Homoscedasticity Slide 21.

$Var(u_i|x_i) = \sigma^2$ (homoscedasticity)

vs.

$Var(u_i|x_i) = \sigma_i^2$ (heteroscedasticity)

Heteroscedasticity Slides 24-25.

$$Var-Cov(u) = \underline{W} = \begin{bmatrix} \sigma_1^2 & 0 & \dots & 0 \\ 0 & \sigma_2^2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \sigma_n^2 \end{bmatrix} \neq \sigma^2 \cdot I$$

$$b = (X^TX)^{-1}X^Ty = (X^TX)^{-1}X^T(X\beta + u) =$$
$$= \underbrace{(X^TX)^{-1}X^TX}_{I}\beta + (X^TX)^{-1}X^Tu =$$
$$= \beta + (X^TX)^{-1}X^Tu$$

$$\left.\begin{aligned} b-\beta &= (X^TX)^{-1}X^Tu \\ (b-\beta)^T &= u^TX\underbrace{(X^TX)^{-1}}_{\text{symmetry}} \end{aligned}\right\} \longrightarrow Var-Cov(b)$$

as $(ABC)^T = C^TB^TA^T$.

Slide 25:

Homoscedasticity, $W = \sigma^2 \cdot I$:

$$\begin{aligned}
Var-Cov(b) &= (X^T X)^{-1} X^T \sigma^2 I \, X (X^T X)^{-1} = \\
&= \sigma^2 \cdot (X^T X)^{-1} \underbrace{X^T X (X^T X)^{-1}}_{I} \\
&= \sigma^2 \cdot (X^T X)^{-1}
\end{aligned}$$

Heteroscedasticity, $W \neq \sigma^2 \cdot I$:

$$Var-Cov(b) = (X^T X)^{-1} X^T W X (X^T X)^{-1}$$

"sandwich" estimator

Slides 25–26:

Least squares estimator:

1. Estimator of the regression coefficients β_j:

$$b = (X^T X)^{-1} X^T y$$ ✓

2. Estimator of the variance of stoch. var. u:

$$s_e^2 = \frac{RSS}{n-k}$$ ✗

3. Estimator of the variance-covariance matrix of regression coefficient estimates:

$$\text{var-cov}(b) = s_e^2 \cdot (X^T X)^{-1}$$ ✗

We also use $e = \hat{u}$.

Slide 32:

Our assumption: $u \sim I\cancel{ID}$

independence identical distribution

Slides 33–34:

Example variables:

- hm1: harmonized money aggregate M1;
- ppr: income of households;
- rvp: interest rate on demand deposits;
- rvv: interest rate on short-term deposits;
- czp: consumer price index.

Autocorrelation

Is about covariance or correlation between stochastic variable u and lagged stochastic variable u, i.e.:

t	u_t	u_{t-1}
1	u_1	—
2	u_2	u_1
3	u_3	u_2
4	u_4	u_3
5	u_5	u_4

Slide 38.

It is a time-series related phenomenon, as it has to do with ordering of observations.