A Note on Generating Functions

We are free to choose whatever generating function we want. We want one that will help us to efficiently sample the parameter space. Therefore we want it to propose candidate new locations that are preferentially in regions with significant probability, rather than candidates that are in regions of very low probability. And we want it to make as large a step as it can without venturing off into very low probability regions.

A common choice of generating function is a Gaussian with mean given by the current location, x_i:

$$g(x_t|x_i) \propto \exp -\{(x_i^\mu - x_t^\mu)(\Sigma^{-1})_{\mu\nu}(x_i^\nu - x_t^\nu)\} \tag{1}$$

where Σ is the generating function covariance matrix and x_t is the candidate new location in the parameter space and μ and ν are the indices of the parameter array; e.g., $\mu = 0$ might correspond to H_0 while $\mu = 1$ might correspond to $\Omega_{\rm m}$.

A convenient choice for Σ is a scaled version of the parameter error covariance matrix:

$$\Sigma_{\mu\nu} = \alpha\langle\delta x_\mu \delta x_\nu\rangle \tag{2}$$

where $\langle ...\rangle$ indicates an ensemble average (average over samples drawn from the posterior), $\delta x_\mu \equiv x_\mu - \langle x_\mu\rangle$, and α is a constant chosen to maximize motion through the parameter space (not too big because then acceptance rate will be low and not too small because the small step size will impede progress through the parameter space). It can be chosen with some experimentation.

But how do you get the paramere error covariance matrix? *If* you already had a chain, you could just estimate it from the chain. Of course, the chain is what you are trying to produce, so presumably you don't have it yet! But, you can do an iterative approach. Choose a diagonal Σ and create a preliminary chain with that. Then use that preliminary chain to estimate $\langle\delta x_\mu \delta x_\nu\rangle$. Use that to form a new Σ and run another chain.