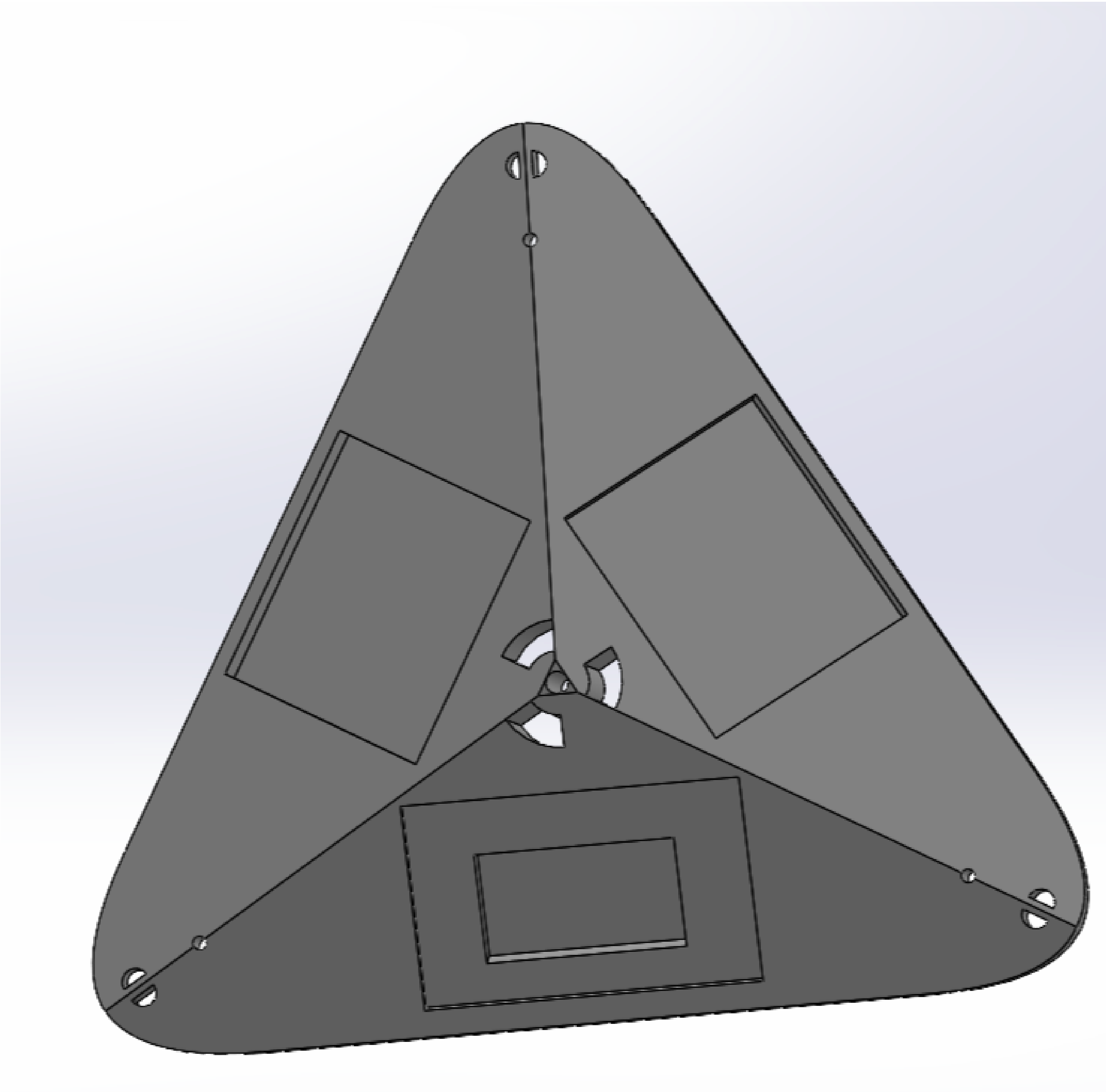
Continuum Robotics

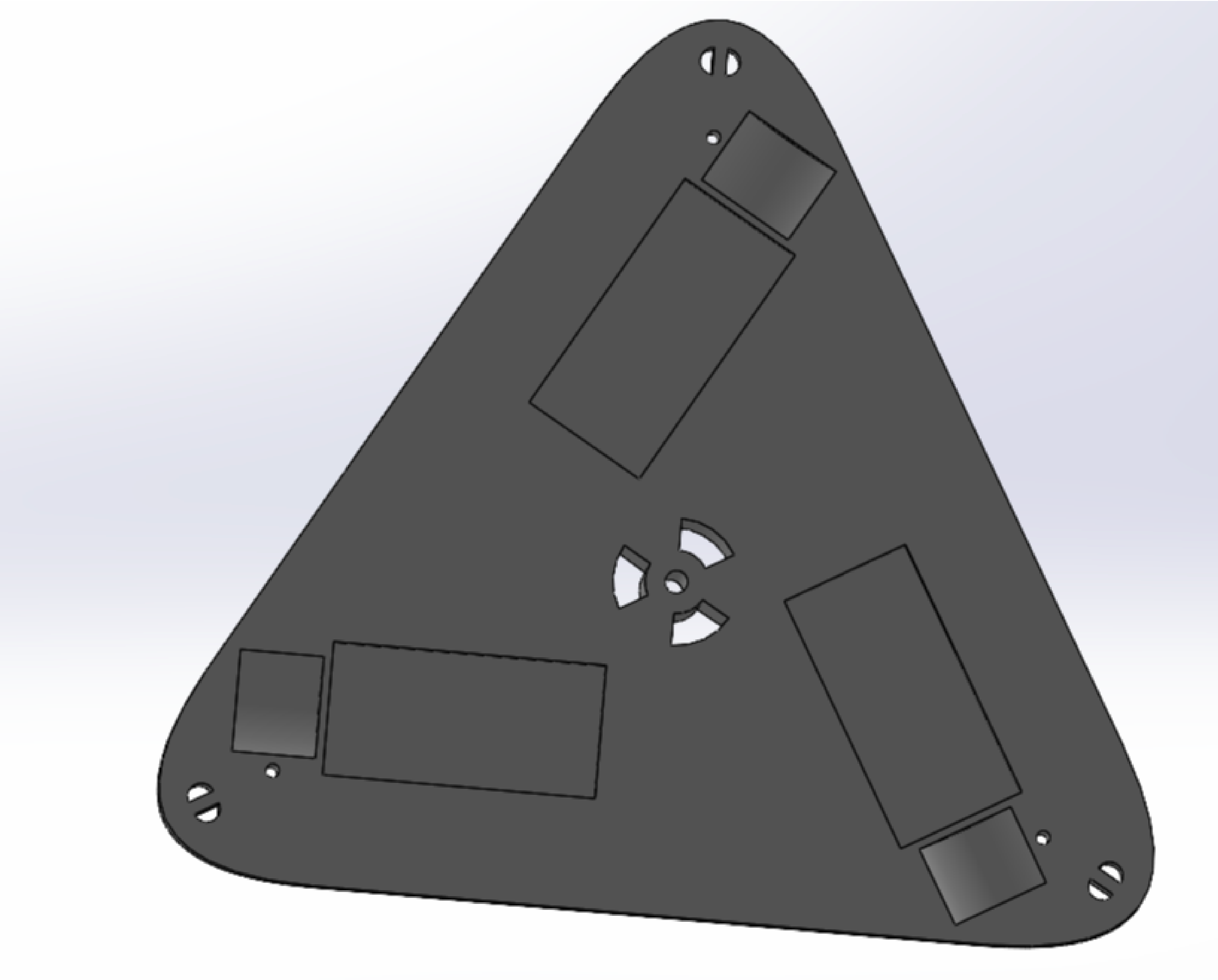
This project consisted of design and control of a novel flexible cable driven robot. The basic design of the robot consists of three motors with 3D printed pulleys attached at its head being used to move the tendons attached to it. The three motors are controlled by two drivers with one being used to control two motors and the second driver being used to control the third motor. A controller is used to vary the rpm as per the input given. The deflection of the tendons is to be measured using an IMU sensor. All these components are housed in a 3D printed triangular segment.

Modeling

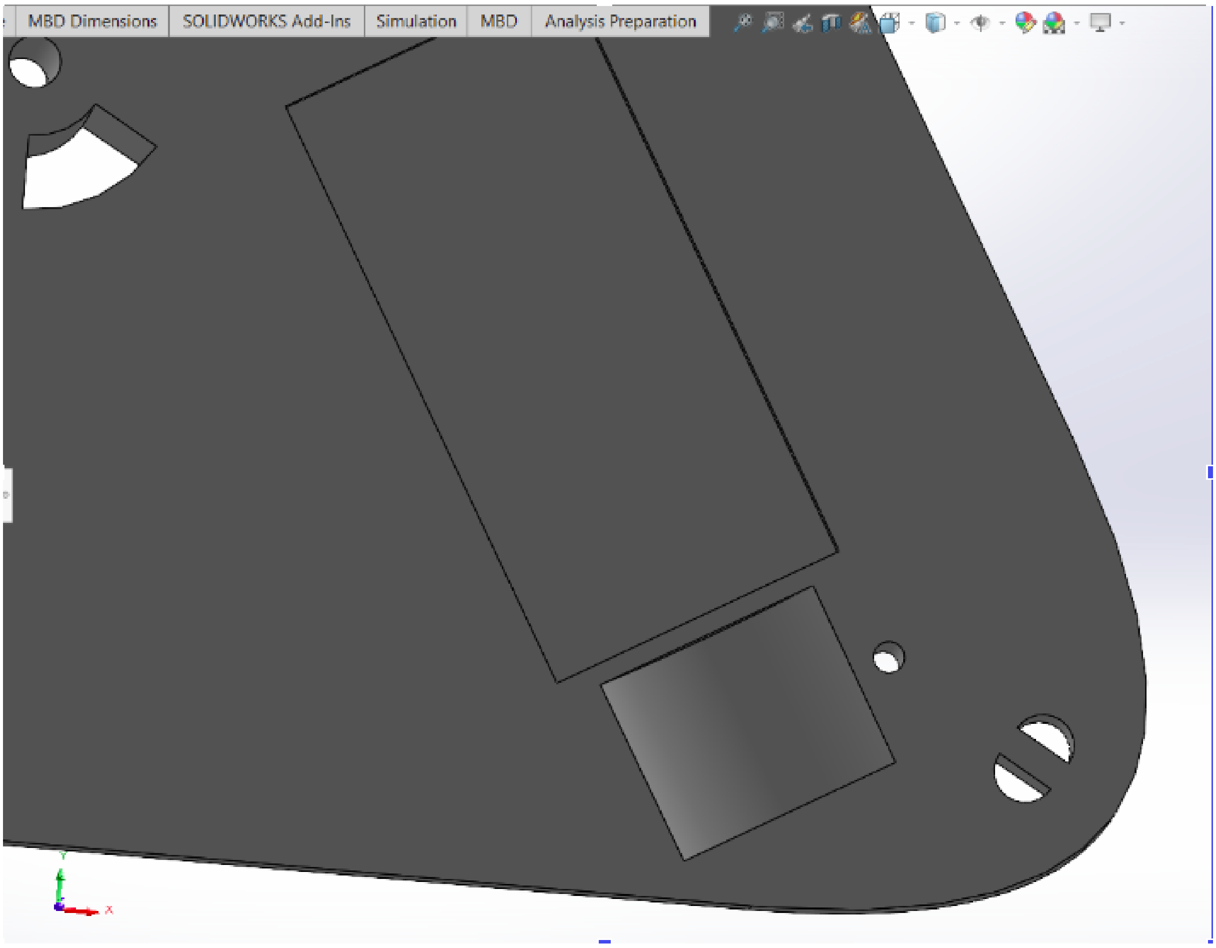
In order to 3D print the model we designed the model on a CAD modeling software. The design is pretty basic, consisting of two triangular segments which house the motors, the drivers and the controllers and one Nitinol wire passing through the center of these two segments.



We chose a triangular segment since the model requires the usage of three motors and a triangular segment would ensure enough spacing for three motors to be housed with minimum usage of material. Three slots have been made on one side of the segment to keep the motors. The motors can be kept in place using glue or a double sided tape.



Two tiny holes are made in the region near the pulley attached to the motor. These holes are used for the tendons to pass through.

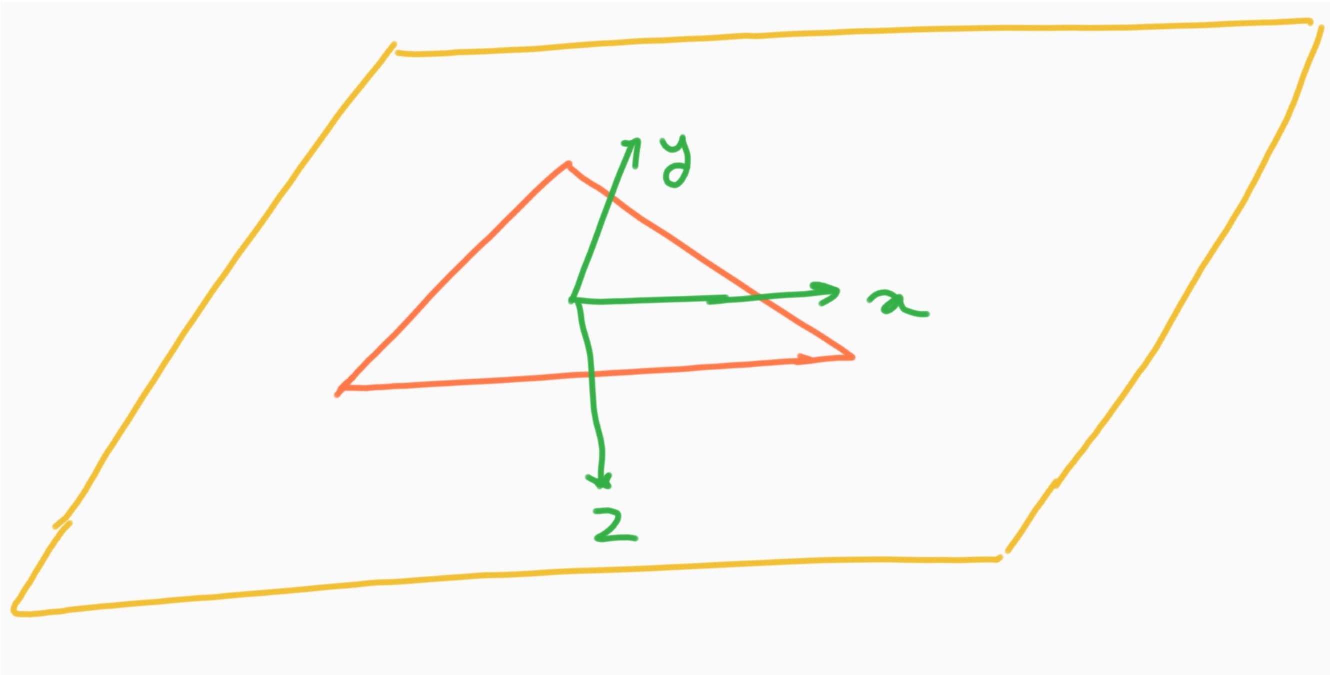


Mathematical Model

Consider a two segment model connected by a Nitinol wire backbone. One of the segment is fixed and is used as a reference to determine the kinematics of the free segment. We use three tendon wires to control the motion.

Coordinate axis

Let the plane of the fixed segment define the x-y plane. The region below be defined by positive z-axis.

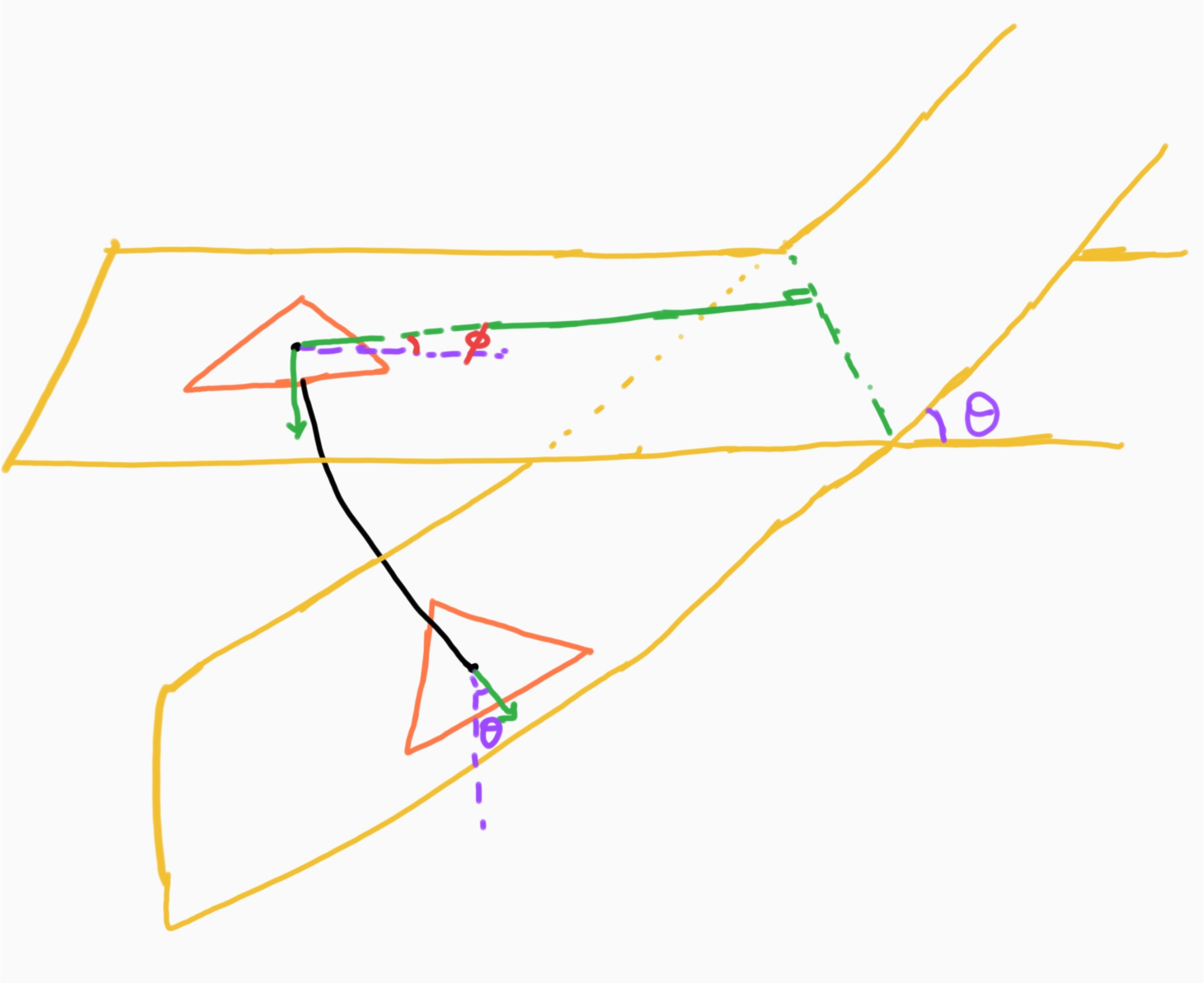


Terminologies

- `l` $\equiv$ the length of backbone between the two segments
- θ $\equiv$ the angle made by the two corresponding planes of the segments. Also the angle made by the free plane.
- ϕ $\equiv$ The angle made by the free segment with the x-axis.
- a $\equiv$ distance of tendon from center.
- d_i $\equiv$ d_1, d_2, d_3, being the respective tendon lengths.

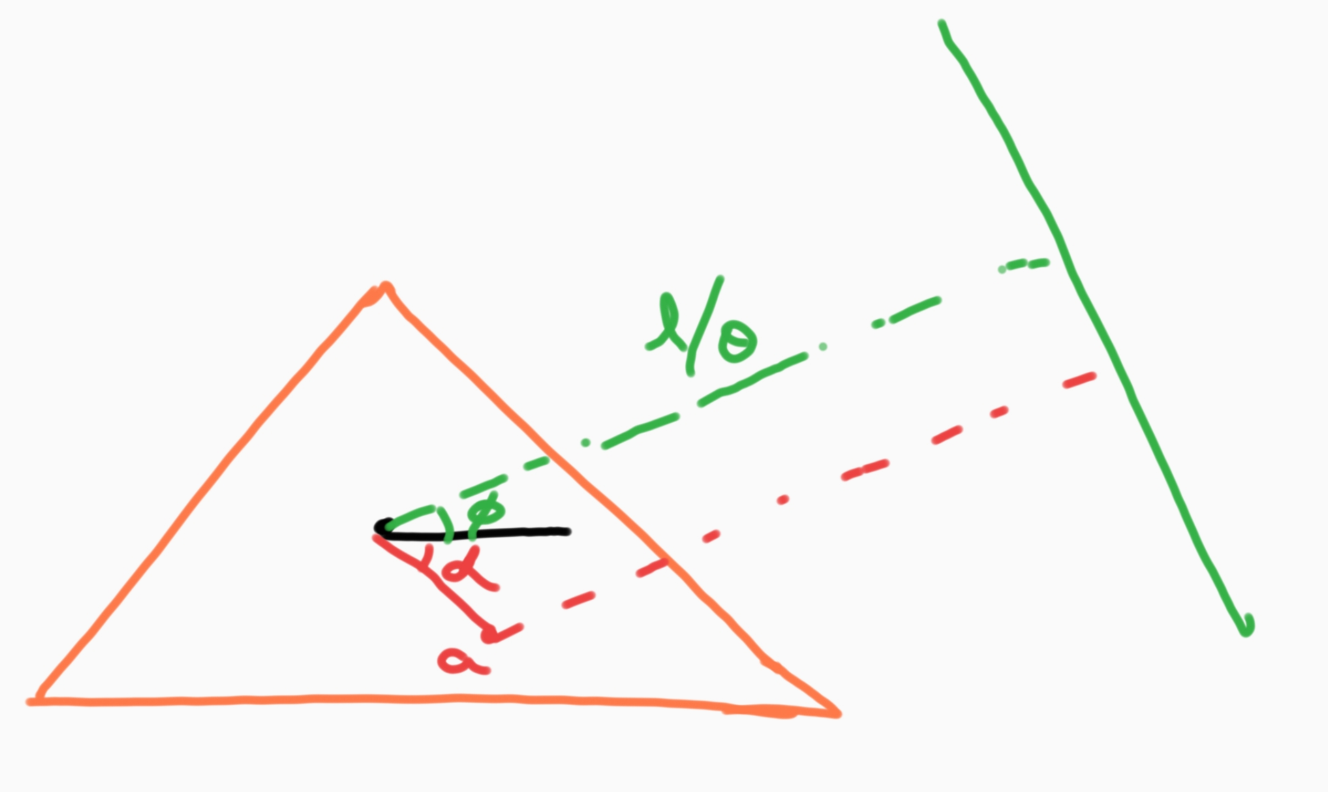
Assumptions

1. Consider that each segment(of height 11.82 mm) is represented by a plane.
2. The backbone does not rotate i.e. the respective position of tendons remain same. There is no twisting.

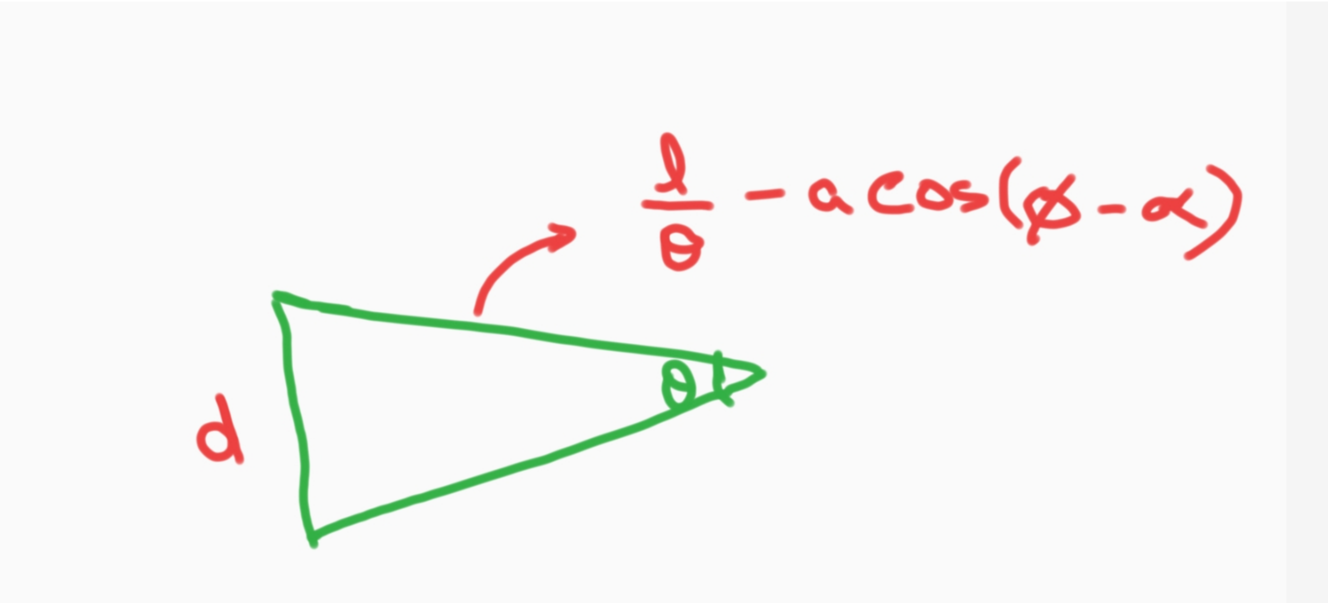


Mathematical Derivation

Consider a point `a` on the fixed segment. An image `a'` is present the free segment.



The isosceles triangle made incorporating the two planes shows that the distance `aa'` is:



$$d = 2\sin\left(\frac{\theta}{2}\right)\left(\frac{l}{\theta} - a\cos\left(\phi - \alpha\right)\right)$$

where,
$\alpha \equiv$ the angle made by the point `a` from the center. The potential values of α are $\left\{\frac{\pi}{3}, \pi, \frac{5\pi}{3}\right\}$.
We can get the values of d_1, d_2 and d_3 by substituting respective values of α.

Now, subtract any two tendon lengths to eliminate $\frac{l}{\theta}$. Let $\angle a > \angle b$.

$$\implies d_a - d_b = 2a\sin\left(\frac{\theta}{2}\right)\left(\cos\left(\phi - \alpha_a\right) - \cos\left(\phi - \alpha_b\right)\right)$$

$$\implies d_a - d_b = 2a\sin\left(\frac{\theta}{2}\right)\left(-2\sin\left(\phi - \frac{\alpha_b + \alpha_a}{2}\right)\sin\left(\frac{\alpha_b - \alpha_a}{2}\right)\right)$$

Since the geometry denote equilateral triangle, $\alpha_a - \alpha_b = \frac{2\pi}{3}$ and $\frac{\alpha_b - \alpha_a}{2}$ denote the respective angle bisectors.

$$\implies d_a - d_b = 2\sqrt{3}a\sin\left(\frac{\theta}{2}\right)\sin\left(\phi - \frac{\alpha_b + \alpha_a}{2}\right)$$

Note

The difference in the length of two tendons is independent of the distance `l` between two concurrent segments.

We now get three equations as follows:

- $a = 2, b = 1, \frac{\alpha_2+\alpha_1}{2} = \frac{\pi}{3}$

$$\implies d_2 - d_1 = 2\sqrt{3}a\sin\left(\frac{\theta}{2}\right)\sin\left(\phi - \frac{\pi}{3}\right)$$

- $a = 3, b = 2, \frac{\alpha_3+\alpha_2}{2} = \pi$

$$\implies d_3 - d_2 = 2\sqrt{3}a\sin\left(\frac{\theta}{2}\right)\sin\left(\phi - \pi\right)$$

- $a = 1, b = 3, \frac{\alpha_1+\alpha_3}{2} = \frac{5\pi}{3}$

$$\implies d_1 - d_3 = 2\sqrt{3}a\sin\left(\frac{\theta}{2}\right)\sin\left(\phi - \frac{5\pi}{3}\right)$$

The above three equations can be used to determine the value of ϕ and θ as functions of d_1, d_2 and d_3.
Let any two equations be represented as,

- $\Delta d_a = b\sin\left(\phi - \beta_a\right)$
- $\Delta d_b = b\sin\left(\phi - \beta_a\right)$
  where,
  $b = 2\sqrt{3}a\sin\left(\frac{\theta}{2}\right)$

Dividing,

$$\implies \frac{\Delta d_a}{\Delta d_b} = \frac{\sin\left(\phi - \beta_a\right)}{\sin\left(\phi - \beta_b\right)}$$

Solving for ϕ,

$$\implies \tan\left(\phi\right) = \frac{\sin\left(\beta_a\right) - \frac{\Delta d_a}{\Delta d_b}\sin\left(\beta_b\right)}{\cos\left(\beta_a\right) - \frac{\Delta d_a}{\Delta d_b}\cos\left(\beta_b\right)}$$

Substituting any two values for Δd, we get,

$$\implies \tan\left(\phi\right) = \frac{\sqrt{3}\left(d_2 - d_3\right)}{\left(d_1 - d_2\right) + \left(d_1 - d_3\right)}$$

Substituting value of ϕ in any equation,

$$\implies b = \frac{2}{\sqrt{3}}\sqrt{d_1^2 + d_2^2 + d_3^2 - d_1 d_2 - d_2 d_3 - d_1 d_3}$$

$\therefore$

$$\implies \sin\left(\frac{\theta}{2}\right) = \frac{1}{3a}\sqrt{d_1^2 + d_2^2 + d_3^2 - d_1 d_2 - d_2 d_3 - d_1 d_3}$$

Conclusion

ϕ and θ can be written as the function of d_1, d_2 and d_3 as,

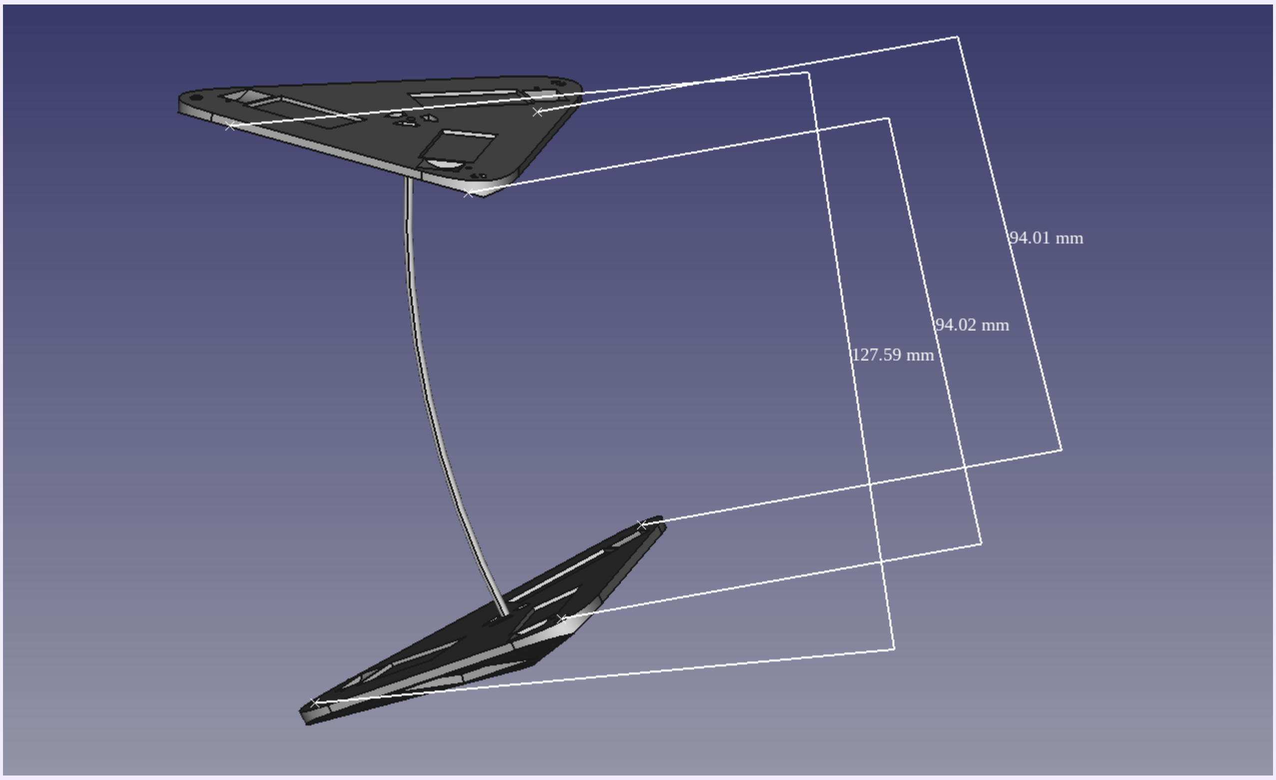
$$\phi = \arctan\left(\frac{\sqrt{3}\left(d_2 - d_3\right)}{\left(d_1 - d_2\right) + \left(d_1 - d_3\right)}\right)$$

Note

When $d_2 = d_3$, the value of α is intuitively zero.

$$\theta = 2\arcsin\left(\frac{1}{3a}\sqrt{d_1^2 + d_2^2 + d_3^2 - d_1 d_2 - d_2 d_3 - d_1 d_3}\right)$$

Example



The geometry made with $\alpha = 0$ and $\theta = \frac{\pi}{6}$ gives $d_1 = 127.59$, $d_2 = 94.01$, $d_3 = 94.02$.
Theoretical solution for θ and α, we get,

$$\alpha \approx 0^\circ$$
$$\theta \approx 32^\circ$$