University of Ljubljana

SEB SCHOOL OF ECONOMICS AND BUSINESS

5. Regression Models with Dummy Explanatory Variables

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Basic definitions

DUMMY VARIABLES

The following names occur for such a variable:

- dummy
- binary
- dihotomous
- indicator
- artificial

Observed unit has the analysed property → Value 1

Observed unit does not have the analysed property → Value 0

Basic definitions

WARNINGS

1. **A categorical variable (attribute) with *m* possible values requires (*m* – 1) dummy variables.**

   "Dummy trap" – Perfect multicollinearity!

2. **Value of the categorical variable that is assigned value 0 for the dummy variable is called the *base* (benchmark, control, reference) *value*.**

3. **Switching the assigned values (0 to 1 and vice versa) for dummy variables does not affect the (absolute) values of regression coefficients.**

Analysis of variance models

ANALYSIS OF VARIANCE (ANOVA) MODELS

A Regression model with one attribute with two possible values

$$y_i = \beta_1 + \beta_2 D_i + u_i$$

$$E(y_i | D_i = 0) = \beta_1$$

$$E(y_i | D_i = 1) = \beta_1 + \beta_2$$

(arithmetic mean)

? $$H_0 : \beta_2 = 0 \ ; \ H_1 : \beta_2 \neq 0$$

i $$|t(b_2)| > t_c$$

Analysis of variance models

B **Regression model with one attribute with several (three) possible values**

$$y_i = \beta_1 + \beta_2 D_{1i} + \beta_3 D_{2i} + u_i$$

$$E(y_i | D_{1i} = 0, D_{2i} = 0) = \beta_1$$

$$E(y_i | D_{1i} = 1, D_{2i} = 0) = \beta_1 + \beta_2$$

$$E(y_i | D_{1i} = 0, D_{2i} = 1) = \beta_1 + \beta_3$$

$H_0 : \beta_2 = 0 \; ; \; H_1 : \beta_2 \neq 0$ $\qquad$ $H_0 : \beta_3 = 0 \; ; \; H_1 : \beta_3 \neq 0$

$|t(b_2)| > t_c$ $\qquad$ $|t(b_3)| > t_c$

Analysis of variance models

C **Regression model with two attributes with two values each**

$$y_i = \beta_1 + \beta_2 D_{1i} + \beta_3 D_{2i} + u_i$$

$$E(y_i | D_{1i} = 0, D_{2i} = 0) = \beta_1$$
$$E(y_i | D_{1i} = 1, D_{2i} = 0) = \beta_1 + \beta_2$$
$$E(y_i | D_{1i} = 0, D_{2i} = 1) = \beta_1 + \beta_3$$
$$E(y_i | D_{1i} = 1, D_{2i} = 1) = \beta_1 + \beta_2 + \beta_3$$

? $H_0 : \beta_2 = 0 \;;\; H_1 : \beta_2 \neq 0$ $\quad$ $H_0 : \beta_3 = 0 \;;\; H_1 : \beta_3 \neq 0$

i $|t(b_2)| > t_c$ $\quad$ $|t(b_3)| > t_c$

Analysis of covariance models

ANALYSIS OF COVARIANCE (ANCOVA) MODELS

D Regression model with one numerical explanatory variable and one attribute with two possible values

$$y_i = \beta_1 + \beta_2 D_i + \beta_3 x_{3i} + u_i$$

$$E(y_i | D_i = 0, x_{3i}) = \beta_1 + \beta_3 x_{3i}$$

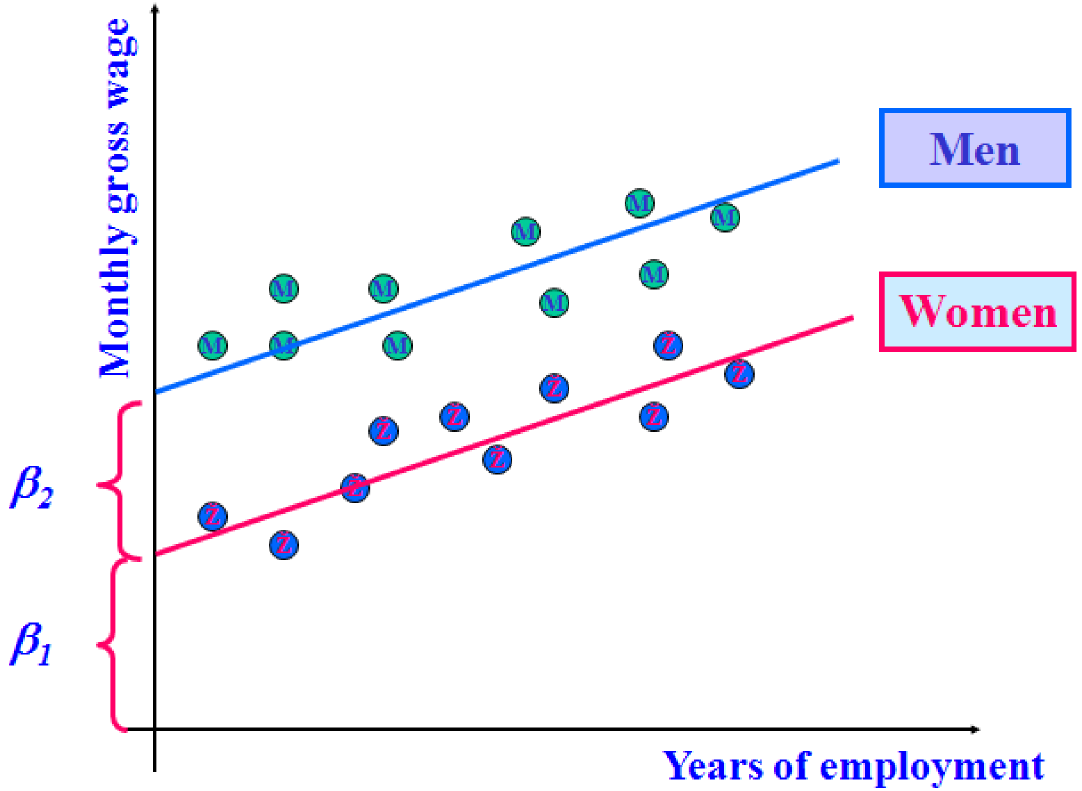
$$E(y_i | D_i = 1, x_{3i}) = (\beta_1 + \beta_2) + \beta_3 x_{3i}$$

(conditional expected value)

$$H_0 : \beta_2 = 0 \; ; \; H_1 : \beta_2 \neq 0$$

$$|t(b_2)| > t_c$$

Analysis of covariance models



Regression coefficient β_2 is also called the *differential intercept coefficient.*

Analysis of covariance models

E **Regression model with one numerical explanatory variable and one attribute with several (three) possible values**

$$y_i = \beta_1 + \beta_2 D_{1i} + \beta_3 D_{2i} + \beta_4 x_{4i} + u_i$$

$$E(y_i | D_{1i} = 0, D_{2i} = 0, x_{4i}) = \beta_1 + \beta_4 x_{4i}$$

$$E(y_i | D_{1i} = 1, D_{2i} = 0, x_{4i}) = (\beta_1 + \beta_2) + \beta_4 x_{4i}$$

$$E(y_i | D_{1i} = 0, D_{2i} = 1, x_{4i}) = (\beta_1 + \beta_3) + \beta_4 x_{4i}$$

? $H_0 : \beta_2 = 0 \; ; \; H_1 : \beta_2 \neq 0$ $\qquad$ $H_0 : \beta_3 = 0 \; ; \; H_1 : \beta_3 \neq 0$

i $|t(b_2)| > t_c$ $\qquad$ $|t(b_3)| > t_c$

Analysis of covariance models

F Regression model with one numerical explanatory variable and two attributes with two values each

$$y_i = \beta_1 + \beta_2 D_{1i} + \beta_3 D_{2i} + \beta_4 x_{4i} + u_i$$

$$E(y_i | D_{1i} = 0, D_{2i} = 0, x_{4i}) = \beta_1 + \beta_4 x_{4i}$$

$$E(y_i | D_{1i} = 1, D_{2i} = 0, x_{4i}) = (\beta_1 + \beta_2) + \beta_4 x_{4i}$$

$$E(y_i | D_{1i} = 0, D_{2i} = 1, x_{4i}) = (\beta_1 + \beta_3) + \beta_4 x_{4i}$$

$$E(y_i | D_{1i} = 1, D_{2i} = 1, x_{4i}) = (\beta_1 + \beta_2 + \beta_3) + \beta_4 x_{4i}$$

? $H_0: \beta_2 = 0 \; ; \; H_1: \beta_2 \neq 0$ $\quad$ $H_0: \beta_3 = 0 \; ; \; H_1: \beta_3 \neq 0$

i $|t(b_2)| > t_c$ $\quad$ $|t(b_3)| > t_c$

Comparison of two regression models

$$y_i = \beta_1 + \beta_2 x_i + u_i$$

a) Chow test

$$F = \frac{(RSS - RSS_1 - RSS_2)/k}{(RSS_1 + RSS_2)/(n_1 + n_2 - 2k)}$$

b) Application of dummy variables

Multiplicative or interaction dummy variable

$$y_i = \beta_1 + \beta_2 D_i + \beta_3 x_i + \beta_4 (D_i x_i) + u_i$$

$$E(y_i | D_i = 0, x_i) = \beta_1 + \beta_3 x_i$$

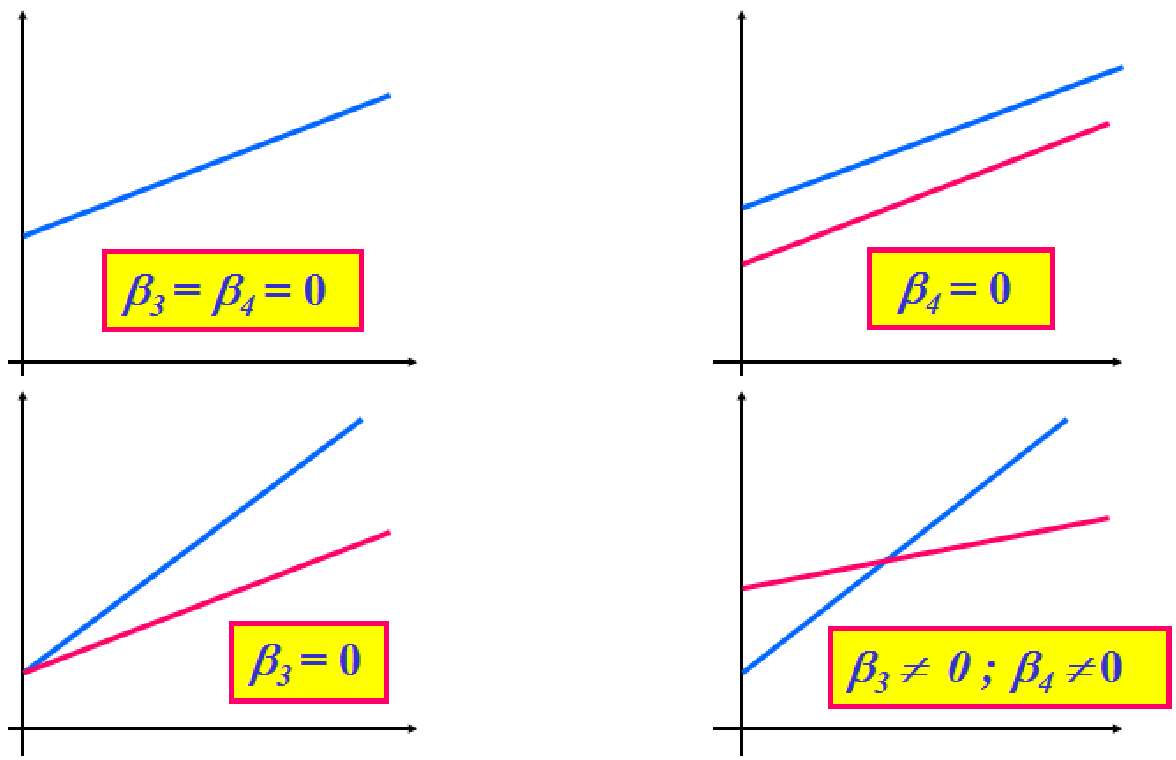
$$E(y_i | D_i = 1, x_i) = (\beta_1 + \beta_2) + (\beta_3 + \beta_4) x_i$$

$$H_0 : \beta_2 = 0 \; ; \; H_1 : \beta_2 \neq 0 \qquad H_0 : \beta_4 = 0 \; ; \; H_1 : \beta_4 \neq 0$$

Comparison of two regression models

Application of one dummy variable offers four possibilities (models)

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 D_t + \beta_4 (x_{2t} D_t) + u_t$$



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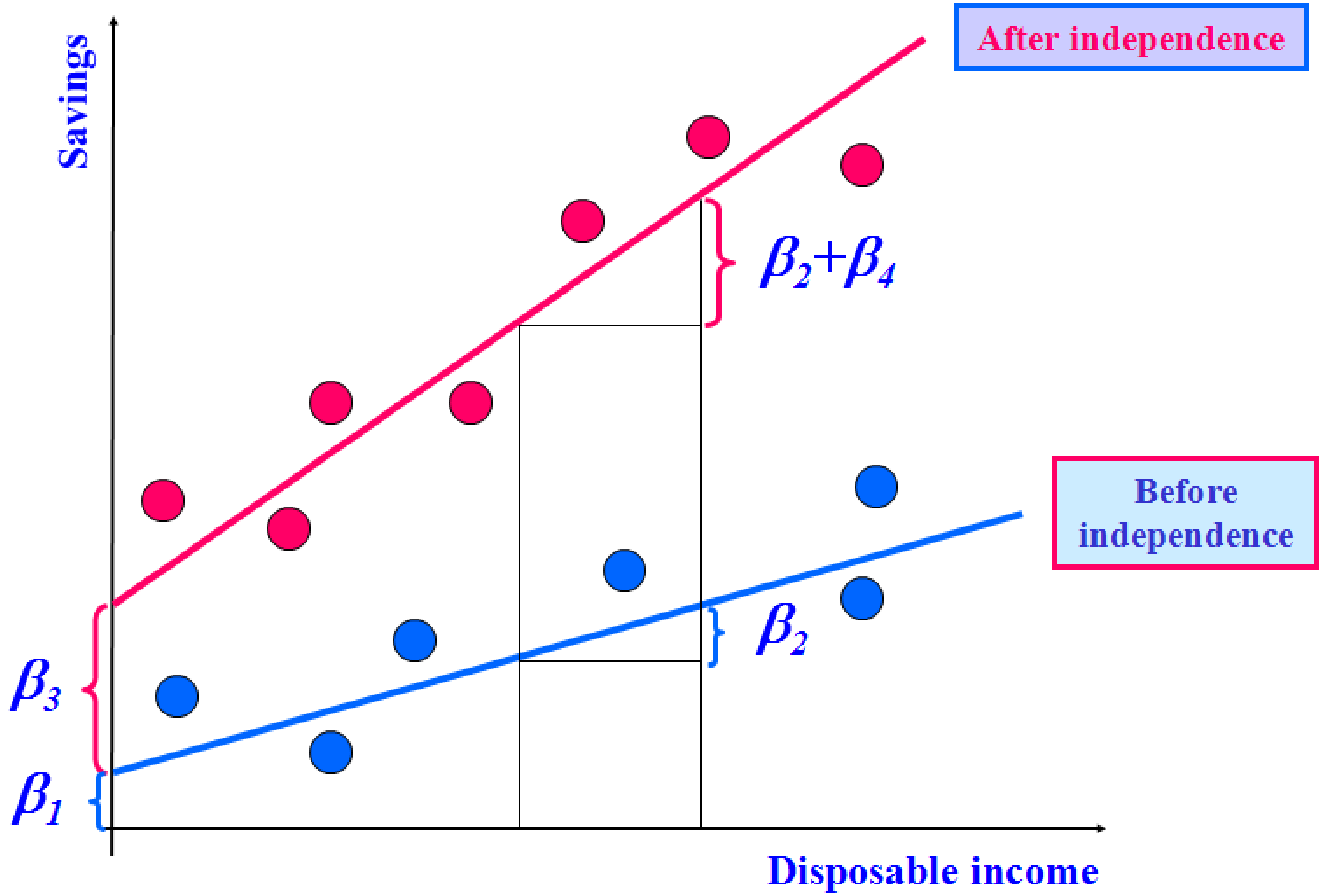
Comparison of two regression models



Comparison of two regression models

Advantages of dummy variables compared to statistical tests (Chow test)

1. **Only one regression model is estimated**

2. **Differences in one or more regression coefficients allowed**

   β_3 – differential intercept coefficient
   β_4 – differential slope coefficient

3. **No degrees of freedom are lost when analysing statistical significance**

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