Quantitative Methods in Finance

Tutorial, Part 4:
Direct, indirect and total effects of an explanatory variable on the dependent variable. Frisch-Waugh-Lovell theorem. Interpretation of the regression coefficients.

Example 1: By applying the method of least squares, the following two regression functions were estimated:

$$\widehat{C} = a_1 + 0.92DI \quad \text{and} \quad \widehat{CPI} = c_1 + 0.78DI \ ,$$

where C represents consumption, CPI is the consumer price index, and DI represents disposable income. Calculate the value of regression coefficient b_2 from the following regression function:

$$\widehat{C} = b_1 + b_2DI + b_3CPI$$
$$t: \quad (4.64) \ (1.95)$$

where $\text{var}(b_3) = 0.09$.

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Example 2: We were analysing how percentage of the population above age 65 (*POP65*) and consumption of cigarettes per capita (*TOBACCO*; number of consumed cigarette packets) affect mortality (*MRT*; number of deaths per 100,000 inhabitants). The sample contains 35 countries (the data are provided in Stata Data file `tobacco.dta`, while the programming code is given in Stata Do file `tobacco-commands.do`) and serves as a basis for econometric estimation of different regression functions, as shown below.

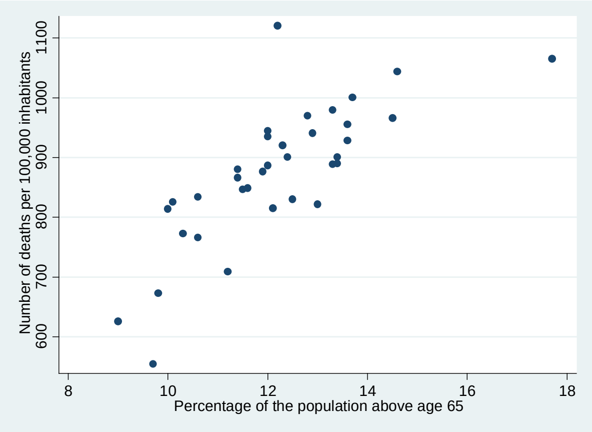
Write down the estimated regression models and interpret the results. What is the meaning of partial regression coefficients in these models?

Computer printout of the results in Stata:

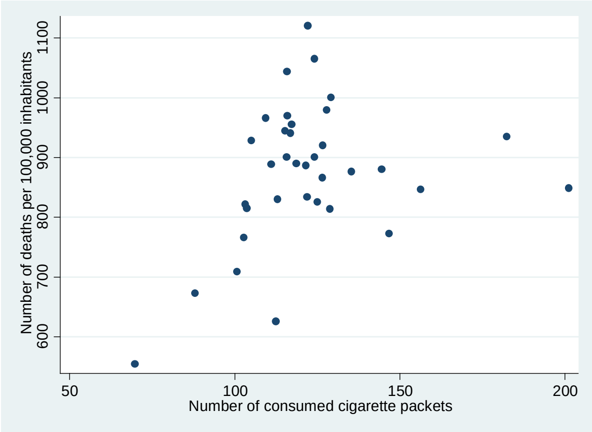
```
. sum mrt pop65 tobacco

    Variable |       Obs        Mean    Std. Dev.       Min        Max
-------------+--------------------------------------------------------
         mrt |        35      874.12    117.4456      554.2     1120.5
       pop65 |        35    12.18286    1.697487          9       17.7
     tobacco |        35    122.2543    23.59086       69.8      201.1
```

```
. scatter mrt pop65
```




```
. scatter mrt tobacco
```




```
. gen lmrt=log(mrt)
. gen lpop65=log(pop65)
. gen ltobacco=log(tobacco)

. regress mrt pop65 tobacco

      Source |       SS       df       MS              Number of obs =      35
-------------+------------------------------           F(  2,    32) =   33.26
       Model |  316665.524     2  158332.762           Prob > F      =  0.0000
    Residual |  152312.816    32   4759.7755           R-squared     =  0.6752
-------------+------------------------------           Adj R-squared =  0.6549
       Total |   468978.34    34  13793.4806           Root MSE      =  68.991

------------------------------------------------------------------------------
         mrt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       pop65 |   52.79328   6.971745     7.57   0.000      38.5923    66.99426
     tobacco |   1.441107    .501654     2.87   0.007     .4192715    2.462943
       _cons |   54.76546   104.3763     0.52   0.603    -157.8422    267.3731
------------------------------------------------------------------------------

. regress lmrt lpop65 ltobacco

      Source |       SS       df       MS              Number of obs =      35
-------------+------------------------------           F(  2,    32) =   47.67
       Model |  .526056623     2  .263028311           Prob > F      =  0.0000
    Residual |  .176566817    32  .005517713           R-squared     =  0.7487
-------------+------------------------------           Adj R-squared =  0.7330
       Total |   .70262344    34  .020665395           Root MSE      =  .07428

------------------------------------------------------------------------------
        lmrt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      lpop65 |   .7856827   .0935545     8.40   0.000     .5951183    .9762471
    ltobacco |   .2834557   .0692842     4.09   0.000     .1423285     .424583
       _cons |   3.449101   .3855704     8.95   0.000     2.663719    4.234482
------------------------------------------------------------------------------

. regress lmrt pop65 tobacco

      Source |       SS       df       MS              Number of obs =      35
-------------+------------------------------           F(  2,    32) =   33.90
       Model |  .477335891     2  .238667945           Prob > F      =  0.0000
    Residual |  .225287549    32  .007040236           R-squared     =  0.6794
-------------+------------------------------           Adj R-squared =  0.6593
       Total |   .70262344    34  .020665395           Root MSE      =  .08391
```

```
------------------------------------------------------------------------------
        lmrt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       pop65 |   .0634508   .0084789     7.48   0.000     .0461798    .0807219
     tobacco |   .0020003   .0006101     3.28   0.003     .0007576    .0032431
       _cons |   5.746105   .1269412    45.27   0.000     5.487535    6.004676
------------------------------------------------------------------------------
```

```
. regress mrt lpop65 ltobacco

      Source |       SS       df       MS              Number of obs =      35
-------------+------------------------------           F(  2,    32) =   43.04
       Model |  341892.883     2  170946.441           Prob > F      =  0.0000
    Residual |  127085.457    32  3971.42053           R-squared     =  0.7290
-------------+------------------------------           Adj R-squared =  0.7121
       Total |   468978.34    34  13793.4806           Root MSE      =  63.019

------------------------------------------------------------------------------
         mrt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      lpop65 |   652.2562   79.37034     8.22   0.000     490.5841    813.9283
    ltobacco |   202.1874   58.77972     3.44   0.002     82.45706    321.9178
       _cons |  -1718.884   327.1125    -5.25   0.000     -2385.19   -1052.577
------------------------------------------------------------------------------
```

```
. regress lmrt pop65 ltobacco

      Source |       SS       df       MS              Number of obs =      35
-------------+------------------------------           F(  2,    32) =   40.79
       Model |  .504679945     2  .252339972           Prob > F      =  0.0000
    Residual |  .197943495    32  .006185734           R-squared     =  0.7183
-------------+------------------------------           Adj R-squared =  0.7007
       Total |   .70262344    34  .020665395           Root MSE      =  .07865

------------------------------------------------------------------------------
        lmrt |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       pop65 |   .0614618   .0079709     7.71   0.000     .0452256     .077698
    ltobacco |   .2987248   .0731981     4.08   0.000     .1496251    .4478244
       _cons |   4.584218   .3565487    12.86   0.000     3.857952    5.310484
------------------------------------------------------------------------------
```

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Example 3: Based on data for 100 companies from the industry of high-tech electronic appliances, we analyse how are sales (*SALES*; in mil. monetary units) related to advertising costs (*ADV*; in 1,000 monetary units), research and development costs (*R&D*; in 1,000 monetary units), and percentage of employees with higher education (*EDUC*).

We estimated the following two regression functions, where the variable ADV^2 represents the squared value of advertising costs:

$$\widehat{SALES} = 12.33 + 0.46ADV - 0.002ADV^2 + 1.203\ln R\&D + 0.004EDUC;$$
$$\widehat{\ln SALES} = 1.78 + 0.125ADV - 0.005ADV^2 + 1.122\ln R\&D + 0.008EDUC.$$

a) Explain, based on the first regression function, the effect on sales of a one-unit change in advertising costs, if research and development costs and percentage of employees with higher education remain unchanged. What is the change in sales, if the advertising costs increase from 4,000 to 5,000 monetary units, while the variables *R&D* and *EDUC* remain unchanged?

b) Interpret the estimates of partial regression coefficients $b_4 = 1.122$ and $b_5 = 0.008$ in the second regression function. What do these estimates represent in the model?

c) Comment on the following statement: “The above regression models are nonlinear in parameters, thus it was not possible to estimate them by using the method of least squares.”

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