

# Hypotheses testing

Bounds of the confidence interval:

$$t_c(n-k, \alpha/2) = 1.99 \quad \alpha = 0.05$$

$\swarrow$  81     $\swarrow$  3     $\swarrow$  0.025

- $b_j - t_c \cdot se(b_j)$
- $b_j + t_c \cdot se(b_j)$

Ch. 2, Slide 69  
Ch. 3, Slide 4

Essential elements of testing:

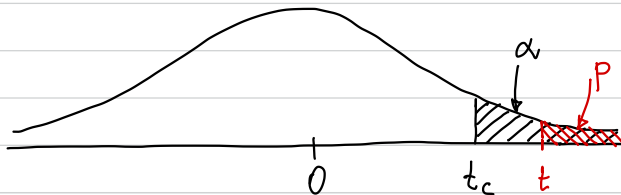
$$t \underset{H_0}{\sim} t_{n-k}$$

Ch. 3, Slide 5

Test statistic vs. p-value

Slide 8

An example of one-tailed t-test in the right tail of the distribution:



$H_0$  is rejected:  $t > t_c$  or  $p \leq \alpha$ .

## "Beta" coefficients or standardised regression coefficients

Slide 10

1) Standardize  $x_j$  and  $y$ :

$$x_{ji}^s = \frac{x_{ji} - \bar{x}_j}{sd(x_j)} ; \forall j = 1, \dots, k$$

$$Var(x_j^s) = 1$$

$$y_i^s = \frac{y_i - \bar{y}}{sd(y)}$$

$$Var(y^s) = 1$$

2) Estimate:

$$\hat{y}_i^s = \underbrace{b_1}_{=0} + \underbrace{\beta a_2}_{\text{standardised or "beta" coefficients}} \cdot x_{2i}^s + \underbrace{\beta a_3}_{\text{standardised or "beta" coefficients}} \cdot x_{3i}^s + \dots + \underbrace{\beta a_k}_{\text{standardised or "beta" coefficients}} \cdot x_{ki}^s$$

3) Compare  $|\beta a_j|$ .

Higher value of  $|\beta a_j|$  means that the variable  $x_j$  has more explanatory power in explaining the variability of  $y$ .

# Statistical inference

Slide 11

- We can commit either the Type I error or the Type II error.
- Our goal is to :
  - minimize the probability of Type I error ( $\alpha \leq 0.05$ ) and at the same time to
  - minimize the probability of Type II error or to maximize the power of the test ( $1 - \beta \geq 0.8$ ).
- Challenges :
  - there is a trade-off between  $\alpha$  and  $\beta$  in practice and
  - in order to regulate  $\beta$ , we would have to know the actual state of nature (1 sample!).
- Reality: in practice we focus on  $\alpha$  and at the same time we try to ensure appropriate power of the test ( $1 - \beta$ ).
- Power of the test depends on :
  - 1) Type of the test;
  - 2) Difference between the actual and the tested value ( $\beta_j$  vs.  $\beta_j$ ) (+);
  - 3) Measurement error (-);
  - ! 4) Sample size (+).

- As we could commit the Type II error, we never accept  $H_0$ , only reject.
- If  $H_0$  is rejected, i.e. a significant result, we make a conclusion from  $H_1$  (i.e.  $\beta_j \neq 0$ ).
- If  $H_0$  is not rejected, i.e. a non-significant result, this means:
  - either no important effect (i.e.  $\beta_j = 0$ )
  - or too little information (could be rejected with more info. or on another sample).
- $H_1$  is neither accepted nor rejected, as its distribution is usually not known explicitly (not the same as that of  $H_0$ ).

## S 14:

- $c$  – vector of parameters of the linear combination of regression coeff. (estimates)
- $r$  – hypothesized value (scalar) from the null hypothesis