

Forward (3 Layers)

$$X = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \quad W_1 = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \quad X \cdot W_1 = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \quad b_1 = \begin{bmatrix} \cdot & \cdot \end{bmatrix}$$

$n \times d$ $d \times h$ $n \times h$ $1 \times h$
 4×3 3×2 4×2 1×2

$$H = X \cdot W_1 + b_1 = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$$

$$A = \text{Relu}(X \cdot W_1 + b_1) = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \times W_2 = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} = A \cdot W_2 = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \quad b_2 = \begin{bmatrix} \cdot & \cdot & \cdot \end{bmatrix}$$

$n \times h$ $h \times d$ $n \times d$ $1 \times d$
 4×2 2×3 4×3 1×3

MNIST input

$$O = A \cdot W_2 + b_2 = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \quad q = \text{Relu}(O) = \begin{bmatrix} \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{bmatrix} \times W_3 = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \quad b_3 = \begin{bmatrix} \cdot & \cdot \end{bmatrix}$$

$n \times d$ $n \times d$ $d \times c$ $1 \times c$
 4×3 4×3 3×2 1×2

$$S = q \cdot W_3 + b_3 = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix} \quad P = \text{softmax}(q \cdot W_3 + b_3) = \begin{bmatrix} \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \\ \cdot & \cdot \end{bmatrix}$$

4×2 $n \times c$ 4×2
 $n \times c$

$$L = -\text{LogLikelihood}(P)$$

Backward (3 Layers)

$$\frac{\partial L}{\partial s} = p - T \quad dp \quad T \in \text{2Hot}_2(\text{label}) \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{matrix} n \times c \\ 4 \times 2 \end{matrix}$$

$$\frac{\partial L}{\partial w_3} = \frac{\partial s}{\partial w_3} \cdot \frac{\partial L}{\partial s} = q^T \cdot (p - T) \quad \begin{matrix} d \times n & n \times c \\ 3 \times 4 & 4 \times 2 \end{matrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{matrix} d \times c \\ 3 \times 2 \end{matrix}$$

$$\frac{\partial L}{\partial b_3} = \frac{\partial s}{\partial b_3} \cdot \frac{\partial L}{\partial s} = 1 \cdot \frac{\partial L}{\partial s} = p - T \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{matrix} n \times c \\ 4 \times 2 \end{matrix}$$

$$\frac{\partial L}{\partial q} = \frac{\partial L}{\partial s} \cdot \frac{\partial s}{\partial q} = (p - T) \cdot W_3^T \quad \begin{matrix} n \times c & c \times d \\ 4 \times 2 & 2 \times 3 \end{matrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} n \times d \\ 4 \times 3 \end{matrix}$$

$$\frac{\partial L}{\partial o} = \frac{\partial q}{\partial o} \cdot \frac{\partial L}{\partial q} = \text{Relu}'(o) \odot \frac{\partial L}{\partial q} \quad \begin{matrix} n \times d & n \times d \\ 4 \times 3 & 4 \times 3 \end{matrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} n \times d \\ 4 \times 3 \end{matrix}$$

$$\frac{\partial L}{\partial w_2} = \frac{\partial o}{\partial w_2} \cdot \frac{\partial L}{\partial o} = A^T \cdot \frac{\partial L}{\partial o} = A^T \cdot \text{Relu}'(o) \odot \frac{\partial L}{\partial q} = \left[A^T \cdot \left[\text{Relu}'(o) \odot \left[(p - T) \cdot W_3^T \right] \right] \right] \quad \begin{matrix} h \times n & n \times d \\ 2 \times 4 & 4 \times 3 \end{matrix} \rightarrow \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} h \times d \\ 2 \times 3 \end{matrix}$$

$$\frac{\partial L}{\partial b_2} = \frac{\partial o}{\partial b_2} \cdot \frac{\partial L}{\partial o} = 1 \cdot \frac{\partial L}{\partial o} = \text{Relu}'(o) \odot \frac{\partial L}{\partial q} = \left[\text{Relu}'(o) \odot \left[(p - T) \cdot W_3^T \right] \right] \quad \begin{matrix} n \times d & n \times d \\ 4 \times 3 & 4 \times 3 \end{matrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} n \times d \\ 4 \times 3 \end{matrix}$$

$$\frac{\partial L}{\partial A} = \frac{\partial o}{\partial A} \cdot \frac{\partial L}{\partial o} = \frac{\partial L}{\partial o} \cdot W_2^T = \left[\text{Relu}'(o) \odot \left[(p - T) \cdot W_3^T \right] \cdot W_2^T \right] \quad \begin{matrix} n \times d & d \times h \\ 4 \times 3 & 3 \times 2 \end{matrix} \quad \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{matrix} n \times h \\ 4 \times 2 \end{matrix}$$

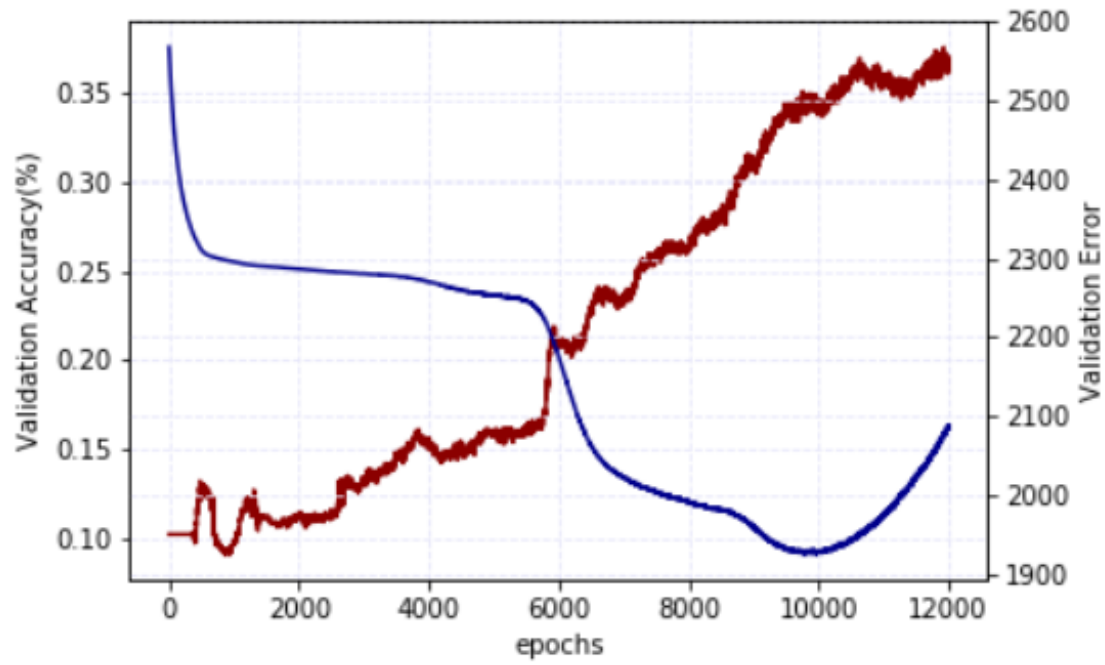
$$\frac{\partial L}{\partial H} = \frac{\partial A}{\partial H} \cdot \frac{\partial L}{\partial A} = f'(H) \odot \frac{\partial L}{\partial A} = \left[f'(H) \odot \left[\text{Relu}'(o) \odot \left[(p - T) \cdot W_3^T \right] \cdot W_2^T \right] \right] \quad \begin{matrix} n \times h & n \times h \\ 4 \times 2 & 4 \times 2 \end{matrix} \rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{matrix} n \times h \\ 4 \times 2 \end{matrix}$$

$$\times: f'(H) = \text{Relu}'(H) = \frac{\partial A}{\partial H} \quad \begin{matrix} da \\ (A'(H)) \end{matrix}$$

$$\frac{\partial L}{\partial w_1} = \frac{\partial H}{\partial w_1} \cdot \frac{\partial L}{\partial H} = X^T \cdot \frac{\partial L}{\partial H} = X^T \cdot f'(H) \odot \frac{\partial L}{\partial A} = \left[X^T \cdot \left[f'(H) \odot \left[\text{Relu}'(o) \odot \left[(p - T) \cdot W_3^T \right] \cdot W_2^T \right] \right] \right] \quad \begin{matrix} d \times n & n \times h \\ 3 \times 4 & 4 \times 2 \end{matrix} \rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad \begin{matrix} d \times h \\ 3 \times 2 \end{matrix}$$

$$\frac{\partial L}{\partial b_1} = \frac{\partial H}{\partial b_1} \cdot \frac{\partial L}{\partial H} = 1 \cdot \frac{\partial L}{\partial H} = f'(H) \odot \frac{\partial L}{\partial A} = \left[f'(H) \odot \left[\text{Relu}'(o) \odot \left[(p - T) \cdot W_3^T \right] \cdot W_2^T \right] \right] \quad \begin{matrix} n \times h & n \times h \\ 4 \times 2 & 4 \times 2 \end{matrix} \quad \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad \begin{matrix} n \times h \\ 4 \times 2 \end{matrix}$$

① 3 Layers 2번째 결과 그래프



나의 최고 validation accuracy : 0.376