MCQ, CQ

Matrix

* Ketab MCQ Sol^n

· History
- Mater (mother)
- J. J. Sylvester, 1850
- Arthur Cayley

· Application
- Quantum Mechanics
- Robotics
- Economics
- Classical Mechanics

What do we intend to learn ?

· addition, substraction, multiplication → Scalar, → Matrix

· classification, formulae

· Sarrus Diagram, Cramer's rule

Adjoint → Concept

- Matrix এর মান হয় না, Determinant এর মান হয়
- [ ] ( ) || || $\left\|\begin{matrix} 2 & 3 \\ 4 & 5 \end{matrix}\right\|$
- ভুক্তি (entry) (i,j) তম ভুক্তি 1.25
- Order/মাত্রা $m \times n \rightarrow m$ by n
- $[a_{ij}]_{m\times n} \pm [b_{ij}]_{m\times n} = [a_{ij} \pm b_{ij}]_{m\times n}$
- $k[a_{ij}]_{m\times n} = [ka_{ij}]_{m\times n}$ ★ $|kA| = k^n|A|$ $A_{n\times n}$
- Multiplication: $A = [\ ]_{m\times n}$ $B = [\ ]_{n\times p}$ $AB = [\ \ ]_{m\times p}$
- $A = B \Rightarrow a_{ij} = b_{ij}$
- Formula

★ $|A^n| = |A|^n$

★ $|AB| = |A||B| = 0$

$|ABCD| = |A||B||C||D|$

$A(B+C) = AB + AC$

$(B+C)A = BA + CA$

বন্টন বিধি: $k(A+B) = kA + kB$

বিনিময় বিধি: $AB = BA$ (not always)

সংযোজন: $(AB)C = A(BC) = ABC$

Classification of Matrix[25]

1. সারি $[a_{ij}]_{1\times n}$

কলাম ভেক্টর

2. কলাম $[a_{ij}]_{m\times 1}$

সারি

3. আয়ত $[a_{ij}]_{m\times n}$; $m\neq n$

4. বর্গ $m=n$

5. অনুভূমিক $c>r$

6. উল্লম্ব $r>c$

7. লম্ব ম্যাট্রিক্স : $AA^T = I \Rightarrow A^T = A^{-1}$

$|A|$

$|A||A^T| = |I| = 1 \Rightarrow |A|^2 = 1$

$|A| = \pm 1$

4.1 কর্ণ: $A = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$ abc $\quad A^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}$

A^{-2}

4.2 স্কেলার: $a=b=c=k$ $\quad k^n$

4.3 অভেদক: $k=1$ $\quad (I)$

$|I| = 1$

8. শূন্য/Null: $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}_{2\times 3}$ $\quad O_{m\times n}$

9. Upper Triangular Matrix :

$\delta_{ij} = 0$

$a_{ij} = 0 \quad i > j$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

10. Lower Triangular Matrix: $\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ $\quad a_{ij} = 0 \quad i < j$

11. Periodic Matrix : $A^{P+1} = A \quad P = \text{period} \rightarrow$

12. Involutory / অভেদঘাতি : $A^2 = I \rightarrow$

13. সমঘাতী / Idempotent : $A^2 = A \quad |A|^2 = |A| \quad |A| = 0, 1$

14. শূন্যঘাতী / Nilpotent : $A^n = 0 \quad n \rightarrow$ index/সূচক

15. Sub Matrix : $\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$

$[2] \quad [3] \qquad \begin{bmatrix} 2 & 3 \end{bmatrix} \quad \begin{bmatrix} 4 & 5 \end{bmatrix}$

$[4] \quad [5] \qquad \begin{bmatrix} 2 \\ 4 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 5 \end{bmatrix}$

16. Matrix এর function: $f(x) = e^x$ $\qquad e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$

$f(A) = e^A$

$A = \begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix}$ $\quad A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\quad A^3 = A^2 \cdot A$ $\quad A^{2,3,4,\dots} = O_{2\times 2}$

$$e^A = I + \frac{A}{1!} + \frac{A^2}{2!} + \frac{A^3}{3!} + \dots = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 6 & -9 \\ 4 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} 7 & -9 \\ 4 & -5 \end{bmatrix}$$

17. ম্যাট্রিক্সের বহুপদী

$$f(x) = a_n x^n + \dots + a_1 x^1 + a_0$$

$$f(A) = a_n A^n + \dots + a_1 A^1 + a_0 I$$

$$f(x) = x^2 - 4x - 5 \qquad A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$f(A)$, A^{-1}

$$A^2 = \begin{bmatrix} & & \\ & & \end{bmatrix}\begin{bmatrix} & & \\ & & \end{bmatrix}$$

$$f(A) = A^2 - 4A - 5I = 0$$

$$|A| \neq 0 \Rightarrow A^{-1}(A^2 - 4A - 5I) = A^{-1}0 = 0$$

$$\Rightarrow A - 4I - 5A^{-1} = 0 \Rightarrow A^{-1} = \frac{1}{5}(A - 4I)$$

Transpose : row $\leftrightarrow$ column   বিম্ব / প্রতিবিম্ব

$(kA \pm B)^T = kA^T \pm B^T$ $(ABCD)^T = D^T C^T B^T A^T$ reversal law

$(A^T)^T = A$ $I^T = I^{-1} = I^n = I$

$(A^{-1})^{-1} = A$ $(ABCD)^{-1} = D^{-1} C^{-1} B^{-1} A^{-1}$ $(A^T)^{-1} = (A^{-1})^T$

$$\begin{bmatrix} a & h & g \\ h & b & f \\ g & f & c \end{bmatrix}$$

প্রতিসম / Symmetric : $A^T = A$

$A^{-1} = \frac{1}{|A|} adj(A)$

অপ্রতিসম / Skew-Symmetric : $A^T = -A$

$$\begin{bmatrix} 0 & h & g \\ -h & 0 & f \\ -g & -f & 0 \end{bmatrix}$$

$$\begin{bmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{bmatrix}^T = -\begin{bmatrix} a_1 & & \\ & & \\ & & \end{bmatrix}$$

$a_1 = -a_1$
$a_5 = -a_5$
$a_9 = -a_9$
$\Rightarrow 0$

Conjugate : $a+ib \leftrightarrow a-ib$

Conjugate matrix : অনুবন্ধী   i এর জায়গায় $(-i)$

$$\begin{bmatrix} 2 & 3-2i \\ 1+2i & i-7 \end{bmatrix} \rightarrow \begin{bmatrix} 2 & 3+2i \\ 1-2i & -i-7 \end{bmatrix} \qquad A \qquad \bar{A} \qquad A^{\theta}$$

Hermitian : $(\bar{A})^T = A$

$$\begin{bmatrix} 2 & 3-i \\ 3+i & 3 \end{bmatrix} \xrightarrow[-]{\theta} \begin{bmatrix} 2 & 3+i \\ 3-i & 3 \end{bmatrix} \xrightarrow{T} \begin{bmatrix} 2 & 3-i \\ 3+i & 3 \end{bmatrix}$$

Skew-Hermitian : $(\bar{A})^T = -A$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \dots \qquad e^1 = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \dots \Rightarrow e - 1 = \frac{1}{1!} + \frac{1}{2!} + \dots$$

$$e^A = I + \frac{A}{1!} + \frac{A^2}{2!} + \dots = I + \frac{A}{1!} + \frac{A}{2!} + \frac{A}{3!} + \dots = I + A(e-1)$$

$A = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \rightarrow$ Idempotent $A^2 = A$

$$A^3 = A^2 . A = A . A = A^2 = A$$

$A \rightarrow$ Idemp. $\quad B = I - A \longrightarrow$ Idemp

$$B^2 = (I - A)(I - A) = I^2 - IA - AI + A^2 = I - A - A + A^2$$

$$= I - A - A + A = I - A \rightarrow$$

$A = \begin{pmatrix} 1 & -3 & -4 \\ -1 & 3 & 4 \\ 1 & -3 & -4 \end{pmatrix} \to$ শূন্যঘাতী

$A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \to A^2 \quad A^3$

দেখাও যে, $A^2 + 2A - 11I = 0 \to A^{-1}$

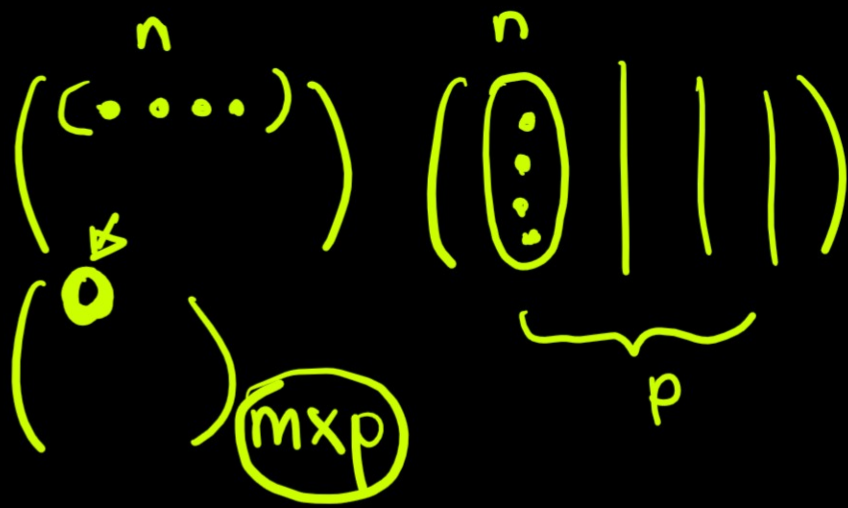
$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

#

$A_{m \times n}$ $\quad$ $B_{n \times p}$

$\# + = (n-1)mp$

$\# * = nmp$



#

$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2a+3b+5c \\ a-7b+9c \\ -8a+17b-16c \end{pmatrix}$

→ a, b, c কে x, y, z এর মাধ্যমে প্রকাশ করা যাবে কি?

যদি যায় তবে প্রকাশ কর।

Solⁿ:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2a+3b+5c \\ a-7b+9c \\ -8a+17b-16c \end{pmatrix} = \overset{A}{\begin{pmatrix} 2 & 3 & 5 \\ 1 & -7 & 9 \\ -8 & 17 & -16 \end{pmatrix}} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

→ $|A| \neq 0$ ∴ প্রকাশ করা যায়

$$A^{-1} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \qquad A^n = A^{n-2} + A^2 - I \quad n \geq 3$$

$$A^{50} u_1 = \begin{bmatrix} 1 \\ 25 \\ 25 \end{bmatrix} \qquad A^{50} u_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad A^{50} u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad u = [u_1 \; u_2 \; u_3]$$

$\rightarrow$ $\text{Trace}(A^{50}) = ?$ $\qquad A^{50} = \begin{bmatrix} 1 & 0 & 0 \\ 25 & 1 & 0 \\ 25 & 0 & 1 \end{bmatrix} \qquad \text{Tr}(A^{50}) = 3$

method of difference

$$\sum_{n=2}^{50} \left(A^n - A^{n-2} \right) = \sum_{n=2}^{50} (A^2 - I) = 25(A^2 - I)$$

$n \in$ even

$$\Rightarrow \quad A^{50} - I = 25A^2 - 25I \quad \Rightarrow \quad A^{50} = 25A^2 - 24I$$

$$\# \quad A = \begin{bmatrix} & & \end{bmatrix} \qquad P \qquad P^{-1} \qquad PAP^{-1} = D = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{bmatrix}$$

$$A^3 = I \qquad A^4 = A \qquad A^5 = A^2 \qquad A^6 = I \qquad D^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & b^n & 0 \\ 0 & 0 & c^n \end{bmatrix}$$

$$\# \quad A^{2025} =$$

$$D^n = (PAP^{-1})^n = PAP^{-1} \overbrace{PAP^{-1}\, PAP^{-1} \ldots PAP^{-1}}^{n-2} PAP^{-1}$$

$$= P A^n P^{-1} \quad \Rightarrow \quad P^{-1} D^n P = P^{-1} P A^n P^{-1} P = A^n$$

$$\# \quad A^n = P^{-1} D^n P \longrightarrow \text{Diagonalization}$$

Method of Difference:

$b_n = a_n - a_{n-1}$

$$b_1 = \cancel{a_1} - a_0$$
$$b_2 = \cancel{a_2} - \cancel{a_1}$$
$$b_3 = \cancel{a_3} - \cancel{a_2}$$
$$\vdots$$
$$b_i = a_i - \cancel{a_{i-1}}$$
$$\vdots$$
$$b_n = a_n - \cancel{a_{n-1}}$$

$$\sum i = \frac{n(n+1)}{2} \qquad \sum i^3 = \left\{\sum i\right\}^2$$

$$\sum i^2 = \frac{n(n+1)(2n+1)}{6} \qquad 9\text{-}10$$

$$\sum_{i=1}^{n} b_i = a_n - a_0$$

বীজগাণিতিক রাশি চক্রক্রমিক রাশি [9-10] Preq. → 2,

আংশিক ভগ্নাংশ HM

অব্যতিক্রমী (Non-singular) : $|A| \neq 0$

ব্যতিক্রমী (Singular) : $|A| = 0$

অনুরাশি : $\begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix}$ Minor

ঐ row
ঐ column } বাদ

সহগুণক/Cofactor : $\begin{bmatrix} A_1 & A_2 \\ B_1 & B_2 \end{bmatrix}$ $(-1)^{r+c}$

$(-1)^{r+c}$

$-b_1$

adjoint/adjugate :

$A \rightarrow \text{Cofactor} \rightarrow \text{Matrix} \xrightarrow{T} \text{adj}(A)$

$A^{-1} = \frac{1}{|A|} \text{adj}(A)$

Theorem: $A\,\mathrm{adj}(A) = \mathrm{adj}(A)\,A = |A| I_n$

$A = [a_{ij}]$

$$a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31} = |A|$$

$$a_{11}A_{12} + a_{21}A_{22} + a_{31}A_{32} = 0$$

cofactor

$[A_{ij}]^T = \mathrm{adj}(A)$

$$A\,\mathrm{adj}(A) = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & \\ \vdots & & & & \\ a_{n1} & \cdots & & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} A_{11} & \cdots & \cdots \\ A_{12} & \cdots & \\ \vdots & & \\ A_{1n} & \cdots & \end{bmatrix}$$

30 sec

$$= \begin{bmatrix} |A| & 0 & \cdots & 0 \\ 0 & |A| & \cdots & 0 \\ 0 & 0 & \ddots & |A| \end{bmatrix} = |A| I_n$$

$$|A\,\mathrm{adj}(A)| = |\underbrace{|A|}\,I_n|$$

$$|kA| = k^n|A|$$

$$\Rightarrow |A|\,|\mathrm{adj}(A)| = |A|^n\,|I_n| = |A^n| = |A|^n$$

$$\Rightarrow |\mathrm{adj}(A)| = |A|^{n-1}$$

$$|\mathrm{adj}(X)| = |X|^{n-1}$$

$$|\mathrm{adj}(\mathrm{adj}(A))| = |\mathrm{adj}(A)|^{n-1} = |A|^{(n-1)^2}$$

$$\left(|A|^{n-1}\right)^{n-1}$$

$$|\underbrace{\mathrm{adj}\ldots\mathrm{adj}}_{k}(A)| = |A|^{(n-1)^k}$$

$A \to$ idempotent $\Rightarrow (A+I)^n = I + (2^n - 1)A$

Sol^n:

$0! = 1 \Leftarrow n! = n(n-1)!$

$$(a+b)^n = \sum_{r=0}^{n} \binom{n}{r} a^{n-r} b^r \qquad \binom{n}{r} = {}^nC_r = \frac{n!}{r!\,(n-r)!}$$

$$(1+x)^n = {}^nC_0 x^0 + {}^nC_1 x^1 + {}^nC_2 x^2 + {}^nC_3 x^3 + \dots + {}^nC_n x^n$$

$$x=1 \Rightarrow 2^n = \underbrace{{}^nC_0}_{1} + \left({}^nC_1 + {}^nC_2 + \dots + {}^nC_n\right) \qquad [A = A^2 = A^3 = \dots = A^n]$$

$$(I+A)^n = {}^nC_0 A^0 + {}^nC_1 A + {}^nC_2 A + \dots + {}^nC_n A$$

$$= I + A\left(\sum_{r=1}^{n} {}^nC_r\right) = I + A(2^n - 1)$$

#

		M	P	C
B	# books	100	125	110
R	ক্রয়মূল্য	60	90	85
S	বিক্রয়মূল্য	70	102	96

3 মাস
26 জুন

Boards + Dhaka

HW

লাভ, $P = S - R = \begin{bmatrix} 10 & 12 & 11 \end{bmatrix}_{1\times 3} = \begin{bmatrix} 10 \\ 12 \\ 11 \end{bmatrix}_{3\times 1}$

$B = \begin{bmatrix} 100 \\ 125 \\ 110 \end{bmatrix}_{3\times 1} = \begin{bmatrix} 100 & 125 & 110 \end{bmatrix}_{1\times 3}$

$\begin{bmatrix} 100 & 125 & 110 \end{bmatrix} \begin{bmatrix} 10 \\ 12 \\ 11 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix} =$

Prerequisite:

$$a^3+b^3+c^3-3abc = (a+b+c)(a^2+b^2+c^2-ab-bc-ca)$$

$$= \frac{1}{2}(a+b+c)\{(a-b)^2+(b-c)^2+(c-a)^2\}$$

$$|AB| = |A||B|$$

1. Prove that, $\begin{vmatrix} \cos(A-P) & \cos(A-Q) & \cos(A-R) \\ \cos(B-P) & \cos(B-Q) & \cos(B-R) \\ \cos(C-P) & \cos(C-Q) & \cos(C-Q) \end{vmatrix} = 0$

2. Prove that, $\begin{vmatrix} a_1x_1+b_1y_1 & a_1x_2+b_1y_2 & a_1x_3+b_1y_3 \\ a_2x_1+b_2y_1 & a_2x_2+b_2y_2 & a_2x_3+b_2y_3 \\ a_3x_1+b_3y_1 & a_3x_2+b_3y_2 & a_3x_3+b_3y_3 \end{vmatrix} = 0$