$\boxed{\text{Example 2}}$

Categorical variable OWNERSHIP $\in$ {private, state, mixed}
↳ 2 dummies:

REF./BASE CATEGORY $\boxed{\text{private}}$ ←

$$D_1 = \begin{cases} 1 ; & \text{state o.} \\ 0 ; & \text{else} \end{cases}$$

$$D_2 = \begin{cases} 1 ; & \text{mixed o.} \\ 0 ; & \text{else} \end{cases}$$

Regression model with dummy variables:

$$\widehat{\ln Q} = b_1 + b_2 \ln L + b_3 \ln K + b_4 D_1 + b_5 D_2 + $$
$$+ b_6 \underbrace{D_1 \ln L} + b_7 \underbrace{D_1 \ln K} + b_8 \underbrace{D_2 \ln L} + b_9 \underbrace{D_2 \ln K}$$

multiplicative dummy variables

If OWNERSHIP = private: $(D_1 = 0, D_2 = 0)$ (base category)

$$\widehat{\ln Q} = b_1 + b_2 \ln L + b_3 \ln K$$

↳ $b_1 = 4.73$ $\quad b_2 = 0.23$ $\quad b_3 = 0.55$

If OWNERSHIP = state: $(D_1 = 1, D_2 = 0)$

$$\widehat{\ln Q} = b_1 + b_2 \ln L + b_3 \ln K + b_4 + b_6 \ln L + b_7 \ln K$$

↳

$b_1 + b_4 = 3.63$
$b_4 = 3.63 - b_1 =$
$= -1.10$

$b_2 + b_6 = 0.53$
$b_6 = 0.53 - b_2 =$
$= 0.30$

$b_3 + b_7 = 0.48$
$b_7 = 0.48 - b_3 =$
$= -0.07$

If OWNERSHIP = mixed : $(D_1 = 0 , D_2 = 1)$

$$\widehat{\ln Q} = b_1 + b_2 \ln L + b_3 \ln K + b_5 + b_8 \ln L + b_9 \ln K$$

$\hookrightarrow$

$b_1 + b_5 = 3.24$	$b_2 + b_8 = 0.18$	$b_3 + b_9 = 0.68$
$b_5 = 3.24 - b_1 =$	$b_8 = 0.18 - b_2 =$	$b_9 = 0.68 - b_3 =$
$= -1.49$	$= -0.05$	$= 0.13$

Single regression model:

$$\widehat{\ln Q} = 4.73 + 0.23 \ln L + 0.55 \ln K - 1.10\, D_1 - 1.49\, D_2 + 0.30\, D_1 \ln L - 0.07\, D_1 \ln K - 0.05\, D_2 \ln L + 0.13\, D_2 \ln K$$

Example 3

categorical variable HHTYPE $\in$ {agri, non-agri, mixed}

$\hookrightarrow$ 2 dummies :

$$D_1 = \begin{cases} 1 & \text{mixed} \\ 0 & \text{else} \end{cases}$$

REF. / BASE CATEGORY $\leftarrow$

$\boxed{\text{agri}}$

$$D_2 = \begin{cases} 1 & \text{non-agri} \\ 0 & \text{else} \end{cases}$$

a) $\widehat{EXP} = b_1 + b_2\, INCOME$

$\underline{b_1 = 43.523}$: The average expenditure for food of a HH without income is on average 43.523 thousand m.u.

$\underline{b_2 = 0.175}$: If disposable income increases by 1000 monetary units the HH expenditure for food on average increases by 175 m.u.

$$\widehat{EXP} = b_1 + b_2 D_1 + b_3 D_2 + b_4 INCOME$$

$b_1 = 31.251$: the average expenditure for food of an agricultural household without income is on average 31.251 thousand m.u.

$b_2 = 4.317$: the expenditure for food of a mixed HH is at given income on average by 4.317 thousand m.u. higher compared to an agricultural HH.

$b_3 = 13.251$: the expenditure for food of a non-agricultural HH is at given income on average by 13.251 thousand m.u. higher compared to an agricultural HH.

$b_4 = 0.163$: If disposable income increases by 1 m.u. the expenditure for food for the same type of household on average increases by 0.163 m.u.

b) $H_0 : \beta_2 = \beta_3 = 0$

$H_1 : \beta_j \neq 0 \; ; \; \exists \; j = 2, 3$

$$F = \frac{(R_B^2 - R_R^2)/(k_B - k_R)}{(1 - R_B^2)/(n - k_B)} = \frac{(0.642 - 0.452)/(4-2)}{(1 - 0.642)/(100-4)} =$$

$$= 25.5$$

$F_c(m_1 = 2, m_2 = 96, \alpha = 0.05) = 3.09$

$F > F_c \Rightarrow$ reject H_0 at $\alpha = 0.05$ and conclude from $H_1 \Rightarrow$ HH type affects EXP

c) $H_0: \beta_2 \leq 0$
$H_1: \beta_2 > 0$

$$t = \frac{b_2 - \beta_{2,0}}{\sqrt{var(b_2)}} = \frac{4.317 - 0}{2.15} = 2.01$$

$t_c(m = n-k = 100-4 = 96, \alpha = 0.05) = 1.67$
$t > t_c \Rightarrow$ reject H_0 at $\alpha = 0.05$ and conclude from H_1 $\Rightarrow$ Yes, we can claim that.

d) To avoid falling into „dummy trap".

One (of m) category is always the base/reference.
We only need two (m-1) dummies to distinguish the other two (m-1) categories from the base one.

HHTYPE	D_1	D_2	~~D_3~~
BASE agri	0	0	~~1~~
mixed	1	0	~~0~~
non-agri	0	1	~~0~~

$D_1 + D_2 + D_3 = 1 \Rightarrow$ perfect multicollinearity!
$\Rightarrow$ not possible to estimate the model

$X^T X$ singular $\rightarrow$
$\det(X^T X) = 0 \rightarrow$
$(X^T X)^{-1}$ not defined $\rightarrow$
$b = (X^T X)^{-1} X^T y = ?!$

Tutorial, Part 9 Distributed-lag models

Example 2

a) The infinite distributed-lag model under Koyck assumption

① $y_t = \alpha + \beta_0 X_t + \cancel{\beta_0 \lambda X_{t-1}} + \cancel{\beta_0 \lambda^2 X_{t-2}} + \dots + u_t$

for previous time-unit, multiplied by λ:

② $\lambda y_{t-1} = \lambda\alpha + \cancel{\beta_0 \lambda X_{t-1}} + \cancel{\beta_0 \lambda^2 X_{t-2}} + \beta_0 \lambda^3 X_{t-3} + \dots + \lambda u_{t-1}$

Subtract ② from ①: (terms cancel out up to ∞)

$$\boxed{y_t = \alpha(1-\lambda) + \beta_0 X_t + \lambda y_{t-1}}$$

$$\hat{y}_t = g_1 + g_2 X_t + g_3 y_{t-1}$$

$$\underset{}{\hat{I}_t} = \underset{a(1-\hat{\lambda})}{2.45} + \underset{b_0}{0.70}\, GDP_t + \underset{\hat{\lambda}}{0.25}\, I_{t-1}$$

b) Koyck model

$$\hat{I}_t = a(1-\hat{\lambda}) + b_0 \, GDP_t + \hat{\lambda} I_{t-1}$$
$$\hat{I}_t = g_1 + g_2 \, GDP_t + g_3 I_{t-1}$$

Impact (short-term) multiplier

$b_0 = g_2 = 0.70$: If in a certain year GDP increases by 1 € the imports on average increase by 0.70 € in the same year.
(short-term marginal propensity to import)

Second intermediate multiplier

$b_2 = b_0 \cdot \hat{\lambda}^2 = g_2 \cdot g_3^2 = 0.70 \cdot 0.25^2 = 0.0438$:
If in a certain year GDP increases by 1 € the imports on average increase by 0.04 € after two years.

Equilibrium (total, long-term) multiplier

$$\sum_{k=0}^{\infty} b_k = b_0 \cdot \sum_{k=0}^{\infty} \hat{\lambda}^k = *$$

Sum of an infinite geometric series:

$$\sum_{k=0}^{\infty} \beta_k = \sum_{k=0}^{\infty} \beta_0 \lambda^k = \beta_0 \sum_{k=0}^{\infty} \lambda^k = \beta_0 \cdot \frac{1}{1-\lambda}$$

$$* = \frac{b_0}{1-\hat{\lambda}} = \frac{g_2}{1-g_3} = \frac{0.70}{1-0{,}25} = 0.93 \quad :$$

If in a certain year GDP increases by 1 €
the imports on average increase by 0,93 €
in the long term.
(long-term propensity to import)

c) Median lag

When 1/2 of the long-term effect of
X_j on Y is reached?

$$\sum_{k=0}^{t-1} \beta_k = \frac{1}{2} \sum_{k=0}^{\infty} \beta_k$$

Sum of a geometric series:

$$\sum_{k=0}^{t-1} \beta_k = \sum_{k=0}^{t-1} \beta_0 \lambda^k = \beta_0 \sum_{k=0}^{t-1} \lambda^k = \beta_0 \frac{1-\lambda^t}{1-\lambda}$$

$$\cancel{\beta_0} \frac{1-\lambda^t}{1-\lambda} = \frac{1}{2} \cancel{\beta_0} \frac{1}{1-\lambda} \qquad / \cdot (1-\lambda)$$

$$1-\lambda^t = \frac{1}{2}$$

$$\lambda^t = \frac{1}{2}$$

$$t \cdot \log \lambda = \log\left(\frac{1}{2}\right) = -\log 2$$

$$t = -\frac{\log 2}{\log \lambda}$$

Median lag: $-\frac{\log 2}{\log \hat{\lambda}} = -\frac{\log 2}{\log g_3} = -\frac{\log 2}{\log 0.25} = \frac{1}{2}$

Half of the long-term impact of GDP on imports is on average reached in half a year.

Example 3

$$\hat{M1}_t = d \cdot b_1 + d \cdot b_2 \, GDP_t + d \cdot b_3 \, IR_t + (1-d) M1_{t-1}$$

$$\hat{M1}_t = g_1 + g_2 \, GDP_t + g_3 \, IR_t + g_4 \, M1_{t-1}$$

(Stata)

$$\hat{M1}_t = 148.03 + 0.0567 \, GDP_t - 11.65 \, IR_t + 0.6255 \, M1_{t-1}$$

$g_4 = 1-d \Rightarrow d = 1 - g_4 = 0.3745$ estimate of the ADJUSTMENT COEFFICIENT

At a certain year 37.45% of the gap between the desired (long-term, $M1^*$) and the actual ($M1$) quantity of money in circulation is closed on average.

$b_1 = \frac{g_1}{d} = 395.25$ estimate of the intercept term of the long-term money demand function

$b_2 = \frac{g_2}{d} = 0.1513$ estimate of the long-term effect of GDP on amount of money

$b_3 = \frac{g_3}{d} = -31.10$ estimate of the long-term effect of IR on amount of money

Long-term money demand function:

$$\hat{M1}^*_t = 395.25 + 0.1513\ GDP_t - 31.10\ IR_t$$

Detection of / correction for AC of order 1

Breusch – Godfrey test

$\underset{p=1}{AR(1)}$: $u_t = \rho_1 u_{t-1} + \varepsilon_t$ $(\rightarrow \text{Stata: } \hat{\rho}_1 = +0.5563)$

$H_0: \rho_1 \leq 0$ (no positive AC of order 1)
$H_1: \rho_1 > 0$ (there is positive AC of order 1)

$\boxed{M_{AR}:}$ $\hat{e}_t = b_1 + b_2 GDP_t + b_3 IR_t + b_4 M1_{t-1} + \hat{\rho}_1 e_{t-1}$

$\hookrightarrow R^2_{AR} = 0.3358$

$LM = (T' - p) \cdot R^2_{AR} = 32 \cdot 0.3358 = 10.746$

$(T' = T - 1 = 34 - 1 = 33$ $\rightarrow$ 1 lost observation due to lag)
in the model for M1

$\chi^2_{c\,(m=p=1,\ \alpha=0.05)} = 3.84$

$LM > \chi^2_c$ $\Rightarrow$ reject H_0 at $\alpha = 0.05$ and conclude from H_1 $\Rightarrow$ there is AC of order 1

Testing up to order $(T' - k - 1 = 33 - 4 - 1 =)$ 28 shows that at $\alpha = 0.05$ $(\alpha = 0.10)$ there is AC of order up to 15 (17).

We employ HAC estimator.