Example 3

a) Graphical assessment of stationarity:
does the time series oscilate around the same level (or trend) or does it wander up and down?

Dickey-Fuller test:

$$\Delta y_t = \gamma y_{t-1} + \nu_t$$

if wandering around 0

Dickey-Fuller test with constant

$$\Delta y_t = \alpha + \gamma y_{t-1} + \nu_t$$

if wandering around a constant ($\neq 0$)

Dickey-Fuller test with constant and trend

$$\Delta y_t = \alpha + \gamma y_{t-1} + \lambda t + \nu_t$$

if wandering around a linear trend

Augmented Dickey-Fuller test
(with constant, trend and lags)

$$\Delta y_t = \alpha + \gamma y_{t-1} + \lambda t + \sum_{s=1}^{m} a_s \Delta y_{t-s} + \nu_t$$

← better more than too few

$(\gamma = \rho - 1)$

$H_0: \rho = 1 \quad (\gamma = 0)$

$H_1: \rho < 1$

Information criteria:

(usualley) MAIC – Modified Akaike IC

SC (Schwarz) – much less complicated in this case

↓ 2 lags:

$$\Delta y_t = \alpha + \gamma y_{t-1} + \lambda t + a_1 \Delta y_{t-1} + a_2 \Delta y_{t-2} + u_t$$

b) • taking first differences

• calculating growth rates: $\ln \frac{y_t}{y_{t-1}}$ OR $\frac{y_t}{y_{t-1}}$

c) AR(I)MA models

AR – AutoRegressive

MA – Moving Average

type of model

$AR(p)$: $y_t = \mu + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + u_t$

$MA(q)$: $y_t = \mu + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \dots + \theta_q u_{t-q}$

→ combination: ARMA (p, q) or ARIMA (p, d, q), where p – AR order, q – MA order, d – order of integration

d – how many times do we have to take first differences to make the time series stationary

Selecting the appropriate p and q orders:

- graphically: correlogram (autocorrelation for MA, partial autocorrelation (effect of all previous lags removed) for AR

- analitically: IC (information criteria)
  → lowest value

⟹ $SBITOP_t$: ARIMA (1, 1, 4)
⟹ $\Delta SBITOP_t$ = d. SBITOP : ARMA (1, 4)
↓

$$y_t = \mu + \phi_1 y_{t-1} + u_t + \theta_1 u_{t-1} + \theta_2 u_{t-2} + \theta_3 u_{t-3} + \theta_4 u_{t-4}$$

d) Out-of-sample forecasting

IRF - Impulse-Response function
→ what happens if we introduce a unit shock to the disturbance (= error) term

Stable system: the shock gradually dies out
- what is the effect of the shock?
- how long does it take to die out?

Tutorial, Part 12

VAR models

Example 1

No. of variables = No. of equations : $g = 3$
No. of lags (the same for each variable): $k = 2$

Matrix notation:

$$Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 Y_{t-2} + u_t$$

$$\begin{bmatrix} y_{1t} \\ y_{2t} \\ y_{3t} \end{bmatrix} = \begin{bmatrix} \beta_{10} \\ \beta_{20} \\ \beta_{30} \end{bmatrix} + \begin{bmatrix} \beta_{11} & \alpha_{11} & \gamma_{11} \\ \beta_{21} & \alpha_{21} & \gamma_{21} \\ \beta_{31} & \alpha_{31} & \gamma_{31} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \\ y_{3t-1} \end{bmatrix} + \begin{bmatrix} \beta_{12} & \alpha_{12} & \gamma_{12} \\ \beta_{22} & \alpha_{22} & \gamma_{22} \\ \beta_{32} & \alpha_{32} & \gamma_{32} \end{bmatrix} \begin{bmatrix} y_{1t-2} \\ y_{2t-2} \\ y_{3t-2} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \\ u_{3t} \end{bmatrix}$$

Explicitly:

$$y_{1t} = \beta_{10} + \beta_{11} y_{1t-1} + \beta_{12} y_{1t-2} + \alpha_{11} y_{2t-1} + \alpha_{12} y_{2t-2} + \gamma_{11} y_{3t-1} + \gamma_{12} y_{3t-2} + u_{1t}$$

$$y_{2t} = \beta_{20} + \beta_{21} y_{1t-1} + \beta_{22} y_{1t-2} + \alpha_{21} y_{2t-1} + \alpha_{22} y_{2t-2} + \gamma_{21} y_{3t-1} + \gamma_{22} y_{3t-2} + u_{2t}$$

$$y_{3t} = \beta_{30} + \beta_{31} y_{1t-1} + \beta_{32} y_{1t-2} + \alpha_{31} y_{2t-1} + \alpha_{32} y_{2t-2} + \gamma_{31} y_{3t-1} + \gamma_{32} y_{3t-2} + u_{3t}$$

$\Rightarrow$ No. of terms to be estimated:
$g \cdot k + 1$ in each equation
$\Rightarrow g(g \times k + 1)$

Example 2

Vector AutoRegressive models:
all variables are endogenous → no need for theory assumptions (but difficult interpretation)

c) Lag-order selection: selection-order criteria (varsoc)
H_0: all coeffs. at that order = 0

$g = 3,\ k = 3 \Rightarrow \underset{\text{intercept}}{g \times k + 1} = 10$ parameters in each equation

H_0: all coeffs. in the equation = 0

d) Wald test for Lag Exclusion (varwle)

H_0: coeffs (for endogenous variables) at given lag are JOINTLY equal to 0

Checking for autoregression (LM test) (varlmar)

H_0: no autocorrelation at lag order

Granger causality (vargranger)

H_0: no Granger causality (all coeffs. of one variable in one equation = 0)

pairwise

the interpretations of relationships between the variables in the VAR model are impossible based on Granger causality. → use IRFs, FEVDs

g^2 IRFs for g variables

IRFs – 1 unit shock
OIRFs – 1 standard deviation shock

IRFs deal with shocks separately – one VAR equation at a time ⇒ quite unrealistic

OIRFs (orthogonalized IRFs) take disturbance correlations into account →
↳ orthogonalized

- only the 1st variable with a potential immediate impact on all other $(g-1)$ variables
- the 2nd variable can have immediate impact just on the remaining $(g-2)$ variables

⋮

⇒ ordering of variables is important

FEVD (Forecast Error Variance Decomposition): share of forecast-error variance explained by shocks of individual variables

e) OIRFs and FEVDs not the same if order of variables is reversed

g)-i) OIRF – orthogonalization of the disturbance term:
- Cholesky decomposition of var-cov matrix Σ

or
- short-run (contemporaneous) / long-run (accumulated) restrictions on the ortogonalization matrix
→ SVAR (structural VAR) model

SVAR: - contemporaneous terms on the RHS, more theoretical
- cannot be estimated without restrictions (as different SVAR models have the same standard form)

not testable

IDENTIFICATION

Expressing for the disturbances from short-run SVAR:

ORIGINAL (var = Σ) ORTHOGONALIZED (var = I_g)

$$A v_t = B \tilde{v}_t$$

$$v_t = A^{-1} B \tilde{v}_t$$ → restrictions on A and B

No. of restrictions: $2g^2 - g(g+1)/2$ (at least)

ALL PARs in A and B FREE PARs IN Σ (symmetric)

according to the theory

Expressing for the disturbances from long-run SVAR:

ACCUMULATED EFFECT

$(A = I_g)$ $$\bar{A} y_t = B \tilde{v}_t \qquad (\bar{A} = I_g - A_1 L - A_2 L^2 - \dots - A_p L^p)$$

$$y_t = \bar{A}^{-1} B \tilde{v}_t = C \tilde{v}_t$$ → restrictions on C

No. of restrictions: $g^2 - g(g+1)/2$ (at least)