University of Ljubljana

SEB SCHOOL OF ECONOMICS AND BUSINESS

6. Distributed–lag Models

Prof. Dr. Miroslav Verbič

miroslav.verbic@ef.uni-lj.si

www.miroslav-verbic.si

EFMD EQUIS ACCREDITED

AACSB ACCREDITED

ASSOCIATION OF MBAS ACCREDITED

Ljubljana, October 2022

Basic definitions

- This topic concerns econometric analysis of **time series**.
- In practice, it takes time for a change in an explanatory variable, Δx_j, to reflect on the dependent variable, Δy.
- This required time is usually more than one time unit (hour, day, week, month, year, etc.). This is especially the case with *higher-frequency data* (such as daily or hourly).

Basic definitions

There are several **groups of reasons** why these effects are not immediate:

- **Psychological** reasons, such as *consumer behaviour* (e.g. increased **income** results in increased **consumption**, but the effects are distributed over time based on habits and financial limitations of individuals);
- **Technological** reasons, such as *technological processes* (e.g. increased **investment** results in enhanced **production**, but this happens only gradually due to technological limitations, such as installing equipment and training);
- **Institutional** reasons, such as *contracts* and *deadlines* (e.g. increased **interest rate** results in increased **deposits**, but this happens only gradually due to fixed deposits that cannot be converted without cost).

Basic definitions

- The fact that these effects are *not* immediate means that the usual static analysis ($x_{jt} \rightarrow y_t$) is *not* appropriate.
- We need to introduce **dynamic analysis** into our modelling framework, i.e. we need to take into account **time lags**.
- These are lagged values of **either** the explanatory variables x_j **or** of the dependent variable y as an explanatory variable (capturing the dynamic nature of all relevant x_j).
- Regression coefficients of those lagged variables capture the adjustment process of y with respect to x_j in time.
- This results in a *class of models* called in the literature the **distributed–lag models**.

Basic definitions

We distinguish between:

- **Finite (lag) distributed–lag models**, where the effect of Δx_{jt} on y_t completes after a certain finite number of time units.

  Also: *models of lagged explanatory variables*.

- **Infinite (lag) distributed–lag models**, where Δx_{jt} affects the current and all future values of y_t without time limitation.

  the effect never ceases to exist

  Also: *autoregression models*

  (analitical solutions of the model include among the explanatory variables also a lagged dependent variable, y_{t-1}).

  on the right handsight we have a lagged dependent variable

Finite (lag) distributed–lag models

The effect lasts for S time periods
↳ it is often difficult to determine S

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \ldots + \beta_s x_{t-s} + u_t$$

The more of these terms we have on the right handsight decrease degrees of freedom ⇒ each lag decreases it by 1.

Not clear in advance, how many lags to take into account.

Adding lags of an explanatory variable decreases degrees of freedom and increases multicollinearity.

We need to pay attention to changes in sign and large changes in values of regression coefficients.

Due to these reasons this model is not used very much.

Infinite (lag) distributed–lag models

The problem is that we don't have infinite data, so we are limited with amount of data.

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \beta_3 x_{t-3} + \ldots + u_t$$

t	β_0	Short-run (impact) multiplier
$t+1$	β_1	Intermediate (interim) multiplier
$t+2$ shifted	β_2	Intermediate (interim) multiplier
$t+3$	β_3	Intermediate (interim) multiplier
$\vdots$	$\vdots$	$\vdots$
Total	$\sum_{k=0}^{\infty} \beta_k$	Long-run (total) multiplier

Note: Subtotals $\beta_0 + \beta_1$, $\beta_0 + \beta_1 + \beta_2$, $\beta_0 + \beta_1 + \beta_2 + \beta_3$ etc. represent the cumulative multipliers for time periods $t+1$, $t+2$, $t+3$ etc.

Cumulative multipliers are partial sums. ($\beta_1 + \beta_2$ for example)

Infinite (lag) distributed–lag models

- There is an obvious issue with this kind of models.
- Namely, one cannot estimate an *infinite* number of parameters (regression coefficients) based on a *finite* sample of (time series) data.
- For analytical solution of the model, we have to assume a certain distribution of effects of x_j on y.
- This is usually done in literature in three ways, resulting in **three** infinite (lag) distributed–lag models: – they are used in practice
  - Koyck model;
  - Partial adjustment model;
  - Adaptive expectations model.

Koyck model

A Geometric scheme of distributed lags

Koyck assumption:

- The effects of all x_{t-k} have the same sign;
- The effects decrease in time at a geometric sequence.

$$\beta_k = \beta_0 \lambda^k \; ; \qquad k = 0, 1, 2, \ldots \; ; \qquad 0 < \lambda < 1.$$

↳ we are interested in λ between 0 and 1

λ – rate of decline (decay)

$(1-\lambda)$ – speed of adjustment

- $\lambda = 0$: no lagged effects over time;
- $\lambda = 1$: no decrease of effects over time.

University of Ljubljana
SEB SCHOOL OF ECONOMICS AND BUSINESS

Koyck model

knjiga 105 - 107

Long-run (total) effect:

$$\sum_{k=0}^{\infty} \beta_k = \sum_{k=0}^{\infty} \beta_0 \lambda^k = \beta_0 \left(1 + \lambda + \lambda^2 + \lambda^3 + \ldots\right) = \beta_0 \frac{1}{1-\lambda}$$

sum of total effect

$$\boxed{s = \frac{a_1}{1-q}}$$

EFMD EQUIS ACCREDITED
AACSB ACCREDITED
ASSOCIATION AMBA ACCREDITED

Koyck model

Koyck transformation of the model:

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \beta_3 x_{t-3} + \ldots + u_t$$

$$\Big\downarrow \; \beta_k = \beta_0 \lambda^k$$

⇒ we plug in the Koyck assumption

$$y_t = \alpha + \beta_0 x_t + \beta_0 \lambda x_{t-1} + \beta_0 \lambda^2 x_{t-2} + \beta_0 \lambda^3 x_{t-3} + \ldots + u_t \quad / \cdot \lambda \;\&\; \mathrm{L}.[\cdot]$$

multiply by lambda and lag it by 1

$$- \quad \lambda y_{t-1} = \lambda \alpha + \beta_0 \lambda x_{t-1} + \beta_0 \lambda^2 x_{t-2} + \beta_0 \lambda^3 x_{t-3} + \ldots + \lambda u_{t-1}$$

$$y_t - \lambda y_{t-1} = \alpha(1-\lambda) + \beta_0 x_t + (u_t - \lambda u_{t-1})$$

→ after doing the multiplication and lagging we subtract them.

Koyck model

This is the last topic in the handbook !!!

$$y_t = \alpha(1-\lambda) + \beta_0 x_t + \lambda y_{t-1} + \underbrace{(u_t - \lambda u_{t-1})}$$

$$y_t = \gamma_1 + \gamma_2 x_t + \gamma_3 y_{t-1} + v_t$$

it has the lag of y on right handsight

Autoregression model!

Koyck model

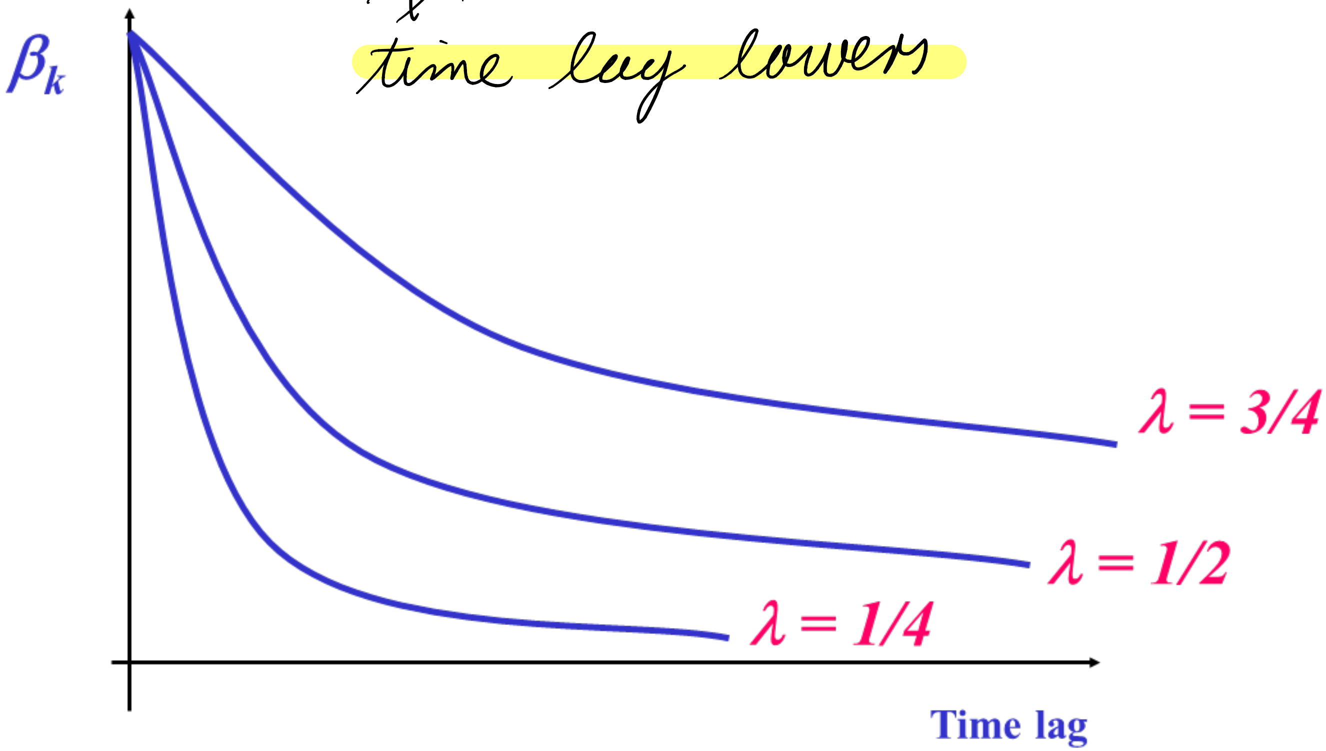
Median lag:

$$-\frac{\log 2}{\log \lambda} \Longleftrightarrow \frac{\ln 0.5}{\ln \lambda}$$

Median lag is the necessary time for the realization of half of the long-run effect of x_j on y.

Koyck model

if lambda decreases the time lag lowers



Partial adjustment model

B **Partial adjustment model (Stock adjustment model)**

We model long-run (equilibrium) level of y:

$$y_t^* = \beta_0 + \beta_1 x_t + u_t$$

that can be a problem

It is not directly observed, but we can assume a gradual adjustment process between the the short-run level y_t and the long-run level y_t^*:

$$\underbrace{y_t - y_{t-1}}_{\text{Actual change}} = \delta\underbrace{(y_t^* - y_{t-1})}_{\text{Desired change}}; \quad 0 < \delta < 1.$$

from one time period to another we are moving towards the equilibrium

Partial adjustment model

δ – coefficient of adjustment

- $\delta = 0$: no adjustment over time;
- $\delta = 1$: immediate full adjustment.

Transformation of the model starts by rewriting the adjustment process:

$$y_t = \delta y_t^* - \delta y_{t-1} + y_{t-1}$$

$$y_t = \delta y_t^* + (1-\delta) y_{t-1}$$

we plug in y^*
from the initial model

Partial adjustment model

Then we plug in $y_t^* = \beta_0 + \beta_1 x_t + u_t$:

$$y_t = \delta(\beta_0 + \beta_1 x_t + u_t) + (1-\delta) y_{t-1}$$

$$y_t = \delta\beta_0 + \underbrace{\delta\beta_1}x_t + \underbrace{(1-\delta)}y_{t-1} + \delta u_t$$

$$y_t = \gamma_1 + \gamma_2 x_t + \gamma_3 y_{t-1} + v_t$$

Autoregression model!

Adaptive expectations model

C Adaptive expectations model (Error learning model)

Now, the explanatory variable takes the expected long-run or equilibrium level x_t^*:

$$y_t = \beta_0 + \beta_1 x_t^* + u_t$$

similar issues as in previous model

x_t^* is not directly observed, but we can assume a gradual adjustment process of expectations about it:

$$\underbrace{x_t^* - x_{t-1}^*}_{\text{Current expectations – Past expectations}} = \gamma(x_t - x_{t-1}^*); \quad 0 < \gamma < 1.$$

Current expectations – Past expectations

in each time period you adapt your expectations (for example if you think a course is easy and you study 1 night before and get a 2, on the second attempt you study for a week just to pass the exam).

Adaptive expectations model

γ – coefficient of expectation

- $\gamma = 0$: expectations are static;
- $\gamma = 1$: expectations realize immediately in full.

The difference between current and past expectations is thus a share of the gap between past expectations and its realization.

Transformation of the model starts by rearranging the adjustment process of expectations:

$$x_t^* = \gamma x_t + (1-\gamma)x_{t-1}^*$$

Adaptive expectations model

Now we plug in current expectations $x_t^* = \gamma x_t + (1-\gamma)x_{t-1}^*$ into our initial model:

$$y_t = \beta_0 + \beta_1 x_t^* + u_t$$

$$y_t = \beta_0 + \beta_1 \gamma x_t + \beta_1 (1-\gamma) x_{t-1}^* + u_t$$

we plug it in there

x_{t-1}^* is still not observed, so we derive it from lagged initial model:

$$y_{t-1} = \beta_0 + \beta_1 x_{t-1}^* + u_{t-1} \quad \Rightarrow \quad x_{t-1}^* = \frac{y_{t-1} - \beta_0 - u_{t-1}}{\beta_1}$$

Adaptive expectations model

Finally, we plug the expression just derived for x^*_{t-1} into our previous expression for y_t:

$$y_t = \beta_0 + \beta_1 \gamma x_t + \beta_1 (1-\gamma) \frac{y_{t-1} - \beta_0 - u_{t-1}}{\beta_1} + u_t$$

$$y_t = \beta_0 + \beta_1 \gamma x_t + (1-\gamma) y_{t-1} - \beta_0 (1-\gamma) - (1-\gamma) u_{t-1} + u_t$$

$$y_t = \beta_0 \gamma + \beta_1 \gamma x_t + (1-\gamma) y_{t-1} + \left[u_t - (1-\gamma) u_{t-1} \right]$$

$$y_t = \xi_1 + \xi_2 x_t + \xi_3 y_{t-1} + v_t$$

Greek letter Xi

Autoregression model!

Infinite (lag) distributed–lag models

WARNINGS

all 3 models are the same

! **Technically, in all three models the same function is estimated (based on sample data, the same values of regression coefficients are obtained).**

! **Theoretical assumptions of the three models are completely different. The regression coefficients are interpreted based on the assumptions of the estimated model.**

! **It is highly likely that the models exhibit (at least) first-order autocorrelation (proper tests should be applied).**

! **By introducing a lagged dependent variable as a regressor we include a stochastic regressor – endogeneity (instrumental variables estimator should be applied).** – $e^2 = 0$ doesn't hold

Example: The file `consumption2.dta` contains data for the USA in the period 1960–1990, namely the values for private consumption (*C*) and disposable income (*Y*). Both variables are expressed in billion USD at prices from the year 1987.

a) Write down and explain the Koyck assumption and with its help derive an autoregressive model from the infinite distributed-lag model.
b) Estimate the autoregressive model from the previous point and, based on the estimates, write down the estimated distributed-lag model. Based on your calculations, explain the impact and long-term multiplier, as well as one intermediate and cumulative multiplier.
c) Check with an appropriate test whether there is first-order autocorrelation present in the analyzed autoregressive model.
d) Write down and explain the partial adjustment model. Estimate the derived autoregressive model and explain the results.
e) Write down and explain the adaptive expectations model. Estimate the derived autoregressive model and explain the results.

a)

Koyck model (geometric scheme of distributed lags)

When estimating the infinite distributed-lag model, we can help ourselves with a variety of assumptions about the distribution of the effects of the explanatory variable on the dependent variable. Koyck assumed that all the effects of the lagged explanatory variable have the same sign and are in time decreasing in a geometric sequence. This means that the values of the regression coefficients of the lagged explanatory variable are geometrically decreasing by moving into the past (i.e. by increasing the lag length). This assumption can be written as follows:

$$\beta_k = \beta_0 \lambda^k; \qquad k = 0, 1, 2, \ldots, \tag{1}$$

where the parameter λ, also referred to as the rate of decline (decay), occupies the values between 0 and 1. The values of the regression coefficients for lags are dependent on the regression coefficient β_0, which represents the effect of the explanatory variable on the dependent variable contemporaneously, and the rate of decline λ. The closer the value of λ is to one (zero), the slower (faster) is the decline of the effects of a lagged explanatory variable on the dependent variable.

Let us now write down the sum of all regression coefficients β_k:

$$\sum_{k=0}^{\infty} \beta_k = \sum_{k=0}^{\infty} \beta_0 \lambda^k = \beta_0 \left(1 + \lambda + \lambda^2 + \lambda^3 + \ldots\right). \tag{2}$$

The sum of the infinite geometric series (the expression in parenthesis) equals $1/(1 - \lambda)$, so that the sum of the regression coefficients, which represents the long-term multiplier in the infinite distributed-lag model, can be calculated by the following formula:

$$\boxed{\sum_{k=0}^{\infty} \beta_k = \beta_0 \frac{1}{1-\lambda}.} \tag{3}$$

Considering expression (1), we can write the infinite distributed-lag model as follows:

$$y_t = \alpha + \beta_0 x_t + \beta_0 \lambda x_{t-1} + \beta_0 \lambda^2 x_{t-2} + \ldots + u_t \,. \tag{4}$$

Estimating parameters of such a model with the least squares method is of course not possible, because the model is not linear in parameters, and additionally, infinitely many parameters would have to be estimated. Therefore, Koyck proposed a solution, the transformation of the model, which we present below.

We first lag model (4) for one time unit:

$$y_{t-1} = \alpha + \beta_0 x_{t-1} + \beta_0 \lambda x_{t-2} + \beta_0 \lambda^2 x_{t-3} + \ldots + u_{t-1} \,. \tag{5}$$

Then we multiply expression (5) with the rate of decline λ and subtract the obtained result from expression (4):

$$\begin{aligned} \lambda y_{t-1} &= \lambda\alpha + \beta_0 \lambda x_{t-1} + \beta_0 \lambda^2 x_{t-2} + \beta_0 \lambda^3 x_{t-3} + \ldots + \lambda u_{t-1}, \\ y_t - \lambda y_{t-1} &= \alpha(1-\lambda) + \beta_0 x_t + (u_t - \lambda u_{t-1}), \end{aligned} \tag{6}$$

or, by some rearrangement:

$$y_t = \alpha(1-\lambda) + \beta_0 x_t + \lambda y_{t-1} + (u_t - \lambda u_{t-1}) \,. \tag{7}$$

With the help of the Koyck transformation we changed the infinite distributed-lag model into an autoregressive model, where the lagged dependent variable acts as an explanatory variable.

Expression (7) can be generalised as:

$$y_t = \gamma_1 + \gamma_2 x_t + \gamma_3 y_{t-1} + v_t \,, \tag{8}$$

wherein the following holds:

$$\gamma_1 = \alpha(1-\lambda); \qquad \gamma_2 = \beta_0; \qquad \gamma_3 = \lambda; \qquad v_t = u_t - \lambda u_{t-1} \,. \tag{9}$$

We call the regression coefficient β_0 the short-term or impact multiplier, which tells us how much on average the dependent variable in the current unit of time changes, if the explanatory variable in the same unit of time increases by one unit of measurement. The expression $\beta_0 \frac{1}{1-\lambda}$ represents the long-term multiplier and tells us by how many units on average the dependent variable changes in the long run, due to the change of the explanatory variable by one unit of measurement.

If we compare the infinite distributed-lags model in general and the model (7), we can see two key simplifications, which occur due to the Koyck transformation:

– in the model (7) only three parameters need to be estimated (α, β in λ), while in the infinite distributed-lag model there are infinitely many regression coefficients,

– furthermore, the problem of multicollinearity is gone, because we replaced variables x_t x_{t-1} x_{t-2} ... with the lagged dependent variable.

In addition to these simplifications we need to take into account the following circumstances when estimating the autoregressive model derived based on the Koyck transformation:

– Given that the variable y_t and consequently the variable y_{t-1} are random variables, a random explanatory variable appears in model (7). As we know from the theory, one of the assumptions of the classical linear regression model requires the explanatory variable to be non-random, or at least independent of the random variable. Therefore, in the case of the autoregressive model it is necessary to determine whether that condition is met.

– In the autoregressive model the random variable v_t appears, defined as $u_t - \lambda u_{t-1}$, which means that its characteristics depend on the assumptions that hold for the random variable u_t. It turns out that the values of the random variable v_t are autocorellated, although the values of the initial random variable u_t are uncorrelated. This means that we have to pay particular attention to the problem of autocorrelation when estimating the autoregressive model.

■

Partial adjustment (stock adjustment) model

The autoregressive model obtained based on the Koyck transformation is a result of purely algebraic derivations, which means that it is not founded based on economic theory. We can eliminate this disadvantage by taking into account the so-called partial adjustment. We proceed from a model in which the desired (equilibrium) or long-term level of the dependent variable is the linear function of an explanatory variable. In this case we get:

$$C_t^* = \beta_0 + \beta_1 Y_t + u_t. \tag{10}$$

Long-term or equilibrium level of personal consumption, e.g., is therefore a linear function of disposable income. Since long-term values of personal consumption C^* are not directly known, we use the partial adjustment model:

$$\boxed{C_t - C_{t-1} = \delta(C_t^* - C_{t-1}), \qquad 0 < \delta \leq 1.} \tag{11}$$

The change in consumption in time unit t equals a share δ (called the adjustment coefficient) of the desired change in consumption during this time unit. If we insert expression (11) into expression (10), we get:

$$C_t - C_{t-1} = \delta(\beta_0 + \beta_1 Y_t + u - C_{t-1})\,; \tag{12}$$

which we can properly rearrange:

$$C_t = \delta\beta_0 + \delta\beta_1 Y_t + (1-\delta)C_{t-1} + \delta u_t\,. \tag{13}$$

In a general form we can write expression (13) as:

$$y_t = \gamma_1 + \gamma_2 x_t + \gamma_3 y_{t-1} + v_t\,, \tag{14}$$

wherein the following holds:

$$\gamma_1 = \delta\beta_0; \qquad \gamma_2 = \delta\beta_1; \qquad \gamma_3 = (1-\delta); \qquad v_t = \delta u_t\,. \tag{15}$$

Model (10) represents the long-term equlibrium consumption function, while the autoregressive model (14) represents the short-term consumption function. The regression coeffcicient β_1 represents the long-term marginal propensity to consume, whereas γ_2 represents the short-term marginal propensity to consume. From the expression (15) it follows that β_1 can be calculated by dividing the short-term marginal propensity to consume with the adjustment coefficient δ. The coefficient tells us what share of the gap between the desired and the actual consumption is eliminated in each time unit.

Adaptive expectations model (error learning model)

When using the adaptive expectations model we will stem from the model in which the dependent variable is a linear function of the expected long-term (equilibrium) level of the explanatory variable. In this case we could write:

$$C_t = \beta_0 + \beta_1 Y_t^* + u_t\,. \tag{16}$$

Personal consumption, e.g., is therefore a linear function of the expected long-term or equilibrium level of disposable income. As expected disposable income Y^* is not directly observed, we use the adaptive expectations model:

$$Y_t^* - Y_{t-1}^* = \gamma(Y_t - Y_{t-1}^*), \qquad 0 < \gamma \le 1. \tag{17}$$

Expectations regarding disposable income are in each unit of time "corrected" by a certain proportion (called the expectation coefficient) of the gap between the actual value of disposable income in the current time unit and its previous expected value.

Example continued (PDF)

d) Partial adjustment model
(explained in PDF, pp. 3–4):

$$\hat{C}_t = \underbrace{d \cdot b_0}_{} + \underbrace{d \cdot b_1}_{} \cdot Y_t + \underbrace{(1-d)}_{} C_{t-1} \qquad \text{(SRM)}$$

$$\hat{C}_t = g_1 + g_2 Y_t + g_3 C_{t-1}$$

We can write down the model (17) as:

$$Y_t^* = \gamma Y_t + (1-\gamma)Y_{t-1}^*, \tag{18}$$

which means that the expected value of disposable income in a given unit of time equals the weighted arithmetic average of the actual value of disposable income in this time unit and its expected value in the previous time unit, wherein γ and $1-\gamma$ are the respective weights.

In the event that γ equals 1, $Y_t^* = Y_t$ holds, which means that the expectations regarding the disposable income are realized promptly and completely. If we have the other extreme that $\gamma = 0$, $Y_t^* = Y_{t-1}^*$ holds, which means that expectations are static and will not change.

If we insert expression (18) into equation (16), we get:

$$C_t = \beta_0 + \beta_1(\gamma Y_t + (1-\gamma)Y_{t-1}^*) + u_t = \beta_0 + \beta_1\gamma Y_t + \beta_1(1-\gamma)Y_{t-1}^* + u_t\,. \tag{19}$$

We write model (16) in the previous time unit:

$$C_{t-1} = \beta_0 + \beta_1 Y_{t-1}^* + u_{t-1} \tag{20}$$

and express the expected value of disposable income Y_{t-1}^*:

$$Y_{t-1}^* = \frac{C_{t-1} - \beta_0 + u_{t-1}}{\beta_1}\,. \tag{21}$$

We insert it into equation (19) and with some derivation we get:

$$C_t = \gamma\beta_0 + \gamma\beta_1 Y_t + (1-\gamma)C_{t-1} + u_t - (1-\gamma)u_{t-1}\,. \tag{22}$$

In a general form we can write expression (22) as:

$$y_t = \xi_1 + \xi_2 x_t + \xi_3 y_{t-1} + v_t\,, \tag{23}$$

wherein the following holds:

$$\xi_1 = \gamma\beta_0; \qquad \xi_2 = \gamma\beta_1; \qquad \xi_3 = (1-\gamma); \qquad \xi_t = u_t - (1-\gamma)u_{t-1}\,. \tag{24}$$

Based on the theory of consumption we define the expected long-term disposable income as the permanent disposable income. Accordingly, the regression coefficient β_1 in model (16) represents the marginal propensity to consume out of disposable income, while the regression coefficient ξ_2 in the autoregressive model (23) represents the marginal propensity to consume out of current income. From expression (24) it follows that β_1 can be calculated by dividing the marginal propensity to consume out of current income by the expectation coefficient γ.

■

a) Done in PDF, pp. 1-3.

b) Koyck model:

$$\hat{C}_t = \underbrace{a(1-\hat{\lambda})}_{g_1} + \underbrace{b_0}_{g_2} Y_t + \underbrace{\hat{\lambda}}_{g_3} C_{t-1} \quad \text{(SRM)}$$

$$\hat{C}_t = g_1 + g_2 Y_t + g_3 C_{t-1}$$

$$\hat{C}_t = -17.898 + 0.4556 Y_t + 0.5211 C_{t-1}$$

PDF, p.6.

- $g_3 = \hat{\lambda} = 0.5211$ estimate of the rate of decline (decay)

The effect of disposable income on consumption in a given year represents on average 52.1% of the effect in the previous year.

12PIT

- $g_2 = b_0 = 0.4556$
  estimate of the short-run multiplier

- $g_1 = a(1-\hat{\lambda}) \Rightarrow a = \frac{g_1}{1-\hat{\lambda}} = \frac{-17.898}{1-0.5211} = -37.372$
  intercept of the infinite (lag) distributed-lag m.

Lagged effects of Y on C:

$b_k = b_0 \cdot \hat{\lambda}^k$ (Koyck assumption)

$b_1 = b_0 \cdot \hat{\lambda} = 0.2374$
$b_2 = b_0 \cdot \hat{\lambda}^2 = 0.1237$
$b_3 = b_0 \cdot \hat{\lambda}^3 = 0.0647$

etc.

Infinite (lag) distributed-lag model:

$$\hat{C}_t = \underset{a}{-37.372} + \underset{b_0}{0.4556} Y_t + \underset{b_1}{0.2374} Y_{t-1} + \underset{b_2}{0.1237} Y_{t-2} + \underset{b_3}{0.0647} Y_{t-3} + \ldots$$

Interpretation of multipliers: — 12PIT

• $b_0 = 0.4556$ estimate of the short-run multiplier (propensity to consume)

If disposable income in a given year increases by 1 USD, then on average, consumption in the same year increases by 0.4556 USD, ceteris paribus.

• $b_2 = 0.1237$ estimate of an intermediate multiplier (one of many)

If disposable income in a given year increases by 1 USD, then on average, consumption after two years increases by 0.1237 USD.

- $b_0 + b_1 + b_2 = 0.8167$

estimate of a cumulative multiplier (one of many)

If disposable income in a given year increases by 1 USD, then on average, consumption in the same year and the next two years cumulatively increases by 0.8167 USD.

- $\sum_{k=0}^{\infty} b_k = \frac{b_0}{1-\hat{\lambda}} = \frac{0.4556}{1-0.5211} = 0.9513$

estimate of the long-run multiplier (propensity to consume)

If disposable income in a given year increases by 1 USD, then on average, consumption in the long run increases by 0.9513 USD.

- $Me = -\frac{\log 2}{\log \hat{\lambda}} = -\frac{\log 2}{\log 0.5211} = 1.06$ estimate of the median lag

Half of the long-run effect of disposable income on consumption realizes on average in 1.06 years.

Example continued (PDF)

d) Partial adjustment model
(explained in PDF, pp. 3–4):

$$\hat{C}_t = \underbrace{d \cdot b_0}_{g_1} + \underbrace{d \cdot b_1}_{g_2} \cdot Y_t + \underbrace{(1-d)}_{g_3} C_{t-1} \qquad \text{(SRM)}$$

$$\hat{C}_t = g_1 + g_2 Y_t + g_3 C_{t-1}$$

Short-run (consumption) function:

$$\hat{C}_t = -17.898 + 0.4556 Y_t + 0.5211 C_{t-1}$$

PDF, p.6.

- $g_2 = 0.4556$
  estimate of the short-run effect
  (propensity to consume)

- $g_3 = 1-d \Rightarrow d = 1 - g_3 = 1 - 0.5211 = 0.4789$
  estimate of the coefficient of adjustment

In a given year, 47.9% of the gap between long-run and actual consumption was eliminated on average.

- $g_2 = d \cdot b_1 \Rightarrow b_1 = \frac{g_2}{d} = 0.9513$
  estimate of the long-run effect
  (propensity to consume)

• $g_1 = d \cdot b_0 \Rightarrow b_0 = \frac{g_1}{d} = -37.372$
intercept of the <u>long-run</u> (consumption) f.

<u>Long-run</u> (consumption) function:

$$\widehat{C_t^*} = b_0 + b_1 Y_t$$
$$\widehat{C_t^*} = -37.372 + 0.9513\, Y_t$$

e) Adaptive expectations model
(explained in **PDF, pp. 4-5**).

Permanent income hypothesis (F. Modigliani):

$$\hat{C}_t = \underbrace{c \cdot b_0}_{} + \underbrace{c \cdot b_1}_{} Y_t + \underbrace{(1-c)}_{} C_{t-1} \qquad \text{(SRM)}$$
$$\hat{C}_t = g_1 + g_2 Y_t + g_3 C_{t-1}$$

Autoregression function
(consumption function of <u>current</u> income):

$$\hat{C}_t = -17.898 + 0.4556 Y_t + 0.5211 C_{t-1}$$

PDF, p.6.

• $g_2 = 0.4556$
estimate of the marginal propensity to consume <u>current</u> income

- $g_3 = 1 - c \Rightarrow c = 1 - g_3 = 1 - 0.5211 = 0.4789$
  estimate of the coefficient of expectation

  Expectations of disposable income in a given year relative to the past year represent on average 47.9% of the gap between the expectations of disposable income in the past year and its realization.

- $g_2 = c \cdot b_1 \Rightarrow b_1 = \frac{g_2}{c} = 0.9513$
  estimate of the marginal propensity to consume permanent income

- $g_1 = c \cdot b_0 \Rightarrow b_0 = \frac{g_1}{c} = -37.372$
  intercept of the adaptive expectation f. (consumption function of permanent income)

Adaptive expectation function (consumption function of permanent income):

$$C_t = b_0 + b_1 Y_t^*$$
$$C_t = -37.372 + 0.9513 Y_t^*$$

Computer printout of the results in Stata:

```
. tsset year
        time variable:  year, 1960 to 1990
                delta:  1 unit

. regress c y

      Source |       SS       df       MS              Number of obs =      31
-------------+------------------------------           F(  1,    29) = 8830.35
       Model |  10704439.9     1  10704439.9           Prob > F      =  0.0000
    Residual |  35154.7449    29  1212.23258           R-squared     =  0.9967
-------------+------------------------------           Adj R-squared =  0.9966
       Total |  10739594.7    30  357986.489           Root MSE      =  34.817

------------------------------------------------------------------------------
           c |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .9224173   .0098161    93.97   0.000     .9023412    .9424935
       _cons |  -39.55193   24.45265    -1.62   0.117    -89.56321    10.45935
------------------------------------------------------------------------------

. regress c y l.c

      Source |       SS       df       MS              Number of obs =      30
-------------+------------------------------           F(  2,    27) = 5761.50
       Model |  9871307.09     2  4935653.55           Prob > F      =  0.0000
    Residual |  23129.8458    27  856.660955           R-squared     =  0.9977
-------------+------------------------------           Adj R-squared =  0.9975
       Total |  9894436.94    29  341187.481           Root MSE      =  29.269

------------------------------------------------------------------------------
           c |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .4555839   .1359322     3.35   0.002      .176674    .7344938
             |
           c |
         L1. |   .5210732   .1502645     3.47   0.002     .2127558    .8293905
             |
       _cons |  -17.89849   23.59184    -0.76   0.455    -66.30495    30.50797
------------------------------------------------------------------------------

. predict ec, resid
(1 missing value generated)                                 30-1

. regress ec l.ec, nocons

      Source |       SS       df       MS              Number of obs =      29
-------------+------------------------------           F(  1,    28) =   12.12
       Model |  6969.43374     1  6969.43374           Prob > F      =  0.0017
    Residual |  16098.1117    28  574.932559           R-squared     =  0.3021
-------------+------------------------------           Adj R-squared =  0.2772
       Total |  23067.5454    29   795.4326            Root MSE      =  23.978

------------------------------------------------------------------------------
          ec |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          ec |
         L1. |   .5491917    .157737     3.48   0.002      .226082    .8723014
------------------------------------------------------------------------------

. scalar rho=_b[l.ec]
. display rho
.54919173
```

Short-run (consumption) function:

$$\hat{C}_t = -17.898 + 0.4556 Y_t + 0.5211 C_{t-1}$$

PDF, p.6.

- $g_2 = 0.4556$
  estimate of the short-run effect (propensity to consume)

- $g_3 = 1 - d \Rightarrow d = 1 - g_3 = 1 - 0.5211 = 0.4789$
  estimate of the coefficient of adjustment

In a given year, 47.9% of the gap between long-run and actual consumption was eliminated on average.

- $g_2 = d \cdot b_1 \Rightarrow b_1 = \frac{g_2}{d} = 0.9513$
  estimate of the long-run effect (propensity to consume)

- $g_1 = d \cdot b_0 \Rightarrow b_0 = \frac{g_1}{d} = -37.372$
  intercept of the long-run (consumption) f.

Long-run (consumption) function:

$$\widehat{C_t^*} = b_0 + b_1 Y_t$$

$$\widehat{C_t^*} = -37.372 + 0.9513 Y_t$$

Autoregression function
(consumption function of current income):

$$\hat{C}_t = -17.898 + 0.4556Y_t + 0.5211C_{t-1}$$

PDF, p.6.

- $g_2 = 0.4556$
  estimate of the marginal propensity to consume current income

- $g_3 = 1 - c \Rightarrow c = 1 - g_3 = 1 - 0.5211 = 0.4789$
  estimate of the coefficient of expectation

Expectations of disposable income in a given year relative to the past year represent on average 47.9% of the gap between the expectations of disposable income in the past year and its realization.

- $g_2 = c \cdot b_1 \Rightarrow b_1 = \frac{g_2}{c} = 0.9513$
  estimate of the marginal propensity to consume permanent income

- $g_1 = c \cdot b_0 \Rightarrow b_0 = \frac{g_1}{c} = -37.372$
  intercept of the adaptive expectation f.
  (consumption function of permanent income)

Adaptive expectation function
(consumption function of permanent income):

$$C_t = b_0 + b_1 Y_t^*$$
$$C_t = -37.372 + 0.9513 Y_t^*$$

```
. regress ec y l.c l.ec

      Source |       SS       df       MS              Number of obs =      29
-------------+------------------------------           F(  3,    25) =    4.17
       Model |  7694.22168     3  2564.74056           Prob > F      =  0.0159
    Residual |  15371.1754    25  614.847017           R-squared     =  0.3336
-------------+------------------------------           Adj R-squared =  0.2536
       Total |  23065.3971    28  823.764182           Root MSE      =  24.796

------------------------------------------------------------------------------
          ec |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           y |   .1350957   .1243967     1.09   0.288     -.121104    .3912955
             |
           c |
         L1. |  -.1487069   .1369076    -1.09   0.288    -.4306733    .1332596
             |
          ec |
         L1. |   .6015586   .1701725     3.53   0.002     .2510817    .9520355
             |
       _cons |  -11.54498    21.9328    -0.53   0.603    -56.71643    33.62646
------------------------------------------------------------------------------

. scalar lm=e(N)*e(r2)

. display lm, invchi2tail(1, 0.05), chi2tail(1, lm)
9.6739036 3.8414588 .00186904

. qui regress c y l.c
. estat bgodfrey, nomiss0

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |          9.674               1                   0.0019
---------------------------------------------------------------------------
                        H0: no serial correlation

. estat bgodfrey

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |          9.797               1                   0.0017
---------------------------------------------------------------------------
                        H0: no serial correlation
```

■

c) Due to the presence of C_{t-1} as a regressor, we employ the Breusch–Godfrey test for first-order autocorrelation.

$AR(1): u_t = \rho_1 u_{t-1} + \varepsilon_t$

$H_0: \rho_1 = 0$ (no first-order AC)
$H_1: \rho_1 \neq 0$ (first-order AC)

$\hat{e}_t = \hat{\rho}_1 e_{t-1}$
$\hat{e}_t = \underbrace{0.5492}_{\hat{\rho}_1 > 0} e_{t-1}$ PDF, p.6.

$H_0: \rho_1 \leq 0$ (no positive first-order AC)
$H_1: \rho_1 > 0$ (positive first-order AC)

PDF, p.7:

$M_{AR}: \hat{e}_t = g_1 + g_2 Y_t + g_3 C_{t-1} + \hat{\rho}_1 e_{t-1}$
$R^2_{AR} = 0.3336$

initial sample size already reduced in SRM due to C_{t-1}

$LM = (T' - 1) \cdot R^2_{AR} = 29 \cdot 0.3336 = \underline{\underline{9.674}}$
$\chi^2_c (m=1, \alpha=0.05) = 3.84$

$LM > \chi^2_c$, we reject H_0 at $\alpha = 0.05$ and conclude from H_1.

We have positive first-order AC present in our distributed-lag model. $\Rightarrow$ we have an issue

University of Ljubljana

SEB SCHOOL OF ECONOMICS AND BUSINESS

6. Distributed–lag Models

Prof. Dr. Miroslav Verbič

miroslav.verbic@ef.uni-lj.si

www.miroslav-verbic.si

EFMD EQUIS ACCREDITED

AACSB ACCREDITED

ASSOCIATION OF MBAS ACCREDITED

Ljubljana, October 2022