Assumptions of the LS estimator

Non-linearity: Slide 23

a) In variables:

$$y_i = \beta_1 + \beta_2 \underline{x_{2i}^2} + u_i$$

$$\Big\downarrow \text{linearization}$$

$$x_{2i}^2 = x_{2i}'$$

✓

b) In parameters:

$$y_i = \beta_1 + \underbrace{\beta_2^2}_{\text{unknown!}} x_{2i} + u_i$$

✗

Fixed explanatory variables:

$$E(X) = X$$
$$E[(X^T X)^{-1} X^T] = (X^T X)^{-1} X^T$$

etc. Slide 24

Heteroscedasticity: Slide 27

$$Var(u_i) = \sigma_i^2$$

Autocorrelation: Slide 29

Is about covariance or correlation between stochastic variable u and lagged stochastic variable u, i.e.:

t	$u_{\overset{i}{t}}$	$u_{\overset{j}{t-1}}$
1	u_1	–
2	u_2	u_1
3	u_3	u_2
4	u_4	u_3
5	u_5	u_4

Assumption 8, example: Slide 33

$y_i = b_1 + b_2 x_{2i} + e_i$, $\text{var}(x_2) = 0$

Then $x_{2i} = \bar{x}_2, \forall i$:

$$b_2 = \frac{\sum_i (x_{2i} - \bar{x}_2)(y_i - \bar{y})}{\sum_i \underbrace{(x_{2i} - \bar{x}_2)^2}_{=0}}$$ not defined

and

$$b_1 = \bar{y} - \underbrace{b_2}_{\text{not defined}} \bar{x}_2$$ not defined.

Assumption 10: Slide 37

Consequences of perfect (multi)collinearity:

1) $\det(X^T X) = 0$;
2) matrix $X^T X$ is singular and thus non-invertible;
3) $b = (X^T X)^{-1} X^T y$ is not defined.

Sample properties of the LS estimator

Unbiasedness: Slide 43

$$b = (X^TX)^{-1}X^Ty = (X^TX)^{-1}X^T(X\beta+u)$$

$$b = \underbrace{(X^TX)^{-1}X^TX}_{I}\beta + (X^TX)^{-1}X^Tu$$

$$b = \beta + (X^TX)^{-1}X^Tu$$

$$E(b) = E(\beta) + E[(X^TX)^{-1}X^Tu]$$

$$E(b) = \beta + (X^TX)^{-1}X^T\underbrace{E(u)}_{=0} = \underline{\beta}$$

fixed regressors

Efficiency: Slide 44

$$b-\beta = (X^TX)^{-1}X^Tu$$

$$(b-\beta)^T = u^TX(X^TX)^{-1}$$

symmetry

$$\text{Var-Cov}(b) = E[(b-\beta)(b-\beta)^T] =$$

$$= E[(X^TX)^{-1}X^Tu \cdot u^TX(X^TX)^{-1}] =$$

$$= (X^TX)^{-1}X^T\underbrace{E(uu^T)}_{\sigma^2\cdot I}X(X^TX)^{-1} =$$

fixed regressors

$$= \sigma^2\cdot(X^TX)^{-1}\underbrace{X^TX(X^TX)^{-1}}_{=I} =$$

$$= \underline{\underline{\sigma^2\cdot(X^TX)^{-1}}}$$

Slide 46:

Assume a trivariate regression model:

$$y_i = b_1 + b_2 x_{2i} + b_3 x_{3i} + e_i \qquad (SRM)$$

Slide 47:

BLUE or MVLUE

"best" (most efficient) "minimum variance"