Example (PDF)

a) Done in PDF, pp. 1-3.

b) Koyck model:

$$\hat{C}_t = a(1-\hat{\lambda}) + b_0 Y_t + \hat{\lambda} C_{t-1} \qquad \text{(SRM)}$$

$$\hat{C}_t = g_1 + g_2 Y_t + g_3 C_{t-1}$$

$$\hat{C}_t = -17.898 + 0.4556 Y_t + 0.5211 C_{t-1}$$

PDF, p. 6.

- $g_3 = \hat{\lambda} = 0.5211$ estimate of the rate of decline (decay)

The effect of disposable income on consumption in a given year represents on average 52.1% of the effect in the previous year.

- $g_2 = b_0 = 0.4556$
estimate of the short-run multiplier

- $g_1 = a(1-\hat{\lambda}) \Rightarrow a = \frac{g_1}{1-\hat{\lambda}} = \frac{-17.898}{1-0.5211} = -37.372$
intercept of the infinite (lag) distributed-lag m.

Lagged effects of Y on C:

$b_k = b_0 \cdot \hat{\lambda}^k$ (Koyck assumption)

$b_1 = b_0 \cdot \hat{\lambda} = 0.2374$
$b_2 = b_0 \cdot \hat{\lambda}^2 = 0.1237$
$b_3 = b_0 \cdot \hat{\lambda}^3 = 0.0647$

etc.

Infinite (lag) distributed-lag model:

$$\hat{C}_t = \overset{a}{-37.372} + \overset{b_0}{0.4556} Y_t + \overset{b_1}{0.2374} Y_{t-1} + \underset{b_2}{0.1237} Y_{t-2} + \underset{b_3}{0.0647} Y_{t-3} + \ldots$$

Interpretation of multipliers:

- $b_0 = 0.4556$ estimate of the short-run multiplier (propensity to consume)

If disposable income in a given year increases by 1 USD, then on average, consumption in the same year increases by 0.4556 USD.

- $b_2 = 0.1237$ estimate of an intermediate multiplier (one of many)

If disposable income in a given year increases by 1 USD, then on average, consumption after two years increases by 0.1237 USD.

- $b_0 + b_1 + b_2 = 0.8167$

estimate of a cumulative multiplier (one of many)

If disposable income in a given year increases by 1 USD, then on average, consumption in the same year and the next two years cumulatively increases by 0.8167 USD.

- $\sum_{k=0}^{\infty} b_k = \frac{b_0}{1-\hat{\lambda}} = \frac{0.4556}{1-0.5211} = 0.9513$

estimate of the long-run multiplier (propensity to consume)

If disposable income in a given year increases by 1 USD, then on average, consumption in the long run increases by 0.9513 USD.

- $Me = -\frac{\log 2}{\log \hat{\lambda}} = -\frac{\log 2}{\log 0.5211} = 1.06$ estimate of the median lag

Half of the long-run effect of disposable income on consumption realizes on average in 1.06 years.

c) Due to the presence of C_{t-1} as a regressor, we employ the Breusch-Godfrey test for first-order autocorrelation.

$AR(1): u_t = \rho_1 u_{t-1} + \varepsilon_t$

$H_0: \rho_1 = 0$ (no first-order AC)
$H_1: \rho_1 \neq 0$ (first-order AC)

$\hat{e}_t = \hat{\rho}_1 e_{t-1}$
$\hat{e}_t = \underbrace{0.5492}_{\hat{\rho}_1 > 0} e_{t-1}$ PDF, p.6.

$H_0: \rho_1 \leq 0$ (no positive first-order AC)
$H_1: \rho_1 > 0$ (positive first-order AC)

PDF, p.7:

$M_{AR}: \hat{e}_t = g_1 + g_2 Y_t + g_3 C_{t-1} + \hat{\rho}_1 e_{t-1}$
$R^2_{AR} = 0.3336$

initial sample size already reduced in SRM due to C_{t-1}

$LM = (T' - 1) \cdot R^2_{AR} = 29 \cdot 0.3336 = \underline{\underline{9.674}}$
$\chi^2_c (m=1, \alpha=0.05) = 3.84$

$LM > \chi^2_c$, we reject H_0 at $\alpha = 0.05$ and conclude from H_1.

We have positive first-order AC present in our distributed-lag model.