Astrometry from CCD Images

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Abstract

Astrometry is used to measure properties of celestial objects such as their position on the sky, their movement and their velocities. In this experiment, we used a charged coupled device(CCD). The CCD mapped the sky on a 2D plane. Observations were made using the Dunlap Institute Telescope located in New Mexico, United States. The object of interest for this experiment is asteroid named 30 Urania. We measured the plate constants of the CCD using a data from NGC7331 in order to correct for the CCD, since the CCD is not an ideal one. The plate constants we calculated to be: a_{11}= 0.9902, a_{12}= -0.0067, a_{13}= 1027.717, a_{21}=0.0068, a_{22}=0.9901 and a_{23}= 1031.749.This way, we will correct for sheerness, distortion and the disorientation of the CCD with the celestial coordinate, and then we applied these plate constants to the data from 30 Urania over the course of four nights in January 2012 and measured the velocity in right ascension and declination directions and the proper motion of this asteroid to be 0.00997 deg/h, 0.02505 deg/h and 0.02696 deg/h, respectively.

1 Introduction

Nowadays, with the advancements of technology, we are able to use computers to do astrometry in order to obtain position and motion of objects in the sky. In this experiment, we will first find the star position using the maxima intensities, then we identify centroids of the stellar sources and use these centroids to compare and match with the stars found from USNO-B1.0 catalog. In the next part, we will use least square fitting technique to find the six constants named plate constants of the CCD which are responsible for the sheerness, distortion and disorientation of the CCD pixels with respect to celestial coordinates. Finally we will use SAOImage DS9 software to detect the asteroid 30 Urania over four nights in January 2012. Using these plate constants, we figure out the position of the asteroid in celestial coordinates and use these data to calculate the velocity and proper motion of this asteroid with respect to the sky.

2 Experimental Procedure

2.1 Equipment

We used the Dunlap Institute Telescope(DIT) which is a 50-cm robotic telescope dedicated to search for optical counterparts of gamma-ray bursts. The telescope was located at New Mexico on Mt. Joy(32:54:10 N +105:31:46 W). The telescope is equipped with a 4096 $\times$ 4096 pixel CCD array and the resultant field of view is approximately 36 $\times$ 36 arc minutes. Pixel size of this CCD is 9μ. This CCD operated in binning mode to collect the data we will be using through this experiment. Binning is useful since it reduces the size of the resultant FITS file and reduces the effect of read-out noise. For 2 $\times$ 2 binning the resultant array is 2048 $\times$ 2048 pixels.

3 Analysis

3.1 CCD Calibration

In order to have good images from the CCD we need to calibrate them. A good image is one that is dark subtracted and divided by flat frames. Therefore, in order to use the data from the CCD, we subtracted the dark data from the actual data. Dark data was taken with the same exposure time as the actual data while the shutter was closed. Dark current is produced due to the noise in the CCD and it might come from the thermal energy of the electrons; therefore, CCDs are kept in cool places in order to reduce the noise. We also divided the dark subtracted data by the flat frames. Flats are morning and evening twilight sky images that are used to normalize the image. Another factor to consider are the bias frames, which are zero second exposures that provide a measure of digital offset when the signal is zero. Here, the bias is assumed to be zero for this long exposure with DIT. Figure 1 shows the NGC7331 star field along with the colour bar with black having zero intensity. We will use this image to find the maxima and centroid points of the stars.

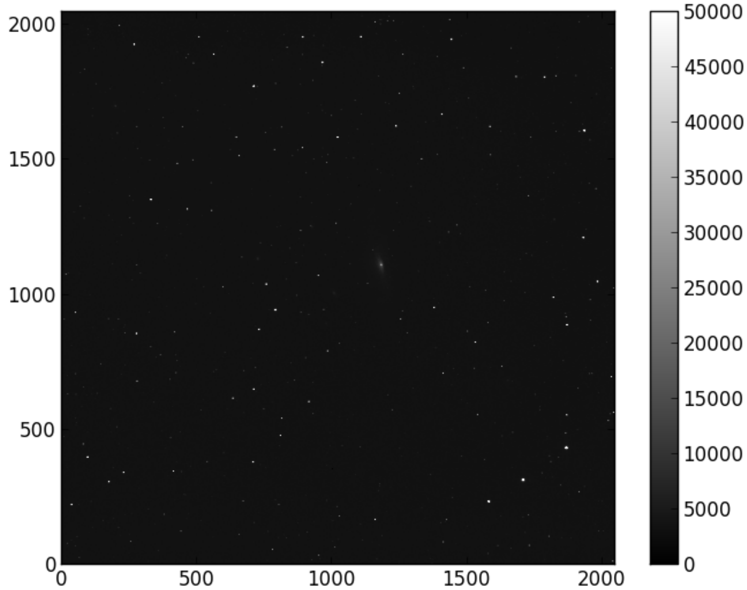

Figure 1: This figure shows the NGC7331. The colour bar shows the intensity of the objects with black having zero intensity. This image is dark subtracted and divided by flat in order to be normalized.

3.2 Detection of Stars

Next step is to find the stars from the CCD image of NGC7331 taken with DIT. The way we did it was to write a python code to check all of the pixels and find the ones that have intensity higher than a specific number(threshold), which for me the threshold was 20000 ADU. The code considered each pixel and then compared it with the pixel on the left, right, bottom and top of it and if its intensity was higher than the adjacent pixel intensities, that pixel was considered as a maxima point.This way the code found all of the maxima points; however, there was a problem. For the larger stars, the code put more than one dot on the object since the object consisted of more than one bright pixel and the code put dots on each of the pixels of the star, so we got a few points on each of the brighter stars. Therefore, we need to find the centroids of these maxima points in order for the code to only point to the centroid of the stars and to put only one dot on each star. In order to calculate the centroids, we modified the code so that it only takes the maxima points and considers a small box around each of the stars and for the maxima points in the specified box finds the centroids. This way we would only have one dot pointing to each star instead of a few. This is shown in figure 1. Figure 2 shows an example of a star centroid. Only one dot is assigned to the star unlike the maxima points that a few points were assigned to each star depending on their intensities. The formulae for calculating the centroids are:

$$< x >= \frac{\Sigma x_i I_i}{I_i} \tag{1}$$

$$< y >= \frac{\Sigma y_i I_i}{I_i} \tag{2}$$

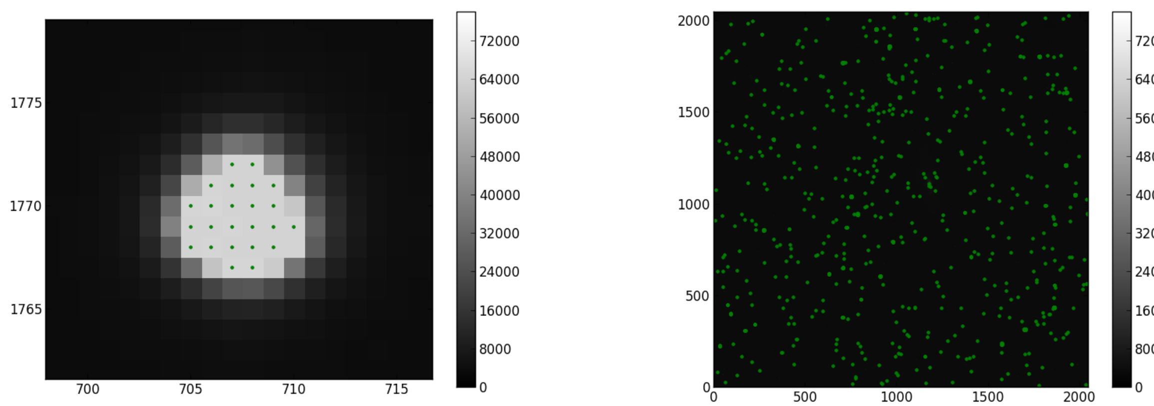

Figure 2: The image on the right shows the maxima points that correspond to the pixels that have intensities higher than 20000 ADU. The image on the left is a random zoomed in part of the right hand side plot in order to show that some of the dots shown on the right image are actually more than one dot. Therefore, we need to find the centroids in order to eliminate the extra dots for one star and only assign one dot to each star.

3.3 Matching Stars and Standard Coordinates

We need to see if the centroids we found from the CCD data are actually stars; therefore, we compared the centroids of NGC7331 found with our code with an image of NGC7331 from USNO-B1.0 Catalog which is the US Naval Observatory. The USNO-B1.0 catalog lists positions, proper motions and magnitudes in various optical bands over the entire sky. The data are from scans of photographic plates from the Palomar 48-inch Schmidth taken during the last 50 years. This catalog provides all-sky coverage down about magnitude 21 and astrometric accuracy of 0.2 arc second at J2000 and is adequate for the purpose of this experiment. Figure 4 shows NGC7331 from USNO catalog both in pixels and in celestial coordinates.

We now have the position of the observed stars from the CCD in pixel x and y and in right ascension and declination from the USNO-B1.0 Catalog. The USNO-B1.0 coordinates

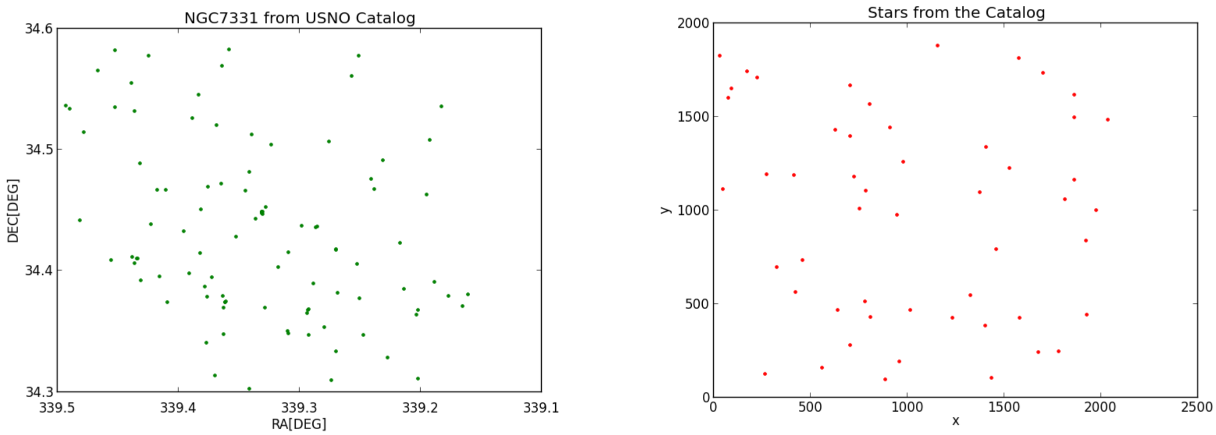

Figure 3: Right.The catalog position of USNO-B1.0 catalog stars within 30 × 30 arc minute square field centred at the positions of NGC7331(RA = 339°.325 ; DEC = 34°.444 at apoch J2000) in sky co-ordinates. Only stars with magnitude greater than 13 are plotted. Left. The position of catalog stars in x and y pixels

are on the celestial sphere, whereas the stars in the CCD image are on a plane, which is the projection of the celestial sphere. Therefore, we need to compare the stars we found with the ones in the catalog. As a result, we need to have a common coordinate system and in order to do so, we need to transform the celestial sphere coordinates of the catalog into plane coordinates; in other words, into pixels. The transformation formulae are given below:

$$X = -\frac{\cos\delta\sin(\alpha-\alpha_0)}{\cos\delta_0\cos\delta\cos(\alpha-\alpha_0)+\sin(\delta)\sin\delta_0} \tag{3}$$

$$Y = -\frac{\sin\delta_0\cos\delta\cos(\alpha-\alpha_0)-\cos\delta_0\sin\delta}{\cos\delta_0\cos\delta\cos(\alpha-\alpha_0)+\sin\delta\sin\delta_0} \tag{4}$$

The values of α_0 and δ_0 are given in the usno catalog. The values we used for α_0 and δ_0 are 5.922338484329758 radians and 0.60115442016539133 radians, respectively. The conversion from (X, Y) coordinates to pixel coordinates (x,y) is straightforward. If f is the focal length of the camera and p is the pixel size, then for an ideal camera we have:

$$x = f(\frac{X}{p}) + x_0 \tag{5}$$

$$y = f(\frac{Y}{p}) + y_0 \tag{6}$$

However, we know that the CCD is not ideal which means that the focal length of the imaging system might not be constant over the field, there might be anamorphic magnification($f_x \neq f_y$) or the CCD might not be oriented in an ideal way(the X-axis might not lie along the declination small circle for instance.) Here we considered the CCD to be ideal as a first guess. f=3435 mm and p=0.018 mm because of the binning of the pixel points. One way to match the stars from the catalog with the ones we found from CCD is to consider a box with a certain radius(we used 15 pixels) and find the difference between the points from catalog and the centroid points we found and take the two stars with the smallest distance to match up together. The result of this is shown in figure 4. As it is seen, the stars from USNO catalog almost perfectly match up with the centroids found from the CCD. In general the CCD is not ideal so we need to take into consideration the effect of a non-ideal CCD. Therefore,the relation between (x,y) coordinates with (X, Y) becomes:

$$x = \frac{f}{p}(X\cos\theta - Y\sin\theta) + x_0$$

(7)

$$y = \frac{f}{p}(X\sin\theta + Y\cos\theta) + y_0$$

(8)

Therefore, we can form a matrix multiplication where X=$(X, Y, 1)$, x=$(x, y, 1)$:

x=TX where

$$T = \begin{pmatrix} (\frac{f}{p})a_{11} & (\frac{f}{p})a_{12} & x_0 \\ (\frac{f}{p})a_{21} & (\frac{f}{p})a_{22} & y_0 \\ 0 & 0 & 1 \end{pmatrix} \tag{9}$$

Thus, there are six unknowns, known as plate constants which we can be found using linear least square. The constant a_{ij} refer to the scale, shear and orientation of the image, and x_0 and y_0 are the pointing offsets in pixels. In order to find the solution to the least-square, we can form the equation a=Bc, where for x we have:

$$a = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix} \tag{10}$$

$$B = \begin{pmatrix} (\frac{f}{p})X_1 & (\frac{f}{p})Y_1 & 1 \\ (\frac{f}{p})X_2 & (\frac{f}{p})Y_2 & 1 \\ \vdots & \vdots & \vdots \\ (\frac{f}{p})X_N & (\frac{f}{p})Y_N & 1 \end{pmatrix}, c = \begin{pmatrix} a_{11} & a_{12} & x_0 \end{pmatrix} \tag{11}$$

We also have a similar equation for y which gives the other three unknowns.(a_{21}, a_{22}, a_{23}) Finally, the elements of matrix c can be found by applying the B transpose and B inverse matrices. We then get: c=$(B^T B)^{-1} B^T a$.

Initially, by matching the stars from the catalog with the stars from the CCD, we can see that some of the stars are off from the other ones(have large residuals), as it is shown in figure 5. However, after removing the points that are off, we get smaller residuals, as shown in figure .

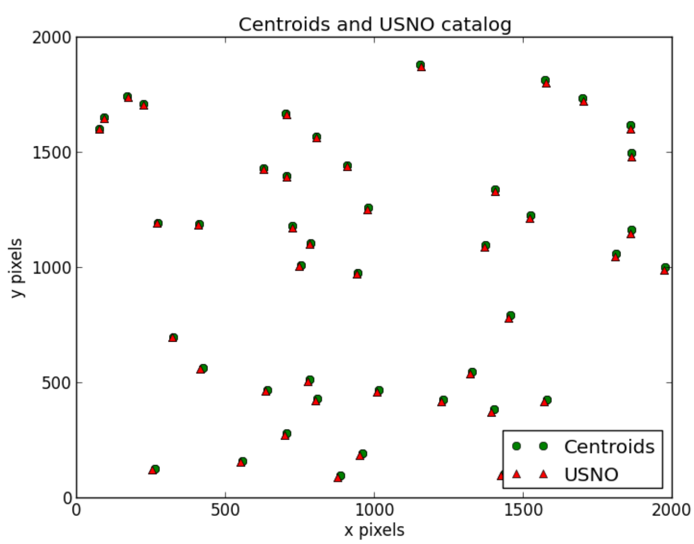

Figure 4: Stars from the USNO-B1.0 catalog and the stars found from the CCD. They almost perfectly match up.

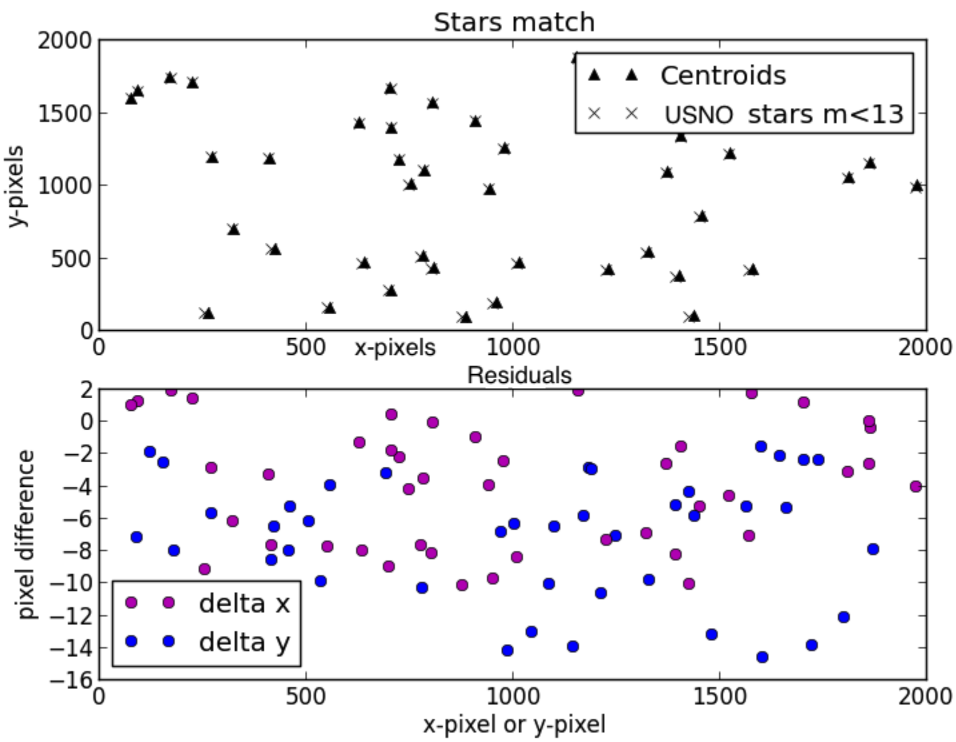

Figure 5: Stars from catalog and from CCD matching, along with their residuals. Some of the points in the residuals are off which means that the range of residuals is a bit high, running from -16 to 2.

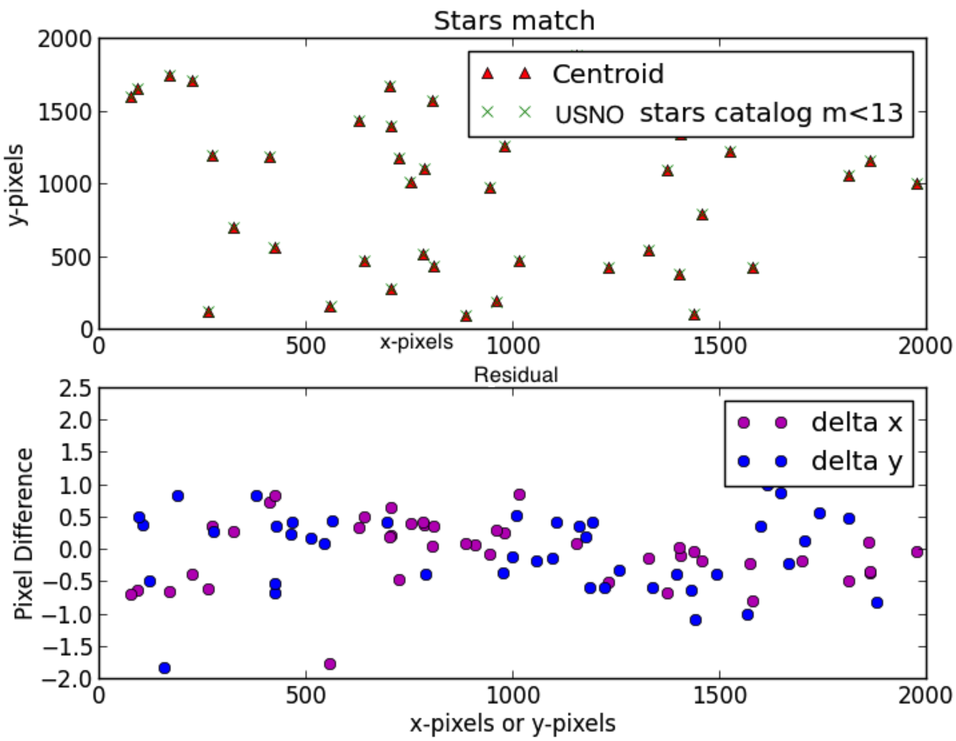

Figure 6: Stars from catalog and from CCD matching, along with their residuals. As it is shown, when the points with high residual are removed, the residuals is pretty small, which means that the catalog and CCD stars match up almost perfectly. The range of residuals runs from -2 to 1.5 pixel which is a good error.

We can also compute the chi squared using the equation below:

$$\chi^2 = (a - Bc)^T (a - Bc) \tag{12}$$

$$\chi^2 = \begin{pmatrix} 1.000106 & -6.7828e-03 & 1.0277e+03 \\ 6.9593e-03 & 1.00005 & 1.03175e+03 \end{pmatrix}, \tag{13}$$

$$\chi^2_{reduced} = \frac{\chi^2}{d.o.f} = \frac{\chi^2}{(stars - 3)} \tag{14}$$

The value for $\chi^2_{reduced}$ is 3.71596726137 and 5.73848317049 for x and y, respectively. The value of $\chi^2_{reduced}$ is close to one, which means the values we got for the six unknowns are close enough to the actual values; therefore, we are getting good results.

Table 1: Plates constants

Unknowns	x	y
	x	y
a_1	0.9902040	0.006890
a_2	-0.0067157	0.9901437
a_3	1027.717	1031.749

4 Detection of Asteroid

Now that we have the plate constants, we can transform the coordinates of the objects detected on the CCD onto the spherical plane. In other words, we are able to find the right ascension and declination of the asteroid using the plate constants we found in previous parts of this experiment. Therefore, we can measure the position of the asteroid on the sky from different nights and calculate its velocity and proper motion through the sky. Data from Asteroid 30 Urania was taken in 5 almost consecutive nights. We used DS9 program in order to detect the asteroid from the background stars. The way we found the asteroid is that we opened the FITS files in DS9 application and log scaled them so that we can better see the stars. Then the way we found the asteroid is that we looked at the pattern of the stars and figured out which bright dot is in one image but not in the others. Therefore, the asteroid is found in all four images. We know the x and y pixel numbers of the position of the asteroid on the CCD from pointing to the asteroid in DS9 and reading off the values of x and y pixels, and the using the plate constants we found previously from NGC7331, we converted the pixel numbers to right ascension and declination coordinate. From the position of the

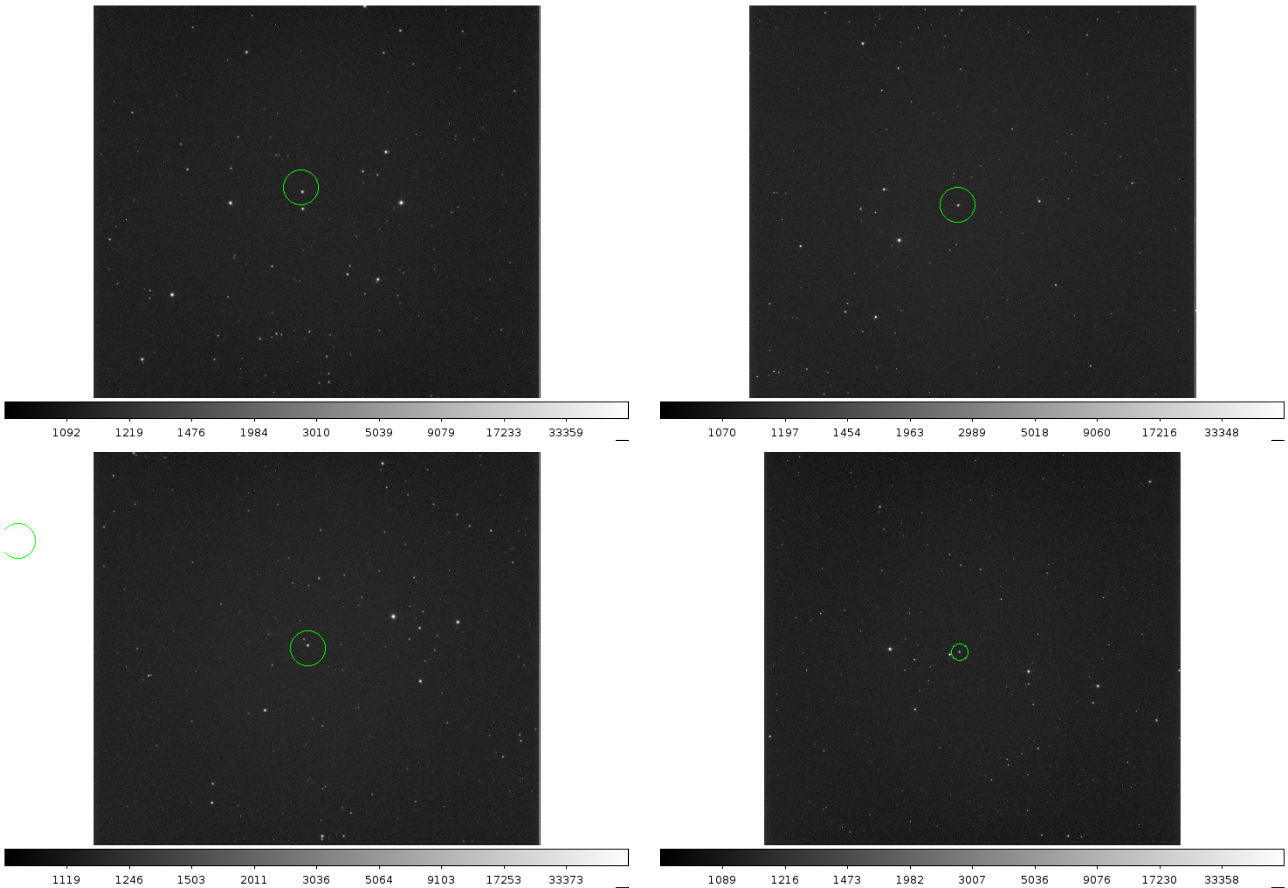

Figure 7: Tracking position of asteroid 30 Urania in the sky. The FITS files were opened using DS9 software. The green circles show the position of the asteroid with respect to the background stars. The way of detection of the asteroid was visually by looking at the fixed background stars and figuring out which point is moving with respect to the background stars. Upper left is data from January 19, 2012, Upper right is the data from January 20, 2012, Lower left is the data from January 22, 2012 and Lower right is the data from january 23, 2012.

asteroid on the sky, we measured the differences in the positions in order to calculate the proper motion. The position results are all listed in table 2. In order to see the movement of the asteroid in the sky, we plotted the position of the asteroid during different times and plotted the line of best fit. This is shown in figure 8. We can use the plate constants found in section 3.3 to convert the position of the asteroid from pixel into right ascension and declination. To do so , we need to solve the following matrix: X=T^{-1}x. The conversion from standard coordinated to celestial coordinated is then given by the following equations:

$$\alpha = tan^{-1}(-\frac{X}{\cos\delta_0 - Y\sin\delta_0}) \tag{15}$$

$$\delta = \sin^{-1}\left(\frac{\sin\delta_0 + Y\cos\delta_0}{(1+X^2+Y^2)^{0.5}}\right) \quad (16)$$

Table 2: Position of asteroid 30 Urania During Different Times

Date	Time	X position	Y position	Right Ascension	Declination
20 Jan 2012	04:28:30	1089.500	977.375	2:57:49.051	+19:14:29.82
21 Jan 2012	04:40:27	1090.375	1039.500	2:58:44.297	+19:16:44.85
23 Jan 2012	05:43:40	1063.130	1008.163	3:00:41.050	+19:21:37.71
24 Jan 2012	04:26:48	1088.602	1038.716	3:01:37.247	+19:24:02.69

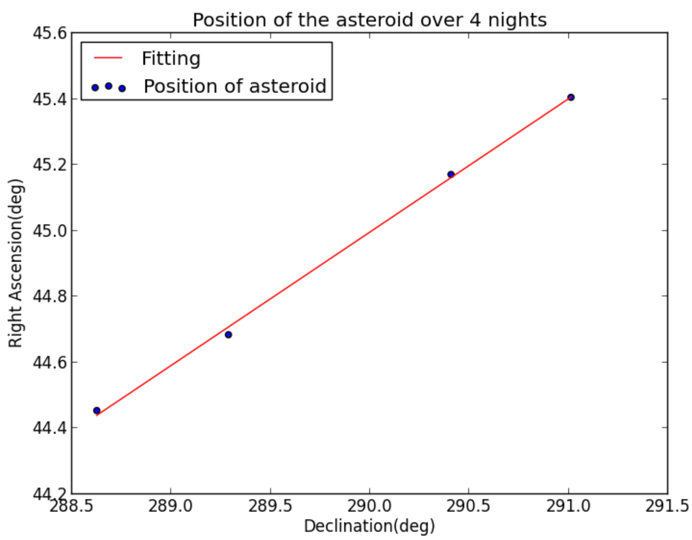

Figure 8: Position of the asteroid 30 Urania over four almost consecutive nights. The detail of the observation time and date is listed in table 2. The positions are in Right ascension and declination calculated using the plate constants we found in previous parts using the data from NGC7331.

Table 3 shows the calculated velocities of the asteroid in declination and right ascension directions, along with their proper motion which is the square root of sum of each of the velocity components squared. If we average the velocities, we end up getting the velocity of the asteroid 30 Urania to be 0.02505 deg/h in declination direction and to be 0.00997 deg/h in the right ascension direction. Therefore, the average proper motion of the asteroid 30 Urania would be 0.02696 deg/h through the sky.

Table 3: Velocities in declination and right ascension direction and the proper motion

Delta Dec(deg)	Delta RA(deg)	Delta time(h)	V_{dec} (deg/h)	V_{ra}(deg/h)	Proper Motion(deg/h)
0.562625	0.230191	24.19916667	0.0232497	0.009512	0.02512
1.22025	0.486471	48.22027778	0.025305743	0.010088515	0.0272426
0.6040833	0.23415383	22.71888889	0.026589473	0.01030657	0.0285172
average values			0.02505	0.00997	0.02696

5 Conclusion

We need to subtract the dark frame from the data we want to use and divide it by flat frames in order to get useful images for analysis. Division by flat frames normalizes the image. In this lab, we learned the technique of astrometry and got to know some of the limiting factors in gathering data from the sky with CCDs. The limiting factors are the ones that make the CCD non-ideal. Some of these limiting factors are distortion of the optical system, pixels not being perfect squares and the CCD might have unknown rotation with respect to celestial coordinates. We learnt how to detect objects from a 2D image from a CCD and how to find the centroids(here we used NGC7331 star field to detect the stars based on a threshold intensity which was 20000 ADU for me). Moreover, we learnt how to query data from USNO-B1.0 catalog which is a catalog listing position and proper motion in various optical bands over the entire sky. We also compared the stars from the catalog with the centroid points we found from the CCD image and matched them. In addition, we learnt how the conversion between the pixel coordinates and celestial coordinates(right ascension and declination) work. Finally, using the mentioned techniques, we were able to detect the movement of the asteroid 30 Urania in the sky using the data taken during 4 almost consecutive nights using DS9 software, and we measured the velocity of this asteroid in both declination and right ascension direction; therefore, we were able to calculate its proper motion through the sky which turned out to be approximately 0.02 degrees per hour.