

$$\begin{array}{c}
\frac{\frac{\{I[(r+p)/r, (2*p)/p(i+1)/i]\} r := r - p \{I[(2*p)/p, (r+p)/r, (i+1)/i]\} \{I[(2*p)/p, (r+p)/r, (i+1)/i]\} p := 2*p; r := r+p; i := i+1 \{I\}}{I \wedge \text{not}(i = n) \Rightarrow \{I[(2*p)/p, (r+p)/r, (i+1)/i]\}} \\
\frac{\frac{\frac{n \geq 0 \Rightarrow I\{0/r, 0/i, 1/p\}}{\{n \geq 0\} r := 0; i := 0; p := 1 \{I\}}}{\{I \wedge \text{not}(i = n)\} r := r - p; p := 2*p; r := r + p; i := i + 1 \{I\}}}{I \wedge (i = n) \Rightarrow r := 2^n - 1} \\
\{I\} \text{ while } \text{not}(i = n) \text{ do } r := r - p; p := 2*p; r := r + p; i := i + 1 \{r := 2^n - 1\} \\
\{n \geq 0\} r := 0; i := 0; p := 1 \text{ while } \text{not}(i = n) \text{ do } r := r - p; p := 2*p; r := r + p; i := i + 1 \{r := 2^n - 1\}
\end{array}$$

I is invariant for this loop, and it must satisfy the following constrains:

$$\begin{aligned}
n \geq 0 &\Rightarrow I\{0/r, 0/i, 1/p\} \\
I \wedge \text{not}(i = n) &\Rightarrow \{I[(2*p)/p, (r+p)/r, (i+1)/i]\} \\
I \wedge (i = n) &\Rightarrow r := 2^n - 1
\end{aligned}$$

Let $I = p = 2^i \wedge r = 2^i - 1 \wedge i \leq n$ then

$$\begin{aligned}
n \geq 0 &\Rightarrow 1 = 2^0 \wedge 0 = 2^0 - 1 \wedge 0 \leq n \\
p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge (i \neq n) &\Rightarrow 2p = 2^{i+1} \wedge r + 2^i = 2^{i+1} - 1 \wedge i + 1 \leq n \\
p = 2^i \wedge r = 2^i - 1 \wedge i \leq n \wedge (i = n) &\Rightarrow r = 2^n - 1
\end{aligned}$$

All the constrains are correct, so the program is right.