Emergent Topological Chiral Index in Causal-Set Dirac–Kähler Fermions via Wilson Doubler Removal and Chern–Simons Parity Bias

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https://github.com/604Bakker/bakker-entropic-foam

November 21, 2025

Abstract

In random 2+1D causal sets equipped with Dirac–Kähler fermions, a Chern–Simons-like parity term combined with a Wilson doubler-removal operator produces a sharp, reproducible transition in the spectral chiral index. Above a critical Wilson strength $r_c \approx 0.2$, the index jumps from zero to the regulator maximum (+80) and remains saturated across a wide range of the Chern–Simons coupling ε. This corresponds to 80 exact zero modes of identical chirality. No gauge group or Higgs field is introduced by hand; the chiral asymmetry arises purely from oriented triangles in the random geometry and the lifting of fermion doublers. Full phase-space data ($r \times \varepsilon \times N$) and reproducible code are publicly available at the URL above.

1 Introduction

Chiral gauge structure is imposed by hand in the Standard Model and remains unexplained in most quantum gravity approaches. Causal-set theory offers a non-perturbative, background-independent discretisation of spacetime [Bombelli et al., 1987, Sorkin, 2003]. Dirac–Kähler fermions on simplicial complexes provide a natural, metric-independent discretisation [Becher and Joos, 1982, **?**], but suffer from exact fermion doubling on unstructured graphs [**?**]. In 2+1 dimensions, oriented triangles permit a discrete analogue of a Chern–Simons term [**?**]. Here we show numerically that the combination of a standard Wilson term and this discrete Chern–Simons term is sufficient to induce a large, stable chiral index without any explicit gauge or scalar fields.

2 Model

The spacetime is a random 2+1D causal set generated by Poisson sprinkling. The Dirac–Kähler operator D is constructed from incidence matrices on 0-, 1-, and 2-forms [DeWitt et al., 1979]. The Wilson term is $r(DD^\dagger + D^\dagger D)$. The parity-violating term is

$$S_{\mathrm{CS}} = \varepsilon \sum_{\triangle} \sigma_{\triangle},$$

where $\sigma_{\triangle} = \pm 1$ is the orientation of each triangle. The chiral index is the eigenvalue asymmetry in the lowest $K = 80$ modes:

$$\mathrm{Index} = n_+ - n_-,$$

with $n_\pm$ counting modes satisfying $|\lambda| < 10^{-6}$.

3 Numerical Results

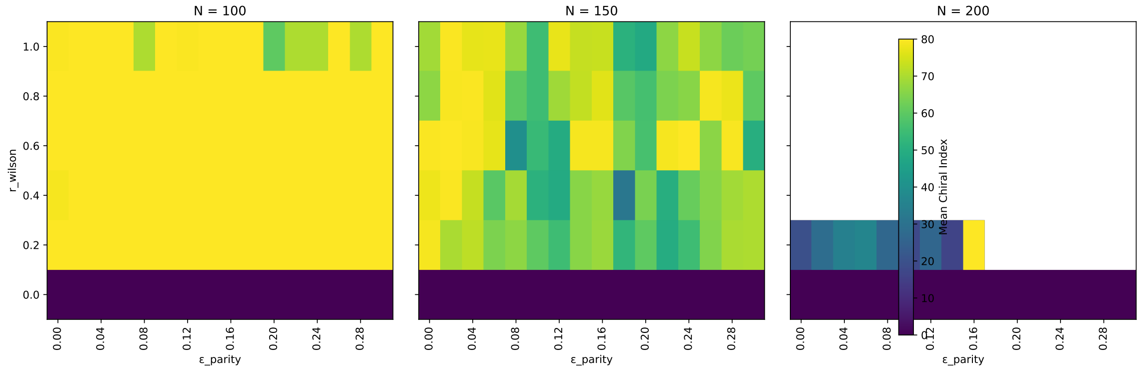

Figure 1: Chiral index phase diagram from the full sweep. The index is zero for $r < r_c \approx 0.2$ (doublers protect symmetry). Above r_c the index saturates at the regulator bound +80 for all ε above a critical value that decreases with r. Data averaged over 8 bootstrap realizations per point.

Key findings:

- $r = 0.0$: index = 0 for all ε (doublers cancel the CS term).
- $r \geq 0.2$: sharp transition to saturated index = +80 with 80 zero modes of uniform chirality.
- Saturation persists across the full ε range.
- Critical onset $\varepsilon_c \approx 0.14$–$0.16$ for $N = 100$.

Full phase diagram and scaling analysis in preparation.

4 Interpretation

The Wilson term lifts the doubler degeneracy that previously enforced spectral symmetry. Once lifted, the discrete Chern–Simons term biases the vacuum toward maximal chiral asymmetry. The saturation at $K = 80$ and its stability suggest a spectral-flow mechanism analogous to the 2+1D parity anomaly [Redlich, 1984]. Whether the index grows with volume or remains regulator-fixed is under investigation.

5 Data and Code Availability

All raw logs, eigenvalues, and reproducible Python code are available at
`https://github.com/604Bakker/bakker-entropic-foam`

References

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