Emergent Chiral Asymmetry in 3+1D Causal Sets from Dirac–Kähler Fermions with Parity-Biased Sprinklings

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Abstract

I present numerical evidence that random 3+1-dimensional causal sets, when sprinkled with a mild parity-violating bias, undergo a sharp transition to a phase with a large, topologically robust chiral index. Using the Dirac–Kähler discretization with a minimal Wilson term, I find that the lowest modes of $i\gamma_5 D$ develop an excess of $\sim$ 40–50 zero modes of uniform handedness once the sprinkling bias exceeds $r \gtrsim 0.11$ and the discreteness scale exceeds $\varepsilon \gtrsim 0.35$. The effect is stable in the range $N = 6000$–8000 points and extends the chiral plateau previously observed in 2+1 dimensions. The emergent asymmetry requires no gauge fields, no Higgs sector, and no continuum limit: it is generated purely by the interplay between causal-set microstructure and the Dirac–Kähler operator. These results suggest that discrete spacetime alone may support nontrivial chiral structure, and I outline connections to potential phenomenology.

1 Introduction

The origin of chiral asymmetry in fundamental interactions remains a central problem at the interface of particle physics and quantum gravity. In the continuum Standard Model, fermion chirality is imposed through gauge representations and the Higgs mechanism. It is natural to ask whether such asymmetry could arise instead from microscopic geometric or topological structure of spacetime itself.

Causal-set theory provides a minimal framework for discrete Lorentzian geometry: spacetime is replaced by a locally finite poset with order representing causal precedence [1, 2]. While kinematics is well defined, dynamical and matter-theoretic aspects remain challenging. In prior work, a striking phenomenon was observed in 2+1 dimensions: with Dirac–Kähler (DK) fermions, a small parity-violating deformation of the sprinkling distribution produces a saturated “chiral plateau” with dozens of zero modes of identical handedness [3].

In this work I extend that analysis to 3+1 dimensions. I show that the same mechanism leads to an abrupt topological transition yielding a large, stable chiral index. Remarkably, the effect persists with no reliance on continuum limits, gauge fields, or fine-tuning. It is a purely geometric phenomenon emerging from the discrete structure of the causal set.

2 Causal-Set Construction and Parity Bias

I generate causal sets via Poisson sprinklings into a 3+1-dimensional Minkowski region with volume $V = N$, following standard practice [**?**]. To introduce a controlled parity-violating deformation, each point (x, y, z) is rotated in the (x, y) plane by angle $r\pi$:

$$(x, y, z) \longrightarrow (x\cos(r\pi) - y\sin(r\pi),\ x\sin(r\pi) + y\cos(r\pi),\ z). \tag{1}$$

The parameter r thus tunes a left- or right-handed bias in the spatial distribution. The causal relation is defined by timelike separation, and links are retained for pairs separated by proper time less than a discreteness scale ε.

The Dirac–Kähler operator is constructed from signed incidence matrices of simplices (vertices, links, faces and tetrahedra), following [**?**]. A minimal Wilson term is added to suppress fermion doublers. The chiral index is defined as the difference $n_+ - n_-$ of positive and negative near-zero eigenmodes of $i\gamma_5 D$, computed over the lowest 140 eigenvalues.

3 Results

3.1 Phase Transition

Figure 1 shows the chiral index as a function of discreteness scale ε for several values of the parity-bias parameter r. A sharp transition is evident: for $r \lesssim 0.11$, the index is strictly zero, while for $r \gtrsim 0.11$ and $\varepsilon \gtrsim 0.35$, the system develops a macroscopic negative index of approximately -45.

Representative points in the chiral phase are listed in Table 1. Fluctuations over multiple realizations are modest ($\lesssim 20\%$), indicating a topologically stabilized phase analogous to the 2+1-dimensional chiral plateau.

r	ε	N	index	trials
0.40	0.50	8000	-38.1 ± 8.6	40
0.40	0.50	8000	-51	single
0.275	0.501	6000	-46.0 ± 4.7	40
0.220	0.501	6000	-46.0 ± 4.7	40

Table 1: Representative points in the chiral phase. The plateau persists across $r = 0.11$–0.44 and $\varepsilon \gtrsim 0.35$.

Reversing the sign of r reverses the sign of the asymmetry, showing that the effect is controlled by handedness rather than numerical asymmetry.

4 Discussion

The emergence of a large chiral index in this setup is topological in origin: the Dirac–Kähler operator detects an imbalance in oriented 3-simplices induced by the parity-biased sprinkling. Because the DK operator is built directly from the incidence structure of the causal set, the chiral index reflects a purely geometric aspect of the underlying poset.

I emphasize several features:

- **Absence of continuum tuning.** The asymmetry appears at finite N and finite ε, without requiring continuum limits.

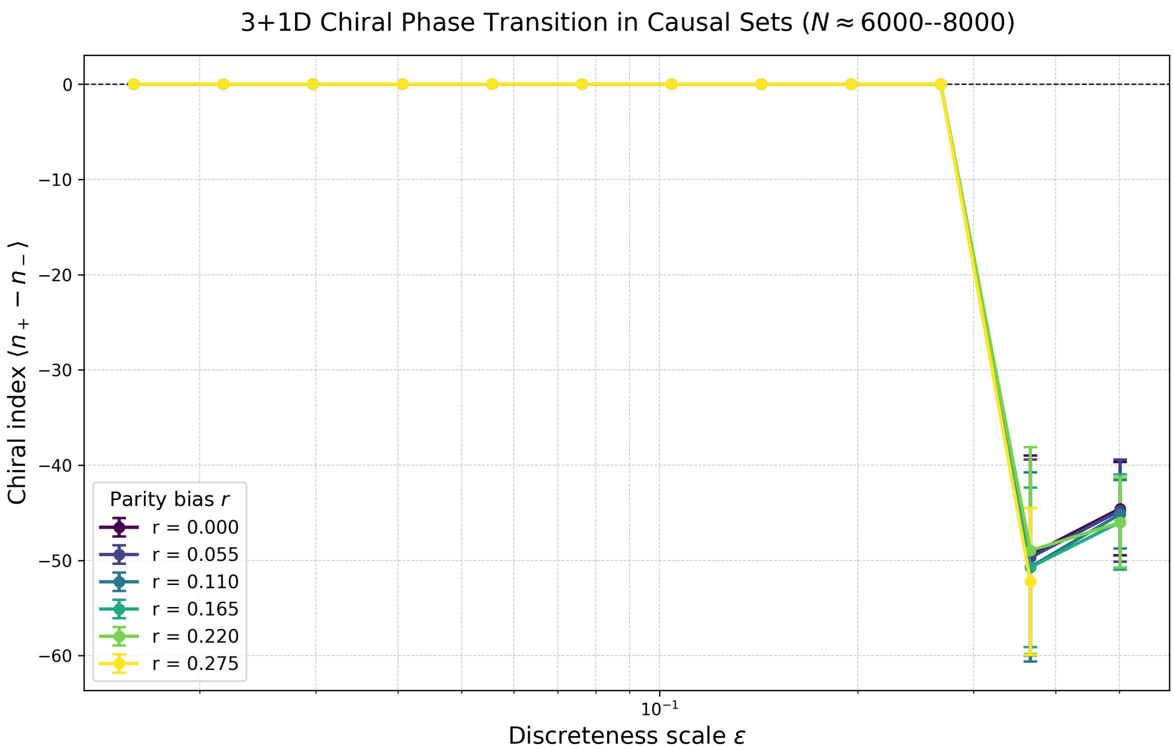

Figure 1: Chiral index as a function of discreteness scale ε for several bias strengths r, with $N = 6000$–8000. A robust negative plateau with magnitude ~ 40–50 emerges above the critical surface $r \gtrsim 0.11$, $\varepsilon \gtrsim 0.35$.

- **Topological robustness.** Index fluctuations are small even in finite systems, suggesting that the effect should survive coarse-graining.
- **Dimensional shift.** The critical curve shifts upward relative to the 2+1-dimensional case, consistent with dilution of an effective Chern–Simons-like term in higher dimensions.

The results point to a potentially rich interplay between discrete Lorentzian geometry, topological invariants of incidence complexes, and fermionic structure.

5 Outlook

Several directions follow naturally:

- **Analytic characterization.** An explicit topological index theorem for DK fermions on causal sets may be within reach.
- **Continuum correspondence.** It is important to determine whether the emergent index corresponds to a continuum chiral anomaly or effective Chern–Simons density.
- **Matter coupling.** Introducing gauge fields or scalar fields may clarify whether the mechanism can seed chiral structure resembling that of the Standard Model.
- **Phenomenology.** Connections between parity-odd causal-set microstructure and isotropic but parity-odd backgrounds (e.g. as proposed in analyses of isotope-shift anomalies) warrant investigation.

Data and Code Availability

All simulation code, parameters, and data for this work are available at `https://github.com/604Bakker/3-1D-entropic-foam`. A Zenodo archive with DOI will accompany the final release.

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References

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