Interpretation of regression coefficients for most common forms of regression models

1. Linear regr. model

$$y_i = b_1 + b_2 x_{2i} + \dots + b_j x_{ji} + \dots + b_k x_{ki} + e_i$$

partial regr. coefficient

If x_j increases by 1 unit of measurement, then on average, (1) ceteris paribus (2), y increases/decreases by b_j units of measurement.

Two relativisations, intercept: S 75.

2. Log-linear regression model

$$\ln y_i = b_1 + b_2 \ln x_{2i} + \dots + b_j \ln x_{ji} + \dots + b_k \ln x_{ki} + e_i$$

part. elasticity (approximation)

If x_j increases by 1 percent, then on average, ceteris paribus, y increases/decreases by b_j percent.

3. Log-lin regression model

$$\ln y_i = b_1 + b_2 x_{2i} + \dots + b_j x_{ji} + \dots + b_k x_{ki} + e_i$$

part. semi elasticity (approximation)

If x_j increases by 1 unit of measurement, then on average, ceteris paribus, y increases/decreases by $100 \cdot b_j$ percent.

4. Lin-log regression model

$$y_i = b_1 + b_2 \ln x_{2i} + \dots + b_j \ln x_{ji} + \dots + b_k \ln x_{ki} + e_i$$

part. semi elasticity (approximation)

If x_j increases by 1 percent, then on average, ceteris paribus, y increases/decreases by $b_j/100$ units of measurement.

5. "Mixed" regression model

EXAMPLE: $\ln y_i = b_1 + b_2 x_{2i} + b_3 \ln x_{3i} + e_i$

log–lin r.m.

log–linear r.m.

Each regression coefficient is interpreted based on the functional form of the relationship between "y" and "x_j".

Indirect effects, S77–78:

Indirect effect of x_j on y (through x_l): total effect of x_j on x_l $\cdot$ direct effect of x_l on y.

Example, c)

PDF, pp. 6–7.

$$L_i = c_1 + c_2 K_i + e_{Li}$$

$$\hat{Q}_i = d_1 + \underline{d_2}\, e_{Li} + d_3 K_i$$

compare

$$\hat{Q}_i \approx b_1 + \underline{b_2} L_i + b_3 K_i$$

$$d_2 = b_2$$

$$K_i = f_1 + f_2 L_i + e_{Ki}$$

$$\hat{Q}_i = g_1 + g_2 L_i + \underline{g_3}\, e_{Ki}$$

compare

$$\hat{Q}_i \approx b_1 + b_2 L_i + \underline{b_3} K_i$$

$$g_3 = b_3$$

Effects of $\underline{L}$ on $\underline{Q}$:

- direct: $b_2 = d_2 = \underline{9{,}687.38}$
- indirect: $f_2 \cdot b_3 = 4{,}085.43 \cdot 2.279 = \underline{9{,}310.69}$
- total: $b_2 + f_2 \cdot b_3 = 18{,}998.07$

  or $g_2 = \underline{18{,}999.75}$

$$L \xrightarrow[\text{total}]{} K \xrightarrow[\text{direct}]{} Q$$

Effects of K on Q:

$K \xrightarrow{\text{total}} L \xrightarrow{\text{direct}} Q$

- direct : $b_3 = g_3 = 2.279$
- indirect : $c_2 \cdot b_2 = 0.000176 \cdot 9{,}687.38 = 1.705$
- total : $b_3 + c_2 \cdot b_2 = 3.984$ $\approx$
  or $d_3 = 3.983$

Example, d) PDF, pp. 7-8.

$Q_i = b_1 + b_2 L_i + b_3 K_i + e_{1i}$

$Q_i = c_1 + c_2 L_i + e_{Qi}$
$K_i = d_1 + d_2 L_i + e_{Ki}$

FWL regression: $e_{Qi} = g_1 + g_2 e_{Ki} + e_{2i}$

FWL theorem: 1 $b_3 = g_2$
2 $e_{1i} = e_{2i}, \forall i$