Comparison of some Non-parametric tests with Parametric counterparts

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CONTENTS

Introduction:

In our daily life everybody make inference. If we see the 'Kash' flower at the riverside we will infer that 'Durga Puja' is coming. In newspaper we can see some article claims that the people who drinks $4-6$ litre of water daily has less digestive problems. Not only human even animals make inference. For example, a monkey knows which fruit is more ripe and eat the riper fruit.

Basically, inference is a logical guess or an opinion that one form based on the information that he has. Statistically, inference means drawing conclusion about the characteristics of a finite population or about the or about the parameter of a probability distribution based on statistical analysis of a random sample. There are two kind of inferences in statistics one is parametric inference and another is non-parametric inference.

A *parametric test* is a test whose model specifies certain conditions about the parameters of the population from which the samples are drawn.

On the other hand, *non-parametric (N.P) test* does not make any assumption regarding the form of the population. Certain assumptions associated with non-parametric tests are fewer and much weaker than those associated with parametric tests. Below we shall give briefly the advantages and disadvantages of non-parametric methods over parametric methods.

Advantages of Non-parametric Methods:

- Non parameteric methods are readily comprehensible, very simple and easy to apply .
- No assumption is made about the form of the frequency function of the parent population from which sampling is done.
- No parametric technique will apply to the data which are measured in categorical scale, while non-parametric methods exist to deal with such data.
- Non-parametric tests are available to deal with the data which are given in ranks or whose seemingly numerical scores have the strength of ranks. For instance. no parametric test can be applied if the scores are given in grades such as A+, A , B, B+, etc.

Disadvantages of Non-parametric Methods:

There are not many disadvantages of non-parametric tests. But most notable one is it is less powerful than parametric test if the assumptions about parental distribution fits the data well.

There are lot of tests (parametric and non-parametric both) for testing of hypothesis. Pitman's efficiency is a well-defined way for comparing those tests.

Objective:

Our objective in this project is to compare different parametric and non-parametric tests for location parameter under single sample and two independent samples set up. We will use asymptotic relative efficiency and power function to compare the test.

Pitman's Asymptotic Relative Efficiency

Suppose we have two test statistics T_n and T_n^*, for a data consisting of n observartions and both statistics are consistent for the test of

$$H_0 : \theta \in \Theta_0 \qquad H_1 : \theta \in \Theta_0^c$$

at the significance level α, that is

$$\lim_{n\to\infty} \text{Power}(T_n) = 1 \qquad \text{and} \qquad \lim_{n\to\infty} \text{Power}(T_n^*) = 1$$

The asymptotic relative effciency(ARE) of T_n relative to T_n^* is the limiting value of the ratio n_2/n_1 where n_1 is the number of observations required by T_n for

$$\text{Power}(T_n) = \text{Power}(T_n^*)$$

based on n_2 observations such that $n_2 \to \infty$.

1 Single Sample Test for Location Parameter:

Let $X_1, X_2, \ldots, X_n$ be a random sample with cumulative distribution function $F_X(\cdot - \theta), \theta \in \mathbb{R}$, where θ is the location parameter. We want to test the null hypothesis

$$H_0 : \theta = \theta_0$$

against the alternative hypothesis $H_1 : \theta > \theta_0$, $H_2 : \theta < \theta_0$ and $H_3 : \theta \neq \theta_0$

If the form of $F(\cdot)$ is unknown, non-parametric tests are used to test H_0. Sign test and Wilcoxon sign-rank test are most popular non-parametric test for location parameter. To conduct these two test we have to assume that $F(\cdot)$ is continuous and symmetric about location parameter.

1.1 Sign Test:

Let us define,

$$Y_i = \begin{cases} 1, & \text{if } X_i > \theta_0 \\ 0, & \text{if } X_i \leqslant \theta_0 \end{cases} \qquad \forall i = 1, 2, \ldots, n.$$

then the test statistic for testing the null hypothesis $H_0 : \theta = \theta_0$ is given by,

$$T_1 = \sum_{i=1}^{n} Y_i$$

and we reject H_0 against

$$H_1 : \theta > \theta_0 \quad \text{if} \quad T_1 \geqslant b_{n,\alpha}$$

$$H_2 : \theta < \theta_0 \quad \text{if} \quad T_1 \leqslant b_{n,\alpha}$$

$$H_3 : \theta \neq \theta_0 \quad \text{if} \quad T_1 \neq b_{n,\frac{\alpha}{2}}$$

where, $b_{n,\alpha} = c_1$ is such that

$$P_{H_0}(T_1 > c_1) \leqslant \alpha$$

where α is the level of significance.

Now, we know that the power of the test that is $P_{H_1}(T_1 > c_1)$ goes to 1 as n goes large. We have find n such that

$$P_{H_1}(T_1 > c_1) \geq 1 - \beta$$

for some $\beta > 0$.

Suppose, the minimum sample size to get $P_{H_1}(T_1 > c_1) \geq 1 - \beta$ is denoted as n_a.

1.2 Wilcoxon Signed-Rank Test:

Let us define, $D_i = X_i - \theta_0, \forall i = 1(1)n$ and

$$Z_i = \begin{cases} 1, & \text{if} \quad D_i > 0 \\ 0, & \text{if} \quad D_i \leqslant 0 \end{cases} \quad \forall i = 1, 2, \ldots, n$$

Now we have to take $|D_i|$ and ranked them as $1, 2, \ldots, n$ from smallest to largest. Suppose R_i is rank of $|D_i|, \forall i = 1(1)n$

Wilcoxon signed-rank test uses the test statistic

$$T_2 = \sum_{i=1}^{n} Z_i \times rank(|D_i|) = \sum_{i=1}^{n} Z_i \times R_i$$

We reject H_0 against

$$H_1 : \theta > \theta_0 \quad \text{if} \quad T_2 \geqslant T_\alpha$$

$$H_2 : \theta < \theta_0 \quad \text{if} \quad T_2 \leqslant T_\alpha$$

$$H_3 : \theta \neq \theta_0 \quad \text{if} \quad T_2 \neq T_{\frac{\alpha}{2}}$$

where, $T_\alpha = c_2$ is such that

$$P_{H_0}(T_2 > c_2) \leqslant \alpha$$

where α is the level of significance.

We have find n such that

$$P_{H_1}(T_2 > c_2) \geq 1 - \beta$$

Suppose, the minimum sample size to get $P_{H_1}(T_2 > c_2) \geq 1 - \beta$ is denoted as n_b.

1.3 t-test:

The parametric counter part for the test for location parameter is student's t-test. The student's t-test will be used if the distribution of the random variable X is known to be normal. But, in practice t-test is use to test for location parameter even if the assumption of normality of the distribution of X does not hold. The Procedure of the t-test can be describe as follows,

$X_1, X_2, \ldots, X_n \sim N(\mu, \sigma^2)$, here μ is location parameter and σ^2 is unknown. Define

$$T_3 = \frac{\sqrt{n} \cdot \bar{X}}{s}$$

where $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ = sample mean and $s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ = sample variance.

To test $H_0 : \mu = \mu_0$ T_3 is used as test statistic.

We reject H_0 against

$$H_1 : \mu > \mu_0 \quad \text{if} \quad T_3 \geqslant \tau_\alpha$$
$$H_2 : \mu < \mu_0 \quad \text{if} \quad T_3 \leqslant \tau_\alpha$$
$$H_3 : \mu \neq \mu_0 \quad \text{if} \quad T_3 \neq \tau_{\frac{\alpha}{2}}$$

where, $\tau_\alpha = c_3$ is such that

$$P_{H_0}(T_3 > c_3) \leqslant \alpha$$

where α is the level of significance.

We have find $n \ni$

$$P_{H_1}(T_3 > c_3) \geq 1 - \beta$$

Suppose, the minimum sample size to get $P_{H_1}(T_3 > c_3) \geq 1 - \beta$ is denoted as n_c.

The above three tests are consistent test. We have used Monte-Carlo simulation to obtain n_a, n_b, n_c and c_1, c_2, c_3.

Let us define $RE_1 = \frac{n_a}{n_b}$ is the Pitman relative efficiency of signed-rank test with respect to sign test and $RE_2 = \frac{n_a}{n_c}$ is the Pitman relative efficiency of t-test with respect to sign test and $RE_3 = \frac{n_b}{n_c}$ is the Pitman relative efficiency of t-test with respect to signed-rank test.

Here are the relative efficiencies followed by the following tables give the minimum sample size required to get the size < 0.05 and the power > 0.95 when $F(\cdot)$ follows normal, Laplace, logistic distributions.

Table for testing $H_0 : F(\cdot) = N(0,1)$ against $H_1 : F(\cdot) = N(1,1)$

Test	Minimum sample size required	Power	Size
Sign test	22	0.953	0.032
Wilcoxon signed-rank test	29	0.971	0.05
t-test	14	0.96	0.05

Table 1.1

Relative efficiencies for testing $H_0 : F(\cdot) = N(0,1)$ **against** $H_1 : F(\cdot) = N(1,1)$**:**
Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.7586207$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.571429$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.071429$

Now to compare the tests with respect to their powers, we find the sample size and compare their respective powers. In the following table 1.1(a), power of the tests are compared if sample size for all the tests are equal to sample size needed for Wilcoxon signed rank test and table 1.1(b) compares the powers of test when the sample size fixed at sample size needed for sign test to have power greater than 0.95 and size less than 0.05.

Test	sample size	Power	Size
Sign test	29	0.969	0.012
Wilcoxon signed-rank test	29	0.971	0.05
t-test	29	1	0.057

Table 1.1a

Test	sample size	Power	Size
Sign test	22	0.953	0.032
t-test	22	0.998	0.046

Table 1.1b

Table for testing $H_0 : F(\cdot) = N(0,1)$ **against** $H_1 : F(\cdot) = N(2,1)$

Test	Minimum sample size required	Power	Size
Sign test	9	0.994	0.034
Wilcoxon signed-rank test	13	0.997	0.046
t-test	5	0.98	0.047

Table 1.2

Relative efficienciesfor testing $H_0 : F(\cdot) = N(0,1)$ **against** $H_1 : F(\cdot) = N(2,1)$**:**
Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.692307692$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.8$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.6$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the results will be

Test	sample size	Power	Size
Sign test	13	0.963	0.001
Wilcoxon signed-rank test	13	0.997	0.046
t-test	13	1	0.063

Table 1.2a

Test	sample size	Power	Size
Sign test	9	0.994	0.034
t-test	9	1	0.067

Table 1.2b

Table for testing $H_0 : F(\cdot) = N(0,1)$ **against** $H_1 : F(\cdot) = N(3,1)$

Test	Minimum sample size required	Power	Size
Sign test	6	0.994	0.02
Wilcoxon signed-rank test	8	0.995	0.05
t-test	4	0.976	0.044

Table 1.3

Relative efficiencies for testing $H_0 : F(\cdot) = N(0,1)$ **against** $H_1 : F(\cdot) = N(3,1)$**:**
Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.75$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.5$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	8	0.989	0.008
Wilcoxon signed-rank test	8	0.995	0.05
t-test	8	1	0.075

Table 1.3a

Test	sample size	Power	Size
Sign test	6	0.994	0.02
t-test	6	1	0.08

Table 1.3b

Table for testing $H_0 : F(\cdot) = N(0,1)$ **against** $H_1 : F(\cdot) = N(4,1)$

Test	Minimum sample size required	Power	Size
Sign test	6	1	0.023
Wilcoxon signed-rank test	8	1	0.049
t-test	3	0.989	0.049

Table 1.4

Relative efficiencies for testing $H_0 : F(\cdot) = N(0,1)$ **against** $H_1 : F(\cdot) = N(4,1)$**:**
Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.75$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 2$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.6667$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	8	1	0.005
Wilcoxon signed-rank test	8	1	0.049
t-test	8	1	0.075

Table 1.4a

Test	sample size	Power	Size
Sign test	6	1	0.023
t-test	6	1	0.08

Table 1.4b

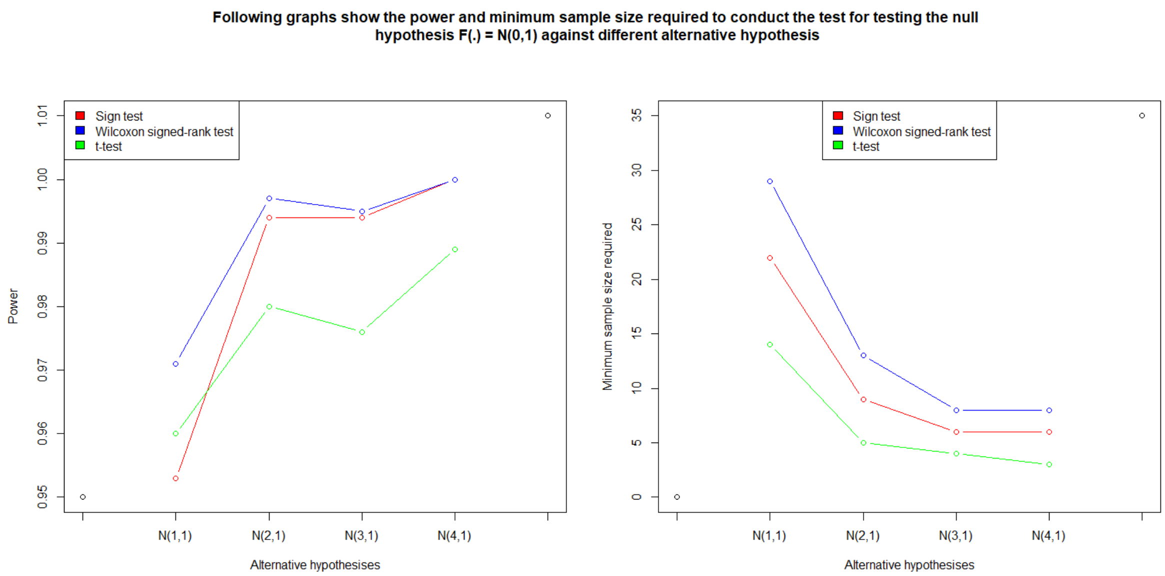

Chart 1.1

Table for testing $H_0 : F(\cdot) = Laplace(0, 1)$ **against** $H_1 : F(\cdot) = Laplace(1, 1)$

Test	Minimum sample size required	Power	Size
Sign test	26	0.962	0.05
Wilcoxon signed-rank test	33	0.976	0.05
t-test	28	0.951	0.029

Table 1.5

Relative efficiencies for testing $H_0 : F(\cdot) = Laplace(0, 1)$ **against** $H_1 : F(\cdot) = Laplace(1, 1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.7878788$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 0.9285714$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 1.1785714$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	33	0.974	0.011
Wilcoxon signed-rank test	33	0.976	0.05
t-test	33	0.984	0.044

Table 1.5a

Test	sample size	Power	Size
Sign test	26	0.962	0.05
t-test	26	0.965	0.049

Table 1.5b

Table for testing $H_0 : F(\cdot) = Laplace(0, 1)$ **against** $H_1 : F(\cdot) = Laplace(2, 1)$

Test	Minimum sample size required	Power	Size
Sign test	12	0.953	0.033
Wilcoxon signed-rank test	16	0.986	0.05
t-test	9	0.973	0.046

Table 1.6

Relative efficiencies for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = Laplace(2,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.75$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.333333$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 1.777778$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	16	0.987	0.013
Wilcoxon signed-rank test	16	0.986	0.05
t-test	16	0.998	0.064

Table 1.6a

Test	sample size	Power	Size
Sign test	12	0.953	0.033
t-test	12	0.995	0.061

Table 1.6b

Table for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = Laplace(3,1)$

Test	Minimum sample size required	Power	Size
Sign test	9	0.991	0.03
Wilcoxon signed-rank test	13	0.997	0.05
t-test	5	0.974	0.05

Table 1.7

Relative efficiencies for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = Laplace(3,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.69231$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.8$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.6$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	13	0.961	0.001
Wilcoxon signed-rank test	13	0.997	0.05
t-test	13	1	0.067

Table 1.7a

Test	sample size	Power	Size
Sign test	9	0.991	0.03
t-test	9	1	0.063

Table 1.7b

Table for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = Laplace(4,1)$

Test	Minimum sample size required	Power	Size
Sign test	9	0.996	0.031
Wilcoxon signed-rank test	10	1	0.05
t-test	5	0.966	0.019

Table 1.8

Relative efficiencies for testing $H_0 : F(\cdot) = Laplace(0, 1)$ **against** $H_1 : F(\cdot) = Laplace(4, 1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.9$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.8$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	10	0.999	0.006
Wilcoxon signed-rank test	10	1	0.05
t-test	10	1	0.063

Table 1.8a

Test	sample size	Power	Size
Sign test	9	0.996	0.031
t-test	9	1	0.063

Table 1.8b

Following graphs show the power and minimum sample size required to conduct the test for testing the null hypothesis F(.) = Laplace(0,1) against different alternative hypothesis

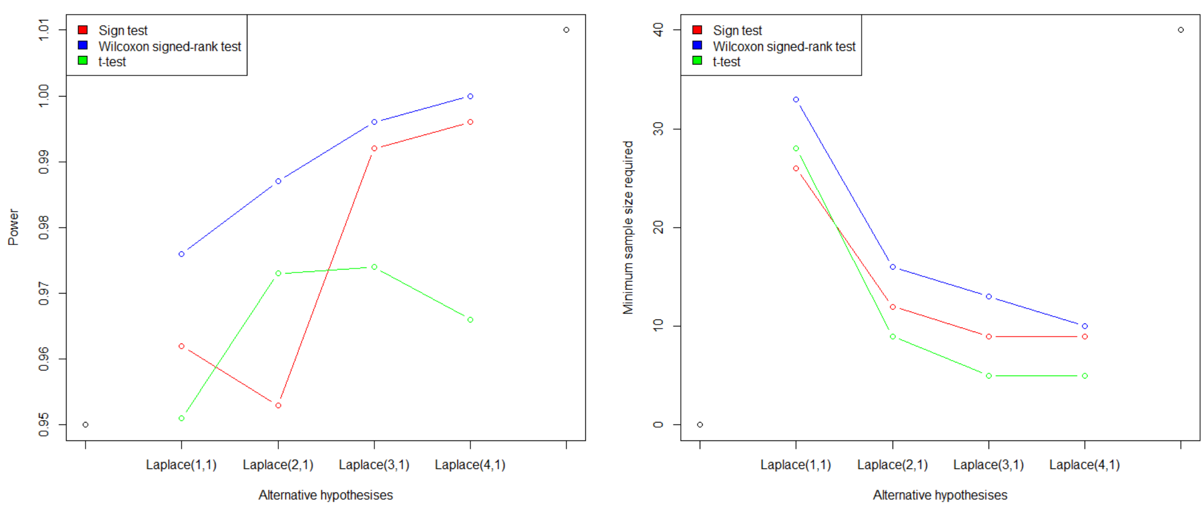

Chart 1.2

Table for testing $H_0 : F(\cdot) = logistic(0, 1)$ **against** $H_1 : F(\cdot) = logistic(1, 1)$

Test	Minimum sample size required	Power	Size
Sign test	51	0.966	0.05
Wilcoxon signed-rank test	57	0.953	0.046
t-test	43	0.952	0.03

Table 1.9

Relative efficiencies for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = logistic(1,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.89474$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.18605$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 1.32558$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	57	0.968	0.03
Wilcoxon signed-rank test	57	0.953	0.046
t-test	57	0.99	0.054

Table 1.9a

Test	sample size	Power	Size
Sign test	51	0.966	0.05
t-test	51	0.988	0.051

Table 1.9b

Table for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = logistic(2,1)$

Test	Minimum sample size required	Power	Size
Sign test	16	0.971	0.041
Wilcoxon signed-rank test	22	0.982	0.047
t-test	13	0.953	0.046

Table 1.10

Relative efficiencies for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = logistic(2,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.72727$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.23077$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 1.69231$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	22	0.966	0.008
Wilcoxon signed-rank test	22	0.982	0.047
t-test	22	0.998	0.057

Table 1.10a

Test	sample size	Power	Size
Sign test	16	0.971	0.041
t-test	16	0.992	0.051

Table 1.10b

Table for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = logistic(3,1)$

Test	Minimum sample size required	Power	Size
Sign test	12	0.984	0.032
Wilcoxon signed-rank test	14	0.989	0.042
t-test	6	0.955	0.049

Table 1.11

Relative efficiencies for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = logistic(3,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.85714$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 2$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.33333$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	14	0.976	0.005
Wilcoxon signed-rank test	14	0.989	0.042
t-test	14	1	0.06

Table 1.11a

Test	sample size	Power	Size
Sign test	12	0.984	0.032
t-test	12	0.999	0.065

Table 1.11b

Table for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = logistic(4,1)$

Test	Minimum sample size required	Power	Size
Sign test	9	0.992	0.036
Wilcoxon signed-rank test	12	0.998	0.05
t-test	6	0.957	0.014

Table 1.12

Relative efficiencies for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = logistic(4,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.75$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.5$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	12	0.985	0.003
Wilcoxon signed-rank test	12	0.998	0.05
t-test	12	1	0.04

Table 1.12a

Test	sample size	Power	Size
Sign test	9	0.992	0.036
t-test	9	1	0.04

Table 1.12b

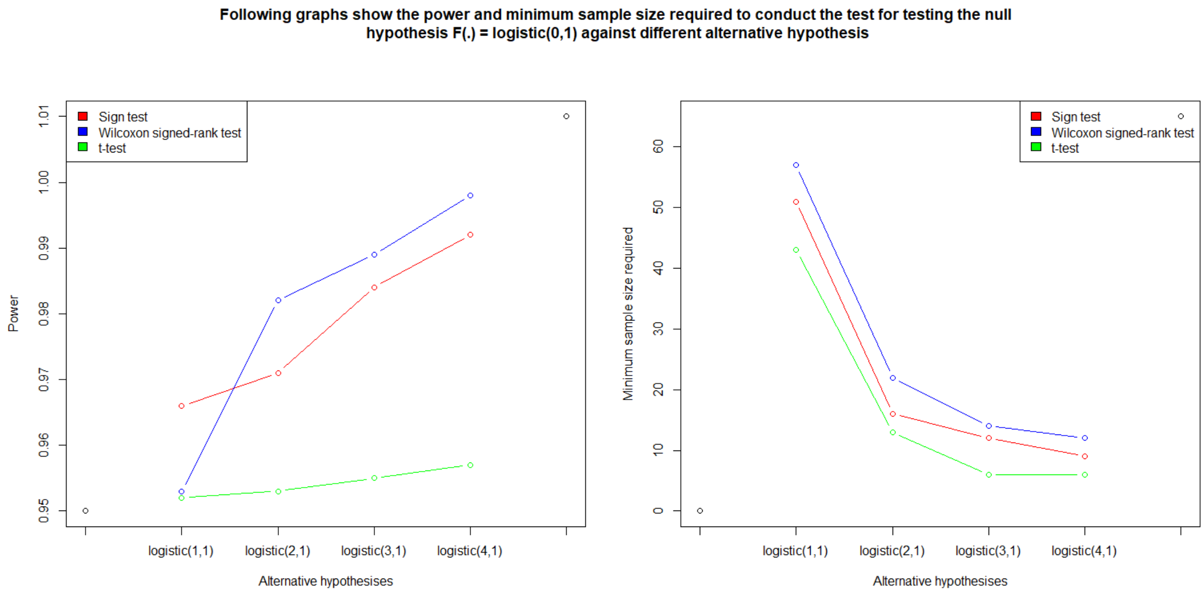

Chart 1.3

Table for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = logistic(1,1)$

Test	Minimum sample size required	Power	Size
Sign test	51	0.971	0.046
Wilcoxon signed-rank test	62	0.951	0.044
t-test	43	0.952	0.032

Table 1.13

Relative efficiencies for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = logistic(1,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.822580645$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.186046512$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 1.441860465$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	62	0.986	0.05
Wilcoxon signed-rank test	62	0.951	0.044
t-test	62	0.992	0.049

Table 1.13a

Test	sample size	Power	Size
Sign test	51	0.971	0.046
t-test	51	0.984	0.054

Table 1.13b

Table for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = logistic(2,1)$

Test	Minimum sample size required	Power	Size
Sign test	17	0.973	0.029
Wilcoxon signed-rank test	22	0.983	0.041
t-test	12	0.959	0.043

Table 1.14

Relative efficiencies for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = logistic(2,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.772727273$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.416666667$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 1.833333333$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	22	0.962	0.007
Wilcoxon signed-rank test	22	0.983	0.041
t-test	22	0.998	0.051

Table 1.14a

Test	sample size	Power	Size
Sign test	17	0.973	0.029
t-test	17	0.996	0.054

Table 1.14b

Table for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = logistic(3,1)$

Test	Minimum sample size required	Power	Size
Sign test	12	0.989	0.021
Wilcoxon signed-rank test	15	0.998	0.043
t-test	6	0.966	0.042

Table 1.15

Relative efficiencies for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = logistic(3,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.8$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 2$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.5$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	15	0.962	0.005
Wilcoxon signed-rank test	15	0.998	0.043
t-test	15	1	0.057

Table 1.15a

Test	sample size	Power	Size
Sign test	12	0.989	0.021
t-test	12	0.999	0.063

Table 1.15b

Table for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = logistic(4,1)$

Test	Minimum sample size required	Power	Size
Sign test	9	0.994	0.015
Wilcoxon signed-rank test	12	0.998	0.048
t-test	6	0.963	0.019

Table 1.16

Relative efficiencies for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = logistic(4,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.75$

Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.5$

Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	12	0.983	0.003
Wilcoxon signed-rank test	12	0.998	0.048
t-test	12	1	0.063

Table 1.16a

Test	sample size	Power	Size
Sign test	9	0.994	0.015
t-test	9	1	0.046

Table 1.16b

Following graphs show the power and minimum sample size required to conduct the test for testing the null hypothesis F(.) = N(0,1) against different alternative hypothesis

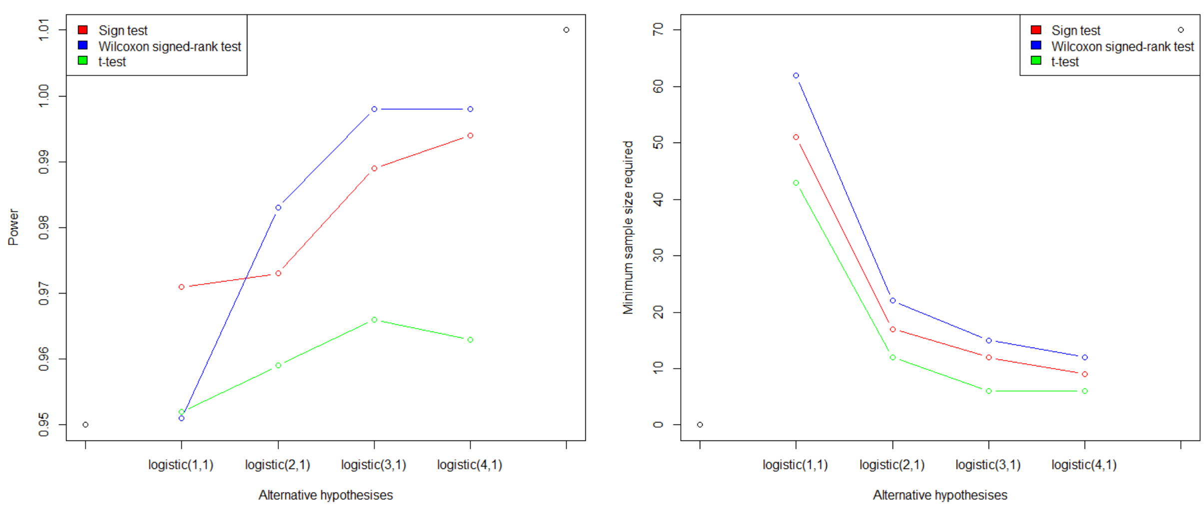

Chart 1.4

Table for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = normal(1,1)$

Test	Minimum sample size required	Power	Size
Sign test	22	0.968	0.021
Wilcoxon signed-rank test	28	0.977	0.045
t-test	14	0.964	0.038

Table 1.17

Relative efficiencies for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = normal(1,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.785714286$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.571428571$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	28	0.979	0.025
Wilcoxon signed-rank test	28	0.977	0.045
t-test	28	0.997	0.03

Table 1.17a

Test	sample size	Power	Size
Sign test	22	0.968	0.021
t-test	22	0.993	0.03

Table 1.17b

Table for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = normal(2,1)$

Test	Minimum sample size required	Power	Size
Sign test	9	0.992	0.018
Wilcoxon signed-rank test	13	0.999	0.045
t-test	5	0.987	0.045

Table 1.18

Relative efficiencies for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = normal(2,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.692307692$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.8$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.6$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	13	0.970	0.003
Wilcoxon signed-rank test	13	0.999	0.045
t-test	13	1	0.03

Table 1.18a

Test	sample size	Power	Size
Sign test	9	0.992	0.018
t-test	9	1	0.042

Table 1.18b

Table for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = normal(3,1)$

Test	Minimum sample size required	Power	Size
Sign test	6	0.997	0.016
Wilcoxon signed-rank test	9	1	0.045
t-test	4	0.987	0.02

Table 1.19

Relative efficiencies for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = normal(3,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.666666667$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.5$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.25$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	9	0.991	0.004
Wilcoxon signed-rank test	9	1	0.045
t-test	9	1	0.04

Table 1.19a

Test	sample size	Power	Size
Sign test	6	0.997	0.016
t-test	6	1	0.047

Table 1.19b

Table for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = normal(4,1)$

Test	Minimum sample size required	Power	Size
Sign test	6	1	0.01
Wilcoxon signed-rank test	8	1	0.03
t-test	3	0.994	0.044

Table 1.20

Relative efficiencies for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = normal(4,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.75$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 2$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.666666667$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	8	1	0.009
Wilcoxon signed-rank test	8	1	0.03
t-test	8	1	0.039

Table 1.20a

Test	sample size	Power	Size
Sign test	6	1	0.01
t-test	6	1	0.047

Table 1.20b

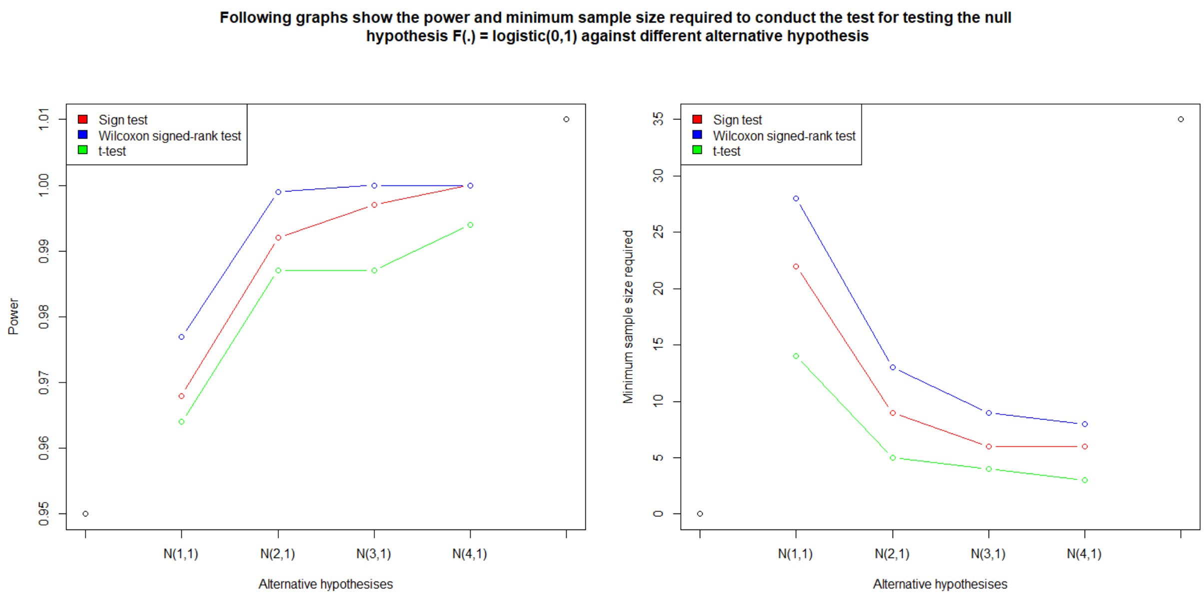

Chart 1.5

Table for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = Laplace(1,1)$

Test	Minimum sample size required	Power	Size
Sign test	26	0.978	0.04
Wilcoxon signed-rank test	34	0.981	0.042
t-test	28	0.957	0.026

Table 1.21

Relative efficiencies for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = Laplace(1,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.764705882$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 0.928571429$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 1.214285714$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	34	0.959	0.014
Wilcoxon signed-rank test	34	0.981	0.042
t-test	34	0.974	0.027

Table 1.21a

Test	sample size	Power	Size
Sign test	26	0.978	0.04
t-test	26	0.939	0.03

Table 1.21b

Table for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = Laplace(2,1)$

Test	Minimum sample size required	Power	Size
Sign test	12	0.964	0.022
Wilcoxon signed-rank test	17	0.995	0.043
t-test	8	0.952	0.048

Table 1.22

Relative efficiencies for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = Laplace(2,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.705882353$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.5$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.125$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	17	0.967	0.007
Wilcoxon signed-rank test	17	0.995	0.043
t-test	17	0.996	0.038

Table 1.22a

Test	sample size	Power	Size
Sign test	12	0.964	0.022
t-test	12	0.987	0.036

Table 1.22b

Table for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = Laplace(3,1)$

Test	Minimum sample size required	Power	Size
Sign test	9	0.986	0.017
Wilcoxon signed-rank test	13	0.998	0.046
t-test	5	0.966	0.045

Table 1.23

Relative efficiencies for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = Laplace(3,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.692307692$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.8$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.6$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	13	0.966	0.001
Wilcoxon signed-rank test	13	0.998	0.046
t-test	13	1	0.038

Table 1.23a

Test	sample size	Power	Size
Sign test	9	0.986	0.017
t-test	9	0.995	0.04

Table 1.23b

Table for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = Laplace(4,1)$

Test	Minimum sample size required	Power	Size
Sign test	9	1	0.018
Wilcoxon signed-rank test	10	0.999	0.034
t-test	5	0.984	0.022

Table 1.24

Relative efficiencies for testing $H_0 : F(\cdot) = Normal(0,1)$ **against** $H_1 : F(\cdot) = Laplace(4,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.9$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.8$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	10	0.996	0.01
Wilcoxon signed-rank test	10	0.999	0.034
t-test	10	1	0.038

Table 1.24a

Test	sample size	Power	Size
Sign test	9	1	0.018
t-test	9	1	0.04

Table 1.24b

Following graphs show the power and minimum sample size required to conduct the test for testing the null hypothesis F(.) = N(0,1) against different alternative hypothesis

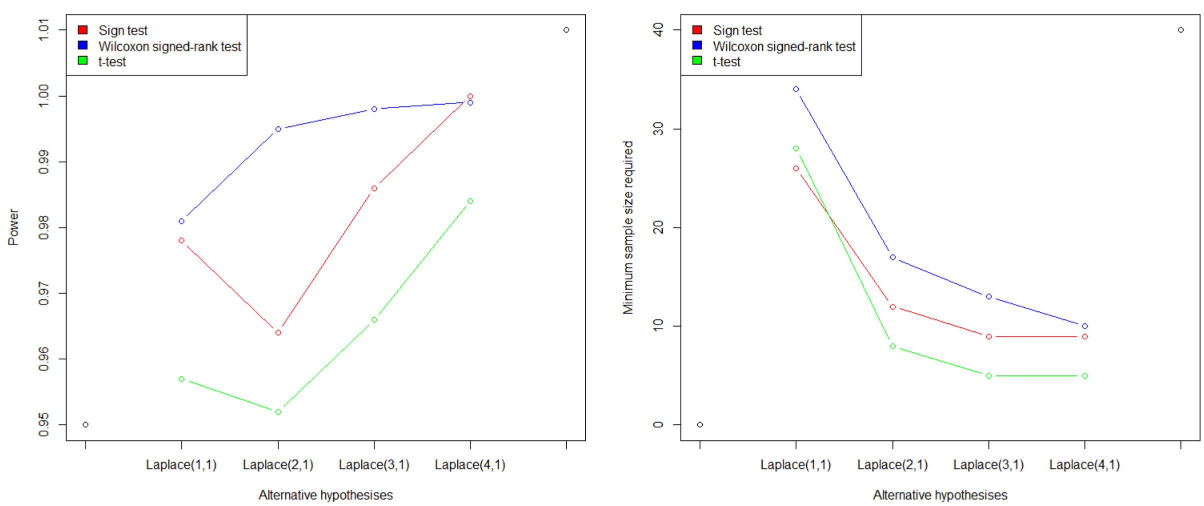

Chart 1.6

Table for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = normal(1,1)$

Test	Minimum sample size required	Power	Size
Sign test	22	0.963	0.027
Wilcoxon signed-rank test	28	0.976	0.043
t-test	14	0.967	0.028

Table 1.25

Relative efficiencies for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = normal(1,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.785714286$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.571428571$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	28	0.97	0.018
Wilcoxon signed-rank test	28	0.976	0.043
t-test	28	1	0.027

Table 1.25a

Test	sample size	Power	Size
Sign test	22	0.963	0.027
t-test	22	0.995	0.027

Table 1.25b

Table for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = normal(2,1)$

Test	Minimum sample size required	Power	Size
Sign test	9	0.988	0.02
Wilcoxon signed-rank test	13	0.999	0.049
t-test	5	0.98	0.05

Table 1.26

Relative efficiencies for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = normal(2,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.692307692$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.8$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.6$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	13	0.971	0.001
Wilcoxon signed-rank test	13	0.999	0.049
t-test	13	1	0.037

Table 1.26a

Test	sample size	Power	Size
Sign test	9	0.988	0.02
t-test	9	1	0.04

Table 1.26b

Table for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = normal(3,1)$

Test	Minimum sample size required	Power	Size
Sign test	6	0.997	0.016
Wilcoxon signed-rank test	9	1	0.03
t-test	4	0.986	0.027

Table 1.27

Relative efficiencies for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = normal(3,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.666666667$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.5$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.25$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	9	0.994	0.001
Wilcoxon signed-rank test	9	1	0.03
t-test	9	1	0.04

Table 1.27a

Test	sample size	Power	Size
Sign test	6	0.997	0.016
t-test	6	1	0.044

Table 1.27b

Table for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = normal(4,1)$

Test	Minimum sample size required	Power	Size
Sign test	6	1	0.023
Wilcoxon signed-rank test	8	1	0.031
t-test	3	0.996	0.031

Table 1.28

Relative efficiencies for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = normal(4,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.75$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 2$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.666666667$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	8	1	0.006
Wilcoxon signed-rank test	8	1	0.031
t-test	8	1	0.037

Table 1.28a

Test	sample size	Power	Size
Sign test	6	1	0.023
t-test	6	1	0.044

Table 1.28b

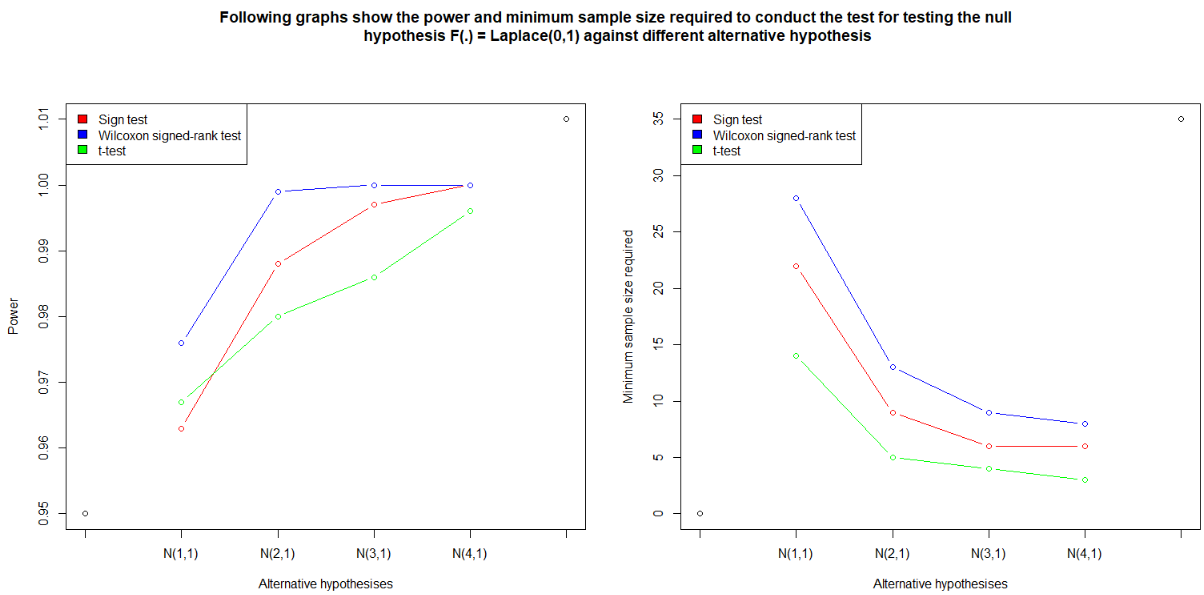

Chart 1.7

Table for testing $H_0 : F(\cdot) = Laplace(0, 1)$ **against** $H_1 : F(\cdot) = logistic(1, 1)$

Test	Minimum sample size required	Power	Size
Sign test	52	0.967	0.026
Wilcoxon signed-rank test	61	0.952	0.047
t-test	42	0.953	0.028

Table 1.29

Relative efficiencies for testing $H_0 : F(\cdot) = Laplace(0, 1)$ **against** $H_1 : F(\cdot) = logistic(1, 1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.852459016$

Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.238095238$

Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 1.452380952$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	61	0.952	0.024
Wilcoxon signed-rank test	61	0.952	0.047
t-test	61	0.982	0.022

Table 1.29a

Test	sample size	Power	Size
Sign test	52	0.967	0.026
t-test	52	0.972	0.024

Table 1.29b

Table for testing $H_0 : F(\cdot) = Laplace(0, 1)$ **against** $H_1 : F(\cdot) = logistic(2, 1)$

Test	Minimum sample size required	Power	Size
Sign test	17	0.968	0.028
Wilcoxon signed-rank test	22	0.987	0.036
t-test	12	0.963	0.034

Table 1.30

Relative efficiencies for testing $H_0 : F(\cdot) = Laplace(0, 1)$ **against** $H_1 : F(\cdot) = logistic(2, 1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.772727273$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.416666667$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 1.833333333$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	22	0.975	0.01
Wilcoxon signed-rank test	22	0.987	0.036
t-test	22	0.995	0.028

Table 1.30a

Test	sample size	Power	Size
Sign test	17	0.968	0.028
t-test	17	0.988	0.029

Table 1.30b

Table for testing $H_0 : F(\cdot) = Laplace(0, 1)$ **against** $H_1 : F(\cdot) = logistic(3, 1)$

Test	Minimum sample size required	Power	Size
Sign test	12	0.992	0.023
Wilcoxon signed-rank test	14	0.996	0.048
t-test	6	0.965	0.036

Table 1.31

Relative efficiencies for testing $H_0 : F(\cdot) = Laplace(0, 1)$ **against** $H_1 : F(\cdot) = logistic(3, 1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.857142857$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 2$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.333333333$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	14	0.976	0.007
Wilcoxon signed-rank test	14	0.996	0.048
t-test	14	0.999	0.033

Table 1.31a

Test	sample size	Power	Size
Sign test	12	0.992	0.023
t-test	12	0.998	0.03

Table 1.31b

Table for testing $H_0 : F(\cdot) = Laplace(0, 1)$ **against** $H_1 : F(\cdot) = logistic(4, 1)$

Test	Minimum sample size required	Power	Size
Sign test	9	0.994	0.024
Wilcoxon signed-rank test	12	1	0.05
t-test	4	0.969	0.047

Table 1.32

Relative efficiencies for testing $H_0 : F(\cdot) = Laplace(0,1)$ **against** $H_1 : F(\cdot) = logistic(4,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.75$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 2.25$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 3$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	12	0.982	0.001
Wilcoxon signed-rank test	12	1	0.05
t-test	12	1	0.03

Table 1.32a

Test	sample size	Power	Size
Sign test	9	0.994	0.024
t-test	9	0.999	0.04

Table 1.32b

Following graphs show the power and minimum sample size required to conduct the test for testing the null hypothesis F(.) = Laplace(0,1) against different alternative hypothesis

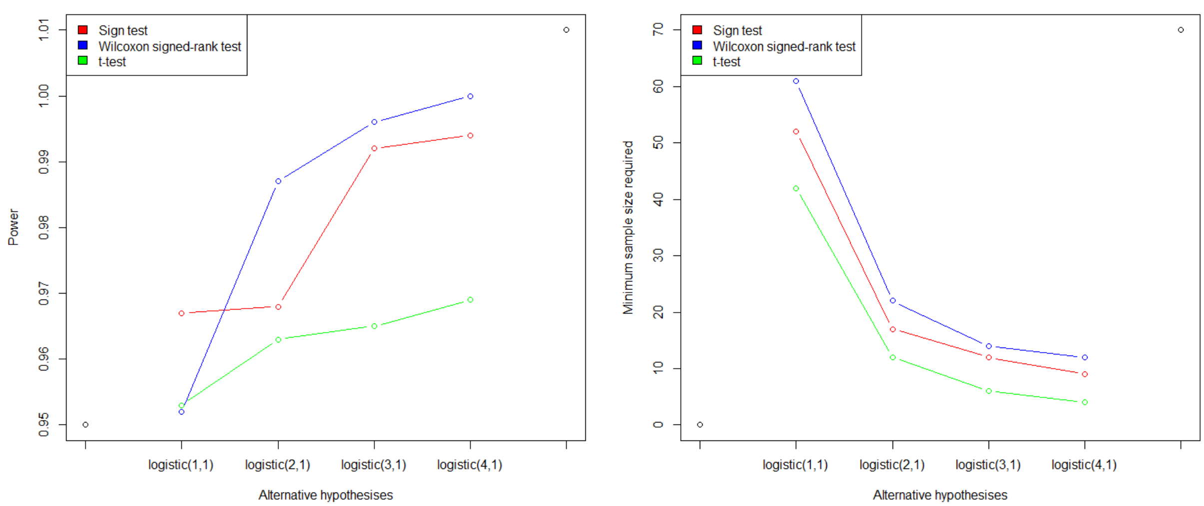

Chart 1.8

Table for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = Laplace(1,1)$

Test	Minimum sample size required	Power	Size
Sign test	27	0.963	0.021
Wilcoxon signed-rank test	34	0.985	0.05
t-test	28	0.952	0.025

Table 1.33

Relative efficiencies for testing $H_0 : F(\cdot) = logistic(0, 1)$ **against** $H_1 : F(\cdot) = Laplace(1, 1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.794117647$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 0.964285714$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 1.214285714$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	34	0.965	0.01
Wilcoxon signed-rank test	34	0.985	0.05
t-test	34	0.972	0.027

Table 1.33a

Test	sample size	Power	Size
Sign test	27	0.963	0.021
t-test	27	0.94	0.03

Table 1.33b

Table for testing $H_0 : F(\cdot) = logistic(0, 1)$ **against** $H_1 : F(\cdot) = Laplace(2, 1)$

Test	Minimum sample size required	Power	Size
Sign test	12	0.969	0.02
Wilcoxon signed-rank test	16	0.996	0.04
t-test	8	0.953	0.039

Table 1.34

Relative efficiencies for testing $H_0 : F(\cdot) = logistic(0, 1)$ **against** $H_1 : F(\cdot) = Laplace(2, 1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.75$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.5$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	16	0.974	0.01
Wilcoxon signed-rank test	16	0.996	0.04
t-test	16	0.995	0.033

Table 1.34a

Test	sample size	Power	Size
Sign test	12	0.969	0.02
t-test	12	0.983	0.034

Table 1.34b

Table for testing $H_0 : F(\cdot) = logistic(0, 1)$ **against** $H_1 : F(\cdot) = Laplace(3, 1)$

Test	Minimum sample size required	Power	Size
Sign test	9	0.987	0.014
Wilcoxon signed-rank test	13	0.999	0.03
t-test	5	0.979	0.044

Table 1.35

Relative efficiencies for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = Laplace(3,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.692307692$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.8$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2.6$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	13	0.961	0.004
Wilcoxon signed-rank test	13	0.999	0.03
t-test	13	1	0.033

Table 1.35a

Test	sample size	Power	Size
Sign test	9	0.987	0.014
t-test	9	0.999	0.038

Table 1.35b

Table for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = Laplace(4,1)$

Test	Minimum sample size required	Power	Size
Sign test	9	1	0.018
Wilcoxon signed-rank test	10	1	0.038
t-test	5	0.977	0.014

Table 1.36

Relative efficiencies for testing $H_0 : F(\cdot) = logistic(0,1)$ **against** $H_1 : F(\cdot) = Laplace(4,1)$**:**

Pitman relative efficiency of signed-rank test with respect to sign test $RE_1 = 0.9$
Pitman relative efficiency of t-test with respect to sign test $RE_2 = 1.8$
Pitman relative efficiency of t-test with respect to signed-rank test $RE_3 = 2$

If we use as much sample as used in Wilcoxon signed-rank test and sign test then the result will be

Test	sample size	Power	Size
Sign test	10	0.998	0.007
Wilcoxon signed-rank test	10	1	0.038
t-test	10	1	0.037

Table 1.36a

Test	sample size	Power	Size
Sign test	9	1	0.018
t-test	9	1	0.038

Table 1.36b

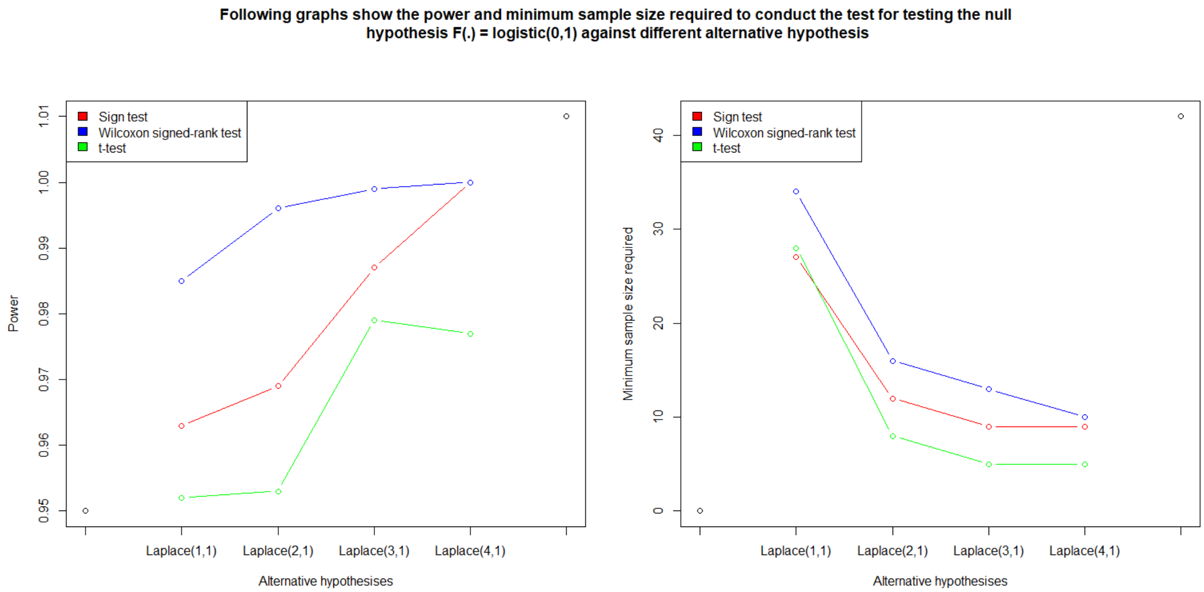

Chart 1.9

2 General Two Sample Test for Location Parameter:

Suppose $X_1, X_2, \ldots, X_m$ be a random sample from a continuous distribution with cumulative distribution function $F_X,\ x \in \mathbb{R}$ and $Y_1, Y_2, \ldots, Y_n$ be another random sample from a continuous distribution with C.D.F. $F_Y,\ y \in \mathbb{R}$. The null hypothesis to be tested here is

$$H_0 : F_Y(x) = F_X(x),\ \forall x$$

against the alternative hypothesis $H_1 : F_Y(x) = F_X(x-\theta)$ for $\theta \in \mathbb{R}$.

We will use *two sample Kolmogrov-Smirnov test* and *Wilcoxon rank sum test* as non-parametric tests to test H_0.

2.1 Two Sample Kolmogrov-Smirnov Test:

Let $X_{(1)}, X_{(2)}, \ldots, X_{(m)}$ and $Y_{(1)}, Y_{(2)}, \ldots, Y_{(n)}$ are two order statistics corresponding to $X_1, X_2, \ldots, X_m$ and $Y_1, Y_2, \ldots, Y_n$ respectively.

The empirical distribution functions are defined as

$$S_m(x) = \begin{cases} 0, \text{ if } x < X_{(1)} \\ \dfrac{k}{m}, \text{ if } X_{(k)} \leqslant x < X_{(k+1)} \\ 1, \text{ if } x \geqslant X_{(k+1)} \end{cases}$$

$$S_n(x) = \begin{cases} 0, \text{ if } x < Y_{(1)} \\ \dfrac{k}{m}, \text{ if } Y_{(k)} \leqslant x < Y_{(k+1)} \\ 1, \text{ if } x \geqslant Y_{(k+1)} \end{cases}$$

The two sample Kolmogrov-Smirnov test is based on the test test statistic

$$D_{m,n} = \sup_{x \in \mathbb{R}} \mid S_m(x) - S_n(x) \mid$$

We reject H_0 against H_1 if $D_{m,n} \geqslant D_{m,n,\alpha}$

where, $D_{m,n,\alpha} = c$ is $\ni$

$$P_{H_0}(D_{m,n} \geqslant c) \leqslant \alpha$$

where α is the level of the significance.

2.2 Willcoxon rank sum test:

Let us define

$$Z_i = \begin{cases} 1, \text{ if in combined ordered arrangment of } X \text{ and } Y, X \text{ appears in } i^{th} \text{ position} \\ 0, \text{if in combined ordered arrangment of } X \text{ and } Y, Y \text{ appears in } i^{th} \text{ position} \end{cases}$$

The Wilcoxon rank sum test is based on the statistic

$$W_N = \sum_{i=1}^{N} iZ_i$$

where $N = m + n$.

We reject H_0 against H_1 if $W_N \geqslant W_{\frac{\alpha}{2},m,n}$ or $W_N < W'_{\frac{\alpha}{2},m,n}$

where

$$\frac{W_{\frac{\alpha}{2},m,n} + W'_{\frac{\alpha}{2},m,n}}{2} = \frac{m * (N+1)}{2}$$

and $W_{\frac{\alpha}{2},m,n} = c$ is $\ni$

$$P_{H_0}(W_n \geqslant c) \leqslant \frac{\alpha}{2}$$

where α is level of significant.

2.3 Two Sample Student's t-test:

Similar test for parametric part is *two sample student's t-test*. This test should be used when the distributions of X and Y are known to be normal. The procedure of the test can be describe as follows,

$X_1, X_2, \ldots, X_m \backsim N(\mu_1, \sigma_1^2)$ and $Y_1, Y_2, \ldots, Y_n \backsim N(\mu_2, \sigma_2^2)$ then, the test statistic will be

$$t = \frac{|\bar{X} - \bar{Y}|}{s \times \sqrt{\dfrac{1}{m} + \dfrac{1}{n}}}$$

where $\bar{X} = \dfrac{1}{m} \sum_{i=1}^{m} X_i$ = sample mean of X, $\bar{Y} = \dfrac{1}{n} \sum_{i=1}^{n} Y_i$ = sample mean of Y and, $s^2 = \dfrac{(m-1)s_x^2 + (n-1)s_y^2}{m+n-2}$ is the pooled variance.

we reject H_0 against H_1 if $t \geqslant t_{m+n-2,\alpha}$

where $t_{m+n-2,\alpha} = c$ is $\ni$

$$P_{H_0}(t \geqslant c) \leqslant \alpha$$

where α is level of significant.

We have used computer simulation to obtain n and m for all of the above three tests.

Then we calculate the relative efficiencies using the formula $RE = \dfrac{m+n}{m^* + n^*}$ (let, n, m are minimum sample size required for A-test and n^*, m^* are minimum sample size required for B-test) for relative efficiency of B-test with respect to A-test.

Here are the relative efficiencies followed by the following tables give the minimum sample size required to get the size < 0.05 and the power > 0.95 when $F(\cdot)$ follows normal, Laplace, logistic distributions.

Table for testing$H_0 : F_Y(\cdot) = N(0,1), F_X(\cdot) = N(0,1)$ **against** $H_1 : F_Y(\cdot) = N(0,1), F_X(\cdot) = N(2,1)$

Test	Minimum sample size required		Power	Size
	m	n		
Kolmogrov-Smirnov test	12	10	0.979	0.039
Wilcoxon rank sum test	28	26	0.999	0.046
t-test	8	8	0.989	0.048

Table 2.1

Relative efficiencies for testing $H_0 : F_Y(\cdot) = N(0,1), F_X(\cdot) = N(0,1)$ **against** $H_1 : F_Y(\cdot) = N(0,1), F_X(\cdot) = N(2,1)$**:**

Relative efficiency of Wilcoxon rank sum test with respect to Kolmogrov-Smirnov test sum test is 0.407407
Relative efficiency of t-test with respect to Kolmogrov-Smirnov test sum test is 1.375
Relative efficiency of t-test with respect to Wilcoxon rank sum test is 3.375

Table for testing $H_0 : F_Y(\cdot) = Laplace(0,1), F_X(\cdot) = Laplace(0,1)$ **against** $H_1 : F_Y(\cdot) = Laplace(0,1), F_X(\cdot) = Laplace(2,1)$

Test	Minimum sample size required		Power	Size
	m	n		
Kolmogrov-Smirnov test	16	13	0.97	0.048
Wilcoxon rank sum test	30	28	0.972	0.043
t-test	13	12	0.963	0.044

Table 2.2

Relative efficiencies for testing $H_0 : F_Y(\cdot) = Laplace(0,1), F_X(\cdot) = Laplace(0,1)$ **against** $H_1 : F_Y(\cdot) = Laplace(0,1), F_X(\cdot) = Laplace(2,1)$
Relative efficiency of Wilcoxon rank sum test with respect to Kolmogrov-Smirnov test sum test is 0.5
Relative efficiency of t-test with respect to Kolmogrov-Smirnov test sum test is 1.16
Relative efficiency of t-test with respect to Wilcoxon rank sum test is 2.23

Table for testing$H_0 : F_Y(\cdot) = logistic(0,1), F_X(\cdot) = logistic(0,1)$ **against** $H_1 : F_Y(\cdot) = logistic(0,1), F_X(\cdot) = logistic(3,1)$**:**

Test	Minimum sample size required		Power	Size
	m	n		
Kolmogrov-Smirnov test	16	13	0.986	0.046
Wilcoxon rank sum test	29	27	0.993	0.049
t-test	11	10	0.98	0.049

Table 2.3

Relative efficiencies for testing $H_0 : F_Y(\cdot) = logistic(0,1), F_X(\cdot) = logistic(0,1)$ **against** $H_1 : F_Y(\cdot) = logistic(0,1), F_X(\cdot) = logistic(3,1)$**:**

Relative efficiency of Wilcoxon rank sum test with respect to Kolmogrov-Smirnov test sum test is 0.517857
Relative efficiency of t-test with respect to Kolmogrov-Smirnov test sum test is 1.38095
Relative efficiency of t-test with respect to Wilcoxon rank sum test is 2.66667

Conclusion:

The main objective of this project was to compare parametric and non-parametric tests using relative efficiency and power function. There is two part in this project. One is single sample test (in this section we used sign test and Wilcoxon signed-rank test as non-parametric test and student's t-test as parametric test) and another one is general two sample test (In this section we used two sample Kolmogorov-Smirnov test and Wilcoxon rank sum test as non-parametric test and two sample student's t-test as parametric test). To perform these tests we generate random samples from normal, logistic and Laplace distribution.

Now the question is which one is the best? Let us discuss one by one.
First single sample test,

- Almost every time it is shown that t-test takes minimum number of sample to give the level 0.05 among all three test, and Wilcoxon signed-rank test takes the most. So, with respect to relative efficiency t-test clearly wins.

- If we talk about power of the tests then at a glance it will be looks like Wilcoxon signed rank test give more power than other two. That is not wrong. But, if we use as much sample as Wilcoxon signed rank test uses on other two test then sometime sign test gives as much power as Wilcoxon signed rank test gives and t-test gives more power than Wilcoxon signed rank test although the size of t-test sometime exceed 0.05.If we relaxed the size a little bit then t-test is superior otherwise Wilcoxon signed rank test got the power.

- If one willing to get a test of lower size sign test will be one then.

Now two sample test, in all cases we have considered t-test gives better results than non-parametric tests.

It is obvious that parametric tests gave better result for any cases.Theoretically, we can use t-test if the parental distribution is normal,but on the basis of this project t-test can be used for other distributions also and hence it is robust under the parental distribution.

Future Direction of Work:

Many different adaptations, tests have been left for the future due of lack of time. There are some ideas that I would like to try. The project mainly focused on location parameter of symmetric distributions.But, the following ideas could be tested:

- We can consider some skewed distributions like gamma distribution, beta distribution, Pareto distribution etc.
- It could be interesting to consider the sample distributions against probability distributions.
- We can use other non-parametric tests like Wilcoxon rank sum test.
- We can perform tests for scale parameter.

References

[1] Gibbons, J. , Chakroborty,S.. *Nonparametric Statistical Inference* Marcel Dekker, Inc

[2] Gun, A.M., Gupta, M.K., Dasgupta, B.*Fundamental of Statistics Vol-2*, The World Press Private Limited, Kolkata

APPENDIX-I

```
##Program for Sign Test
m<-1000
n<-NULL
mu0<-0
mu1<-1
n1<-NULL
c<-NULL
size<-NULL
power<-NULL

##Generating Sample from Laplace Distribution
rlaplace<-function(n,mu,sigma){
u<-runif(n)
w<-NULL
for(i in 1:n)
{
if(u[i]<0.5)
w[i]<-mu+sigma*log(2*u[i])
else
w[i]<-mu-sigma*log(2*(1-u[i]))
}
w
}

##Simulated Size
Size<-function(m,n,c,mu0)
{
z<-NULL
for( i in 1:m)
{
x<-rlaplace(n,mu0,1)
y<-x[x>mu0]
z[i]<-length(y)
}
d<-z[z>=c]
S<-length(d)/m
S
}

##Simulated Power
Pow<-function(m,n,c,mu0,mu1)
{
z<-NULL
```

```
for(i in 1:m)
{
x<-rlaplace(n,mu1,1)
y<-x[x>mu0]
z[i]<-length(y)
}
d1<-z[z>=c]
SP<-length(d1)/m
SP
}

##Trial and Error Method
for(j in 1:50)
{
n[1]<-500
for(i in 1:10000000)
{
c[j]<-i
if(Size(m,n[j],c[j],mu0)<=0.05)
break
}

for(i in 1:10000000)
{
n1[j]<-i
if(Pow(m,n1[j],c[j],mu0,mu1)>=0.95)
break
}
size[j]<-Size(m,n1[j],c[j],mu0)
power[j]<-Pow(m,n1[j],c[j],mu0,mu1)
n[j+1]<-n1[j]
}
data.frame(n1,c,power,size)
```

APPENDIX-II

```
##Program for Kolmogorav-Smirnov Test
m<-1000
mu0<-0
mu1<-1
c<-NULL
n<-NULL
n0<-NULL
n1<-NULL
n2<-NULL
power=NULL
size=NULL

##Simulated size
Size<-function(m,n1,n2,c,mu0)
{
w1=NULL
w2=NULL
D=NULL
for(l in 1:m)
{
x=rnorm(n1,mu0,1)
y=rnorm(n2,mu0,1)
z=c(x,y)
sn<-sort(z)
sm<-sort(z)
r<-rank(sn)
for(j in 1:(n1+n2))
for(i in 1:n2)
for(k in 1:n1)
{
if(sn[j]==y[i])(w1[i]=r[j])
if(sm[j]==x[k])(w2[k]=r[j])
}

s1=c(1:n2)/n2
s2=c(1:n1)/n1
{
if(min(y)<min(x))
{
sn[sort(w1)]=s1
for(i in sort(w2))
sn[i]=sn[i-1]
```

```
sm[sort(w2)]=s2
for(i in sort(w1[w1>1]))
{
sm[1]=0
sm[i]=sm[i-1]
}}
else
{
sn[sort(w2)]=s2
for(i in sort(w1))
sn[i]=sn[i-1]

sm[sort(w1)]=s1
for(i in sort(w2[w2>1]))
{
sm[1]=0
sm[i]=sm[i-1]
}}}
D[l]=max(abs(sm-sn))
}
d<-D[D>=c]
SS<-length(d)/m
SS
}

##Simulated power
Pow<-function(m,n1,n2,c,mu0,mu1)
{
w1=NULL
w2=NULL
D=NULL
for(l in 1:m)
{
x=rnorm(n1,mu0,1)
y=rnorm(n2,mu1,1)
z=c(x,y)
sn<-sort(z)
sm<-sort(z)
r<-rank(sn)
for(j in 1:(n1+n2))
for(i in 1:n2)
for(k in 1:n1)
{
if(sn[j]==y[i])(w1[i]=r[j])
```

```
if(sm[j]==x[k])(w2[k]=r[j])
}

s1=c(1:n2)/n2
s2=c(1:n1)/n1
{
if(min(y)<min(x))
{
sn[sort(w1)]=s1
for(i in sort(w2))
sn[i]=sn[i-1]

sm[sort(w2)]=s2
for(i in sort(w1[w1>1]))
{
sm[1]=0
sm[i]=sm[i-1]
}}
else
{
sn[sort(w2)]=s2
for(i in sort(w1))
sn[i]=sn[i-1]

sm[sort(w1)]=s1
for(i in sort(w2[w2>1]))
{
sm[1]=0
sm[i]=sm[i-1]
}}}
D[l]=max(abs(sm-sn))
}
d<-D[D>=c]
SP<-length(d)/m
SP
}

##Trial and Error Method
for( j in 1:25)
{
n[1]<-25
n0[1]<-30
h<-n[1]/n0[1]
for(i in 1:100000000)
```

```
{
c[j]<-i
if(Size(m,n[j],n0[j],(c[j]/(n1*n2)),mu0)<=0.05)
break
}
for(i in 2:10000000)
{
n1[j]<-i
n2[j]<-floor(n1[j]*h)
if(Pow(m,n1[j],n2[j],(c[j]/(n1*n2)),mu0,mu1)>=0.95)
break
}
size[j]<-Size(m,n1[j],n2[j],(c[j]/(n1*n2)),mu0)
power[j]<-Pow(m,n1[j],n2[j],(c[j]/(n1*n2)),mu0,mu1)
n[j+1]<-n1[j]
n0[j+1]<-n2[j]
}

data.frame(n1,n2,c,power,size)
```