

# Vector autoregression

## Cholesky decomposition

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decomposes a symmetric positive definite matrix  $A$  into the product of a lower triangular matrix  $L$  and its transpose:

$$A = L \cdot L^T$$

### Example:

- $A$  is symmetric:  $A = A^T$
- $A$  is positive definite:  
 $x^T A x > 0$  for any non-zero vector  $x$

$$A = \begin{bmatrix} 4 & 12 & -16 \\ 12 & 37 & -43 \\ -16 & -43 & 98 \end{bmatrix}$$

Cholesky decomposition produces:

$$L = \begin{bmatrix} 2 & 0 & 0 \\ 6 & 1 & 0 \\ -8 & 5 & 3 \end{bmatrix} \quad \text{and} \quad L^T = \begin{bmatrix} 2 & 6 & -8 \\ 0 & 1 & 5 \\ 0 & 0 & 3 \end{bmatrix}$$

## Short-run SVAR model

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For  $g=3$ :

- **at most**  $3 \cdot 4 / 2 = 6$  parameters are estimated in matrices A and B combined;
- **at least**  $2 \cdot 3^2 - 6 = 12$  restrictions are needed.

## Long-run SVAR model

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For  $g=3$ :

- **at most**  $3 \cdot 4 / 2 = 6$  parameters are estimated in matrix  $\pi C$ .
- **at least**  $3^2 - 6 = 3$  restrictions are needed.

## Example 2

Ad g)

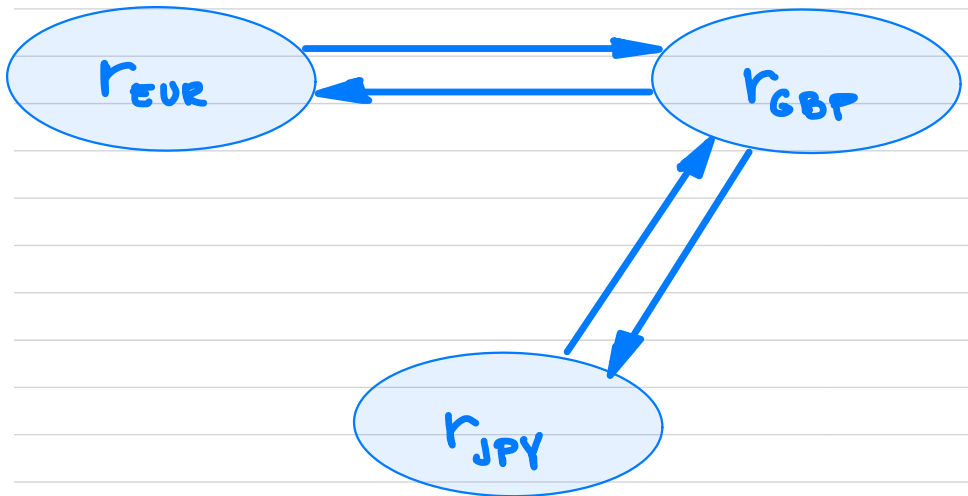
PDF, pp. 18-19.

Granger causality Wald test:

$H_0$ : "excluded" variable does **not** Granger cause the "equation" variable

$H_1$ : "excluded" variable Granger causes the "equation" variable

Scheme of Granger causality test results:



Consequently, the pound-dollar rate ( $r_{GBP}$ ) reacts the fastest.