Practical 2 - Probability calculus

(some simple exercises.)

Today, we focus on discrete random variables (RVs).

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①

Given: ·) RVs $X_1, X_2, \dots, X_n, W$, each taking values in $\{x_1, x_2, x_3, x_4\}$ (4 possible outcomes).

·) Their joint distr. $P(X_1, \dots, X_n, W)$

Task: ·) Find a math. expression for the marginal distr. $P(W)$.

·) How many additive terms does this expression involve? (Computational costs in $\mathcal{O}(\,.\,)$?)

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②

Given: ·) green box with 5 apples & 2 plums (/ red)

·) black box with 2 apples & 3 plums (/ blue)

Task: We choose randomly a box and then randomly pick a fruit out of it.

RVs { X ... box
RVs { Y ... Fruit

·) Compute all conditional probabilities $P(Y = \text{fruit} \mid X = \text{box})$, $\text{box} \in \{g, b\}$, $\text{fruit} \in \{a, p\}$.

·) Compute the marg. distr. $P(X)$, $P(Y)$.

·) Assume, an apple has been picked. What are the prob. that it has been picked from the (i) green (ii) black box? (Hint: use Bayes rule).

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(3)

Given: ·) Two binary RVs X, Y with following joint distr.:

X \ Y	0	1
0	0.28	0.52
1	0.07	0.13

Task: ·) Show that X and Y are statistically independent.
