ d. 3 and 2
- ☐ e. 2 and -2



$$17 = 5 \cdot 3 + 2 \implies 17 \bmod 5 = 2$$

$$-17 = 5(-4) + 3 \implies -17 \bmod 5 = 3$$

Question **11**

Incorrect

Mark 0.00 out of 1.00

Find $\gcd(2^{30}, 8^{11})$.

- ☐ a. 8
- ☐ b. None of the others
- ☐ c. 8^{11}
- ☐ d. 2^{30}
- ☒ e. $2^{30} \cdot 8^{11}$



Question **12**

Correct

Mark 1.00 out of 1.00

Give a big-O estimate for the number of additions used in the following algorithm:

```
procedure sum(n: positive integer)
```

```
  s := 0
```

```
  for i := 1 to n
```

```
    for j := 1 to n
```

```
      s := s + i + j
```

- ☐ a. $O(2n)$
- ☒ b. $O(n^2)$
- ☐ c. $O(n)$
- ☐ d. $O(\log n)$



Question **13**

Correct

Mark 1.00 out of 1.00

If $f(x) = (2x + 3) \bmod 26$ is a good coding function.

- ☒ a. No. $f(0) = f(13)$, and hence f is not a bijection (no inverse function for decryption)
- ☐ b. Yes. $f(x)$ is a bijection and therefore it is a good coding function



Your answer is correct.

Question **14**

Incorrect

Mark 0.00 out of 1.00

If $a = -37 \bmod 7$ and $b = 37 \bmod 7$, what is the value of $a + b$?

- ☒ a. 4
- ☐ b. 0
- ☐ c. 7
- ☐ d. None of the others
- ☐ e. 2



Question **15**

Correct

Mark 1.00 out of 1.00

Give a big-O estimate for $(x^2 + x \log x) \cdot (2x + 3)$.

- ☒ a. $O(x^3)$
- ☐ b. None of the others
- ☐ c. $O(x^2)$
- ☐ d. $O(x \log x)$
- ☐ e. $O(x^2 \log x)$

Question **16**

Correct

Mark 1.00 out of 1.00

Encrypt the message VA using the function $f(p) = (p + 7) \bmod 26$.

- ☐ a. AL
- ☐ b. None of the others
- ☒ c. CH
- ☐ d. DM
- ☐ e. BK



Question 17

Correct

Mark 1.00 out of 1.00

Given the bubble sort algorithm.

ALGORITHM 4 The Bubble Sort.

```
procedure bubblesort( $a_1, \dots, a_n$  : real numbers with  $n \geq 2$ )  
for  $i := 1$  to  $n - 1$   
    for  $j := 1$  to  $n - i$   
        if  $a_j > a_{j+1}$  then interchange  $a_j$  and  $a_{j+1}$   
    { $a_1, \dots, a_n$  is in increasing order}
```

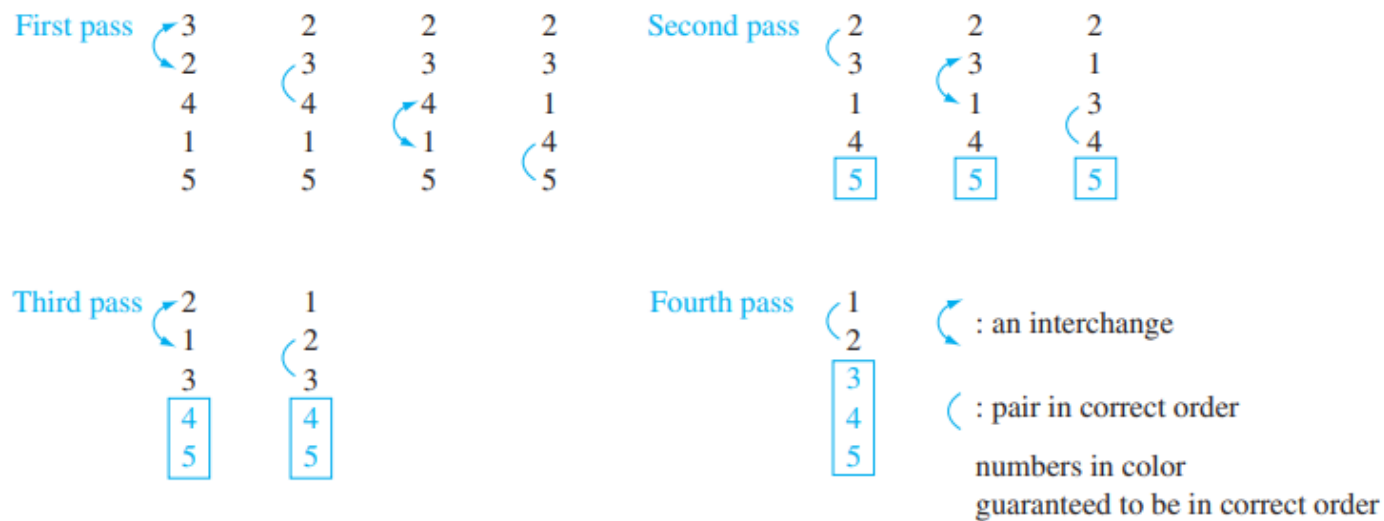
Use the bubble sort to put 3, 2, 4, 1, 5 into increasing order.

Showing the lists obtained after the second pass ($i = 2$).

- ☐ a. 3, 1, 2, 4, 5
- ☐ b. 1, 2, 3, 4, 5
- ☐ c. 2, 1, 3, 5, 4
- ☒ d. 2, 1, 3, 4, 5
- ☐ e. None of these



Your answer is correct.

Question **18**

Correct

Mark 1.00 out of 1.00

The password for a quiz was encrypted using the function $f(p) = (p + 5) \bmod 26$. If the result is "GZ", find the quiz password.

- ☐ a. TV
- ☐ b. BT
- ☐ c. None of the others
- ☐ d. CV
- ☒ e. BU



Question 19

Correct

Mark 1.00 out of 1.00

Find the best big-oh function for the function

$$f(n) = 1 + 4 + 7 + \dots + (3n + 1).$$

- ☐ a. $O(n)$
- ☒ b. $O(n^2)$
- ☐ c. $O(n^3)$
- ☐ d. $O(1)$



Your answer is correct.

Question **20**

Incorrect

Mark 0.00 out of 1.00

Give the best big-oh estimate for the function

$$f(n) = 1^2 + 2^2 + 3^2 + \dots + n^2$$

- ☐ a. $O(n^3)$
- ☒ b. $O(n^2 \log n)$
- ☐ c. None of these
- ☐ d. $O(n^4)$
- ☐ e. $O(n^2)$



Your answer is incorrect.

Question **21**

Correct

Mark 1.00 out of 1.00

How many positive integers less than 5 are **relatively prime** to 5?

- ☐ a. 1
- ☐ b. 3
- ☐ c. 2
- ☐ d. 5
- ☒ e. 4

Question **22**

Correct

Mark 1.00 out of 1.00

A sequence of pseudo-random numbers are generated using $x_{n+1} = (3x_n + 5) \bmod 31$ with seed $x_0 = 2$.
Find x_1 , x_2 , and x_3 .

- ☐ a. $x_1 = 11, x_2 = 7, x_3 = 13$
- ☐ b. None of the others
- ☐ c. $x_1 = 8, x_2 = 29, x_3 = 30$
- ☐ d. $x_1 = 11, x_2 = 1, x_3 = 8$
- ☒ e. $x_1 = 11, x_2 = 7, x_3 = 26$



Question **23**

Correct

Mark 1.00 out of 1.00

A pseudorandom sequence $\{x_n\}$ is generated by:

$$x_0 = 1, x_{n+1} = (3x_n + 5) \bmod 17 \text{ if } n \geq 0.$$

Find x_0, x_1, x_2, x_3 .

- ☒ a. 1, 8, 12, 7
- ☐ b. None of the others
- ☐ c. 1, 8, 10, 12
- ☐ d. 8, 12, 7, 9
- ☐ e. 12, 7, 9, 15



Question **24**

Correct

Mark 1.00 out of 1.00

Find the integer x such that $5x \equiv 1 \pmod{13}$ and $0 \leq x \leq 12$.

- ☐ a. 3
- ☐ b. None of the others
- ☐ c. 5
- ☐ d. 7
- ☒ e. 8

Question **25**

Correct

Mark 1.00 out of 1.00

Which of the following statements is/are TRUE:

- (i) $9n^2 + 9n + 99$ is $O(n^2)$
- (ii) $n \log n$ is $O(n)$

- ☒ a. (i) only
- ☐ b. Both (i) and (ii)
- ☐ c. (ii) only
- ☐ d. Neither (i) nor (ii)



Question **26**

Correct

Mark 1.00 out of 1.00

Convert 19 to a base 3 expansion.

- ☐ a. $(61)_3$
- ☐ b. $(21)_3$
- ☒ c. $(201)_3$
- ☐ d. None of the others
- ☐ e. $(102)_3$



Question **27**

Correct

Mark 1.00 out of 1.00

Given the Euclidean algorithm.

ALGORITHM 1 The Euclidean Algorithm.**procedure** $gcd(a, b: \text{positive integers})$ $x := a$ $y := b$ **while** $y \neq 0$ $\quad r := x \bmod y$ $\quad x := y$ $\quad y := r$ **return** $x \{gcd(a, b) \text{ is } x\}$

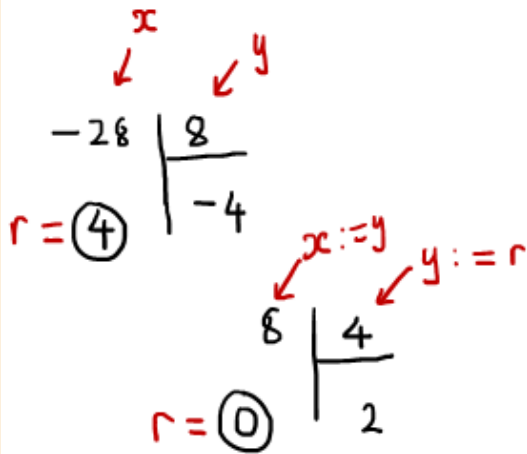
Use the Euclidean algorithm to find $gcd(28, 8)$.

How many **divisions** are required?

- ☒ a. 2
- ☐ b. 3
- ☐ c. 5
- ☐ d. 4
- ☐ e. None of these



Your answer is correct.



Question 28

Incorrect

Mark 0.00 out of 1.00

Find the best big- O function for $\sum_{j=1}^n (j^3 + j)$

- ☐ a. $O(n^4)$
- ☒ b. $O(n)$
- ☐ c. $O(n^3)$
- ☐ d. $O(n^2)$

✗

Your answer is incorrect.

Question **29**

Correct

Mark 1.00 out of 1.00

If a and b are two distinct primes, the ab^2 has ___ positive divisors.

- ☒ a. 6
- ☐ b. 5
- ☐ c. 2
- ☐ d. 4
- ☐ e. 3



