Example 1: Derive the lest squares estimator formula for estimation of regression coefficients in case of the following regression function:

$$y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + u_i .$$

It is a trivariate regression model, i.e. a model with three explanatory variables, which contains the constant term ($x_{1i} = 1, \forall i$).

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Example 2: We have data available for variables y, x_2 and x_3. We observe each variable eight times, as shown in the table below.

i	1	2	3	4	5	6	7	8
y_i	2	2	1	5	–4	1	4	1
x_{2i}	1	2	0	–1	1	–1	–2	0
x_{3i}	–1	–1	2	–4	3	0	2	–1

Estimate the partial regression coefficients for this particular example, if the regression function is of the following form:

$$y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + u_i .$$

Perform the calculation with the use of matrix algebra; first manually, and then also in Stata software (the programming code is given in Stata Do file `estimation-commands.do`).

Computer printout of the results in Stata:

```
. matrix y=(2\2\1\5\-4\1\4\1)
. matrix list y

y[8,1]
    c1
r1   2
r2   2
r3   1
r4   5
r5  -4
r6   1
r7   4
r8   1

. matrix X=(1,1,-1\1,2,-1\1,0,2\1,-1,-4\1,1,3\1,-1,0\1,-2,2\1,0,-1)
. matrix list X

X[8,3]
    c1  c2  c3
r1   1   1  -1
r2   1   2  -1
r3   1   0   2
r4   1  -1  -4
r5   1   1   3
r6   1  -1   0
r7   1  -2   2
r8   1   0  -1
```

```
. matrix XX=(X)'*X
. matrix list XX

symmetric XX[3,3]
    c1  c2  c3
c1   8
c2   0  12
c3   0   0  36

. matrix Xy=(X)'*y
. matrix list Xy

Xy[3,1]
     c1
c1   12
c2  -12
c3  -27

. matrix XXinv=invsym(XX)
. matrix list XXinv

symmetric XXinv[3,3]
           c1         c2         c3
c1       .125
c2          0  .08333333
c3          0          0  .02777778

. matrix b=XXinv*Xy
. matrix list b

b[3,1]
      c1
c1   1.5
c2    -1
c3  -.75
```

■