Model estimation

Example: Production functions PDF a,b

Theory: $Q = f(L, K)$

product, labour, capital

Data:

- Q – value added in 1.000 m.u.
- L – average number of employees
- K – sum of tangible and intangible assets in 1.000 m.u.

Model 1: Linear production function

$$Q_i = \underbrace{\beta_1 + \beta_2 L_i + \beta_3 K_i}_{E(Q_i|L_i,K_i)} + u_i \qquad \text{(PRM)}$$

$$Q_i = \underbrace{b_1 + b_2 L_i + b_3 K_i}_{\hat{Q}_i} + e_i \qquad \text{(SRM)}$$

Model 2: Cobb-Douglas production function

$$Q_i = \beta_1 \cdot L_i^{\beta_2} \cdot K_i^{\beta_3} \cdot u_i \quad / \ln \qquad \text{(PRM)}$$

$\downarrow$ linearization

$$\ln Q_i = \ln \left[\beta_1 \cdot L_i^{\beta_2} \cdot K_i^{\beta_3} \cdot u_i \right]$$

$$\ln Q_i = \ln \beta_1 + \ln L_i^{\beta_2} + \ln K_i^{\beta_3} + \ln u_i$$

$$\underbrace{\ln Q_i}_{LQ_i} = \underbrace{\ln \beta_1}_{\beta_1'} + \beta_2 \underbrace{\ln L_i}_{LL_i} + \beta_3 \underbrace{\ln K_i}_{LK_i} + \underbrace{\ln u_i}_{u_i'}$$

$$LQ_i = \underbrace{b_1 + b_2 LL_i + b_3 LK_i}_{\widehat{LQ}_i} + e_i \qquad \text{(SRM)}$$

Linearized Cobb-Douglas production function.
Regression coefficients now represent elasticities.

Stata: PDF, pp. 1–3.

- data exploration;
- logarithmic transformation;
- scatter plots.

Model 1

Estimation by regress and interpretation

1) Model statistics: PDF, pp. 3–4.

n

F, $p(F)$ } Statistical sign. of the model as a whole

R^2

$\bar{R}^2$

Se

2) ANOVA table:

mean squares: SS / df

ESS df: $k-1$ EMS

RSS df: $n-k$ RMS $= S_e^2$

TSS df: $n-1$ TMS

3) Parameter estimates:

b_j, $se(b_j)$, $\underbrace{t_j, \; p_j}$, 95% conf. interval

$H_0: \beta_j = 0$

$H_1: \beta_j \neq 0$

"e+05" means "$\cdot 10^5$", "e-05" means "$\cdot 10^{-5}$".

• $b_2 = 9687.4$

$p(b_2) = 0.009 < \alpha = 0.05$, we reject $H_0: \beta_2 = 0$ and conclude that $\beta_2 \neq 0$.

The value is non-random / stat. significant.

Interpretation:

Regression coefficient

If x_j increases by 1 unit of measurement, then on average, ceteris paribus, y increases / decreases by b_j units of measurement.

If the average number of employees increases by 1 employee, then on average, ceteris paribus, value added increases (+) by 9.7 mill. m.u. (it's in 1000 m.u.)

• $b_3 = 2.279$

$p(b_3) = 0.003 < \alpha = 0.05$

If the sum of tangible and intangible assets increases by 1000 m.u., then on average, ceteris paribus, value added increases (+) by 2279 m.u.

Calculation by using matrix algebra:

$b = (X^T X)^{-1} X^T y$ PDF, p.4.

Model 2

PDF, pp.5–6.

Estimation by regress and interpretation

- $b_2 = 0.96$

  $p(b_2) = 0.000 < \alpha = 0.05$

  Interpretation:

  Elasticity

  If x_j increases by 1 percent, then on average, ceteris paribus, y increases/decreases by b_j percent.

  If the average number of employees increases by 1 percent, then on average, ceteris paribus, value added increases by 0.96 percent.
  (+)

- $b_3 = 0.19$

  $p(b_3) = 0.006 < \alpha = 0.05$

If the sum of tangible and intangible assets increases by 1 percent, then on average, ceteris paribus, value added increases by (+) 0.19 percent.

Calculation by using matrix algebra:

For homework.