Autocorrelation

$Var-Cov(u) = \underline{W} \neq \sigma^2 \cdot I$ Slides 44–45.

Example, AR(1):

$$W = \frac{1}{1-\rho^2} \cdot \begin{bmatrix} \sigma^2 & \rho^1 & \cdots & \rho^{T-1} \\ \rho^1 & \sigma^2 & \cdots & \rho^{T-2} \\ \rho^2 & \rho^1 & \cdots & \rho^{T-3} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^{T-1} & \rho^{T-2} & \cdots & \sigma^2 \end{bmatrix}$$

ρ – coefficient of first-order autocorrelation

Slide 45:

No autocorrelation, $W = \sigma^2 \cdot I$:

$$Var-Cov(b) = (X^TX)^{-1}X^T\sigma^2 I\, X(X^TX)^{-1} =$$
$$= \sigma^2 \cdot (X^TX)^{-1} \underbrace{X^TX(X^TX)^{-1}}_{I}$$
$$= \sigma^2 \cdot (X^TX)^{-1}$$

Autocorrelation, $W \neq \sigma^2 \cdot I$:

$$Var-Cov(b) = (X^TX)^{-1}X^TWX(X^TX)^{-1}$$

"sandwich" estimator

Slides 45–46:

Least squares estimator:

(1.) Estimator of the regression coefficients β_j:

$$b = (X^T X)^{-1} X^T y$$ ✓

(2.) Estimator of the variance of stoch. var. u:

$$s_e^2 = \frac{RSS}{n-k}$$ ✗

(3.) Estimator of the variance-covariance matrix of regression coefficient estimates:

$$\text{var-cov}(b) = s_e^2 \cdot (X^T X)^{-1}$$ ✗

Slide 52:

Our assumption: $u \sim \cancel{IID}$

independence identical distribution

HAC: heteroscedasticity and autocorrelation consistent (estimator of variance).

Slides 53–55:

Example, variables:

- hm1: harmonized money aggregate M1;
- ppr: income of households;
- rvp: interest rate on demand deposits;
- rvv: interest rate on short-term deposits;
- czp: consumer price index.

Regression models with dummy explanatory variables

Dummy trap:

gender	D_1	D_2
M	1	0
F	0	1

Slide 2.

$D_{1i} + D_{2i} = 1, \forall i$

One can write: $D_{1i} = 1 - D_{2i}$ or $D_{2i} = 1 - D_{1i}$, resulting in perfect multicollinearity.

Types of regression models with dummy explanatory variables

(A)
- y – gross wage
- gender (male, female): $D = \begin{cases} 0; & \text{female} \\ 1; & \text{male} \end{cases}$

(B)
- y – gross wage
- education (elementary school, college, university):

$D_1 = \begin{cases} 0; & \text{other} \\ 1; & \text{college} \end{cases}$ $\quad D_2 = \begin{cases} 0; & \text{other} \\ 1; & \text{university} \end{cases}$

C
- y – savings
- gender (male, female):

$$D_1 = \begin{cases} \underline{0}; & \text{female} \\ 1; & \text{male} \end{cases}$$

- labour force participation (employed, unemployed):

$$D_2 = \begin{cases} \underline{0}; & \text{unemployed} \\ 1; & \text{employed} \end{cases}$$

D
- y – gross wage
- gender (male, female): $D = \begin{cases} \underline{0}; & \text{female} \\ 1; & \text{male} \end{cases}$
- x_3 – years of employment

E
- y – gross wage
- education (elementary school, college, university):

$$D_1 = \begin{cases} \underline{0}; & \text{other} \\ 1; & \text{college} \end{cases} \qquad D_2 = \begin{cases} \underline{0}; & \text{other} \\ 1; & \text{university} \end{cases}$$

- x_4 – years of employment

F
- y – savings
- gender (male, female):

$$D_1 = \begin{cases} 0; & \text{female} \\ 1; & \text{male} \end{cases}$$

- marital status (single, married):

$$D_2 = \begin{cases} 0; & \text{single} \\ 1; & \text{married} \end{cases}$$

- x_4 – years of employment