AQM Assignment One

Ada Ziemyte, Shalva, Anders Riis

Contents

a. ARIMA Models

> *Identify, estimate and diagnostic check ARIMA models for each of the following variables: ln_inv, ln_inc, and ln_consump.*

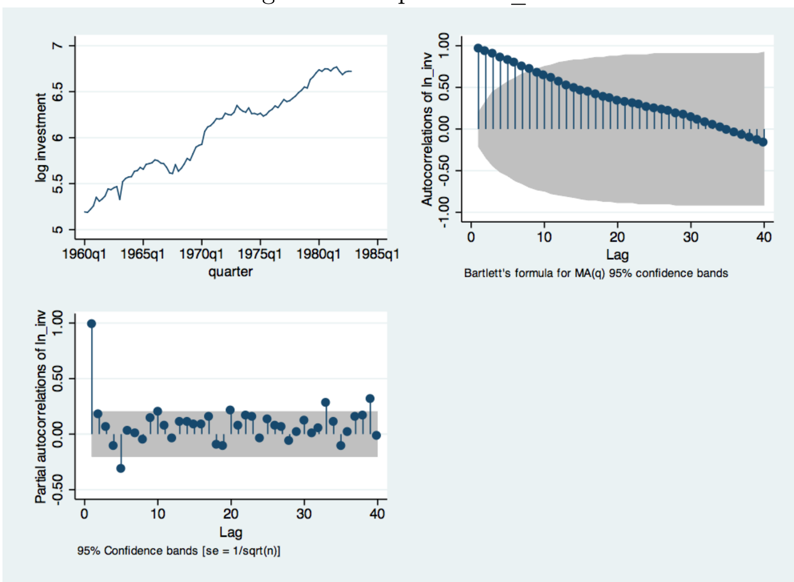
To identity an ARIMA model we must check for stationarity, correct for stationarity and examine the graphs for the autocorrelations and partial autocorrelations.[1]

a.i. ARIMA model for *ln_inv*

As can be seen in figure 1 on the following page, the ACF decays monotonically. That the ACF decays slowly indicates that the data is non-stationary, i.e. that we have a non-stationary mean.[2] Stationarity is when we have a flat looking series without trend.[3] The scatterplot in figure 1 on the next page also indicates

[1] http://www.polsci.wvu.edu/duval/ps791c/Notes/Stata/arimaident.html
[2] http://www.polsci.wvu.edu/duval/ps791c/Notes/Stata/arimaident.html
[3] http://www.itl.nist.gov/div898/handbook/pmc/section4/pmc442.htm

Figure 1: Graphs for *ln_ inv*

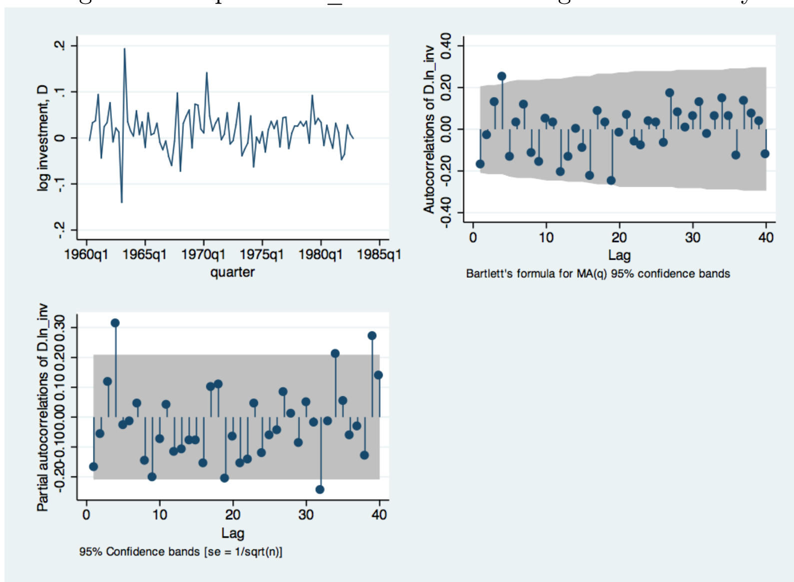


that the series is non-stationary as we can see a clear trend. The PACF cuts off after one lag (see figure 1), so we probably have an AR(1) process. However, because of non-stationarity we first attempt to correct for this by taking the first difference.

Now in figure 2 on the next page we see that the scatterplot is flatter, portraying stationarity. The ACF decays in an alternating or oscillating fashion, probably portraying some sort of MA as it seems to cut off (becomes zero) after four values or after zero values ($q = 0$). The PACF never seems to cut off ($p = 0$), although it can be seen as decaying. Thus, we have evidence of an ARIMA(0,1,0) model:

```
 . arima ln_inv, arima(0,1,0)

(setting optimization to BHHH)
Iteration 0:   log likelihood =  154.00497
Iteration 1:   log likelihood =  154.00497

ARIMA regression

Sample:  1960q2 - 1982q4                        Number of obs      =         91
                                                Wald chi2(.)       =          .
Log likelihood =   154.005                      Prob > chi2        =          .

------------------------------------------------------------------------------
             |                 OPG
    D.ln_inv |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ln_inv       |
```

Figure 2: Graphs for *ln_ inv* after correcting for stationarity




```
       _cons |   .0167964   .0047276     3.55   0.000     .0075304    .0260623
-------------+----------------------------------------------------------------
      /sigma |    .044543   .0020718    21.50   0.000     .0404823    .0486037
------------------------------------------------------------------------------
Note: The test of the variance against zero is one sided, and the two-sided confidence interval is truncated at z
```

It does not make sense to do an `armaroot` test as we have no AR or MA parameters in the model.

Another way of interpreting the graphs in figure 2 is that we have an ARIMA(4,1,4) model, the results of which are:

```
 . arima ln_inv, arima(4,1,4)

(setting optimization to BHHH)
Iteration 0:   log likelihood =  151.19416
Iteration 1:   log likelihood =   153.8349
Iteration 2:   log likelihood =  155.72441
Iteration 3:   log likelihood =  158.06115
Iteration 4:   log likelihood =  158.78522
(switching optimization to BFGS)
Iteration 5:   log likelihood =  160.42741
Iteration 6:   log likelihood =  161.31532
Iteration 7:   log likelihood =  161.57824
Iteration 8:   log likelihood =  162.63005
Iteration 9:   log likelihood =  162.99665
Iteration 10:  log likelihood =  163.47528
Iteration 11:  log likelihood =  163.81731
Iteration 12:  log likelihood =  164.09801
Iteration 13:  log likelihood =  164.17117
```

```
Iteration 14:  log likelihood =  164.23693
(switching optimization to BHHH)
Iteration 15:  log likelihood =  164.28633
Iteration 16:  log likelihood =  164.28633  (backed up)
Iteration 17:  log likelihood =  164.28633  (not concave)
Iteration 18:  log likelihood =  164.29517
Iteration 19:  log likelihood =  164.29517  (backed up)
(switching optimization to BFGS)
Iteration 20:  log likelihood =  164.29519  (backed up)
Iteration 21:  log likelihood =  164.29682
Iteration 22:  log likelihood =  164.30153
Iteration 23:  log likelihood =  164.30185
Iteration 24:  log likelihood =   164.3047
Iteration 25:  log likelihood =  164.30508
Iteration 26:  log likelihood =  164.30518
Iteration 27:  log likelihood =  164.30518

ARIMA regression

Sample:  1960q2 - 1982q4                        Number of obs      =        91
                                                Wald chi2(7)       =    361.32
Log likelihood =  164.3052                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
    D.ln_inv |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ln_inv       |
       _cons |   .0170483   .0066675     2.56   0.011     .0039803    .0301164
-------------+----------------------------------------------------------------
ARMA         |
          ar |
         L1. |    .250352   .3175816     0.79   0.431    -.3720964    .8728004
         L2. |  -.6410894   .3055835    -2.10   0.036    -1.240022   -.0421568
         L3. |   .6959524   .2326489     2.99   0.003     .2399689    1.151936
         L4. |  -.1234277    .371418    -0.33   0.740    -.8513935    .6045381
             |
          ma |
         L1. |  -.4332859   .3581174    -1.21   0.226    -1.135183    .2686114
         L2. |   .7828925   .3674864     2.13   0.033     .0626323    1.503153
         L3. |  -.7487543          .        .       .            .           .
         L4. |   .5336774    .347815     1.53   0.125    -.1480275    1.215382
-------------+----------------------------------------------------------------
      /sigma |   .0387658   .0050688     7.65   0.000     .0288311    .0487004
------------------------------------------------------------------------------
Note: The test of the variance against zero is one sided, and the two-sided confidence interval is truncated at z
```

To test for stationarity we use the -armaroots- command to know whether the absolute value of the roots of the characteristic equation are smaller than one, which is required for stationarity. As we can see in the following table all the roots are smaller than one in absolute value:

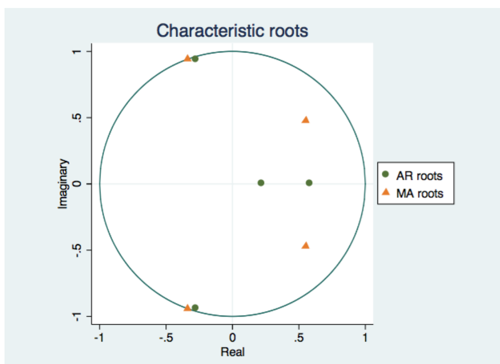
```
. armaroots

  Characteristic roots of AR-polynomial
 +-------------------------------------------------+
```

Characteristic roots	Modulus	Period
-.2762973 + .9377075i	.977566	3.38
-.2762973 - .9377075i	.977566	3.38
.5804232	.580423	
.2225233	.222523	

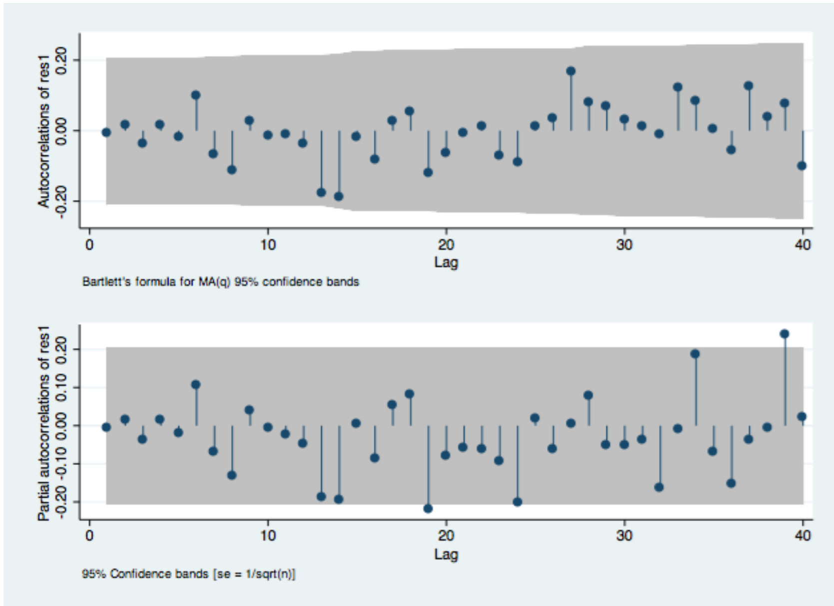
Characteristic roots of MA-polynomial

Characteristic roots	Modulus	Period
-.3382539 + .941054i	.999999	3.28
-.3382539 - .941054i	.999999	3.28
.5548968 + .47751503i	.730533	8.87
.5548968 - .47751503i	.730533	8.87

Graphically we can also confirm that none of the roots are larger than one in absolute value:



We now turn to the ACF and the PACF for the residuals to confirm that these have no distinct pattern:

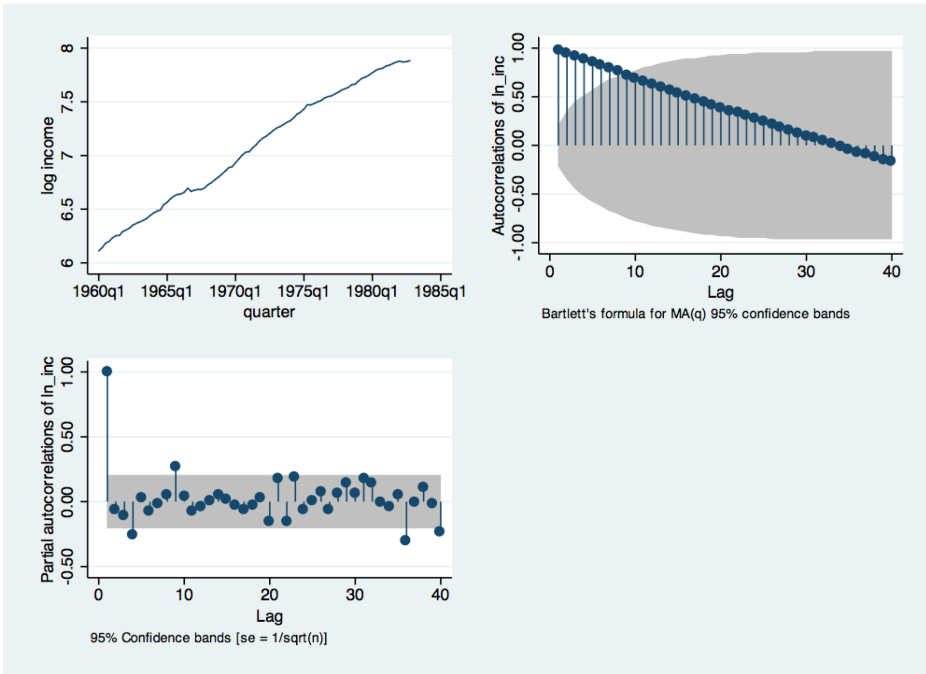


The graph above indeed seems to confirm that the residuals are white noise.
The Portmanteau statistics confirms this by yielding:

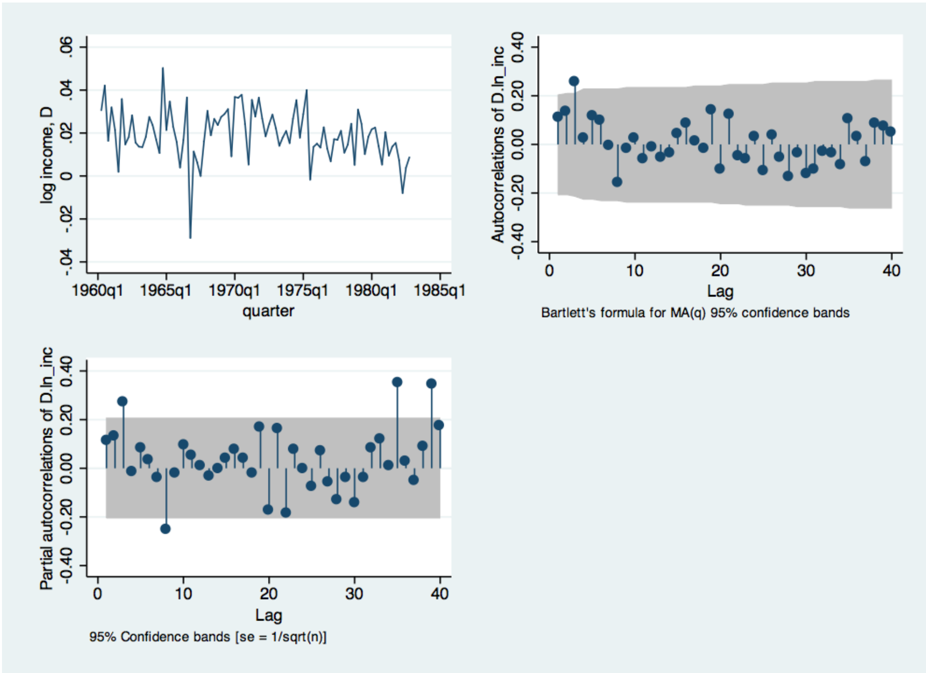
```
. wntestq res1

Portmanteau test for white noise
---------------------------------------
 Portmanteau (Q) statistic =    30.0852
 Prob > chi2(40)           =     0.8728
```

a.ii. ARIMA model for *ln_ inc*

We now turn to the variable *ln_ inc.*



The graphs shown above again demonstrate that we probably have non-stationary data, so we do a first difference transformation again.



The results of this first difference transformation can be seen in the graphs above. Now the data does not portray a non-stationary mean, so we decide that $d = 1$. Again the case is more ambiguous for the ACF and PACF. We could again conclude that the ACF never cuts off, so we have a MA(0), where $q = 0$, and the PACF never cuts off (becomes zero), so $p = 0$. Estimating an ARIMA(0,1,0) model gives the results:

```
 . arima ln_inc, arima(0,1,0)

(setting optimization to BHHH)
Iteration 0:   log likelihood =  274.34425
Iteration 1:   log likelihood =  274.34425

ARIMA regression

Sample:  1960q2 - 1982q4                          Number of obs      =        91
                                                  Wald chi2(.)       =         .
Log likelihood =  274.3443                        Prob > chi2        =         .

------------------------------------------------------------------------------
             |                 OPG
    D.ln_inc |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ln_inc       |
       _cons |    .019464   .0012907    15.08   0.000     .0169343    .0219938
-------------+----------------------------------------------------------------
      /sigma |   .0118704   .0006448    18.41   0.000     .0106066    .0131342
```

```
------------------------------------------------------------------------------
Note: The test of the variance against zero is one sided, and the two-sided confidence interval is truncated at z
```

Another possibility is however to interpret the ACF and the PACF cuts off after three after significant values so we have an ARIMA(3,1,3) model.

```
 . arima ln_inc, arima(3,1,3)

(setting optimization to BHHH)
Iteration 0:   log likelihood =  270.31907
Iteration 1:   log likelihood =  274.68144
Iteration 2:   log likelihood =  276.35436
Iteration 3:   log likelihood =  277.54725
Iteration 4:   log likelihood =    278.059
(switching optimization to BFGS)
Iteration 5:   log likelihood =  278.30896
Iteration 6:   log likelihood =  278.77259
Iteration 7:   log likelihood =  278.78656
Iteration 8:   log likelihood =  279.11859
Iteration 9:   log likelihood =   279.5552
Iteration 10:  log likelihood =   279.5552  (backed up)
Iteration 11:  log likelihood =  279.70686  (backed up)
Iteration 12:  log likelihood =  280.14331
Iteration 13:  log likelihood =  280.39292
Iteration 14:  log likelihood =   280.4006
(switching optimization to BHHH)
Iteration 15:  log likelihood =  280.40117
Iteration 16:  log likelihood =  280.40177  (backed up)
Iteration 17:  log likelihood =   280.4201
Iteration 18:  log likelihood =  280.42064
Iteration 19:  log likelihood =  280.43726
(switching optimization to BFGS)
Iteration 20:  log likelihood =  280.49572
Iteration 21:  log likelihood =  280.67651
Iteration 22:  log likelihood =  280.74026
Iteration 23:  log likelihood =   280.7466
Iteration 24:  log likelihood =  281.03547
Iteration 25:  log likelihood =  281.05958
Iteration 26:  log likelihood =  281.06647
Iteration 27:  log likelihood =  281.06818
Iteration 28:  log likelihood =  281.06845
Iteration 29:  log likelihood =  281.06846

ARIMA regression

Sample:  1960q2 - 1982q4                        Number of obs      =        91
                                                Wald chi2(6)       =     47.81
Log likelihood =  281.0685                      Prob > chi2        =    0.0000

------------------------------------------------------------------------------
             |                 OPG
    D.ln_inc |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ln_inc       |
       _cons |   .0193814   .0022928     8.45   0.000     .0148877    .0238752
-------------+----------------------------------------------------------------
```

```
ARMA         |
          ar |
         L1. |   .3524409   .3992522     0.88   0.377    -.4300791    1.134961
         L2. |   .5973378   .1728554     3.46   0.001     .2585474    .9361281
         L3. |  -.3721901   .3633335    -1.02   0.306    -1.084311    .3399305
             |
          ma |
         L1. |  -.2371601   480.2177    -0.00   1.000    -941.4465    940.9722
         L2. |  -.6004014   593.9994    -0.00   0.999    -1164.818    1163.617
         L3. |   .6367674   305.7404     0.00   0.998    -598.6033    599.8769
-------------+----------------------------------------------------------------
      /sigma |   .0108757    2.61138     0.00   0.498            0    5.129086
------------------------------------------------------------------------------
Note: The test of the variance against zero is one sided, and the two-sided confidence interval is truncated at z
```

The Portmanteau statistics yield...

a.iii. ARIMA model for *ln_ consump*

Identifying an ARIMA model for the variable *ln_ consump* we again first look at some graphs in figure 3 on the next page. Again we have to take the first diffference to correct for stationarity. After this is done we see in the lower pane that we have a stationary mean.

Estimating an ARIMA(?,1,?) gives...

b. Dynamic Linear Model

Formulate and estimate a dynamic linear model with four lags for ln_ consump with ln_ inc and ln_ inv as explanatory variables.

The model can be written as

$$y_t = a_0 + a_1 y_{t-1} + a_2 y_{t-2} + a_3 y_{t-3} + a_4 y_{t-4} + b_0 ln\_inc_t + c_0 ln\_inv_t + u_t, \quad (1)$$

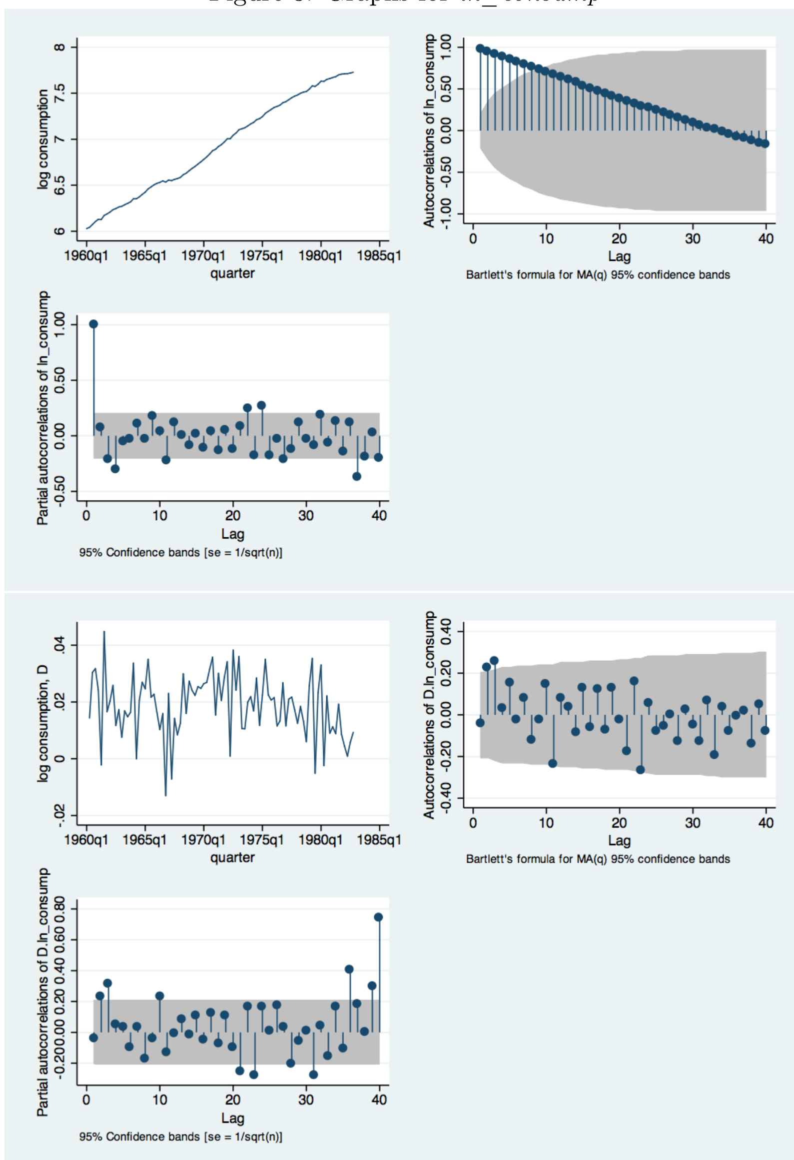
where y is *ln_consump*.[4]

The results of a dynamic linear model with four lags for *ln_ consump* are:

```
. reg ln_consump L(1/4).ln_consump ln_inc ln_inv

      Source |       SS       df       MS              Number of obs =      88
-------------+------------------------------           F(  6,    81) =55970.25
       Model |  22.3174282     6  3.71957137           Prob > F      =  0.0000
    Residual |  .005382954    81  .000066456           R-squared     =  0.9998
-------------+------------------------------           Adj R-squared =  0.9997
       Total |  22.3228112    87  .256584037           Root MSE      =  .00815

------------------------------------------------------------------------------
  ln_consump |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
```

[4]See AQM lecture notes chapter 3, p. 18.

Figure 3: Graphs for *ln_ consump*



```
  ln_consump |
         L1. |   .4046597   .1056179     3.83   0.000     .1945131    .6148062
         L2. |   .2783657   .1128595     2.47   0.016     .0538107    .5029207
         L3. |   .1180614    .111389     1.06   0.292    -.1035678    .3396905
         L4. |  -.1543535   .0870547    -1.77   0.080     -.327565    .0188579
             |
      ln_inc |   .3326671   .0489638     6.79   0.000     .2352445    .4300897
      ln_inv |    .009699   .0120582     0.80   0.424    -.0142931    .0336911
       _cons |   .0492794   .0134815     3.66   0.000     .0224554    .0761033
------------------------------------------------------------------------------
```

Mis-specification tests are:

```
Ramsey RESET test using powers of the fitted values of ln_consump
       Ho:  model has no omitted variables
                  F(3, 78) =      6.02
                  Prob > F =      0.0010

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of ln_consump

         chi2(1)      =     1.10
         Prob > chi2  =   0.2944

White's test for Ho: homoskedasticity
         against Ha: unrestricted heteroskedasticity

         chi2(27)     =     30.03
         Prob > chi2  =    0.3130

Cameron & Trivedi's decomposition of IM-test

---------------------------------------------------
              Source |       chi2     df      p
---------------------+-----------------------------
  Heteroskedasticity |      30.03     27    0.3130
            Skewness |       6.37      6    0.3831
            Kurtosis |       4.75      1    0.0294
---------------------+-----------------------------
               Total |      41.14     34    0.1863
---------------------------------------------------

----------------------------------------------------------
 (n = 88) |        D-H     P-value        asy.     P-value
----------+-----------------------------------------------
Residuals |     6.1568      0.0460      3.6067      0.1647
----------------------------------------------------------

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |          2.096               1                   0.1476
---------------------------------------------------------------------------
                        H0: no serial correlation
```

```
LM test for autoregressive conditional heteroskedasticity (ARCH)
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |          1.558               1                   0.2120
---------------------------------------------------------------------------
         H0: no ARCH effects      vs.  H1: ARCH(p) disturbance
```

c. Error Correction Model

*Reformulate the model in b. as an error correction model; store the log-likelihood value using the command –*estimates store llu-*; and subject the model to mis-specification testing.*

Writing equation 1 on page 10 as an error correction model yields:

$$\Delta y_t = a_0 + \alpha_1 \Delta y_{t-1} + \alpha_2 \Delta y_{t-2} + \alpha \Delta_3 y_{t-3} + \beta_0 \Delta ln\_inc_t + \gamma_0 \Delta ln\_inv_t + a y_{t-1} + u_t,$$

again where y is $ln\_consump$.[5] Running the regression gives:

```
      Source |       SS       df       MS              Number of obs =      88
-------------+------------------------------           F(  6,    81) =   11.06
       Model |  .004719858     6  .000786643           Prob > F      =  0.0000
    Residual |  .005759951    81  .000071111           R-squared     =  0.4504
-------------+------------------------------           Adj R-squared =  0.4097
       Total |  .010479809    87  .000120458           Root MSE      =  .00843

------------------------------------------------------------------------------
 dln_consump |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 dln_consump |
         L1. |  -.2673194    .087738    -3.05   0.003    -.4418905   -.0927483
         L2. |   .1660194   .0843342     1.97   0.052    -.0017792     .333818
         L3. |   .2591583   .0865452     2.99   0.004     .0869605    .4313562
             |
     dln_inc |   .4554999   .0815985     5.58   0.000     .2931446    .6178553
     dln_inv |    .062451   .0202667     3.08   0.003     .0221265    .1027754
             |
   ln_consump |
         L1. |  -.0004039   .0018446    -0.22   0.827    -.0040741    .0032664
             |
       _cons |    .008395   .0138116     0.61   0.545    -.0190858    .0358757
------------------------------------------------------------------------------
```

The results of mis-specification tests are:

```
Ramsey RESET test using powers of the fitted values of dln_consump
```

[5] See answer to assignment 4, IQM, p. 13.

```
       Ho:  model has no omitted variables
                 F(3, 78) =      1.19
                 Prob > F =      0.3193

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of dln_consump

         chi2(1)      =     0.47
         Prob > chi2  =   0.4911

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: L.dln_consump L2.dln_consump L3.dln_consump dln_inc dln_inv L.ln_consump

         chi2(6)      =     7.60
         Prob > chi2  =   0.2686

White's test for Ho: homoskedasticity
         against Ha: unrestricted heteroskedasticity

         chi2(27)     =     38.39
         Prob > chi2  =    0.0719

Cameron & Trivedi's decomposition of IM-test

---------------------------------------------------
              Source |       chi2     df      p
---------------------+-----------------------------
  Heteroskedasticity |      38.39     27    0.0719
            Skewness |       9.38      6    0.1532
            Kurtosis |       0.65      1    0.4218
---------------------+-----------------------------
               Total |      48.42     34    0.0519
---------------------------------------------------

----------------------------------------------------------
 (n = 88) |        D-H     P-value        asy.     P-value
----------+-----------------------------------------------
Residuals |     2.5948      0.2732      2.4639      0.2917
----------------------------------------------------------

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |          4.266               1                   0.0389
       2     |          8.164               2                   0.0169
       3     |          9.789               3                   0.0205
       4     |         10.731               4                   0.0298
---------------------------------------------------------------------------
                        H0: no serial correlation
LM test for autoregressive conditional heteroskedasticity (ARCH)
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |          3.184               1                   0.0743
```

```
     2     |          8.175              2                   0.0168
     3     |          1.331              3                   0.7217
     4     |          0.949              4                   0.9175
---------------------------------------------------------------------------
         H0: no ARCH effects      vs.  H1: ARCH(p) disturbance
```

The Ramsey RESET test and the two Breusch-Pagan tests for heteroskedasticity do not reveal any problems as we have p-values comfortably above, say, a 10% significance level. However, the White's test for homoskedasticity is rejected at a 10% significance level. Moreover, the Breusch-Godfrey LM test for autocorrelation shows some problems as the null hypothesis of no serial correlation is rejected for all the lags at the 5% significance level.

d. Model Specification

> *Reduce the model in successive steps by dropping insignificant variables (subjecting each step to mis-specification testing) in order to obtain a parsimonious model.*

We now drop the first lag of *ln_ consump* in the error correction model above. This gives us the following result:

```
      Source |       SS       df       MS              Number of obs =      88
-------------+------------------------------           F(  5,    82) =   13.42
       Model |  .004716449     5   .00094329           Prob > F      =  0.0000
    Residual |  .005763359    82  .000070285           R-squared     =  0.4501
-------------+------------------------------           Adj R-squared =  0.4165
       Total |  .010479809    87  .000120458           Root MSE      =  .00838

------------------------------------------------------------------------------
 dln_consump |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
 dln_consump |
         L1. |  -.2657825   .0869475    -3.06   0.003    -.4387488   -.0928161
         L2. |   .1683854    .083152     2.03   0.046     .0029696    .3338012
         L3. |   .2603634   .0858671     3.03   0.003     .0895463    .4311805
             |
     dln_inc |   .4581345   .0802364     5.71   0.000     .2985187    .6177503
     dln_inv |   .0626535   .0201277     3.11   0.003      .022613     .102694
       _cons |   .0054381   .0028747     1.89   0.062    -.0002807    .0111569
------------------------------------------------------------------------------
```

Now we see that we do not have insignificant variables.

The mis-specification tests of this model are:

```
Ramsey RESET test using powers of the fitted values of dln_consump
       Ho:  model has no omitted variables
                 F(3, 79) =      1.20
                 Prob > F =      0.3151
```

```
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: fitted values of dln_consump

         chi2(1)      =     0.57
         Prob > chi2  =   0.4492

Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
         Ho: Constant variance
         Variables: L.dln_consump L2.dln_consump L3.dln_consump dln_inc dln_inv

         chi2(5)      =     4.26
         Prob > chi2  =   0.5122

White's test for Ho: homoskedasticity
         against Ha: unrestricted heteroskedasticity

         chi2(20)     =     25.95
         Prob > chi2  =    0.1675

Cameron & Trivedi's decomposition of IM-test

---------------------------------------------------
              Source |       chi2     df      p
---------------------+-----------------------------
  Heteroskedasticity |      25.95     20    0.1675
            Skewness |       9.69      5    0.0846
            Kurtosis |       0.62      1    0.4322
---------------------+-----------------------------
               Total |      36.25     26    0.0871
---------------------------------------------------

----------------------------------------------------------
 (n = 88) |        D-H     P-value        asy.     P-value
----------+-----------------------------------------------
Residuals |     2.7414      0.2539      2.6979      0.2595
----------------------------------------------------------

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |          4.309               1                   0.0379
       2     |          8.027               2                   0.0181
       3     |          9.487               3                   0.0235
       4     |         10.267               4                   0.0362
---------------------------------------------------------------------------
                        H0: no serial correlation
LM test for autoregressive conditional heteroskedasticity (ARCH)
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |          3.173               1                   0.0748
       2     |          8.125               2                   0.0172
       3     |          1.381               3                   0.7100
       4     |          0.998               4                   0.9101
```

```
------------------------------------------------------------------------------
        H0: no ARCH effects      vs.  H1: ARCH(p) disturbance
```

We now test whether we can exclude the last lag, but the -testparm- command does lead to rejection, so we keep the third lag.

```
 ( 1)  L3.dln_consump = 0

       F(  1,    82) =    9.19
            Prob > F =    0.0032
```

e. Comparison of Models

> *Save the log-likelihood value of the parsimonious model (–estimates store llr-) and perform a likelihood ratio test comparing the model in c. with the model in d. (-lrtest llu llr-). You may wish to save the log-likelihood value at each step of the reduction using different names with the estimates command at each step.*

```
 . lrtest llu llr

Likelihood-ratio test                                 LR chi2(1)  =      0.05
(Assumption: llr nested in llu)                       Prob > chi2 =    0.8195
```

Can't reject that the models are different...

f. VAR(4) Model

> *Estimate a VAR(4) for the three variables dln_consump, dln_inc, and dln_inv, that is, for the first difference of ln_inv, ln_inc, and ln_consump. Then determine the minimum number of lags required for the VAR, and subject it to mis-specification testing.*

First we start with a VAR(4) model:

```
 . var dln_consump dln_inc dln_inv, lags(1/4) dfk

Vector autoregression

Sample:  1961q2 - 1982q4                        No. of obs      =        87
Log likelihood =  738.3533                      AIC             = -16.07709
FPE            =  2.11e-11                      HQIC            = -15.63197
Det(Sigma_ml)  =  8.53e-12                      SBIC            = -14.97168

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
dln_consump          13     .009863   0.3108   33.37069   0.0008
```

```
dln_inc             13     .011582   0.1728    15.4603   0.2172
dln_inv             13     .043414   0.1950   17.92655   0.1179
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
dln_consump  |
 dln_consump |
         L1. |  -.4186234   .1462402    -2.86   0.004    -.7052489   -.1319979
         L2. |  -.1654575   .1601944    -1.03   0.302    -.4794328    .1485178
         L3. |   .0699281    .148281     0.47   0.637    -.2206974    .3605536
         L4. |    .025482   .1316879     0.19   0.847    -.2326216    .2835856
             |
     dln_inc |
         L1. |   .2971221   .1203449     2.47   0.014     .0612505    .5329937
         L2. |   .3767172   .1288433     2.92   0.003     .1241891    .6292454
         L3. |   .2181382   .1315844     1.66   0.097    -.0397625    .4760389
         L4. |   .0940014   .1226229     0.77   0.443    -.1463351    .3343378
             |
     dln_inv |
         L1. |   .0043691   .0261169     0.17   0.867    -.0468191    .0555573
         L2. |   .0395284   .0274742     1.44   0.150      -.01432    .0933769
         L3. |   .0087278   .0281372     0.31   0.756    -.0464201    .0638758
         L4. |   -.025073   .0268227    -0.93   0.350    -.0776445    .0274986
             |
       _cons |   .0076971   .0039049     1.97   0.049     .0000436    .0153507
-------------+----------------------------------------------------------------
dln_inc      |
 dln_consump |
         L1. |   .1913047   .1717152     1.11   0.265     -.145251    .5278603
         L2. |  -.0049856   .1881002    -0.03   0.979    -.3736552    .3636841
         L3. |  -.0085702   .1741116    -0.05   0.961    -.3498226    .3326822
         L4. |   .0246611    .154628     0.16   0.873    -.2784042    .3277263
             |
     dln_inc |
         L1. |  -.0722509   .1413089    -0.51   0.609    -.3492113    .2047094
         L2. |   .0380503   .1512877     0.25   0.801    -.2584682    .3345688
         L3. |   .1734287   .1545064     1.12   0.262    -.1293983    .4762557
         L4. |  -.0531727   .1439838    -0.37   0.712    -.3353757    .2290303
             |
     dln_inv |
         L1. |   .0480724   .0306665     1.57   0.117    -.0120328    .1081776
         L2. |   .0582109   .0322602     1.80   0.071     -.005018    .1214397
         L3. |    .016095   .0330387     0.49   0.626    -.0486597    .0808498
         L4. |  -.0028715   .0314952    -0.09   0.927    -.0646011     .058858
             |
       _cons |    .011433   .0045852     2.49   0.013     .0024462    .0204198
-------------+----------------------------------------------------------------
dln_inv      |
 dln_consump |
         L1. |   .4213112   .6436848     0.65   0.513    -.8402878     1.68291
         L2. |   .4410982   .7051051     0.63   0.532    -.9408823    1.823079
         L3. |  -.0088644   .6526676    -0.01   0.989    -1.288069    1.270341
         L4. |  -.5482858   .5796322    -0.95   0.344    -1.684344    .5877724
             |
     dln_inc |
```

```
         L1. |   .4098625   .5297049     0.77   0.439    -.6283401    1.448065
         L2. |  -.1649085   .5671112    -0.29   0.771    -1.276426     .946609
         L3. |   .0542709   .5791765     0.09   0.925    -1.080894    1.189436
         L4. |  -.2581422   .5397318    -0.48   0.632    -1.315997    .7997127
             |
     dln_inv |
         L1. |  -.2678892   .1149551    -2.33   0.020    -.4931972   -.0425813
         L2. |  -.0702267   .1209292    -0.58   0.561    -.3072437    .1667902
         L3. |   .1621356   .1238476     1.31   0.190    -.0806013    .4048725
         L4. |   .3186896   .1180618     2.70   0.007     .0872927    .5500865
             |
       _cons |   .0071406   .0171878     0.42   0.678    -.0265469    .0408281
------------------------------------------------------------------------------
. varsoc

   Selection-order criteria
   Sample:  1961q2 - 1982q4                     Number of obs      =        87
  +---------------------------------------------------------------------------+
  |lag |    LL      LR      df    p      FPE       AIC      HQIC      SBIC    |
  |----+----------------------------------------------------------------------|
  |  0 |  696.398                      2.4e-11  -15.9402  -15.9059  -15.8552* |
  |  1 |  711.682  30.568    9  0.000  2.1e-11  -16.0846  -15.9477* -15.7445  |
  |  2 |  724.696  26.028    9  0.002  1.9e-11* -16.1769* -15.9372  -15.5817  |
  |  3 |  729.124  8.8557    9  0.451  2.1e-11  -16.0718  -15.7294  -15.2215  |
  |  4 |  738.353  18.458*   9  0.030  2.1e-11  -16.0771   -15.632  -14.9717  |
  +---------------------------------------------------------------------------+
   Endogenous:  dln_consump dln_inc dln_inv
    Exogenous:  _cons
```

Althout first lag of *ln_ inv* is significant...

We then determine the lag structure by starting with three lags:

```
 . var dln_consump dln_inc dln_inv, lags(1/3) dfk

Vector autoregression

Sample:  1961q1 - 1982q4                           No. of obs      =        88
Log likelihood =  737.5558                         AIC             = -16.08081
FPE            =  2.09e-11                         HQIC            = -15.74057
Det(Sigma_ml)  =  1.05e-11                         SBIC            = -15.23627

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
dln_consump          10      .00973   0.2953   32.68978   0.0002
dln_inc              10     .011309   0.1800   17.12733   0.0468
dln_inv              10     .045251   0.1096   9.599902   0.3838
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
dln_consump  |
 dln_consump |
         L1. |  -.3930788   .1385722    -2.84   0.005    -.6646753   -.1214823
         L2. |  -.1281016   .1441055    -0.89   0.374    -.4105431      .15434
         L3. |   .1015717   .1278624     0.79   0.427     -.149034    .3521773
```

```
             |
     dln_inc |
         L1. |   .2940007   .1174905     2.50   0.012     .0637236    .5242778
         L2. |   .3438497   .1205479     2.85   0.004     .1075802    .5801192
         L3. |   .1810283   .1187276     1.52   0.127    -.0516734    .4137301
             |
     dln_inv |
         L1. |   .0034602   .0256856     0.13   0.893    -.0468826    .0538031
         L2. |   .0437956   .0264529     1.66   0.098    -.0080512    .0956423
         L3. |   .0168297   .0263789     0.64   0.523     -.034872    .0685314
             |
       _cons |   .0091472   .0034369     2.66   0.008     .0024111    .0158833
-------------+----------------------------------------------------------------
dln_inc      |
 dln_consump |
         L1. |   .1923453    .161054     1.19   0.232    -.1233148    .5080055
         L2. |  -.0339177    .167485    -0.20   0.840    -.3621824     .294347
         L3. |  -.0434862   .1486067    -0.29   0.770      -.33475    .2477776
             |
     dln_inc |
         L1. |  -.0815851    .136552    -0.60   0.550    -.3492222     .186052
         L2. |   .0624807   .1401055     0.45   0.656     -.212121    .3370824
         L3. |   .1984851   .1379898     1.44   0.150      -.07197    .4689402
             |
     dln_inv |
         L1. |   .0476297   .0298528     1.60   0.111    -.0108808    .1061401
         L2. |   .0598946   .0307446     1.95   0.051    -.0003637    .1201529
         L3. |   .0155151   .0306586     0.51   0.613    -.0445747    .0756048
             |
       _cons |    .011234   .0039945     2.81   0.005      .003405     .019063
-------------+----------------------------------------------------------------
dln_inv      |
 dln_consump |
         L1. |   .4585587    .644436     0.71   0.477    -.8045127     1.72163
         L2. |   .4443132   .6701688     0.66   0.507    -.8691936     1.75782
         L3. |   -.199181   .5946297    -0.33   0.738    -1.364634    .9662718
             |
     dln_inc |
         L1. |   .3711541   .5463946     0.68   0.497    -.6997597    1.442068
         L2. |   .2534399   .5606132     0.45   0.651    -.8453418    1.352222
         L3. |   .3569366   .5521478     0.65   0.518    -.7252532    1.439126
             |
     dln_inv |
         L1. |  -.2598316   .1194521    -2.18   0.030    -.4939533   -.0257098
         L2. |  -.1253296   .1230204    -1.02   0.308    -.3664451     .115786
         L3. |   .0466697   .1226763     0.38   0.704    -.1937714    .2871107
             |
       _cons |  -.0100135   .0159833    -0.63   0.531    -.0413401    .0213131
------------------------------------------------------------------------------
. varsoc

   Selection-order criteria
   Sample:  1961q1 - 1982q4                     Number of obs      =        88
  +---------------------------------------------------------------------------+
  |lag |    LL      LR      df    p      FPE       AIC      HQIC      SBIC    |
  |----+----------------------------------------------------------------------|
  |  0 |  703.801                       2.4e-11  -15.9273  -15.8933  -15.8428* |
```

```
|  1 |  719.587  31.572    9  0.000  2.1e-11  -16.0815  -15.9454* -15.7437  |
|  2 |  732.981  26.787*   9  0.002  1.9e-11* -16.1814* -15.9432  -15.5902  |
|  3 |  737.556  9.1506    9  0.423  2.1e-11  -16.0808  -15.7406  -15.2363  |
+---------------------------------------------------------------------------+
 Endogenous:  dln_consump dln_inc dln_inv
  Exogenous:  _cons
```

The -varsoc- command shows us that we can drop the third lag, so we try with two lags:

```
. dln_consump dln_inc dln_inv, lags(1/2) dfk

Vector autoregression

Sample:  1960q4 - 1982q4                          No. of obs      =        89
Log likelihood =  742.2131                        AIC             = -16.20704
FPE            =  1.84e-11                        HQIC            = -15.97035
Det(Sigma_ml)  =  1.15e-11                        SBIC            = -15.61983

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
dln_consump          7     .009938   0.2400   25.88962   0.0002
dln_inc              7     .011224   0.1514   14.62996   0.0233
dln_inv              7     .044295   0.1051   9.633771   0.1409
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
dln_consump  |
 dln_consump |
         L1. |  -.2845172   .1274068    -2.23   0.026    -.5342299   -.0348044
         L2. |  -.1159776   .1270233    -0.91   0.361    -.3649386    .1329834
             |
     dln_inc |
         L1. |   .2893204    .112313     2.58   0.010      .069191    .5094497
         L2. |   .3664341   .1089872     3.36   0.001     .1528231    .5800451
             |
     dln_inv |
         L1. |   .0027381   .0255556     0.11   0.915    -.0473499    .0528261
         L2. |   .0497402    .025462     1.95   0.051    -.0001644    .0996447
             |
       _cons |   .0123795   .0029602     4.18   0.000     .0065776    .0181813
-------------+----------------------------------------------------------------
dln_inc      |
 dln_consump |
         L1. |   .3050571   .1438994     2.12   0.034     .0230195    .5870947
         L2. |   .0490208   .1434662     0.34   0.733    -.2321677    .3302094
             |
     dln_inc |
         L1. |  -.1232543   .1268517    -0.97   0.331     -.371879    .1253704
         L2. |   .0209769   .1230954     0.17   0.865    -.2202857    .2622394
             |
     dln_inv |
         L1. |   .0433473   .0288637     1.50   0.133    -.0132244    .0999191
         L2. |   .0616319    .028758     2.14   0.032     .0052673    .1179965
```

```
             |
       _cons |   .0125949   .0033434     3.77   0.000      .006042    .0191478
-------------+----------------------------------------------------------------
dln_inv      |
 dln_consump |
         L1. |   .6520473   .5678885     1.15   0.251    -.4609936    1.765088
         L2. |   .5980687    .566179     1.06   0.291    -.5116217    1.707759
             |
     dln_inc |
         L1. |   .3374819   .5006109     0.67   0.500    -.6436975    1.318661
         L2. |   .1827302   .4857871     0.38   0.707    -.7693951    1.134855
             |
     dln_inv |
         L1. |  -.2725654   .1139085    -2.39   0.017    -.4958218   -.0493089
         L2. |  -.1340503   .1134912    -1.18   0.238    -.3564891    .0883884
             |
       _cons |  -.0099191   .0131944    -0.75   0.452    -.0357798    .0159415
------------------------------------------------------------------------------
. varsoc

   Selection-order criteria
   Sample:  1960q4 - 1982q4                     Number of obs      =        89
  +---------------------------------------------------------------------------+
  |lag |    LL      LR      df    p      FPE       AIC      HQIC      SBIC    |
  |----+----------------------------------------------------------------------|
  |  0 |  712.092                      2.4e-11  -15.9346  -15.9008  -15.8508* |
  |  1 |  728.167   32.15    9  0.000  2.1e-11  -16.0936  -15.9584  -15.7581  |
  |  2 |  742.213  28.092*   9  0.001  1.8e-11*  -16.207* -15.9704* -15.6198  |
  +---------------------------------------------------------------------------+
   Endogenous:  dln_consump dln_inc dln_inv
    Exogenous:  _cons
. varlmar

   Lagrange-multiplier test
  +--------------------------------------+
  | lag  |      chi2    df   Prob > chi2 |
  |------+-------------------------------|
  |   1  |    8.8693     9     0.44942   |
  |   2  |   10.9722     9     0.27762   |
  +--------------------------------------+
   H0: no autocorrelation at lag order
. varnorm

   Jarque-Bera test
  +--------------------------------------------------------+
  |          Equation |           chi2   df  Prob > chi2 |
  |-------------------+------------------------------------|
  |       dln_consump |          10.245   2    0.00596   |
  |           dln_inc |           1.145   2    0.56398   |
  |           dln_inv |           7.029   2    0.02976   |
  |               ALL |          18.419   6    0.00527   |
  +--------------------------------------------------------+

   Skewness test
  +--------------------------------------------------------+
  |          Equation | Skewness   chi2   df  Prob > chi2 |
  |-------------------+------------------------------------|
```

```
  |       dln_consump | -.73238    7.956   1    0.00479   |
  |           dln_inc | -.10013    0.149   1    0.69975   |
  |           dln_inv |  .32757    1.592   1    0.20709   |
  |               ALL |            9.697   3    0.02133   |
  +-------------------------------------------------------+

   Kurtosis test
  +-------------------------------------------------------+
  |          Equation | Kurtosis   chi2   df  Prob > chi2 |
  |-------------------+-----------------------------------|
  |       dln_consump |  3.7855    2.288   1    0.13036   |
  |           dln_inc |  2.4816    0.997   1    0.31810   |
  |           dln_inv |  4.2109    5.437   1    0.01971   |
  |               ALL |            8.722   3    0.03322   |
  +-------------------------------------------------------+
   dfk estimator used in computations
. varwle

   Equation: dln_consump
  +-----------------------------------+
  | lag |    chi2      df  Prob > chi2 |
  |-----+-----------------------------|
  |   1 |  7.675739     3     0.053    |
  |   2 |  17.32727     3     0.001    |
  +-----------------------------------+

   Equation: dln_inc
  +-----------------------------------+
  | lag |    chi2      df  Prob > chi2 |
  |-----+-----------------------------|
  |   1 |  9.628048     3     0.022    |
  |   2 |  5.914459     3     0.116    |
  +-----------------------------------+

   Equation: dln_inv
  +-----------------------------------+
  | lag |    chi2      df  Prob > chi2 |
  |-----+-----------------------------|
  |   1 |  7.873887     3     0.049    |
  |   2 |  2.993971     3     0.393    |
  +-----------------------------------+

   Equation: All
  +-----------------------------------+
  | lag |    chi2      df  Prob > chi2 |
  |-----+-----------------------------|
  |   1 |  44.06615     9     0.000    |
  |   2 |  28.28163     9     0.001    |
  +-----------------------------------+
```

g. Final Model

Finally, drop individual variables from each of the three equations until you end up with a parsimonious model.

We now drop individual variables from each equation until we end up with the following parsimonious model with eight constraints:

```
 . var dln_consump dln_inc dln_inv, lags(1/2) dfk constraint (1 2 3 4 5 6 7 8)
Estimating VAR coefficients

Iteration 1:   tolerance =  .08076175
Iteration 2:   tolerance =  .00197295
Iteration 3:   tolerance =   .0000493
Iteration 4:   tolerance =  1.529e-06
Iteration 5:   tolerance =  5.099e-08


Vector autoregression

Sample:  1960q4 - 1982q4                           No. of obs      =        89
Log likelihood =  739.5869                         AIC             = -16.14802
FPE            =  2.18e-15                         HQIC            = -15.91133
Det(Sigma_ml)  =  1.63e-15                         SBIC            = -15.56081

Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
dln_consump           5     .009606   0.2292   37.83738   0.0000
dln_inc               4      .01085   0.1394   15.67358   0.0013
dln_inv               4     .042994   0.0849   9.673415   0.0216
----------------------------------------------------------------

 ( 1)  [dln_consump]L.dln_inv = 0
 ( 2)  [dln_inc]L2.dln_inc = 0
 ( 3)  [dln_inv]L2.dln_inc = 0
 ( 4)  [dln_inc]L2.dln_consump = 0
 ( 5)  [dln_inv]L.dln_inc = 0
 ( 6)  [dln_inc]L.dln_inc = 0
 ( 7)  [dln_inv]L2.dln_inv = 0
 ( 8)  [dln_consump]L2.dln_consump = 0
------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
dln_consump  |
 dln_consump |
         L1. |  -.2560975   .1075375    -2.38   0.017    -.4668671   -.0453279
         L2. |   2.70e-18   1.79e-17     0.15   0.880    -3.24e-17    3.78e-17
             |
     dln_inc |
         L1. |   .2899923   .0825004     3.52   0.000     .1282945    .4516901
         L2. |    .286996   .0723422     3.97   0.000     .1452079    .4287841
             |
     dln_inv |
         L1. |   1.12e-19   5.57e-18     0.02   0.984    -1.08e-17    1.10e-17
         L2. |   .0515305   .0226578     2.27   0.023     .0071221     .095939
             |
       _cons |   .0112223   .0024842     4.52   0.000     .0063534    .0160913
-------------+----------------------------------------------------------------
dln_inc      |
 dln_consump |
         L1. |    .235463   .1110663     2.12   0.034     .0177771    .4531489
         L2. |  -3.32e-17   2.73e-17    -1.22   0.224    -8.67e-17    2.03e-17
             |
```

```
    dln_inc |
        L1. |  -3.35e-17   2.71e-17    -1.24   0.215    -8.66e-17    1.95e-17
        L2. |  -1.73e-17   1.61e-17    -1.07   0.282    -4.88e-17    1.42e-17
            |
    dln_inv |
        L1. |   .0472669   .0230169     2.05   0.040     .0021546    .0923791
        L2. |   .0629894   .0263325     2.39   0.017     .0113786    .1146003
            |
      _cons |   .0127585   .0024014     5.31   0.000     .0080519    .0174651
-------------+----------------------------------------------------------------
dln_inv      |
 dln_consump |
        L1. |   .8894705   .4463473     1.99   0.046     .0146459    1.764295
        L2. |   .7532321   .4187462     1.80   0.072    -.0674953    1.573959
            |
    dln_inc |
        L1. |  -8.84e-17   1.10e-16    -0.80   0.421    -3.04e-16    1.27e-16
        L2. |  -8.48e-17   2.14e-16    -0.40   0.692    -5.05e-16    3.35e-16
            |
    dln_inv |
        L1. |    -.25624   .1045915    -2.45   0.014    -.4612356   -.0512443
        L2. |  -2.63e-17   2.54e-17    -1.04   0.300    -7.62e-17    2.35e-17
            |
      _cons |  -.0097218   .0125614    -0.77   0.439    -.0343416     .014898
------------------------------------------------------------------------------
. varlmar

   Lagrange-multiplier test
  +--------------------------------------+
  | lag  |      chi2    df   Prob > chi2 |
  |------+-------------------------------|
  |   1  |  -7.1e+02     9     1.00000   |
  |   2  |  -7.2e+02     9     1.00000   |
  +--------------------------------------+
   H0: no autocorrelation at lag order
. varnorm

   Jarque-Bera test
  +--------------------------------------------------------+
  |          Equation |           chi2   df  Prob > chi2 |
  |--------------------+-----------------------------------|
  |       dln_consump |          11.036   2    0.00401   |
  |           dln_inc |           0.869   2    0.64764   |
  |           dln_inv |          12.553   2    0.00188   |
  |               ALL |          24.457   6    0.00043   |
  +--------------------------------------------------------+

   Skewness test
  +--------------------------------------------------------+
  |          Equation | Skewness   chi2   df  Prob > chi2 |
  |--------------------+-----------------------------------|
  |       dln_consump | -.74579    8.250   1    0.00407   |
  |           dln_inc | -.17547    0.457   1    0.49915   |
  |           dln_inv |  .42156    2.636   1    0.10446   |
  |               ALL |           11.343   3    0.01001   |
  +--------------------------------------------------------+
```

Kurtosis test

Equation	Kurtosis	chi2	df	Prob > chi2
dln_consump	3.8666	2.785	1	0.09514
dln_inc	2.6666	0.412	1	0.52091
dln_inv	4.6353	9.917	1	0.00164
ALL		13.114	3	0.00440

dfk estimator used in computations