Credit Default Swap (spread) and Actuarial Spread

CRI Internal Training

March 2019
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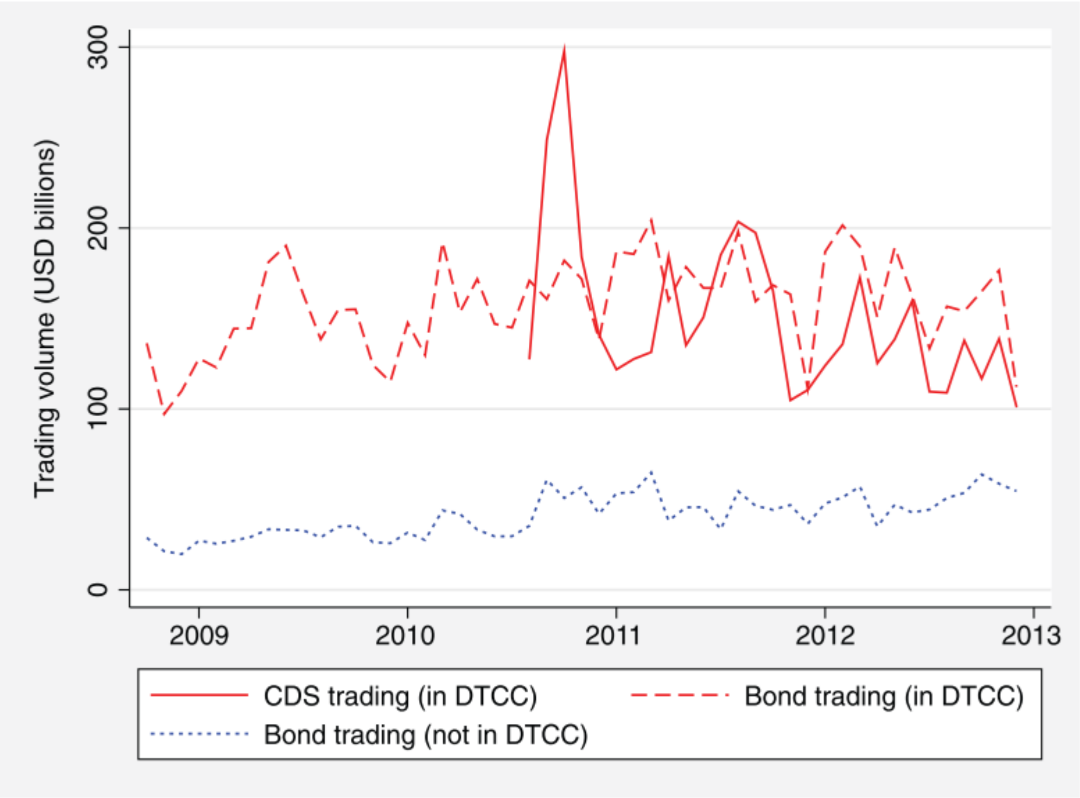
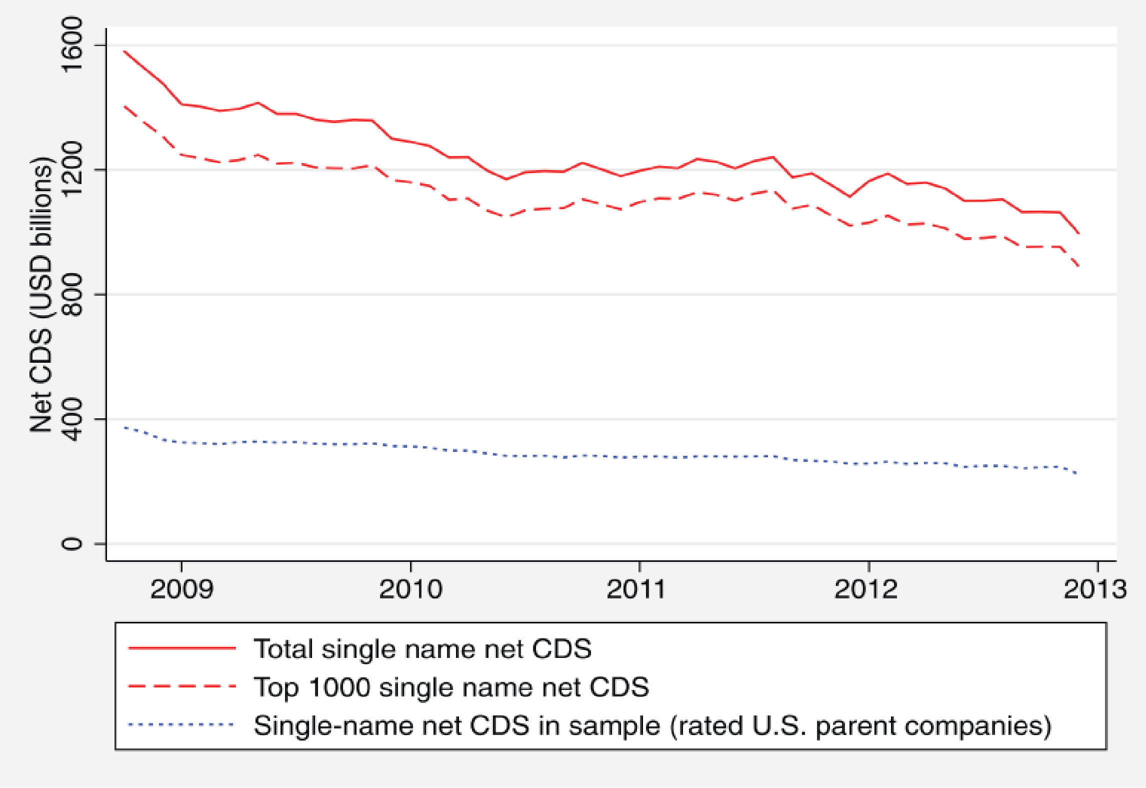
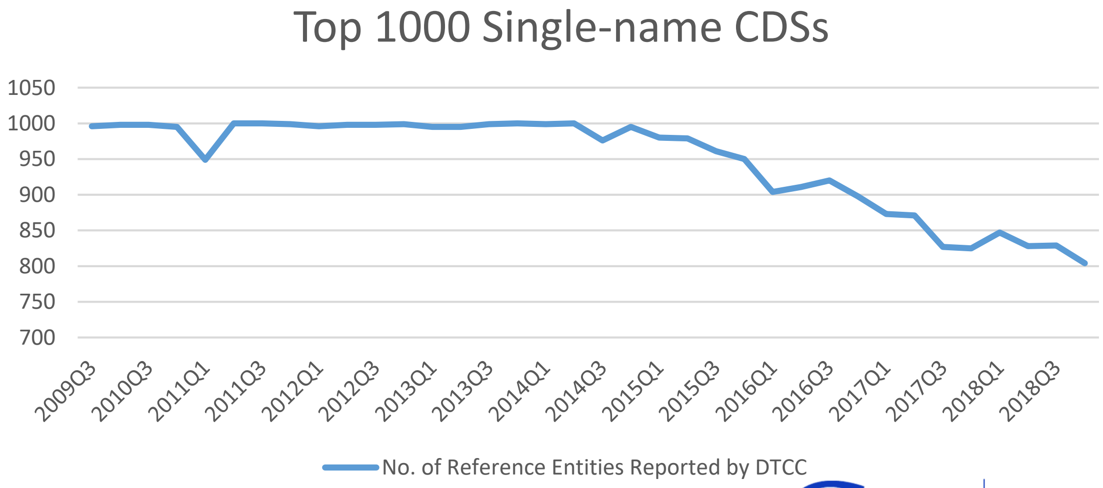
Bond, CDS, and AS – Information Components

- Corporate bond yield spread (= bond yield – Rf):
  - Default event risk (and recovery rate)
  - Common and Firm-specific factors risk
  - Liquidity risk
  - Tax effect
  - …
- CDS spread (par spread, price):
  - Credit risk of reference entity (and recovery rate)
  - Counter-party risk
  - Liquidity risk
  - …
- AS (actuarial spread) determinants:
  - Credit risk of reference entity (and recovery rate)

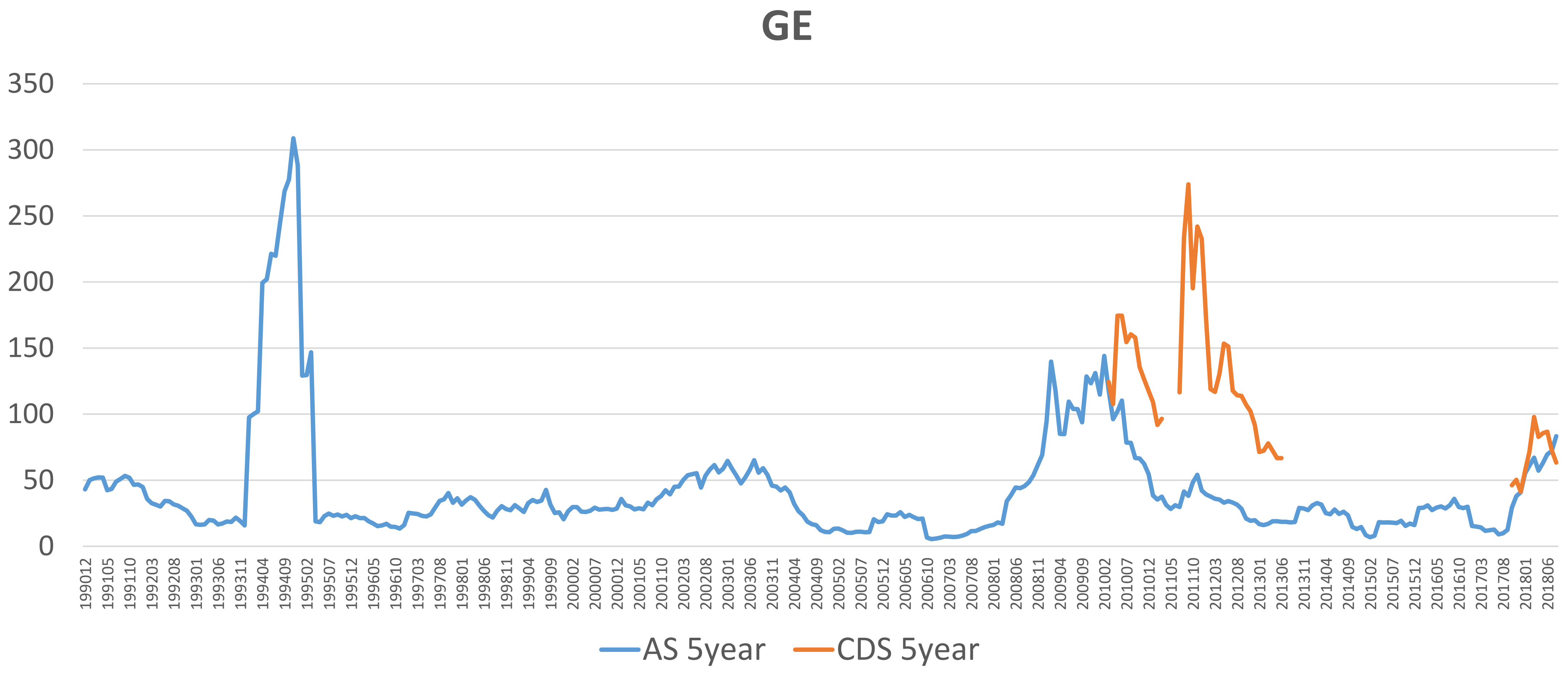




- Oehmke, Martin, and Adam Zawadowski, 2017, The anatomy of the CDS market, *Review of Financial Studies* 30, 80–119.



General Electricity – CDS vs AS



Bond Yield Spread & CDS Spread

Table 2

Test of the relationship between 5-year CDS spreads and 5-year bond yields using the assumption that the risk-free rate is (i) the Treasury rate, r_T, and (ii) the swap rate, r_S

	a	b	Standard error of residuals	Adjusted R^2
Eq. (4): risk-free rate is the Treasury rate	0.12	1.10	0.250	0.941
	(0.070)	(0.014)		
Eq. (5): risk-free rate is the swap rate	0.09	0.972	0.203	0.961
	(0.059)	(0.010)		

The CDS spread and all rates are measured in percentage points. Standard errors are shown in parentheses.

- Hull, John, Mirela Predescu, and Alan White, 2004, The relationship between credit default swap spreads, bond yields, and credit rating announcements, *Journal of Banking and Finance* 28, 2789–2811.

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Proxy CDS Curves with AS

- CDS are commonly used for risk benchmarking in **credit risk management** in general and in **accounting practice** in particular, where the latter pertains to accounting reporting standards on credit exposures (IFRS-9 for international firms and CECL for US firms).

- Duan, Jin-Chuan, 2017, Proxy CDS Curves for Individual Corporates Globally.
  - Use 29 repressors (empowered by modern big-data analytics – zero-norm penalty regression) to predict USD-denominated CDS premiums of different tenors (less than 500 corporate names) and then generate proxy CDS curves for corporates without liquid or traded CDS.

Table 1: Single-regressor R^2

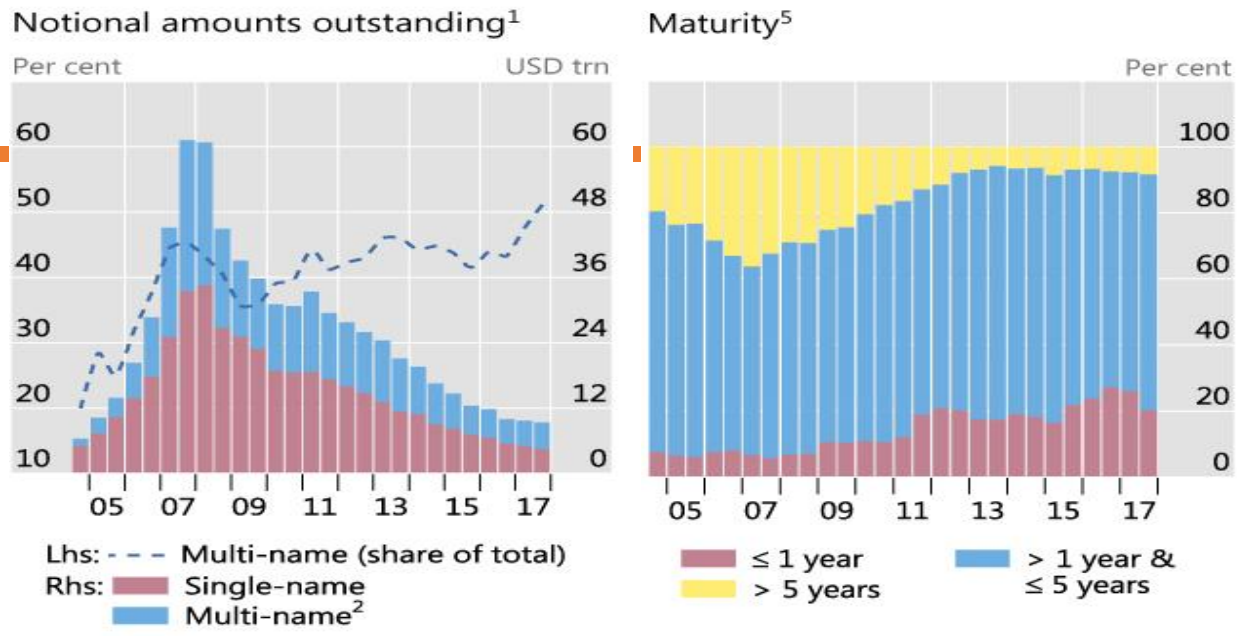
	R^2
CDS(bps)	
logCDS	
Regressor	
logAS	0.4860
logASlevel	0.4718
logAStrend	0.0333
DTDlevel	0.3819
DTDtrend	0.0268

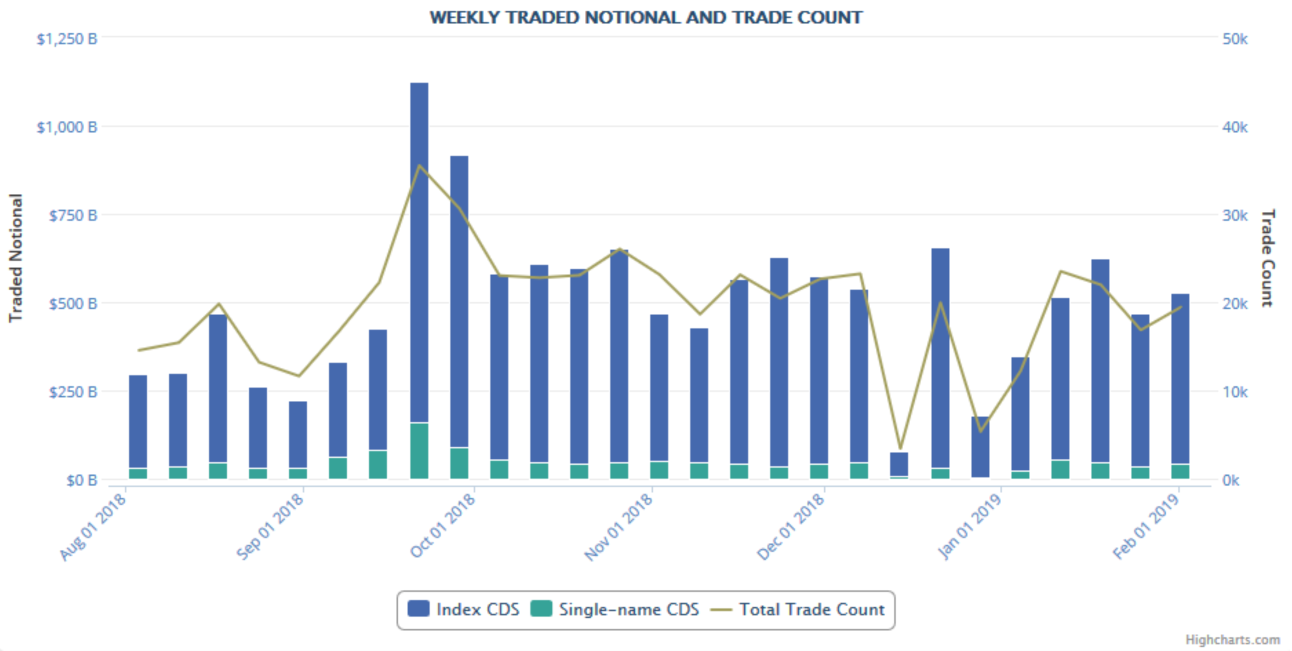
Table 4: R^2 of the proxy CDS model for the whole sample and various subcategories

	R^2	# of Reference Corporates	# of Data
Whole sample	80.89%	405	141,918
US	81.41%	309	121,840
Non-US	77.29%	96	20,078
Financial	81.36%	73	20,818
Non-Financial	80.48%	332	121,100
Investment grade	74.02%	370	118,559
High yield	71.38%	138	23,359
Senior debt	80.79%	404	141,826
Subordinated debt	93.49%	1	92
Pre-financial crisis	75.14%	244	17,696
Post-financial crisis	81.25%	395	124,222
Pre-CDS Big Bang	81.20%	372	40,529
Post-CDS Big Bang	80.60%	374	101,389
Tenor(1 year)	78.72%	354	25,548
Tenor(2 years)	79.93%	319	20,432
Tenor(3 years)	78.54%	356	26,740
Tenor(4 years)	77.17%	314	20,750
Tenor(5 years)	77.78%	404	48,448

CDS History

- The first CDS were traded by J.P. Morgan in 1995.
- The outstanding notional value:
  - 1996: $40 billion
  - 2001: $1.2 trillion
  - 2004: $4.8 trillion
  - 2007: $61.2 trillion
  - 2017: $9.4 trillion
- The sharp drop from 2007 to 2017:
  - During the Great Financial Crisis (GFC) and its aftermath this was driven by compression,
  - whereas in recent years it appears to have been driven by the rise of central clearing.





(Source: BIS Quarterly Review (December 2010); ISDA SwapsInfo)

ISDA "Big Bang" (8 April 2009)

- Before 'Big Bang':
  - CDSs were traded at par (i.e., zero cost of entry).
  - CDSs were quoted by par spread, which makes the clean PV of the CDS zero.
  - For all protection buyers, the cost of entry is zero and they only needs to pay regular coupon payments.

- After 'Big Bang': standard CDS contracts.
  - Standard coupon rates: 100 or 500bps in North America, and 25, 100, 500 or 1000bps in Europe. In combination with standard contract sizes, help to equalize the size of cash flows across contracts. (featured with upfront payment)
  - Standard dates of payments and maturity: 20th of March, June, September, and December (IMM dates).
  - Full first coupons: in combination with standard contract sizes, equalize the first coupons on different contracts.
  - Determinations committees: consistent treatment of contracts in the light of credit or succession events.
  - Standard effective dates: all outstanding contracts on a given *reference entity* affected by the same events.

(Source: BIS Quarterly Review (December 2010); OpenGamma Quantitative Research, 2013)

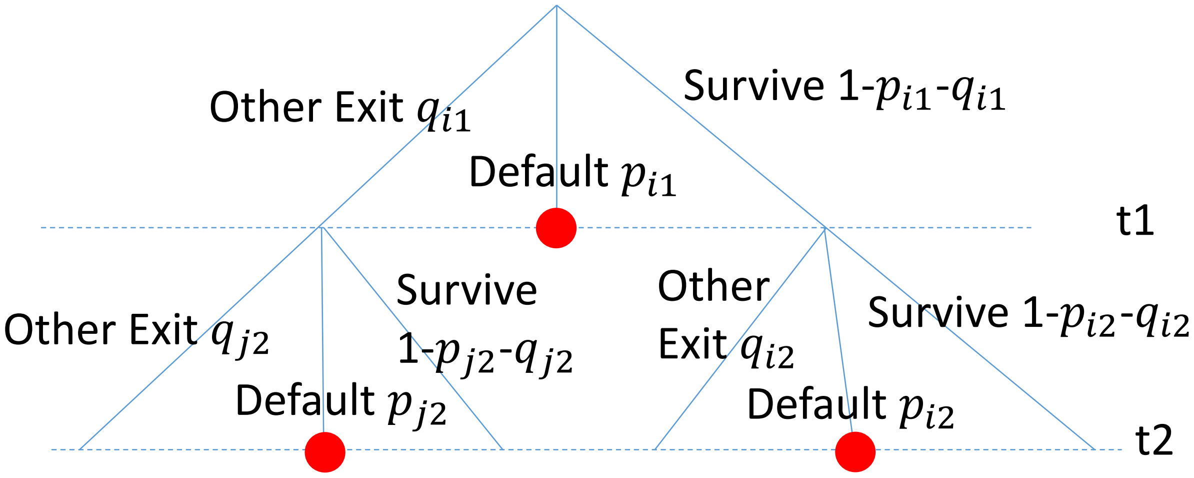
CDS & AS – how are the spreads determined?

- No arbitrage: PV of protection = PV of premium

→ If we assume zero cost to entry, then get par spread (fixed premium per quarter).

- CDS par spread:
  - Actual price determined by the interaction of market participants, based on the standard model (ISDA) and their expectations on credit curve (risk neutral PDs) and recovery rate, among other things;
- Actuarial par spread:
  - Estimated number based on the standard model (ISDA) and NUS-CRI physical PDs and POEs (other exit) and assumed constant recovery rate over time.

An Illustration



Cash Flow	Protection payment	Premium	Probability of default
If default at t1	N(1-R) *δ_1	0	p_{i1}
If survive t1 & default at t2	N(1-R) * ($\delta_1+\delta_2$)	Nc/2 *δ_1	(1-p_{i1}-q_{i1})*p_{i2}
If other exit t1 & default at t2	N(1-R) * ($\delta_1+\delta_2$)	Nc/2 *δ_1	$q_{i1}*p_{j2}$
No defaults at all	0	Nc/2 * ($\delta_1+\delta_2$)	(1-p_{i1}-q_{i1})*(1-p_{i2}) + q_{i1}*(1-p_{j2})

$$N(1-R)*\left[\delta_1*p_{i1}+(\delta_1+\delta_2)*(1-p_{i1}-q_{i1})*p_{i2}+(\delta_1+\delta_2)\ *q_{i1}*p_{j2}\right]$$
$$= Nc/2*\delta_1*(1\text{-}p_{i1}\text{-}q_{i1})*p_{i2} + Nc/2*\delta_1*q_{i1}*p_{j2} + Nc/2*(\delta_1+\delta_2)*[(1\text{-}p_{i1}\text{-}q_{i1})*(1\text{-}p_{i2}) + q_{i1}*(1\text{-}p_{j2})]$$

- N: notional value.
- c: coupon rate (annualized, assuming semi-annual payment (quarterly payment in practice)).
- R: recovery rate.
- p_{i1}: conditional PD of entity i in period 1.
- q_{i1}: condition probability of other exit of entity i in period 1 (cannot be canceled out if accrual period needs to be considered, see Appendix B in Duan (2014)).
- δ_t: discount factor at the end of period t.

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AS Calculation

- AS is par spread, i.e., assuming zero cost of entry.
- Frequency of credit event: daily
- Inputs:
  - Tenor: up to 5 years.
  - Discount curve: zero-rate curve
    - simple boostrap (following ISDA), using USD dominated Libor (over night, 1 week, 1M, 2M, 3M, 6M, and 12M) and Swap rates (1Y, 2Y, 3Y, 4Y, 5Y, 6Y).
  - Credit curve: NUS-CRI PD and Other Exit term structure (forward 1-day PD and POE).
  - Recovery rate: assumed constant over time.
  - Day count convention:
    - DCC payments: Act/360 (~ A in the paper Duan (2014))
    - DCC curves: Act/365
    - Accrual period: ISDA standard CDS conversion (except for the first accrual period)

Table 1. The 5-year CDS payment dates, accrual periods, and number of days in each period for the trade date of November 16, 2011.

Payment #	Payment Date	Accrual Start Date (inclusive)	Accrual End Date (inclusive)	# of Days
1	20.12.2011	20.09.2011	19.12.2011	33[a]
2	20.03.2012	20.12.2011	19.03.2012	91
3	20.06.2012	20.03.2012	19.06.2012	92
4	20.09.2012	20.06.2012	19.09.2012	92
5	20.12.2012	20.09.2012	19.12.2012	91
6	20.03.2013	20.12.2012	19.03.2013	90
7	20.06.2013	20.03.2013	19.06.2013	92
8	20.09.2013	20.06.2013	19.09.2013	92
9	20.12.2013	20.09.2013	19.12.2013	91
10	20.03.2014	20.12.2013	19.03.2014	90
11	20.06.2014	20.03.2014	19.06.2014	92
12	22.09.2014	20.06.2014	21.09.2014	94
13	22.12.2014	22.09.2014	21.12.2014	91
14	20.03.2015	22.12.2014	19.03.2015	88
15	22.06.2015	20.03.2015	21.06.2015	94
16	21.09.2015	22.06.2015	20.09.2015	91
17	21.12.2015	21.09.2015	20.12.2015	91
18	21.03.2016	21.12.2015	20.03.2016	91
19	20.06.2016	21.03.2016	19.06.2016	91
20	20.09.2016	20.06.2016	19.09.2016	92
21	20.12.2016	20.09.2016	20.12.2016	92

Note: [a]This value reflects the number of days in the first accrual period that is applicable to this CDS contract, i.e., from November 17, 2011 (trade date plus one day) to December 19, 2011 inclusive.

Source: Duan, 2014. Actuarial Par Spread and Empirical Pricing of CDS by Decomposition.

Risk-neutral PDs

$$E_t^Q[(1-R_\tau)D_t(\tau-t)1_{\{t<\tau\le t'_k\}}]$$

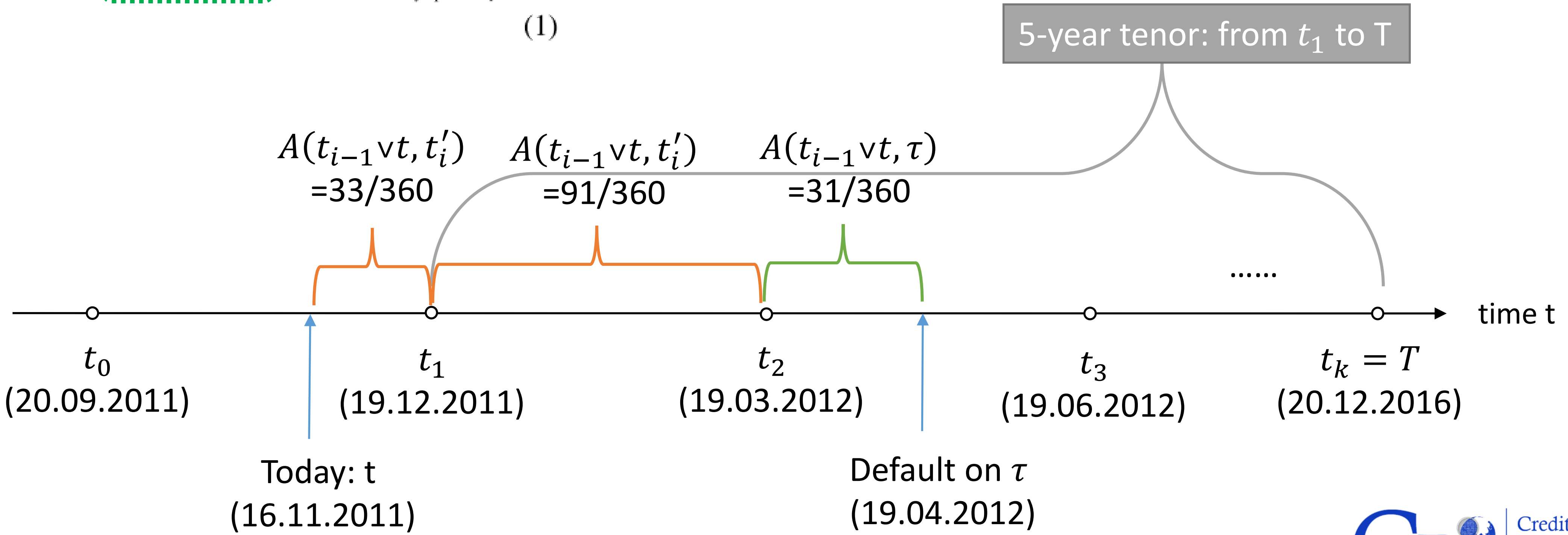
Protection leg

$$= S_t(T-t)\sum_{i=1}^{k}\{\boxed{A(t_{i-1}\vee t, t'_i)}$$

Premium leg

$$\times E_t^Q[D_t(t_i-t)1_{\{t'_i<\tau\}}]$$

$$+ E_t^Q[\boxed{A(t_{i-1}\vee t, \tau)}D_t(\tau-t)1_{\{t'_{i-1}<\tau\le t'_i\}}]\}, \tag{1}$$



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Continuous-time Compounding Rf & Physical PDs

$$S_t^{(a)}(T-t)$$

$$= \frac{(1-\bar{R}_t)\boxed{E_t^P\left[e^{-r_t(0,\tau-t)(\tau-t)}1_{\{t<\tau\leq t_k'\}}\right]}}{\sum_{i=1}^k\left\{A(t_{i-1}\vee t, t_i')e^{-r_t(0,t_i-t)(t_i-t)}\boxed{E_t^P\left[1_{\{t_i'<\tau\}}\right]}\right.}$$

$$\left.+\boxed{E_t^P\left[A(t_{i-1}\vee t,\tau)e^{-r_t(0,\tau-t)(\tau-t)}1_{\{t_{i-1}'<\tau\leq t_i'\}}\right]}\right\} \quad (4)$$

$$E_t^P\left[e^{-r_t(0,\tau-t)(\tau-t)}1_{\{t<\tau\leq t_k'\}}\right]$$

$$= \int_0^{t_k'-t} e^{-[r_t(0,s)+\psi_t(0,s)]s} f_t(s)ds$$

$$+\int_0^{t_k'-t} e^{-[r_t(0,s)+\psi_t(0,s)]s} h_t(s)1_{\{Sub\}}$$

$$\times P_t^*(s, t_k'-t)ds \quad (8)$$

$$E_t^P\left(1_{\{t_i'<\tau\}}\right) = 1 - P_t(0, t_i'-t; r_t(0,u) = 0$$

$$\text{for } 0\leq u\leq t_i'-t). \quad (7)$$

$$E_t^P\left[A(t_{i-1}\vee t,\tau)e^{-r_t(0,\tau-t)(\tau-t)}1_{\{t_{i-1}'<\tau\leq t_i'\}}\right]$$

$$= \int_{t_{i-1}'\vee t}^{t_i'} A(t_{i-1}\vee t, s)e^{-[r_t(0,s-t)+\psi_t(0,s-t)](s-t)}$$

$$\times f_t(s-t)ds$$

$$+\int_{t_{i-1}'\vee t}^{t_i'} A(t_{i-1}\vee t, s)e^{-[r_t(0,s-t)+\psi_t(0,s-t)](s-t)}$$

$$\times h_t(s-t)1_{\{Sub\}}P_t^*(s-t, t_i'-t)ds.$$

(9)

$f_t(s)$ and $h_t(s)$ are the time-t forward default and other-exit intensities with the forward starting time of t+s.

Define: $\psi_t(s,q) = \frac{\int_s^q [f_t(u)+h_t(u)]du}{q-s}$ for s≤q.

Then PD over $[t, t_i']$ = $\int_0^{t_i'-t} e^{-\psi_t(0,u)u} f_t(u)du$, where $e^{-\psi_t(0,u)u} = 1 - PD_t(0,u) - POE_t(0,u)$ i.e., given survival till u.

Note: s in the definitions are different from in equations (7-9).

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Forward Intensity Functions

- $f_t(s)$ and $h_t(s)$ are the time-t forward default and other-exit intensities with the forward starting time of t+s.

$$f_t(s) = \exp\{\alpha_0(s) + \alpha_1(s)x_{1,t} + \cdots + \alpha_{12}(s)x_{12,t}\}, \quad (10)$$

$$h_t(s) = \exp\{\beta_0(s) + \beta_1(s)x_{1,t} + \cdots + \beta_{12}(s)x_{12,t}\}. \quad (11)$$

- The coefficients as functions of forward starting time are constrained by the Nelson–Siegel function of four or three parameters, depending on whether the covariate is stochastic or not.
    - Daily discretization is employed, which means Δs = 1/365.

$$\begin{aligned} &\alpha_i(s; \varrho_{i,0}, \varrho_{i,1}, \varrho_{i,2}, d_i) \\ &\quad = \varrho_{i,0} + \varrho_{i,1}\frac{1-\exp(-s/d_i)}{s/d_i} \\ &\quad + \varrho_{i,2}\left[\frac{1-\exp(-s/d_i)}{s/d_i} - \exp(-s/d_i)\right] \\ &\quad \text{for } i = 0, 1, 2, \ldots, 12 \text{ and } \varrho_{i,0} = 0 \\ &\quad \text{for } i = 1, 2, \ldots, 12. \end{aligned} \quad (12)$$

- Get NS parameters from NUS-CRI system, with bootstrapped discount curve and day counts ready, we can calculate AS with equation (4).

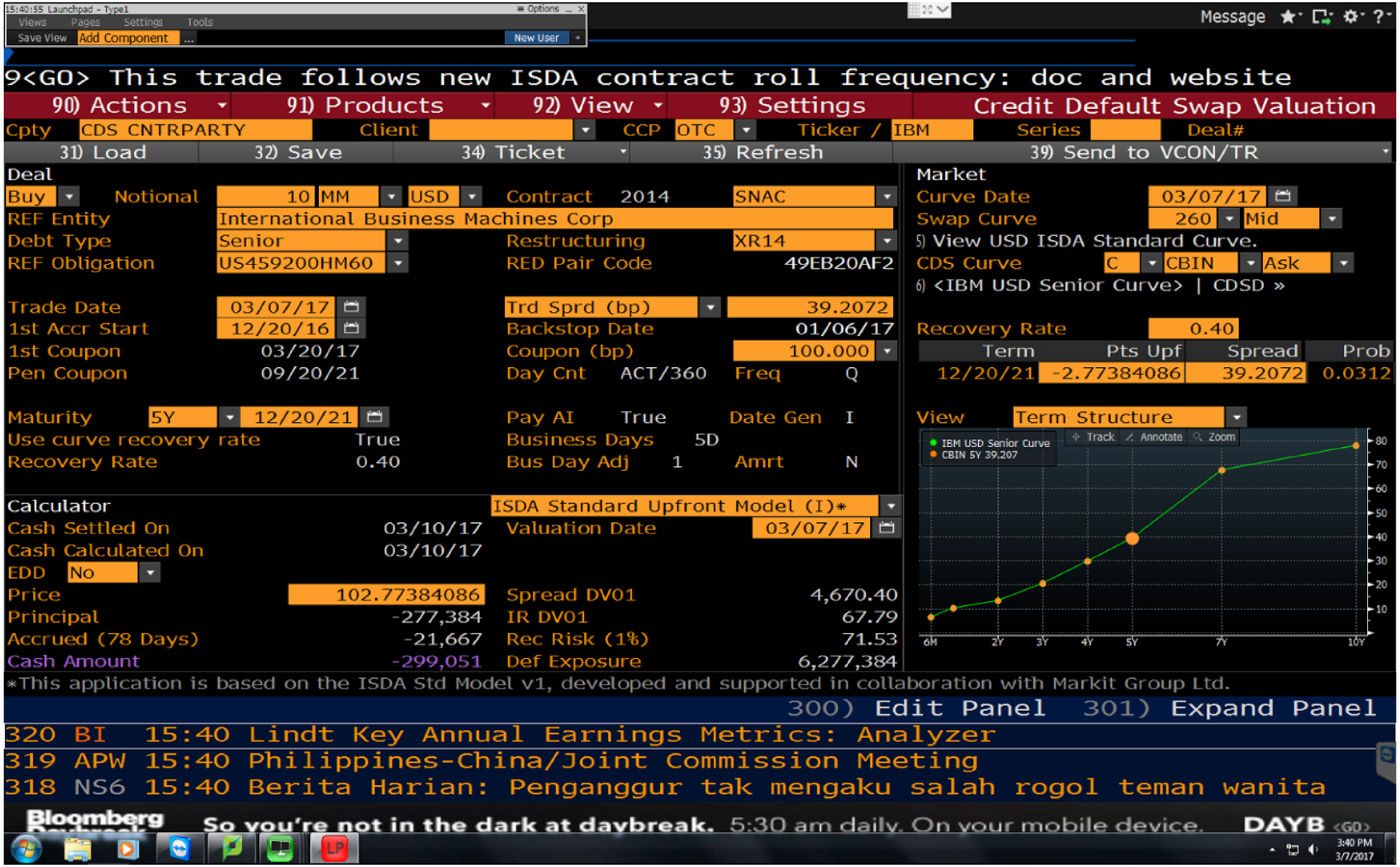
Recovery Rate

- Measured as the bond's trading price one month after default.
- "Predicting Recovery Rate at the Time of Corporate Default" by J.C. Duan and R.C. Hwang, 2018 (Novermber).
  - Bimodal distribution: over 30% of the recovery rates at 1 and close to 7% at 0, higher than the occurrence frequency at any other recovery rate (with data from from Moody's Ultimate Recovery Database for 3,827 defaulted debts spanning 1990-2012).
  - Debt attributes known from the issuance time and industry distress level at the time of default are both significant in predicting recovery rate.

Table 2: Distributions of recovery rates and each of the six predictors. The sample contains 3827 recovery rates and the values of the six predictors from December 1990 to 2012. The six predictors are ID, DC, IR, CT, IT, and UT with definitions in Table 1.

Variable	Mean	Std	5%	25%	50%	75%	95%
Panel A: Recovery rate							
$R_i \in [0,1]$	0.551	0.385	0.000	0.178	0.581	1.000	1.000
($R_i = 1$ with 30.36%; $R_i = 0$ with 6.69%)							

Bloomberg CDS tool: CDSW



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Swaps Market

- Interest Rate Derivatives

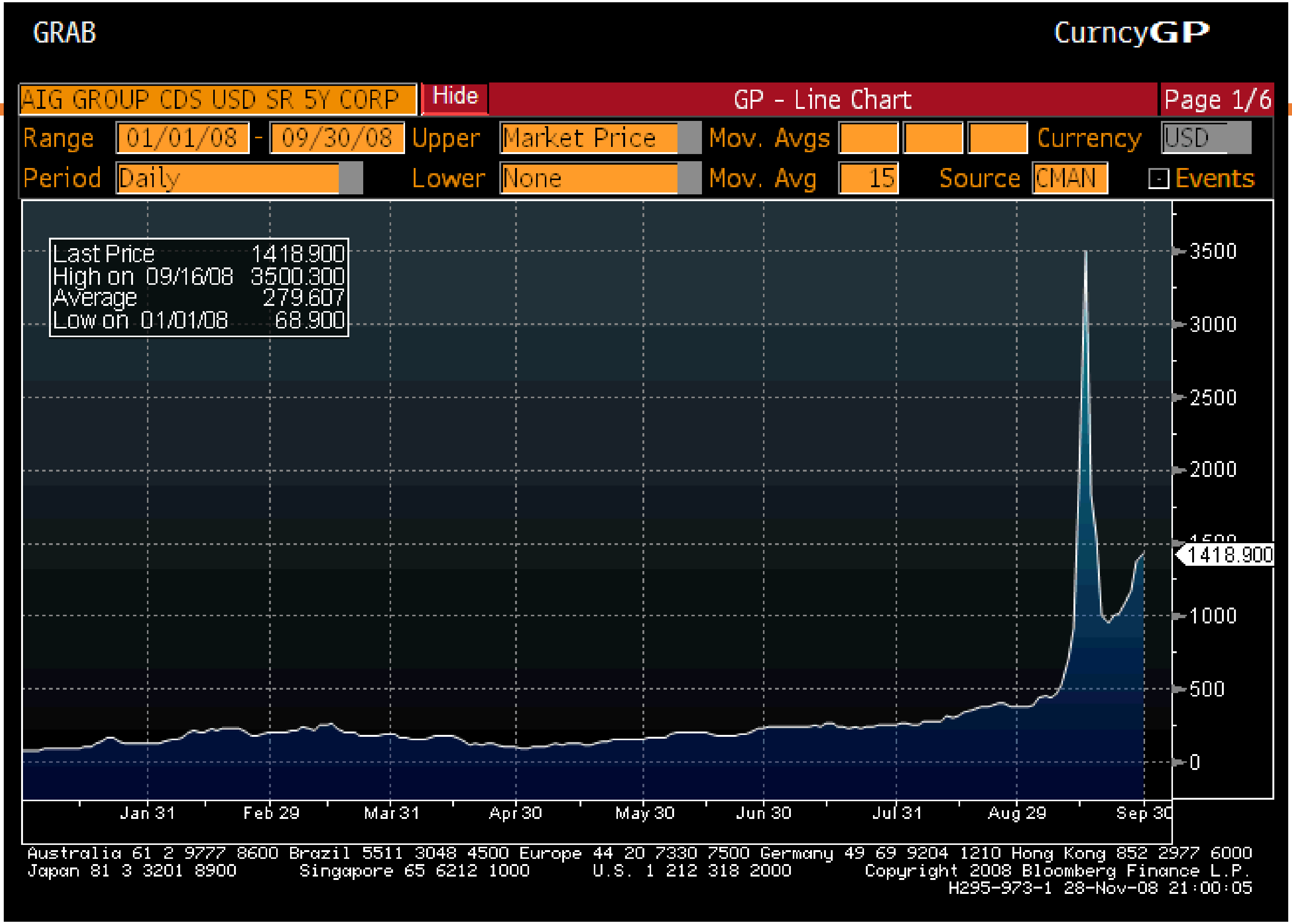
Week Ending	March 8, 2019		March 9, 2018			
	Notional (US$ trillions)	Trade Count (thousands)	Notional (US$ trillions)	Trade Count (thousands)	Change in Notional	Change in Trade Count
IRD Total	5.31	31.4	5.29	26.7	0.3%	18%
Fixed-for-Floating IRS	1.79	21.0	1.37	17.0	31%	23%
FRAs	1.64	4.6	2.06	4.3	-20%	7%
OIS	1.28	1.7	1.14	1.5	12%	11%
Other	0.59	4.1	0.73	3.9	-18%	7%
Cleared	4.73	26.6	4.70	21.7	1%	23%
ON SEF	3.07	19.3	3.04	16.2	1%	19%

- Credit Derivatives

Week Ending	March 8, 2019		March 9, 2018			
	Notional (US$ trillions)	Trade Count (thousands)	Notional (US$ trillions)	Trade Count (thousands)	Change in Notional	Change in Trade Count
Credit Derivatives Total	0.17	5.0	0.16	4.4	7%	13%
CDX HY	0.03	1.5	0.02	1.1	8%	33%
CDX IG	0.07	1.3	0.06	1.0	25%	32%
iTraxx Europe	0.04	1.1	0.03	0.6	16%	75%
Other	0.04	1.1	0.05	1.7	-22%	-34%
Cleared	0.14	4.1	0.13	3.7	5%	12%
ON SEF	0.14	4.0	0.13	3.5	6%	13%

Source: ISDA SwapsInfo, 11 March 2019.

Thank you very much for attention!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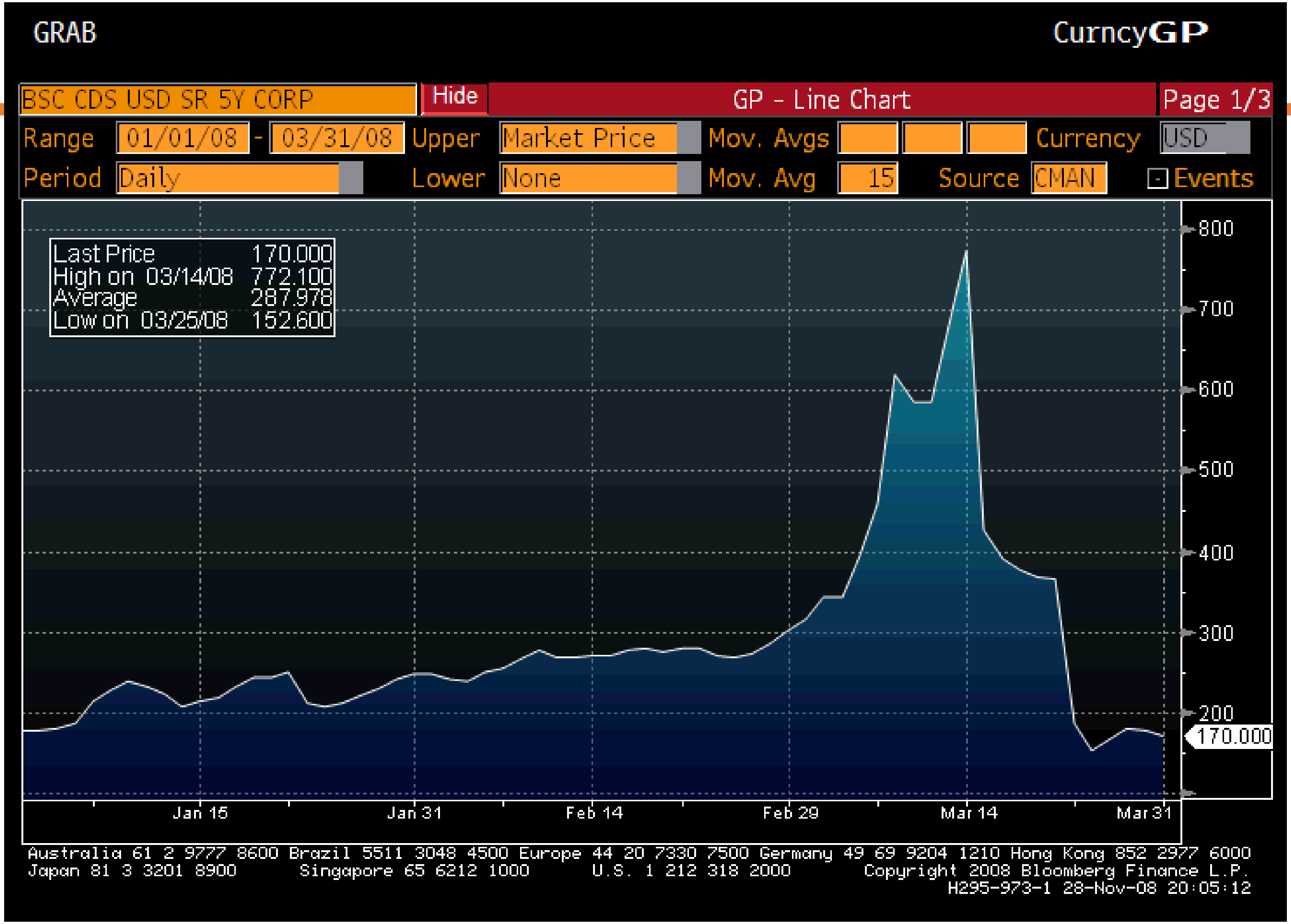

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