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# Bayesian Learning, 6 hp

## Computer lab 2

You can use any programming language for the labs, but my hints, help and solutions will be in R.
You are supposed to work and submit your labs in pairs, but do make sure that both of you are contributing.
The **deadline** for this lab is **November** 21.

1. *Linear and polynomial regression*
   The data set JapanTemp.dat contains daily temperatures (in Celcius degrees) at some japanese location over the course of a year. The response variable is *temp* and the covariate is

$$time = \frac{\text{the number of days since beginning of year}}{365}.$$

   (a) Perform a Bayesian analysis of a quadratic regression

$$temp = \beta_0 + \beta_1 \cdot time + \beta_2 \cdot time^2 + \varepsilon, \; \varepsilon \overset{iid}{\sim} N(0, \sigma^2)$$

   using the conjugate prior. Set the prior hyperparameters $\mu_0$, $\Omega_0$, $\nu_0$ and $\sigma_0^2$ to sensible values. You may not be an expert in japanese temperatures, and I don't expect any deep expert knowledge, but do come up with something. You may simplify by assuming that $\Omega_0$ is a diagonal matrix, if you want.
   [Hint: it may be useful as a preliminary exploratory step to use the `lm()` command. The command `lm(y ~ x + I(x^2))` fits a quadratic model using plain least squares]

   (b) One way to check if a suggested prior is reasonable is to simulate draws from the joint prior of all parameters and for every draw compute the regression curve. This gives a collection of regression curves, one for each draw from the prior. Do the curves look reasonable? If not, change the prior hyperparameters until the collection of prior regression curves do agree with your prior beliefs about the regression curve.
   [Hint: the R package `mvtnorm` will be handy.]

   (c) Perform sensitivity analysis with respect to some or all of the prior hyperparameters. That is, what is the effect on the posterior from changing the prior?

   (d) Write a program that simulates from the joint posterior distribution of $\beta_0$, $\beta_1$,$\beta_2$ and $\sigma^2$. Try it out on the model in a).
   [Hint: the R package `mvtnorm` will be handy.]

(e) It is of interest to locate the day with the highest expected temperature (that is, the *time* where $E(temp|time)$ is maximal). Let's call this value $\tilde{x}$. Use the simulations in d) to simulate from the posterior distribution of $\tilde{x}$.

(f) Say now that you want to estimate a polynomial model of order 7, but you are worried that higher order terms may not be needed, and may ruin your pretty little quadratic model. Suggest a suitable prior that mitigates this potential problem.

May Bayes be with you!