Subpixel Edge Detection Based on Edge Gradient Directional Interpolation and Zernike moment

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1 ABSTRACT

In the paper, we propose a new subpixel edge detection approach based on edge gradient directional interpolation and Zernike orthogonal moment. Due to the interpolation of the edge neighbors, the proposed method is able to enrich the edge information and ensure that there is only one edge inside the moment template. The enriching of edge information not only increases the detection accuracy of the moment based method but also increases the number of detected subpixel edge points. Experimental results show that the proposed method can greatly increase the edge detection accuracy and avoid the edge interference when there are complex edges in the image.

KEYWORDS: Image processing, Subpixel, Edge detection, Orthogonal moments, Zernike moments, Interpolation

2 INTRODUCTION

Edge detection is a basic task in image processing. Traditional gradient-based edge detection methods can only locate pixel-level edges. In general, the edge detection accuracy is improved by increasing the sampling frequency. However, the maximum sampling rate is limited by physical device conditions and cannot be increased indefinitely. Morover, in practical industrial applications, cost is also an important unavoidable consideration. This problem has led to the birth of subpixel edge detection algorithms. The subpixel edge detection can be understood as a method of improving the detection accuracy by image processing algorithms that can make the edge accuracy less than one pixel under the condition that the hardware condition of the camera system is not changed.

The early edge detection algorithm is based on gradients, and the edge points are located by calculating the local gradient or the maximum value of the derivative. The most typical ones are the *Sobel* [1] , *Prewitt* [2] and *Robert* operator [3]. Later, Canny [4] summed up the previous work and proposed a method based on non-maximum suppression and morphological continuity operations to find the optimal edge. It has achieved good results and is widely used in nowadays. Other edge detection algorithms include *Malik*'s edge detection method [5], which is excellent, but complicated. Other pixel-level edge detection algorithms can also be found in the reference [6–8].For subpixel edge detection, the current popular methods include fitting based method, moment based method, and interpolation based method. The core idea of the interpolation based method is to interpolate the gray value of the pixel or the derivative of the gray value to increase the edge information [10]. The quadratic interpolation algorithm is simple and the hardware is easy to implement. Especially when the diffusion function of the optical system is symmetrical, the detection accuracy of the interpolation edge is high. However, the disadvantage is that it is susceptible to noise.The fitting method is a method for solving the subpixel edge position by fitting hypothetical edge model gray values, typically ,the edge model based on Gaussian functions proposed by *Ye* [9] et al.The core idea of the moment based method [11, 14] is to use the moment information of the image to solve the edge parameters. There are many kinds of moments, such as space moments, Zernike moments, and so on. The Zernike moment method [13] which was used by *Ghosal* firstly requires only 3 masks to be calculated. The amount of calculation is much smaller than that of the spatial moment [12], and the accuracy is also high. Later, [15, 16] Qu and Hwang S K proposed an improved algorithm to improve subpixel positioning accuracy by increasing the Zernike moment order or increasing the template size.

At present, the detection of subpixels by the method of moments has been proved to be a simple and accurate method. Even so, there is no universally applicable method in practical application, and there is still much room for improvement in accuracy [13]. Moment-based modeling is based on the assumption that the image edge is an ideal step edge. In order to increase the detection accuracy, an effect and simple method is to increase the template size of the moment [15] [16]. However, when there are complicated edges inside the template, there edges can interfere to each other, leading to a mis-location of the edge. Since it is difficult to ensure that there is only one edge within a template, the edge needs to be re-modeled if there are more than one edge inside the template. However, after analysis, it is not only difficult to re-model the edge universally, but it is also difficult to obtain an accurate analytical solution. Re-modeling the image edge by strict edge parameters is also difficult to find as many as subpixel edge points. Furthermore, the subsequent edge filtering by the threshold based on the strict edge parameter re-modeling is also very complicated. With the rapid acceleration of hardware computing, more accurate methods are readily available.

By summarizing the advantages and disadvantages of subpixel edge detection algorithms, this paper proposes an improved method. First, the coarse position of the edges is obtained based on pixel-level, and the neighborhood of the edge points is traced. In order to increase the edge accuracy while avoiding the problems mentioned above, this paper proposes to enrich the edge information by interpolating the edge neighbors in their gradient directions. Due to the interpolation of the edge neighbors, the region for the moment template to convolute is enlarged. Consequently, it is able to ensure that there is only one edge inside the template, which can effectively avoid the interference of the edges. Meanwhile, rich edge information not only increases the accuracy of the detection of the moment based method but also increases the number of detected subpixel edge points. Moreover, if the Canny pixel-level edge location operation is performed on the neighbors after interpolation, double edges can be detected, which means that subpixel edges with thickness characteristics can obtained.. Finally, the large-scale mask Zernike moment method is used to detect the neighbor's edge and derive the subpixel-level edge parameters.

The structure of this paper is as follows.Section 3 briefly introduces the principle of Zernike moments. In Section 4, the subpixel edge detection of Zernike moments based on the 7*7 mask is discussed, and the theoretical derivation is given. Section 5 discusses the method of gradient interpolation in the neighborhood of the edge point. In Section 6, two experiments are designed to verify the accuracy of the edge subpixel edge location, the complex edge location capability. Finally, some conclusions are given in Section 7.

3 ZERNIKE IMAGE MOMENT

In the continuous domain,it can be proved that a function can be described by a combination of polynomials within unit cicle,i.e. $x^2+y^2 \leq 1$. The Zernike polynomial is an orthogonal complete polyomial, so the function can be described without redundancy. Zernike moment of order n and repetition m for an image $f(x,y)$ is definded as [13] [17]

$$Z_{nm} = \frac{n+1}{\pi} \iint_{x^2+y^2\leq 1} f(x,y)V^*_{nm}(\rho,\theta)dxdy \tag{1}$$

Where $n \geq m$ and n is a non-negative integer. In an discrete image, Z_{nm}can be expressed as follow when $x^2+y^2 \leq 1$

$$Z_{nm} = \sum_x \sum_y f(x,y)V^*_{nm}(\rho,\theta) \tag{2}$$

The Zernike polynomials V_{nm} can be shown as

$$V_{nm} = R_{nm}(\rho)e^{im\theta}, |\rho| \leq 1 \tag{3}$$

where $R_{nm}(\rho)$ is a radial. When *n-m* is an odd number, $R_{nm}(\rho) = 0$. When *n-m* is an even number, $R_{nm}(\rho)$ can be expressed as

$$R_{nm}(\rho) = \sum_{k=0}^{(n-|m|)/2} \frac{(-1)^k(n-k)!}{k!(\frac{n+|m|}{2}-k)!(\frac{n-|m|}{2}-k)!}\rho^{n-2k} \tag{4}$$

Combining Eq. (2) (3) to calculate the Zernike polynomials [15], as shown in the following Table **??**

Therefore we can find Zernike polynomials V_{nm} by looking up tables, and According to Eq. (2) we can calculate the Zernike moments of the image $f(x,y)$ easily. In addition, the Zernike moment has an important property [13] that is rotation invariance, i.e.

$$Z'_{nm} = Z_{nm}e^{-im\phi} \tag{5}$$

where Z'_{nm} is the Zernike moments of rotated image and Z_{nm} is the original image's Zernike moments. Eq. (5) shows the changes in Zernike moment after image rotation.i.e. the magnitude of the Zernike moment of the rotated image ϕ and the original image is not changed.

4 ZERNIKE MOMENT-BASED EDGE DECTECTION

To locate subpixel edge locations,establish a 2-D unit circle model with an ideal step edge on it is established. As the Fig.1 shows, k is the height of the step,h the height of the background, l is the distance between the edge and the center of the unit circle. When the ideal edge line is rotated ϕ , it will be perpendicular to the x axis. Next we will use Zernike moments to calculate this set of edge position parameters i.e. h, k, l.

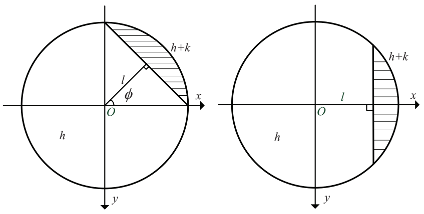


Fig. 1: Ideal model of subpixel edge detection

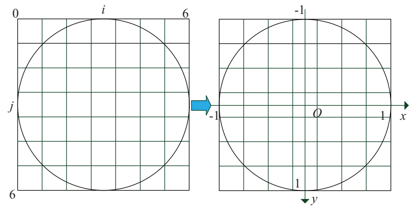


Fig. 2: Zernike 7*7 mask model

Now assuming this model is $f(x,y)$, its rotated Zernike moments Z'_{nm} [15] is calculated according to Eq.(2) and Eq. (5). Solve the equation about h, k, l and get

$$k = \frac{3Z'_{11}}{2(1-l_2^2)^{3/2}} \tag{6}$$

$$h = \frac{1}{\pi}\left[Z_{00} - \frac{k\pi}{2} + k\arcsin(l_2) + kl_2\sqrt{1-l_2^2}\right] \tag{7}$$

$$l = \frac{l_1+l_2}{2} = \frac{1}{2}\left[\sqrt{\frac{5Z'_{40}+3Z'_{20}}{8Z'_{20}}} + \sqrt{\frac{5Z'_{11}+Z'_{11}}{6Z'_{11}}}\right] \tag{8}$$

$$\phi = \arctan\left[\frac{Im[Z_{n1}]}{Re[Z_{n1}]}\right] \tag{9}$$

As mentioned above, we derive the edge paremeters h, k, l and ϕ. So, the subpixel location of the edge within the unit circle can be marked as

$$\begin{bmatrix} x_s \\ y_s \end{bmatrix} = \begin{bmatrix} x + l\cos(\phi) \\ y + l\sin(\phi) \end{bmatrix} \tag{10}$$

where (x,y) is the center of the unit circle.However, in actual discrete images,we use the N*N range of pixels to approximate the unit circle.as shown in Fig. 2. The unit circle is divided into 7*7 areas. The above-mentioned calculation results are based on the 7*7 area within the unit circle. We put the pixel values of each point on the actual image 7*7 area into the unit circle to calculate the subpixel edge parameters. But actually it is impossible to divide a pixel into 7*7 neighborhoods on the image. So the edge position parameter l derived from the unit circle is finally multiplied by the scaling factor $N/2$. Only in this way can the subpixel position of the edge of the image be detected.

$$Subpixel = \begin{bmatrix} x_s \\ y_s \end{bmatrix} = \begin{bmatrix} x + \frac{lN}{2}\cos(\phi) \\ \\ y + \frac{lN}{2}\sin(\phi) \end{bmatrix} \tag{11}$$

From the above discussion, it is known that by deriving the M_{n1} of the image, ϕ can be calculated by Eq. (9) .Then, we derive the other Zernike moments of the image and get the rotated image Zernike moments [13] . Finally, we obtain the edge parameters h, k, l by Eq. (7)) (6) (8). Therefore, it is very important to calculate the Zernike moments in the N*N field of the image, which will determine the accuracy of the entire detection.

To approximate the Zernike moments of the image, we construct a series of masks which can be derived by

$$M_{nm(i,j)} = \iint_{\Omega_{i,j}=C\cap S_{i,j}} V^*_{nm}(\rho,\theta)dxdy \tag{12}$$

The Zernike polynomials are obtained by looking up the Table **??**, and the masks for calculating the Zernike moments of image are obtained through *matlab* calculation. Convolve the *N*N* field at each point of the image with the masks to get the M_{nm}.

Finally, we can get the parameters h, k, l and ϕ of the edge by putting the M_{nm} into Eq. (6) (8) (7) (9). By setting the characteristic thresholds l_T, k_T shown as Eq. (13) (14), we can filter out the specific edge position or guarante detection accuracy.

$$l \leq l_T \tag{13}$$

$$k \geq k_T \tag{14}$$

where l_T must be less than 0.5 pixel and k_T can be flexibly adjusted according to different situations.

5 SUBPIXEL EDGE DETECTION BASED ON DIRECTIONAL INTERPOLATION

Different from the discussion in the continuous domain, due to the discreteness of the image, the detection of the subpixel edge is bound to have errors.Using Zernike moments for edge subpixel locating, the main reason for the error is the M_{nm} mask size. If the image only has simple and sparse edges, the large mask can bring high precision.However, as mentioned in Section 1, if the image has complex edges near to each other, there will be errors in the edge detection. i.e.

$$Accuracy \propto N \tag{15}$$

Also, the ability to portray details is also poor when there is less edge information in the mask coverage area.

In order to overcome this limitation, this paper proposes an improved algorithm in subpixel edge detection. The principle is to interpolate the edge according to the its gradient direction to enrich its position information, so that the large-scale Zernike moment mask can exert its accurate edge positioning capability.

Canny algorithm to is applied to approximate the edge points. The gradient of the edge point *f(x,y)* can be expressed as

$$gradf(x,y) = \nabla f(x,y) = \{\frac{\partial f(x,y)}{\partial x}, \frac{\partial f(x,y)}{\partial y}\} \tag{16}$$

Gradient direction interpolation of the edge point field can be expressed as

$$x_t = \lfloor C_t \frac{\partial f(x,y)}{\partial x} \rfloor, y_t = \lfloor C_t \frac{\partial f(x,y)}{\partial y} \rfloor \tag{17}$$

In Eq. 17 , x_t , y_t denotes the extension times in x-direction and y-direction of the edge point in N*N domain, respectively. C_t is a constant, and the $\lfloor\rfloor$ symbol denotes rounding up.

The bilinear interpolation method is applied to extent the edge points. As shown in Fig.4 if the image after interpolating is subjected to *Canny* detection, dual-edge information can be obtained. By the mapping information with the original edge, the dual-edge information of the original edge can be found.This will further increase the detection depth of the edge subpixels and obtain more abundant subpixel information of the original edge.

$$Subpixel = \begin{bmatrix} x_s \\ y_s \end{bmatrix} = \begin{bmatrix} x/x_t + \frac{lN}{2x_t}\cos(\phi) \\ \\ y/y_t + \frac{lN}{2y_t}\sin(\phi) \end{bmatrix} \tag{18}$$

As shown in Fig.4 if the image after interpolating is subjected to *Canny* detection, dual-edge information can be obtained. By the mapping information with the original edge, the double-edge information of the original edge can be found. This will further increase the detection depth of the edge subpixels and obtain more abundant subpixel information of the original edge.

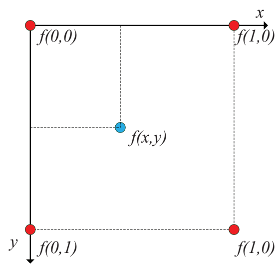


Fig. 3: Bilinear interpolation

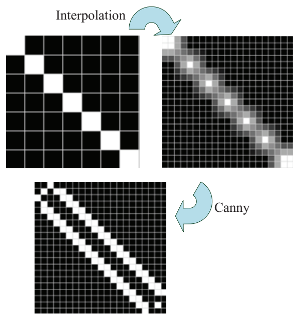


Fig. 4: Separation of double edge

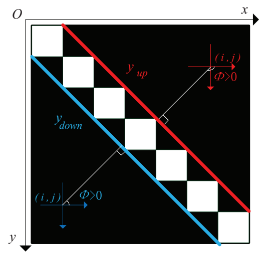


Fig. 5: 7*7 field true subpixel double-edge location model

6 EXPERIMENTAL AND SIMULATION RESULTS

In this section, we will describe the proposed edge detection scheme for experimental verification and analysis. Using Intel Core i7-4710MQ (2.50GHz) by Visual Studio 2017 (64-bit).

To verify the localization ability of this algorithm in the subpixel points in the edge point 7*7 field, we simulated the algorithm based on *OpenCV3.0(C++)*. Six 7*7 images was created by *OpenCV3.0* function *line* .Fig.5 shows a 7*7 real subpixel double edge location model.

As shown in Fig. 6, Each image represents an ideal edge condition in the edge point 7*7 field. Respectively, (a) represents 0° edge, (b) represents 37°, (c) represents 45°, (d) represents 53°, (e) represents 90°, (f) represents $\arctan(1/3)$.

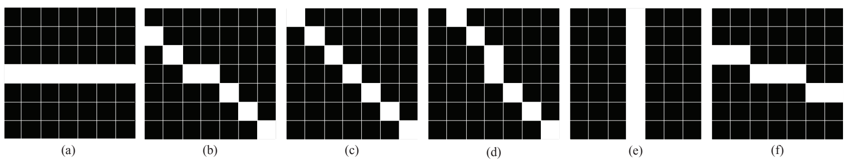
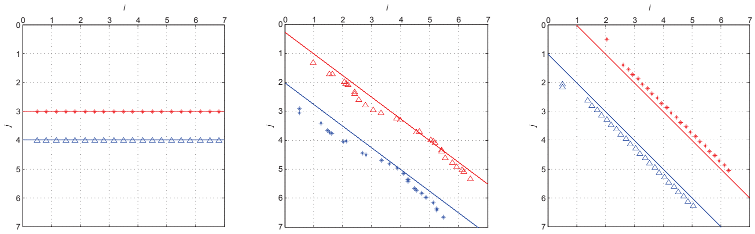


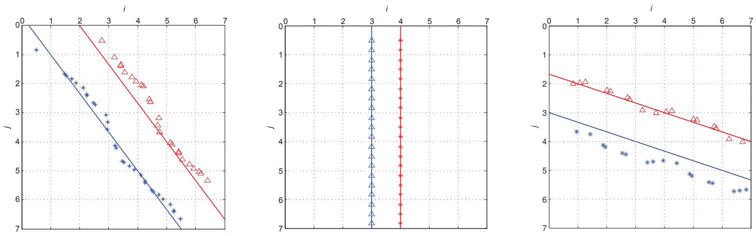
Fig. 6: Edge pixel level 7*7 field

The subpixel localization algorithm proposed in this paper is used to perform subpixel detection on the six images shown in Fig. 6. The threshold of l, k for algorithm execution is set to $k_T = 150, l_T = \sqrt{2}/14$. The detected subpixel points and the actual edge lines are marked in the same figure which shown in Fig. 7 from (a) to (f). In Fig. 7 (a) (b) (c) (d) (f), the blue line indicates the $\phi < 0$ true edge, and the red line indicates the $\phi > 0$ edge. The blue * indicates the detected subpixel edge point of $\phi < 0$, and the red $\triangle$ symbol indicates the detected edge point of $\phi > 0$ .In Fig. 7 (e) The red line marks the true edge of $\phi = 180°$, and the blue line marks the true edge of $\phi = 0°$. Experimental results show that the algorithm proposed in this paper can control the subpixel edge positioning accuracy of 0.4 pixels in the 7*7 edge point neighborhood while *Ghosal* algorithm can not detect any subpixel edge point.

In order to test the ability of the algorithm to locate complex and dense edges, the *Lena* image was employed. In Fig. 8 (b), the edge detected by the *Canny* algorithm is the reference point for this experiment. As shown in Fig. 8 (c) (d), by comparing the localization performance of the algorithm and the *Ghosal* [13] algorithm in the complex edge part of the *Lena* image, it is found that the algorithm has better detail characterization capabilities.In addition, we can see from the experimental results that the algorithm can detect more edge points and have better continuity. As can be seen from Fig. 8 (e)-(g), the algorithm's ability to locate at the dual edge is better than the glsoal algorithm.



(a) 0°edge:$x_t = 1, y_t = 3$ (b) 37°edge:$x_t = 4, y_t = 3$ (c) 45°edge:$x_t = 3, y_t = 3$



(d) 53°edge:$x_t = 3, y_t = 4$ (e) 90°edge:$x_t = 3, y_t = 1$ (f) $arctan(1/3)$ edge: $x_t = 6, y_t = 2$

Fig. 7: Comparison of subpixel edge detection results and actual edge position

7 CONCLUSION

In this paper, we propose a subpixel detection method based on Zernike moment edge gradient direction interpolation. The proposed method first performs the *Canny* edge detector to locate the coarse position on the original image. A gradient-direction interpolation is then perform on the neighbors of the edge points. Finally, the Zernike moment method is used to perform subpixel edge location in the 7*7 domain and the edge points are mapped back to the original image. Through this method, we have achieved accurate positioning of su-pixel edges under the condition of large-sized templates, and are suitable for the detection of complex edges. Furthermore, by performing pixel-level edge detection on the interpolated edge neighbors, the Zernike moment method can also be used to accurately locate the double edges and obtain the contour information of the subpixel edges.

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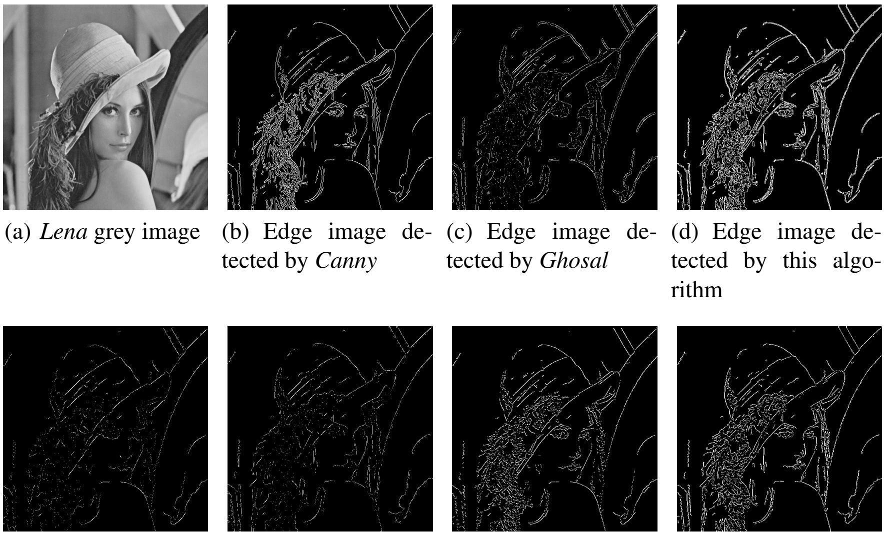

(a) *Lena* grey image

(b) Edge image detected by *Canny*

(c) Edge image detected by *Ghosal*

(d) Edge image detected by this algorithm

(e) Edge image detected by *Ghosal* $\phi < 0$

(f) Edge image detected by *Ghosal* $\phi > 0$

(g) Edge image detected by this algorithm $\phi < 0$

(h) Edge image detected by this algorithm $\phi > 0$

Fig. 8: Comparision of edge detection results for the *Lena* images

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