Quantitative Methods in Finance

Tutorial, Part 5:
Testing hypotheses about regression coefficients and their linear combinations.

Example 1: We gathered a sample of data for 32 European countries for the year 2003. We have the following variables available (the data are provided in Stata Data file `health.dta`, while the programming code is given in Stata Do file `health-commands-t05.do`):

- life expectancy at birth (*LIFE*; in years);
- health expenditure per capita (*EXP*; in U.S. dollars);
- percentage of smokers among adults (*TOBACCO*);
- consumption of alcohol per capita (*ALCO*; in litres of distilled spirits).

Estimate the linear regression model: $LIFE_i = \beta_1 + \beta_2 EXP_i + \beta_3 TOBACCO_i + u_i$ and answer the questions below by applying the appropriate procedures of hypotheses testing.

a) Does the percentage of smokers among adults affect the life expectancy at birth?
b) Can we claim that an increase of health expenditure per capita by 100 U.S. dollars extends the life expectancy at birth by more than two months (given that the percentage of smokers among adults remains unchanged)?
c) Test the null hypothesis that a decrease of life expectancy at birth due to an increased percentage of smokers among adults by one percentage point can be »compensated« by an increase of health expenditure per capita of 100 U.S. dollars.

Computer printout of the results in Stata:

Estimation of the model for Exercises a, b and c

```
. regress life exp tobacco

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  2,    29) =   31.97
       Model |  385.751827     2  192.875914           Prob > F      =  0.0000
    Residual |   174.97295    29  6.03354999           R-squared     =  0.6880
-------------+------------------------------           Adj R-squared =  0.6664
       Total |  560.724777    31   18.087896           Root MSE      =  2.4563

------------------------------------------------------------------------------
        life |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         exp |   .0023323   .0003709     6.29   0.000     .0015736    .0030909
     tobacco |  -.2503555   .0889983    -2.81   0.009    -.4323774   -.0683335
       _cons |   79.62409   2.796632    28.47   0.000     73.90433    85.34384
------------------------------------------------------------------------------
```

Testing hypotheses by applying the t-test in scalar notation (Exercises a, b and c)

```
. estat vce

Covariance matrix of coefficients of regress model

        e(V) |        exp     tobacco       _cons
-------------+------------------------------------
         exp |  1.376e-07
     tobacco |  9.960e-06    .0079207
       _cons | -.00048708  -.24150696   7.8211512

. display 2*ttail(29, abs(-2.81))
.00878239

. display ttail(29, 1.78)
.04277883

. display 2*ttail(29, abs(-0.16))
.87399033
```

Testing hypotheses by applying the t-test in matrix notation (Exercises b and c)

```
. matrix beta=(e(b))'
. matrix list beta

beta[3,1]
                 y1
    exp   .00233227
tobacco  -.25035545
  _cons   79.624085

. matrix vcm=e(V)
. matrix list vcm

symmetric vcm[3,3]
                exp     tobacco       _cons
    exp   1.376e-07
tobacco   9.960e-06    .0079207
  _cons  -.00048708  -.24150696   7.8211512

. matrix c=(100\0\0)
. matrix list c

c[3,1]
     c1
r1  100
r2    0
r3    0

. matrix varcb=c'*vcm*c
. matrix list varcb

symmetric varcb[1,1]
           c1
c1  .00137602

. scalar secb=sqrt(varcb[1,1])
. display secb
.03709481

. matrix tb=(c'*beta-0.167)/secb
```

```
. matrix list tb

symmetric tb[1,1]
           c1
r1  1.7853439

. display ttail(29, 1.7853439)
.04233458

. matrix c=(100\1\0)
. matrix list c

c[3,1]
     c1
r1  100
r2    1
r3    0

. matrix varcb=c'*vcm*c
. matrix list varcb

symmetric varcb[1,1]
           c1
c1  .01128871

. scalar secb=sqrt(varcb[1,1])
. display secb
.10624835

. matrix tc=(c'*beta-0)/secb
. matrix list tc

symmetric tc[1,1]
            c1
r1  -.16121158

. display 2*ttail(29, abs(-.16121158))
.87304462
```

Testing a hypothesis by applying the F-test (Exercise c)

```
. gen x_transf=exp-100*tobacco

. regress life x_transf

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  1,    30) =   66.05
       Model |   385.59502     1   385.59502           Prob > F      =  0.0000
    Residual |  175.129757    30  5.83765858           R-squared     =  0.6877
-------------+------------------------------           Adj R-squared =  0.6773
       Total |  560.724777    31   18.087896           Root MSE      =  2.4161

------------------------------------------------------------------------------
        life |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
    x_transf |   .0023683   .0002914     8.13   0.000     .0017732    .0029634
       _cons |   79.18374   .5903236   134.14   0.000     77.97814    80.38934
------------------------------------------------------------------------------

. display Ftail(1,29,0.026)
.87301842
```

```
. regress life exp tobacco

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  2,    29) =   31.97
       Model |  385.751827     2  192.875914           Prob > F      =  0.0000
    Residual |   174.97295    29  6.03354999           R-squared     =  0.6880
-------------+------------------------------           Adj R-squared =  0.6664
       Total |  560.724777    31   18.087896           Root MSE      =  2.4563

------------------------------------------------------------------------------
        life |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         exp |   .0023323   .0003709     6.29   0.000     .0015736    .0030909
     tobacco |  -.2503555   .0889983    -2.81   0.009    -.4323774   -.0683335
       _cons |   79.62409   2.796632    28.47   0.000     73.90433    85.34384
------------------------------------------------------------------------------

. test 100*exp+tobacco=0

 ( 1)  100*exp + tobacco = 0

       F(  1,    29) =    0.03
            Prob > F =    0.8730
```

■

Example 2: We analyse an aggregate consumption function for the period 1965–1989, which has the following specification (the data are provided in Stata Data file `consumption.dta`, while the programming code is given in Stata Do file `consumption-commands.do`):

$$C_t = \beta_1 + \beta_2 Y_t + \beta_3 ST_t + \beta_4 TR_t + \beta_5 FT_t + u_t,$$

where C represents the consumption of households, Y denotes personal incomes, ST represents social transfers, TR denotes other transfers, and FT represents foreign transfers (all in mil. monetary units and in 1972 prices).

Verify with the trinity of econometric tests whether we can exclude other transfers and foreign transfers from the above model specification.

Computer printout of the results in Stata:

Estimation of the model

```
. regress c y st tr ft

      Source |       SS       df       MS              Number of obs =      25
-------------+------------------------------           F(  4,    20) =  130.82
       Model |  3.69668581     4  .924171451           Prob > F      =  0.0000
    Residual |  .141293465    20  .007064673           R-squared     =  0.9632
-------------+------------------------------           Adj R-squared =  0.9558
       Total |  3.83797927    24  .159915803           Root MSE      =  .08405

------------------------------------------------------------------------------
           c |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .3981591   .1938749     2.05   0.053    -.0062569     .802575
```

```
          st |   1.628632   .5127575     3.18   0.005      .559039    2.698226
          tr |   7.506952   6.290407     1.19   0.247    -5.614608    20.62851
          ft |   .4450372   .5246166     0.85   0.406    -.6492939    1.539368
       _cons |   .3635363   .1152333     3.15   0.005     .1231638    .6039088
------------------------------------------------------------------------------
```

Testing a reduction of the model with the F-test

```
. regress c y st

      Source |       SS       df       MS              Number of obs =      25
-------------+------------------------------           F(  2,    22) =  253.82
       Model |  3.67855587     2  1.83927794           Prob > F      =  0.0000
    Residual |    .1594234    22  .007246518           R-squared     =  0.9585
-------------+------------------------------           Adj R-squared =  0.9547
       Total |  3.83797927    24  .159915803           Root MSE      =  .08513

------------------------------------------------------------------------------
           c |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .6648524   .0872809     7.62   0.000     .4838429    .8458619
          st |   .9483866    .283467     3.35   0.003      .360512    1.536261
       _cons |   .2653599   .0937114     2.83   0.010     .0710144    .4597054
------------------------------------------------------------------------------

. display ((.1594234-.141293465)/2)/(.141293465/(25-5))
1.2831404

. display Ftail(2,20,1.2831404)
.29901989

. qui regress c y st tr ft
. test tr=ft=0

 ( 1)  tr - ft = 0
 ( 2)  tr = 0

       F(  2,    20) =    1.28
            Prob > F =    0.2990
```

Testing a reduction of the model with the LM-test

```
. regress c y st

      Source |       SS       df       MS              Number of obs =      25
-------------+------------------------------           F(  2,    22) =  253.82
       Model |  3.67855587     2  1.83927794           Prob > F      =  0.0000
    Residual |    .1594234    22  .007246518           R-squared     =  0.9585
-------------+------------------------------           Adj R-squared =  0.9547
       Total |  3.83797927    24  .159915803           Root MSE      =  .08513

------------------------------------------------------------------------------
           c |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .6648524   .0872809     7.62   0.000     .4838429    .8458619
          st |   .9483866    .283467     3.35   0.003      .360512    1.536261
       _cons |   .2653599   .0937114     2.83   0.010     .0710144    .4597054
------------------------------------------------------------------------------

. predict test, resid
```

```
. regress test y st tr ft

      Source |       SS       df       MS              Number of obs =      25
-------------+------------------------------           F(  4,    20) =    0.64
       Model |  .018129935     4  .004532484           Prob > F      =  0.6391
    Residual |  .141293467    20  .007064673           R-squared     =  0.1137
-------------+------------------------------           Adj R-squared = -0.0635
       Total |  .159423401    24  .006642642           Root MSE      =  .08405

------------------------------------------------------------------------------
        test |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |  -.2666934   .1938749    -1.38   0.184    -.6711094    .1377226
          st |   .6802458   .5127575     1.33   0.200    -.3893476    1.749839
          tr |   7.506952   6.290407     1.19   0.247    -5.614608    20.62851
          ft |   .4450372   .5246166     0.85   0.406    -.6492939    1.539368
       _cons |   .0981764   .1152333     0.85   0.404    -.1421961     .338549
------------------------------------------------------------------------------

. scalar lm=25*0.1137
. display lm
2.8425

. display chi2tail(2,lm)
.24141206
```

Testing a reduction of the model with the LR-test

```
. regress c y st tr ft

      Source |       SS       df       MS              Number of obs =      25
-------------+------------------------------           F(  4,    20) =  130.82
       Model |  3.69668581     4  .924171451           Prob > F      =  0.0000
    Residual |  .141293465    20  .007064673           R-squared     =  0.9632
-------------+------------------------------           Adj R-squared =  0.9558
       Total |  3.83797927    24  .159915803           Root MSE      =  .08405

------------------------------------------------------------------------------
           c |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .3981591   .1938749     2.05   0.053    -.0062569     .802575
          st |   1.628632   .5127575     3.18   0.005      .559039    2.698226
          tr |   7.506952   6.290407     1.19   0.247    -5.614608    20.62851
          ft |   .4450372   .5246166     0.85   0.406    -.6492939    1.539368
       _cons |   .3635363   .1152333     3.15   0.005     .1231638    .6039088
------------------------------------------------------------------------------

. scalar llb=e(ll)
. display llb
29.223937

. display -25/2*(ln(2*_pi)+ln(0.141293465/25)+1)
29.223937

. regress c y st

      Source |       SS       df       MS              Number of obs =      25
-------------+------------------------------           F(  2,    22) =  253.82
       Model |  3.67855587     2  1.83927794           Prob > F      =  0.0000
    Residual |    .1594234    22  .007246518           R-squared     =  0.9585
-------------+------------------------------           Adj R-squared =  0.9547
       Total |  3.83797927    24  .159915803           Root MSE      =  .08513
```

```
------------------------------------------------------------------------------
           c |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .6648524   .0872809     7.62   0.000     .4838429    .8458619
          st |   .9483866    .283467     3.35   0.003      .360512    1.536261
       _cons |   .2653599   .0937114     2.83   0.010     .0710144    .4597054
------------------------------------------------------------------------------

. scalar llr=e(ll)
. display llr
27.714881

. display -25/2*(ln(2*_pi)+ln(0.1594234/25)+1)
27.714881

. display 2*(llb-llr)
3.0181129

. display chi2tail(2, 2*(llb-llr))
.22111852

. qui regress c y st tr ft
. estimates store mb

. qui regress c y st
. estimates store mr

. lrtest mr mb

Likelihood-ratio test                                 LR chi2(2)  =      3.02
(Assumption: mr nested in mb)                         Prob > chi2 =    0.2211
```

■