**Example:** We analyse production functions for 81 manufacturing companies in the computer manufacturing industry for a given year (the data are provided in Stata Data file `production.dta`, while the programming code is given in Stata Do file `production-commands-l05.do`). We have cross-section data available for the following variables:

- value added as a proxy for the product ($Q$; in 1,000 monetary units);
- average number of employed workers as a proxy for labour ($L$);
- sum of tangible and intangible assets as a proxy for capital ($K$; in 1,000 monetary units).

a) Explore the data using different Stata commands. By using the scatter plots, examine the relationships of the linear and the Cobb-Douglas production function.
b) Estimate the linear and the Cobb-Douglas production function by the least squares estimator. Interpret the results of both models. Also, calculate the regression coefficients from the computer printouts manually.
c) Study direct, indirect and total effects of explanatory variables on the dependent variable for both production functions. What do the regression coefficients $b_j$ represent?
d) Study the validity of the Frisch-Waugh-Lovell theorem on the case of explanatory variable capital in the model of linear production function.

***Computer printout of the results in Stata:***

*a) Data exploration*

```
. describe

Contains data from production.dta
  obs:            81
 vars:             4
 size:           810
-------------------------------------------------------------------------------
              storage  display     value
variable name   type   format      label      variable label
-------------------------------------------------------------------------------
n               byte   %8.0g                  Observation
q               long   %8.0g                  Value added
l               byte   %8.0g                  Average number of employed workers
k               long   %8.0g                  Tangible and intangible assets
-------------------------------------------------------------------------------
Sorted by:

. inspect q l k

q:  Value added in 1000 monetary units          Number of Observations
-------------------------------------      ------------------------------
                                           Total   Integers   Nonintegers
|  #                          Negative         -          -          -
|  #                          Zero             -          -          -
|  #                          Positive        81         81          -
|  #                                         -----      -----       -----
|  #                          Total           81         81          -
|  #   .   .   .   .          Missing          -
+----------------------      -----
1514            2312943                      81
  (81 unique values)
```

```
l:  Average number of employed workers          Number of Observations
------------------------------------------      ------------------------------
                                                Total   Integers   Nonintegers
|  #                          Negative              -          -           -
|  #                          Zero                  -          -           -
|  #                          Positive             81         81           -
|  #                                              -----      -----       -----
|  #                          Total                81         81           -
|  #   .   .   .   .          Missing               -
+----------------------                         -----
1                     58                          81
  (22 unique values)

k:  Tangible and intangible assets in 1,000     Number of Observations
------------------------------------------- ------------------------------
                                                Total   Integers   Nonintegers
|  #                          Negative              -          -           -
|  #                          Zero                  -          -           -
|  #                          Positive             81         81           -
|  #                                              -----      -----       -----
|  #                          Total                81         81           -
|  #   .   .   .   .          Missing               -
+----------------------                         -----
38                344297                        81
  (81 unique values)
```

**. sum**

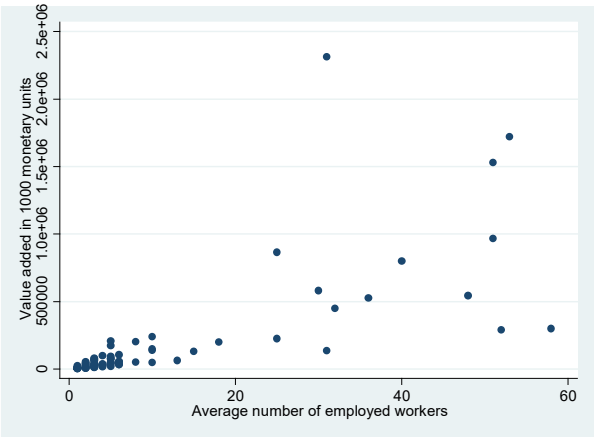
```
    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
           n |        81          41    23.52658          1         81
           q |        81    180519.3    388072.4       1514    2312943
           l |        81    10.39506    14.81442          1         58
           k |        81    40226.93    71409.35         38     344297
```

**. list q l k**

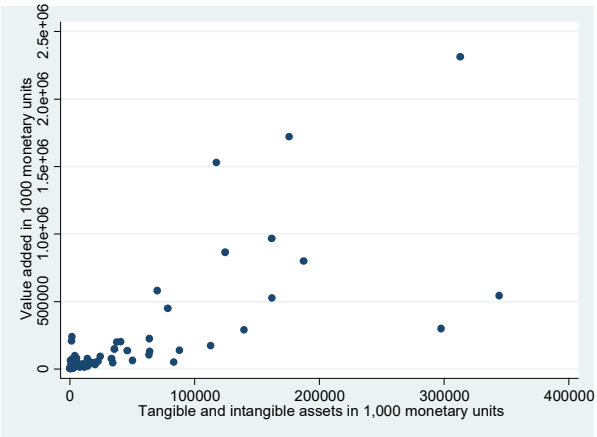
```
     +-----------------------+
     |       q    l        k |
     |-----------------------|
  1. |    1514    1       43 |
  2. |    2106    1      266 |
  3. |    2758    1      639 |
  4. |    4117    1       38 |
  5. |    4243    1      656 |
     |-----------------------|
  6. |    4553    1     1597 |
  7. |    4649    1      287 |
  8. |    4963    1      560 |
  9. |    5749    1      962 |
 10. |    5807    2     2499 |
     |-----------------------|
 11. |    6267    1      418 |
 12. |    6353    2      855 |

...

 74. |  544607   48   344297 |
 75. |  580309   30    70084 |
     |-----------------------|
 76. |  798583   40   187301 |
 77. |  864224   25   124547 |
 78. |  965788   51   161713 |
 79. | 1.5e+06   51   117635 |
 80. | 1.7e+06   53   175943 |
     |-----------------------|
 81. | 2.3e+06   31   313144 |
     +-----------------------+
```

```
. sort q

. gen lq=log(q)
. gen ll=log(l)
. gen lk=log(k)
```

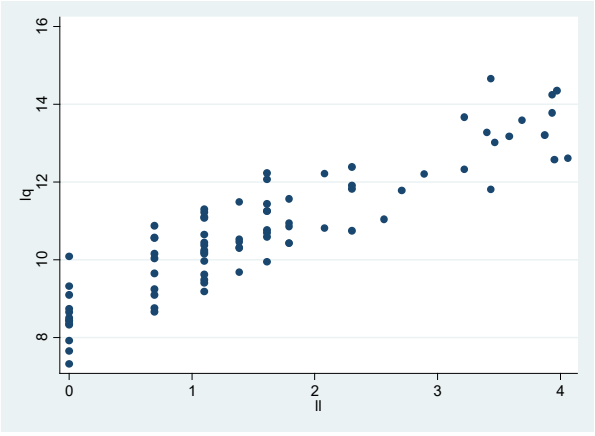
```
. twoway scatter q l
```



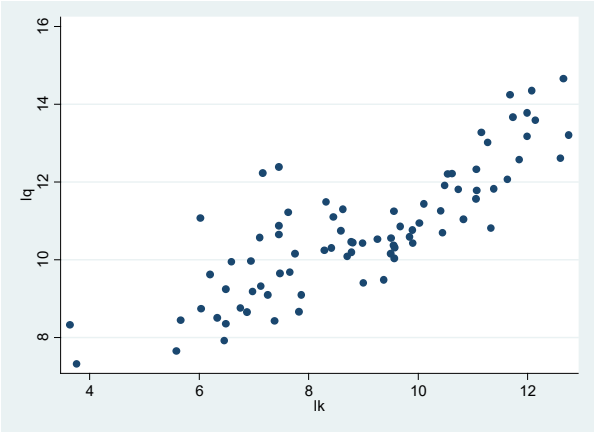

```
. twoway scatter q k
```




```
. twoway scatter lq ll
```




```
. twoway scatter lq lk
```




*b) Estimation of production functions*

```
. regress q l k
```

```
      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  2,    78) =   52.90
       Model |  6.9350e+12     2  3.4675e+12           Prob > F      =  0.0000
    Residual |  5.1130e+12    78  6.5551e+10           R-squared     =  0.5756
-------------+------------------------------           Adj R-squared =  0.5647
       Total |  1.2048e+13    80  1.5060e+11           Root MSE      =  2.6e+05

------------------------------------------------------------------------------
           q |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           l |   9687.383   3640.852     2.66   0.009     2439.003    16935.76
           k |    2.27941   .7553228     3.02   0.003      .775678    3.783142
       _cons |  -11875.29   34865.13    -0.34   0.734    -81286.43    57535.85
------------------------------------------------------------------------------
```

```
. gen cons=1
```

```
. mkmat cons l k, matrix(xlin)
. matrix list xlin

xlin[81,3]
       cons       l       k
 r1       1       1      43
 r2       1       1     266

...

r81       1      31  313144

. matrix xxlin=(xlin)'*xlin
. matrix list xxlin

symmetric xxlin[3,3]
           cons          l          k
cons         81
   l        842      26310
   k    3258381  1.056e+08  5.390e+11

. mkmat q, matrix(ylin)
. matrix list ylin

ylin[81,1]
           q
 r1     1514
 r2     2106

...

r81  2312943

. matrix xylin=(xlin)'*ylin
. matrix list xylin

xylin[3,1]
              q
cons   14622065
   l  4.856e+08
   k  2.213e+12

. matrix xxlininv=invsym(xxlin)
. matrix list xxlininv

symmetric xxlininv[3,3]
            cons           l           k
cons   .01854393
   l  -.00067176   .00020222
   k   1.951e-08  -3.556e-08   8.703e-12

. matrix blin=xxlininv*xylin
. matrix list blin

blin[3,1]
              q
cons  -11875.29
   l  9687.3835
   k  2.2794101
```

```
. regress lq ll lk

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  2,    78) =  190.75
       Model |  178.261263     2  89.1306313           Prob > F      =  0.0000
    Residual |    36.44752    78  .467275898           R-squared     =  0.8302
-------------+------------------------------           Adj R-squared =  0.8259
       Total |  214.708783    80  2.68385978           Root MSE      =  .68358

------------------------------------------------------------------------------
          lq |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
          ll |   .9645479   .1199229     8.04   0.000     .7257997    1.203296
          lk |   .1885438   .0673358     2.80   0.006     .0544886     .322599
       _cons |   7.546026   .4617465    16.34   0.000      6.62676    8.465293
------------------------------------------------------------------------------

. mkmat cons ll lk, matrix(xlog)
. matrix list xlog

xlog[81,3]
          cons         ll         lk
 r1          1          0  3.7612002
 r2          1          0  5.5834961

...

r81          1  3.4339871  12.654418

. matrix xxlog=(xlog)'*xlog
. matrix list xxlog

symmetric xxlog[3,3]
           cons         ll         lk
cons         81
  ll  127.86665  314.02997
  lk  725.93231  1314.3448  6861.7146

. mkmat lq, matrix(ylog)
. matrix list ylog

ylog[81,1]
            lq
 r1  7.3225102
 r2  7.6525459

...

r81  14.654032

. matrix xylog=(xlog)'*ylog
. matrix list xylog

xylog[3,1]
            lq
cons  871.43169
  ll  1515.5936
  lk  8039.3867

. matrix xxloginv=invsym(xxlog)
. matrix list xxloginv

symmetric xxloginv[3,3]
            cons          ll          lk
cons   .45628254
  ll   .08194953   .03077734
  lk  -.06396946  -.01456514   .00970328
```

```
. matrix blog=xxloginv*xylog
. matrix list blog

blog[3,1]
            lq
cons  7.5460264
  ll  .96454793
  lk  .18854381
```

*c) Direct, indirect and total effects*

```
. regress l k

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  1,    79) =  201.49
       Model |  12612.2633     1  12612.2633           Prob > F      =  0.0000
    Residual |  4945.09471    79  62.5961355           R-squared     =  0.7183
-------------+------------------------------           Adj R-squared =  0.7148
       Total |   17557.358    80  219.466975           Root MSE      =  7.9118

------------------------------------------------------------------------------
           l |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           k |   .0001758   .0000124    14.19   0.000     .0001512    .0002005
       _cons |   3.321905   1.010492     3.29   0.002     1.310571    5.333239
------------------------------------------------------------------------------

. predict resl, res

. regress q resl k

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  2,    78) =   52.90
       Model |  6.9350e+12     2  3.4675e+12           Prob > F      =  0.0000
    Residual |  5.1130e+12    78  6.5551e+10           R-squared     =  0.5756
-------------+------------------------------           Adj R-squared =  0.5647
       Total |  1.2048e+13    80  1.5060e+11           Root MSE      =  2.6e+05

------------------------------------------------------------------------------
           q |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
        resl |   9687.383   3640.852     2.66   0.009     2439.003    16935.76
           k |   3.982756   .4008578     9.94   0.000      3.18471    4.780803
       _cons |   20305.28   32700.13     0.62   0.536    -44795.69    85406.25
------------------------------------------------------------------------------

. regress q l k

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  2,    78) =   52.90
       Model |  6.9350e+12     2  3.4675e+12           Prob > F      =  0.0000
    Residual |  5.1130e+12    78  6.5551e+10           R-squared     =  0.5756
-------------+------------------------------           Adj R-squared =  0.5647
       Total |  1.2048e+13    80  1.5060e+11           Root MSE      =  2.6e+05

------------------------------------------------------------------------------
           q |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           l |   9687.383   3640.852     2.66   0.009     2439.003    16935.76
           k |    2.27941   .7553228     3.02   0.003      .775678    3.783142
       _cons |  -11875.29   34865.13    -0.34   0.734    -81286.43    57535.85
------------------------------------------------------------------------------
```

```
. regress k l

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  1,    79) =  201.49
       Model |  2.9304e+11     1  2.9304e+11           Prob > F      =  0.0000
    Residual |  1.1490e+11    79  1.4544e+09           R-squared     =  0.7183
-------------+------------------------------           Adj R-squared =  0.7148
       Total |  4.0794e+11    80  5.0993e+09           Root MSE      =   38137

------------------------------------------------------------------------------
           k |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           l |   4085.427   287.8158    14.19   0.000     3512.544     4658.31
       _cons |  -2241.338   5187.195    -0.43   0.667    -12566.19    8083.516
------------------------------------------------------------------------------

. predict resk, res

. regress q l resk

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  2,    78) =   52.90
       Model |  6.9350e+12     2  3.4675e+12           Prob > F      =  0.0000
    Residual |  5.1130e+12    78  6.5551e+10           R-squared     =  0.5756
-------------+------------------------------           Adj R-squared =  0.5647
       Total |  1.2048e+13    80  1.5060e+11           Root MSE      =  2.6e+05

------------------------------------------------------------------------------
           q |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           l |   18999.75   1932.239     9.83   0.000     15152.95    22846.54
        resk |    2.27941   .7553228     3.02   0.003     .7756781    3.783142
       _cons |  -16984.22      34824    -0.49   0.627    -86313.49    52345.05
------------------------------------------------------------------------------

. regress q l k

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  2,    78) =   52.90
       Model |  6.9350e+12     2  3.4675e+12           Prob > F      =  0.0000
    Residual |  5.1130e+12    78  6.5551e+10           R-squared     =  0.5756
-------------+------------------------------           Adj R-squared =  0.5647
       Total |  1.2048e+13    80  1.5060e+11           Root MSE      =  2.6e+05

------------------------------------------------------------------------------
           q |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           l |   9687.383   3640.852     2.66   0.009     2439.003    16935.76
           k |    2.27941   .7553228     3.02   0.003      .775678    3.783142
       _cons |  -11875.29   34865.13    -0.34   0.734    -81286.43    57535.85
------------------------------------------------------------------------------
```

*d) Frisch-Waugh-Lovell theorem*

```
. regress q l k

      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  2,    78) =   52.90
       Model |  6.9350e+12     2  3.4675e+12           Prob > F      =  0.0000
    Residual |  5.1130e+12    78  6.5551e+10           R-squared     =  0.5756
-------------+------------------------------           Adj R-squared =  0.5647
       Total |  1.2048e+13    80  1.5060e+11           Root MSE      =  2.6e+05
```

```
------------------------------------------------------------------------------
           q |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           l |   9687.383   3640.852     2.66   0.009     2439.003    16935.76
           k |    2.27941   .7553228     3.02   0.003      .775678    3.783142
       _cons |  -11875.29   34865.13    -0.34   0.734    -81286.43    57535.85
------------------------------------------------------------------------------
```

**. predict resid1, res**

**. regress q l**

```
      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  1,    79) =   87.69
       Model |  6.3380e+12     1  6.3380e+12           Prob > F      =  0.0000
    Residual |  5.7100e+12    79  7.2278e+10           R-squared     =  0.5261
-------------+------------------------------           Adj R-squared =  0.5201
       Total |  1.2048e+13    80  1.5060e+11           Root MSE      =  2.7e+05

------------------------------------------------------------------------------
           q |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           l |   18999.75   2028.962     9.36   0.000      14961.2     23038.3
       _cons |  -16984.22   36567.22    -0.46   0.644    -89769.45    55801.01
------------------------------------------------------------------------------
```

**. predict resqfw, res**

**. regress k l**

```
      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  1,    79) =  201.49
       Model |  2.9304e+11     1  2.9304e+11           Prob > F      =  0.0000
    Residual |  1.1490e+11    79  1.4544e+09           R-squared     =  0.7183
-------------+------------------------------           Adj R-squared =  0.7148
       Total |  4.0794e+11    80  5.0993e+09           Root MSE      =   38137

------------------------------------------------------------------------------
           k |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
           l |   4085.427   287.8158    14.19   0.000     3512.544     4658.31
       _cons |  -2241.338   5187.195    -0.43   0.667    -12566.19    8083.516
------------------------------------------------------------------------------
```

**. predict reskfw, res**

**. regress resqfw reskfw**

```
      Source |       SS       df       MS              Number of obs =      81
-------------+------------------------------           F(  1,    79) =    9.22
       Model |  5.9698e+11     1  5.9698e+11           Prob > F      =  0.0032
    Residual |  5.1130e+12    79  6.4721e+10           R-squared     =  0.1046
-------------+------------------------------           Adj R-squared =  0.0932
       Total |  5.7100e+12    80  7.1375e+10           Root MSE      =  2.5e+05

------------------------------------------------------------------------------
      resqfw |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      reskfw |    2.27941    .750527     3.04   0.003     .7855237    3.773297
       _cons |   .0005777   28267.12     0.00   1.000     -56264.3     56264.3
------------------------------------------------------------------------------
```

**. predict resid2, res**

```
. list resid1 resid2
```

| | resid1 | resid2 |
|---|---|---|
| 1. | 3603.892 | 3603.892 |
| 2. | 3687.584 | 3687.583 |
| 3. | 3489.364 | 3489.364 |
| 4. | 6218.29 | 6218.289 |
| 5. | 4935.614 | 4935.614 |
| 6. | 3100.689 | 3100.688 |
| 7. | 6182.716 | 6182.716 |
| 8. | 5874.438 | 5874.437 |
| 9. | 5744.114 | 5744.114 |
| 10. | -7388.722 | -7388.722 |
| 11. | 7502.114 | 7502.113 |
| 12. | -3095.372 | -3095.372 |
| 13. | -4511.34 | -4511.34 |
| 14. | 7905.542 | 7905.541 |
| 15. | -9865.388 | -9865.387 |
| 16. | 1406.51 | 1406.51 |
| 17. | 10504.76 | 10504.76 |
| 18. | -23468.89 | -23468.89 |
| 19. | -30861.5 | -30861.5 |
| 20. | -3090.771 | -3090.77 |
| 21. | 4126.923 | 4126.924 |
| 22. | -15702.03 | -15702.03 |

...

| | | |
|---|---|---|
| 73. | -180972.1 | -180972.1 |
| 74. | -693306.2 | -693306.2 |
| 75. | 141812.6 | 141812.6 |
| 76. | -3972.844 | -3972.85 |
| 77. | 350021 | 350021 |
| 78. | 114996.5 | 114996.5 |
| 79. | 778072.3 | 778072.3 |
| 80. | 816731.7 | 816731.7 |
| 81. | 1310726 | 1310726 |
| Mean | -.0010632 | -.0011936 |

■