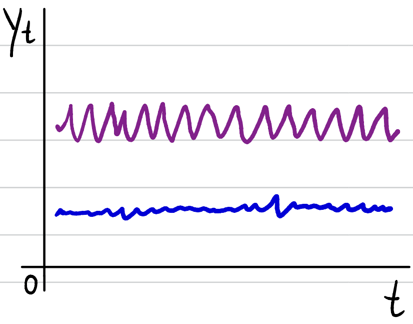
Stationarity

Slides 4–5:

An "ideal" situation:



Slide 12:

When $\rho = 1$ or $\gamma = 0$, we get from the Dickey–Fuller (DF) equation:

$$\Delta y_t = 0 \cdot y_{t-1} + v_t$$
$$\Delta y_t = v_t$$
$$y_t - y_{t-1} = v_t$$
$$y_t = y_{t-1} + v_t$$

random walk (inherently non-stationary)

Slide 18:

Approximation of growth rate of y_t:

$$\Delta \ln y_t = \ln y_t - \ln y_{t-1}$$

Approximation of percentage changes in y_t:

$$100 \cdot \Delta \ln y_t = 100 \cdot (\ln y_t - \ln y_{t-1})$$

Example 1

Ad a) & b) PDF, pp. 2–4.

When $|\hat{\tau}| > |\tau_c|$ or $p < \alpha$, we reject the unit root (non-stationarity).

Ad d) PDF, p. 5.

Continuously compounded percentage returns:

$$r_y = 100 \cdot \Delta \ln y_t$$
$$r_y = 100 \cdot (\ln y_t - \ln y_{t-1})$$
$$r_y = 100 \cdot \ln\left(\frac{y_t}{y_{t-1}}\right)$$

ARMA (Box–Jenkins) models

Slide 23:

We distinguish among 3 types of processes:

1) AR processes;
2) MA processes;
3) ARMA processes.

AR(p) process

Slide 23

$$y_t = \mu + \sum \phi_i y_{t-i} + u_t$$
$$y_t = \mu + \sum \phi_i L^i y_t + u_t$$
$$y_t - \sum \phi_i L^i y_t = \mu + u_t$$
$$y_t - \phi_1 L y_t - \phi_2 L^2 y_t - \dots - \phi_p L^p y_t = \mu + u_t$$
$$y_t \underbrace{(1 - \phi_1 L - \phi_2 L^2 - \dots - \phi_p L^p)}_{\phi(L)} = \mu + u_t$$

$$\phi(L) y_t = \mu + u_t$$

EXAMPLE 1:

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(random walk process)

$$y_t = y_{t-1} + u_t$$
$$y_t - y_{t-1} = u_t$$
$$y_t - L y_t = u_t$$
$$y_t (1 - L) = u_t$$

Characteristic equation:

$$1 - z = 0$$
$$z = 1 \quad \times$$

Non-stationary (for stationarity, z should be above 1).

EXAMPLE 2:

$$y_t = 3y_{t-1} - 2.75\,y_{t-2} + 0.75\,y_{t-3} + u_t$$
$$y_t - 3y_{t-1} + 2.75\,y_{t-2} - 0.75\,y_{t-3} = u_t$$
$$y_t - 3Ly_t + 2.75L^2 y_t - 0.75L^3 y_t = u_t$$
$$y_t\left(\underline{1 - 3L + 2.75L^2 - 0.75L^3}\right) = u_t$$

Characteristic equation:

$$1 - 3z + 2.75z^2 - 0.75z^3 = 0$$
$$(1-z)(1-1.5z)(1-0.5z) = 0$$

$z_1 = \underline{1}$ ✗
$z_2 = \underline{\tfrac{2}{3}}$ ✗
$z_3 = \underline{2}$ ✓

Non-stationary
(for stationarity, all z_j should be above 1).

MA(q) process — Slide 26

$$y_t = \mu + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \dots + \theta_q u_{t-q}$$
$$y_t = \mu + u_t + \theta_1 L u_t + \theta_2 L^2 u_t + \dots + \theta_q L^q u_t$$
$$y_t = \mu + u_t\,(\underbrace{1 + \theta_1 L + \theta_2 L^2 + \dots + \theta_q L^q}_{\theta(L)})$$

$$y_t = \mu + \theta(L)\,u_t$$