Started on	Wednesday, 19 October 2022, 7:36 AM
State	Finished
Completed on	Wednesday, 19 October 2022, 8:17 AM
Time taken	41 mins 14 secs
Marks	30.00/30.00
Grade	**10.00** out of 10.00 (**100**%)
Feedback	Excellent!

Question **1**

Correct

Mark 1.00 out of 1.00

Each user has a password 6 characters long where each character is an uppercase letter, a lowercase letter, or a digit. Each password must contain at least one digit.
How many possible passwords are there?

a. $(26 + 10)^6 - 26^6$

b. $(26 + 26 + 10)^6 - (26 + 26)^6$ ✔

c. None of these

d. $(26 + 26 + 10)^6 - 10^6$

e. $(26 + 26 + 10)^6 - (26 + 10)^6$

Question 2

Correct

Mark 1.00 out of 1.00

Find 2^{100} mod 15

- a. 1 ✔
- b. None of the other choices is correct
- c. 2
- d. 8
- e. 4

Question 3

Correct

Mark 1.00 out of 1.00

Given the recursive definition of a set A:

BASIS STEP. $1 \in A, 4 \in A, 5 \in A$

RECURSIVE STEP. $x \in A \rightarrow x+3 \in A$

Let B be the set $\{x \in A \mid x < 15\}$. Find B.

a. None of the other choices is correct

b. B = {1, 4, 5, 7, 8}

c. B = {1, 4, 5, 7, 8, 10, 11, 13, 14}

d. B = {1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}

e. B = {1, 4, 5}

Question **4**

Correct

Mark 1.00 out of 1.00

Consider the set A defined recursively by

$1 \in A; 3 \in A,$

$x \in A \rightarrow x + 4 \in A$

Find {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} - A

- a. {2, 4, 6, 8, 10} ✔
- b. None of these
- c. {2, 4, 5, 6, 7, 8, 9, 10}
- d. {1, 3, 5, 7, 9}

Question 5

Correct

Mark 1.00 out of 1.00

Suppose you have 23 books (10 novels, 8 history books, and 5 math books). Assume that all 23 books are different. In how many ways can you
1) put the 23 books in a row on a shelf?
2) put the 23 books in a row on a shelf if the novels are on the left, the math books are in the middle, and the history books are on the right?

a. 23!, 10! + 5! + 8!

b. None of these

c. 23!, 10 × 5 × 8

d. 23!, 10! × 5! × 8!

Question **6**

Correct

Mark 1.00 out of 1.00

Suppose you wish to use the Principle of Mathematical Induction to prove
P(n): $1 \times 1! + 2 \times 2! + 3 \times 3! + \ldots + n \times n! = (n + 1)! - 1$ for all integers $n > 0$.
Write P(3).

a. 23
b. None of these
c. 3×3!
d. $1 \times 1! + 2 \times 2! + 3 \times 3! = 4! - 1$
e. $1 \times 1! + 2 \times 2! + 3 \times 3!$

Question 7

Correct

Mark 1.00 out of 1.00

1) Find the number of subsets of S = {1, 2, 3, 4, 5, 6, 7,} that contain the number 5.
2) Find the number of subsets of S = {1, 2, 3, 4, 5, 6, 7,} that do not contain the number 5.

- a. $2^6, 2^6$
- b. None of these
- c. $2^6, 2^7 - 1$
- d. $1 + 2^6, 2^7 - 1$

Question 8

Correct

Mark 1.00 out of 1.00

How many one-to-one functions are there from the set with three elements to a set with six elements?

- a. 18
- b. 30
- c. None of the other choices is correct
- d. 120
- e. 6!

Question 9

Correct

Mark 1.00 out of 1.00

Give a recursive definition with initial condition(s) of the sequence $a_1 = 16$, $a_2 = 13$, $a_3 = 10$, $a_4 = 7$, ...

a. $a_1 = 16$, $a_n = 3 - a_{n-1}$, for n > 1

b. None of these

c. $a_1 = 16$, $a_n = a_{n-1} - 3$ for n > 1

d. $a_1 = 16$, $a_{n+1} = a_n - 3$, for n > 1

Question **10**

Correct

Mark 1.00 out of 1.00

Consider the following procedure

procedure *T*(n: non-negative integer)

if *n* < 3 **then** *return n*

else return {n + *T(n-1) - T(n-2)*}

Find T(3), T(4), T(5)

- a. 4, 7, 6
- b. None of these
- c. 5, 6, 7
- d. 4, 5, 6
- e. 4, 6, 7 ✔

Question 11

Correct

Mark 1.00 out of 1.00

Give as good a big-oh estimate as possible for $(n\log n + n)(n^2 + 1)$.

a. $o(n^2)$

b. $O(n^3)$

c. $O(n^2 \log n)$

d. None of these

e. $O(n^3 \log n)$

Question 12

Correct

Mark 1.00 out of 1.00

Suppose that a "word" is any string of six letters of the alphabet, with repeated letters allowed.

1) How many words end with KA?

2) How many words begin with K and end with A?

a. 26^4, 26^4

b. 26^4, $2 \cdot 26^4$

c. 26^4, $2 \cdot 26^5 - 26^4$

d. None of these

e. $4 \cdot 26$, $4! \cdot 26^2$

Question 13

Correct

Mark 1.00 out of 1.00

Give a recursive definition of the set

A = {1, 5, 25, 125, 625, ...}

Which one is true?

(i) $1 \in \mathrm{A}; x \in A \rightarrow 5x \in A$
(ii) $1 \in \mathrm{A}; x \in A \rightarrow 5^x \in A$

a. Only (i) ✔

b. Only (ii)

c. Neither

d. Both

Question **14**

Correct

Mark 1.00 out of 1.00

Which of the following integers is **congruent** to -37 modulo 7?

- a. 57
- b. 40 ✔
- c. 37
- d. 49
- e. None of these

Question **15**

Correct

Mark 1.00 out of 1.00

Which of the following is a **recursive definition** of the function F(n) = 1 - 3n, n = 1, 2, 3, ... ?

- a. F(1) = -2 and F(n) = -F(n-1) - 3 for n > 1
- b. F(1) = -2 and F(n) = -F(n-1) + 3 for n > 1
- c. F(1) = -2 and F(n+1) = F(n) - 3 for n > 1
- d. None of the other choices is correct
- e. F(1) = -2 and F(n) = F(n-1) - 3 for n > 1 ✔

Question 16

Correct

Mark 1.00 out of 1.00

Consider the bubble sort algorithm

procedure *bubblesort*($a_1, \ldots, a_n$: real numbers with $n \geq 2$)
for $i := 1$ **to** $n - 1$
  for $j := 1$ **to** $n - i$
    if $a_j > a_{j+1}$ **then** interchange a_j and a_{j+1}
{$a_1, \ldots, a_n$ is in increasing order}

Use the bubble sort to put 3, 2, 4, 1, 5 into increasing order. What is the order of the numbers **after the second pass** (i = 2)?

a. 3, 1, 2, 4, 5
b. None of the other choices is correct
c. 1, 2, 3, 4, 5
d. 2, 3, 1, 4, 5
e. 2, 1, 3, 4, 5

Question 17

Correct

Mark 1.00 out of 1.00

Given the function

$$A(m,n) = \begin{cases} 2n & \text{if } m = 0 \\ 0 & \text{if } m \geq 1 \text{ and } n = 0 \\ 2 & \text{if } m \geq 1 \text{ and } n = 1 \\ A(m-1, A(m,n-1)) & \text{if } m \geq 1 \text{ and } n \geq 2 \end{cases}$$

Find A(2, 2)

- a. 1
- b. 4 ✔
- c. 2
- d. 8
- e. None of these

Question **18**

Correct

Mark 1.00 out of 1.00

Consider all bit strings of length 8.

1) How many begin with 010?

2) How many begin with 01 and end with 10?

- a. 2^5, $2.2^5 - 2^4$
- b. None of these
- c. 2^5, 2^4 ✔
- d. 10, 8

Question **19**

Correct

Mark 1.00 out of 1.00

How many strings of **four ASCII characters** contain the character @ at least once?
[Note: There are 128 different ASCII characters.]

- a. None of the other choices is correct
- b. 4*128 + 3*128 + 2*128 + 128
- c. $128^4 - 127^4$ ✔
- d. $128^4 - 1$

Question 20

Correct

Mark 1.00 out of 1.00

Suppose that a "word" is any string of seven letters of the alphabet, with repeated letters allowed.

1) How many words have no vowels?

2) How many words begin with a vowel and end with a vowel?

- a. 5.26^6, 25.26^5
- b. 21^7, 25.26^5 ✔
- c. 21^7, $5.26^6 + 5.26^6$
- d. None of these

Question 21

Correct

Mark 1.00 out of 1.00

Determine whether the integers in the set {21, 34, 55} are pairwise relatively prime.

(That is, we need each pair of (21, 34), (34, 55), (21, 55) to be relatively prime)

- a. No, because gcd(21, 55) $\neq$ 1
- b. No, because gcd(21, 34) $\neq$ 1
- c. No, because gcd(34, 55) $\neq$ 1
- d. Yes ✔

Question 22

Correct

Mark 1.00 out of 1.00

Give a recursive definition with initial condition(s) of
f(n) = (n - 1)!, n = 1, 2, 3, ...

- a. f(1) = 1, and f(n) = (n-1)f(n-1) for n > 1
- b. f(1) = 1, and f(n) = (n-1)f(n-2) for n > 1
- c. None of these
- d. f(1) = 0, and f(n) = (n-1)f(n-1) for n > 1
- e. f(1) = 1, and f(n) = nf(n-1) for n > 1

Question 23

Correct

Mark 1.00 out of 1.00

Estimate the best big-oh for the function

$$\frac{6n+4n^5-4}{7n^2-3}$$

a. $O(n^2)$

b. $O(n^3)$

c. $O(n^5)$

d. $O(n^4)$

e. None of the other choices is correct

Question 24

Correct

Mark 1.00 out of 1.00

Suppose that a “word” is any string of seven letters of the alphabet, with repeated letters allowed.

1) How many words begin with the letter K?

2) How many words begin with A or end with B?

a. None of these

b. 26^6, $26^6 + 26^6 - 26^5$ ✔

c. 26^6, $26^6 - 26^5$

d. $6!.26$, $5!.26^2$

e. 26^6, 2.26^6

Question 25

Correct

Mark 1.00 out of 1.00

Consider two set A, B such that $|A| = 13$, $|A - B| = 10$ and $|B - A| = 2$. Find $|B|$.

a. 5 ✔

b. 8

c. 6

d. 7

Question 26

Correct

Mark 1.00 out of 1.00

Give the best big-oh estimate for the function

f(n) = 1 + 2 + 3 + ... + n

- a. $O(n^4)$
- b. None of these
- c. $O(n^2)$ ✔
- d. $O(n^3)$
- e. O(n)

Question 27

Correct

Mark 1.00 out of 1.00

Find a_1, a_2, a_3, a_4 if a_n is recursively defined by:

BASIS STEP. $a_1 = 3$, $a_2 = 2$

RECURSIVE STEP. $a_n = 4a_{n-1} - 3a_{n-2}$ for $n > 2$

- a. 3, 2, -1, -10 ✔
- b. 3, 2, -1, -7
- c. None of the other choices is correct
- d. 3, 2, 6, 18
- e. 2, 3, -1, -7

Question 28

Correct

Mark 1.00 out of 1.00

The integers $a_1, a_2, \ldots, a_n$ are *pairwise relatively prime* if $\gcd(a_i, a_j) = 1$ whenever $1 \leq i < j \leq n$.

Suppose 21, 25, m are pairwise relative prime. Select m from the following integers or say that no such m.

- a. 6
- b. 5
- c. 7
- d. 8 ✔
- e. No such m

Question 29

Correct

Mark 1.00 out of 1.00

Find a + b if

a = -37 mod 7

and b = 37 mod 7

- a. 10
- b. -1
- c. 0
- d. 7 ✔
- e. None of these

Question 30

Correct

Mark 1.00 out of 1.00

To prove the $2^n < n!$, for $n \geq 4$ we use mathematical induction.

Make the correct order for the following proof.

1. Suppose for every $k \geq 4, 2^k < k!$
2. Therefore, the statement is true for every $n \geq 4$.
3. $2^4 < 4!$
4. We have

$$2^{k+1} = 2 \cdot 2^k < 2 \cdot k! < (k+1)k! = (k+1)!$$

a. None of the other choices is correct

b. 3, 1, 4, 2

c. 3, 1, 2, 4

d. 1, 3, 2, 4

e. 3, 2, 1, 4