DACS 2101

Assignment 2

Gowshikrajan
senthilkumar

60307390

1. $x \oplus y$ ①

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

~~DNF~~

DNF:

$x \oplus y = \bar{x}y + x\bar{y}$

x	y	$\overline{x \oplus y}$
0	0	1
0	1	0
1	0	0
1	1	1

CNF:

$\overline{x \oplus y} = \bar{x}\bar{y} + xy$

$x \oplus y = \overline{\bar{x}\bar{y} + xy}$

$= (x+y)(\bar{x}+\bar{y})$

a) $x \oplus 0 = \bar{x} \cdot 0 + x \cdot \bar{0} = x \cdot \bar{0} = x$

b) $x \oplus 1 = \bar{x} \cdot 1 + x \cdot \bar{1} = \bar{x} + x \cdot 0 = \bar{x}$

c) $x \oplus x = (x+x)(\bar{x}+\bar{x}) = x \cdot \bar{x} = 0$

d) $x \oplus \bar{x} = (x+\bar{x})(\bar{x}+x) = (1) \cdot (1) = 1$

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a) $F(x, y, z) = 1 \iff x = 0$.

x	y	z	$F(x,y,z)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

~~$F(x,y,z) = x\bar{y}\bar{z} + x\bar{y}z + xy\bar{z} + xyz$~~

$F(x,y,z) = \bar{x}\bar{y}\bar{z} + x\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz$

b)

x	y	z	$F(x,y,z)$
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

~~$F(x,y,z) = xy\bar{z} + xyz$~~

$$F(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z$$

c) $x+y=0$

x	y	z	$F(x,y,z)$
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

$$F(x,y,z) = \bar{x}\bar{y}\bar{z} + \bar{x}\bar{y}z$$

d) $xyz=0$

x	y	z	$F(x,y,z)$
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

~~$F(x,y,z) = xyz$~~

$$F(x,y,z) = \bar{x}\bar{y}z + \bar{x}y\bar{z} + \bar{x}yz + x\bar{y}\bar{z} + x\bar{y}z + xy\bar{z} + \cancel{\bar{x}yz} + xyz$$

3. a) $x+y+z$

$= \overline{\bar{x}\bar{y}\bar{z}}$

b) $x+\bar{y}(\bar{x}+z)$

$= \overline{x\cdot(y+\overline{\bar{x}+z})}$

$= \overline{x\cdot(\overline{\bar{y}\cdot(\bar{x}+z)})}$

$= \overline{x\cdot(\overline{\bar{y}\cdot(\overline{x\bar{z}})})}$

c) $\overline{x+\bar{y}}$

$= \overline{\overline{\bar{x}y}}$

$= \bar{x}y$

d) $\bar{x}(x+\bar{y}+\bar{z})$

$= \bar{x}\cdot(\overline{\bar{x}yz})$

Part 2: sets

1. a) set of people who speak English with australian accent is subset of people who speak English.
(2nd is subset of 1st)

b) 2nd is subset of 1st

~~c) 2nd is subset of 1st~~

c) neither is subset

d) 1st is subset of 2nd

2. a) T

b) T

c) F

d) T

e) T

f) T

g) F

h) F

3. a) $A \cup C = B \cup C \rightarrow A = B$

Let $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{1, 3\}$

$A \cup C = \{1, 2, 3\}$

$B \cup C = \{1, 2, 3\}$

$A \cup C = B \cup C$ but $A \neq B$.

$\therefore$ Not possible from proof by counterexample

b) $A \cap C = B \cap C \rightarrow A = B$

Let $A = \{1, 2\}$, $B = \{2, 3\}$, $C = \{2\}$

$A \cap C = \{2\}$

$B \cap C = \{2\}$

$A \cap C = B \cap C$ but $A \neq B$

$\therefore$ Not possible from proof by counterexample

c) $(A \cup C = B \cup C) \land (A \cap C = B \cap C) \rightarrow A = B$

Assume $A \neq B \land (A \cup C = B \cup C \land A \cap C = B \cap C)$

$A \neq B$ means, without loss of generality, $\exists x \in A\,(x \notin B)$. Let that element be c [Existential instantiation].

$c \in A \cup C \equiv c \in B \cup C$. Since $c \notin B$, $c \in C$. [From $c \in B \cup C \equiv c \in B \lor c \in C$]

Therefore, since $c \in A \land c \in C$,

$c \in A \cap C \equiv c \in B \cap C \equiv c \in B \land c \in C$.

However, we know $c \notin B$. This is a <u>contradiction</u>.

$\therefore (A \cup C = B \cup C) \land (A \cap C = B \cap C) \rightarrow A = B$ has to be true.

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Part 3

7 6 5 4 3 2 1 0

to generate: 60307390

i_0	i_1	i_2	s_1	s_2	s_3	s_4	s_5	s_6	s_7	
0	0	0	0	1	1	1	1	1	1	[0]
0	0	1	1	1	1	1	0	0	1	[9]
0	1	1	0	0	1	1	0	0	1	[7]
0	1	0	1	0	1	1	0	1	1	[3]
1	1	0	0	1	1	1	1	1	1	[0]
1	1	1	1	1	1	0	1	1	1	[6]
1	0	1	1	0	1	1	0	1	1	[3]
1	0	0	0	1	1	1	1	1	1	[0]

s_1:

$i_0 \backslash i_1 i_2$	00	01	11	10
0	0	1	0	1
1	0	1	1	0

s_2:

$i_0 \backslash i_1 i_2$	00	01	11	10
0	1	1	0	0
1	1	0	1	1

s_5:

$i_0 \backslash i_1 i_2$	00	01	11	10
0	1	0	0	0
1	1	0	1	1

s_6:

$i_0 \backslash i_1 i_2$	00	01	11	10
0	1	0	0	1
1	1	1	1	1

s_3/ s_7:

$i_0 \backslash i_1 i_2$	00	01	11	10
0	1	1	1	1
1	1	1	1	1

$s_1 = \bar{i}_1 i_2 + i_0 i_2 + \bar{i}_0 i_1 \bar{i}_2$

$\cancel{s_2 = i_1 i_2}$

$s_2 = \bar{i}_1 \bar{i}_2 + \bar{i}_0 \bar{i}_1 + i_0 i_1$

$s_3 = 1$

$s_4 = \bar{i}_0 + \bar{i}_1 + \bar{i}_2$

$s_5 = \bar{i}_1 \bar{i}_2 + i_0 i_1$

$s_6 = i_0 + \bar{i}_0 \bar{i}_2$

$s_7 = 1$

s_4:

$i_0 \backslash i_1 i_2$	00	01	11	10
0	1	1	1	1
1	1	1	0	1