Number Systems and Number Systems Conversion

Motivation:

A digital circuit is one that is built with devices with two well-defined states. Such circuits can process information represented in binary form. Systems based on digital circuits touch all aspects our present day lives. The present-day home products including electronic games and appliances, communication and office automation products, computers with a wide range of capabilities, and industrial instrumentation and control systems, electromedical equipment, and defense and aerospace systems are heavily dependent on digital circuits.

We all use numbers to perform several tasks in our daily lives:

- to communicate
- to perform tasks
- to quantify
- to measure
- Numbers have become symbols of the present era

Learning Objectives:

- Number systems: Binary, octal, and hexa-decimal number systems
- Explain how a number with one radix is converted into a number with another radix.
- Summarize the advantages of using different number systems.

Number System Types:

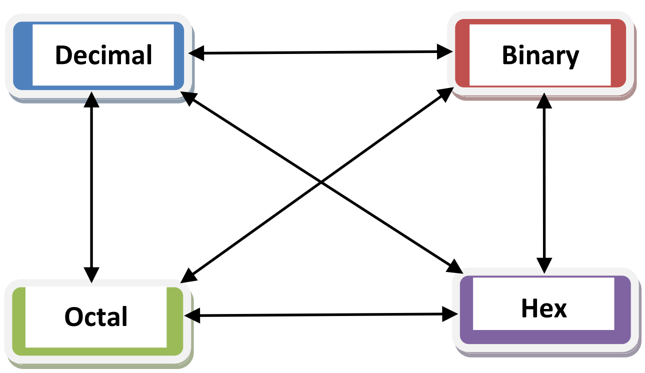
The weighted-positional system based on the use of radix 10 is the most commonly used numbering system in most of the transactions and activities of today's world. However, the advent of computers brought the binary system, using the radix 2, into extensive use. With the radix value of 2, the binary number system requires very long strings of 1s and 0s to represent a given number. Some of the problems associated with handling large strings of binary digits may be eased by grouping them into three digits for octal numbers and four digits for hexadecimal numbers.

Number system	Radix or Base	Digits
Decimal	10	0-9
Binary	2	0-1
Octal	8	0-7
Hexadecimal	16	0-9, 10-15 (A-F)

In the octal number system, the digits will have one of the following eight values 0, 1, 2, 3, 4, 5, 6 and 7. In the hexadecimal system we have one of the sixteen values 0 through 15. However, the decimal values from 10 to 15 will be represented by alphabet A (=10), B (=11), C (=12), D (=13), E (=14) and F (=15).
Any integer > 2 can serve as the radix. A number like $(357)_5$ is incorrect.

Number Systems Conversion

In general, conversion between numbers with different radices cannot be done by simple substitutions. Such conversions would involve arithmetic operations. Let us work out procedures for converting a number in any radix to any other radix, and vice versa.



Conversion of decimal numbers to numbers with radix r:

Decimal to Binary:

Example: $(156)_{10} = ()_2$

Q: Quotient R: Remainder

	Q	R
156 ÷ 2	78	0
78 ÷ 2	39	0
39 ÷ 2	19	1
19 ÷ 2	9	1
9 ÷ 2	4	1
4 ÷ 2	2	0
2 ÷ 2	1	0
1 ÷ 2	0	1

$(156)_{10} = (10011100)_2$

Example: $(0.8125)_{10} = ()_2$

Multiplication	Resultant Integer Part
0.8125 x 2 = 1.6250	1
0.625 x 2 = 1.250	1
0.25 x 2 = 0.50	0
0.5 x 2 = 1.0	1
0 x 2 = 0	0

Now, write these resultant integer part in top to bottom order, this will be 0.11010 which is equivalent binary fractional number of decimal fractional 0.8125.

Decimal to Octal: $(678)_{10} = (\ )_8$

	Q	**R**
678 ÷ 8	84	6
84 ÷ 8	10	4
10 ÷ 8	1	2
1 ÷ 8	0	1

$(678)_{10} = (1246)_8$

Decimal to Hexadecimal: $(678)_{10} = (\ )_{16}$

	Q	**R**
678 ÷ 16	42	6
42 ÷ 16	2	A
2 ÷ 16	0	2

$(678)_{10} = (2A6)_{16}$

Conversion to decimal numbers from numbers with radix r:

Binary to Decimal:

Example: $(11100110)_2 = ()_{10}$

$(11100110)_2 = 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^5 + 0 \times 2^4 + 0 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 0 \times 2^0$

$= 128 + 64 + 32 + 0 + 0 + 4 + 2 + 0 = (230)_{10}$

Example: $(101.101)_2 = ()_{10}$
$(101.101)_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3}$
$= 4 + 0 + 1 + 0.5 + 0 + 0.125$
$= 5 + 0.5 + 0.125 = (5.625)_{10}$

Octal to Decimal:
Example: $(331)_8 = ()_{10}$
$(331)_8 = 3 \times 8^2 + 3 \times 8^1 + 1 \times 8^0 = 192 + 24 + 1 = (217)_{10}$
$(33.56)_8 = 3 \times 8^1 + 3 \times 8^0 + 5 \times 8^{-1} + 6 \times 8^{-2} = (27.71875)_{10}$

Hexadecimal to Decimal:
Example: $(D9)_{16} = ()_{10}$
$(D9)_{16} = 13 \times 16^1 + 9 \times 16^0 = 208 + 9 = (217)_{10}$
$(E5.A)_{16} = 14 \times 16^1 + 5 \times 16^0 + 10 \times 16^{-1} = (229.625)_{10}$

Binary to Octal: Conversion from binary to an octal number requires grouping the binary digits into groups of three.

$(11011001)_2 = (011\ 011\ 001) = (331)_8$

$(111101.01101)_2 = (111\ 101.011\ 010) = (75.32)_8$

Note that adding a leading zero does not alter the value of the number. Similarly, for grouping the digits in the fractional part of a binary number, trailing zeros may be added without changing the value of the number.

Binary to Hexadecimal: Conversion from binary to a hex number requires grouping the binary digits into groups of four.
$(11011001)_2 = (1101\ 1001) = (D9)_{16}$

$(001100101.110111)_2 = (0000\ 0110\ 0101.1101\ 1100) = (065.DC)_{16} = (65.DC)_{16}$

Octal to Binary: $(331)_8 = ()_2$
$(331)_8 = (011\ 011\ 001) = (11011001)_2$

Hexadecimal to Binary: $(D9)_{16} = ()_2$
$(D9)_{16} = (1101\ 1001) = (11011001)_2$

Octal to Hexadecimal OR Hexadecimal to Octal: This can easily be achieved through straight binary system. For example, any octal number first, may be converted to straight binary system and then to the hexadecimal system. Similar approach may be used for Hexadecimal to Octal conversion.