

## Quantitative Methods in Finance

### **Tutorial, Part 12:** *Vector autoregressive models.*

**Example 1:** Consider the following vector autoregressive model:

$$y_t = \beta_0 + \sum_{i=1}^k \beta_i y_{t-i} + u_t,$$

where  $y_t$  is a  $g \times 1$  vector of variables determined by  $k$  lags of all  $g$  variables in the system,  $u_t$  is a  $g \times 1$  vector of disturbance terms,  $\beta_0$  is a  $g \times 1$  vector of constant term coefficients, and  $\beta_i$  are  $g \times g$  matrices of coefficients on the  $i$ -th lag of  $y$ .

If  $g = 3$  and  $k = 2$ , write out all the equations of the VAR model in full. ■

**Example 2:** In the data set `stockexchange.dta`, there are data on three stock exchange indices: DAX30, FTSE100, and S&P500. Those trading days that did not contain index price information for all three stock exchange indices were left out. The programming code is given in Stata Do file `stockexchange-commands.do`.

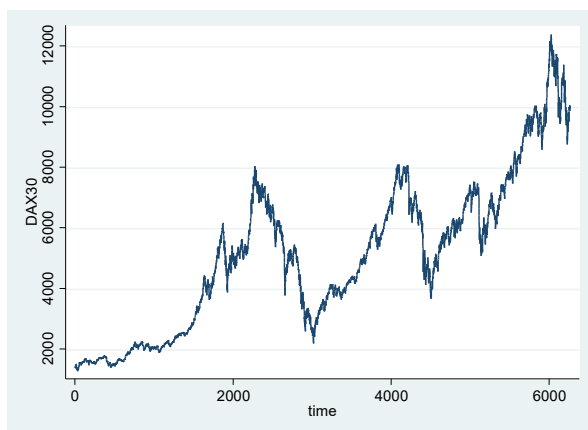
- Inspect the three time series visually. Do they look stationary? Perform the Augmented Dickey-Fuller (ADF) test on each of the three time series.
- Construct a set of growth rates. Check for stationarity of the newly generated time series.
- Estimate the appropriate vector autoregressive model based on the growth rates of the three stock exchange indices.
- Based on the estimated VAR model, perform all the appropriate diagnostics, Granger causality tests, and model interpretation.
- Perform sensitivity analysis on the ordering of model variables by reversing the ordering.
- Employ the estimated VAR model in order to compute dynamic forecasts of the growth rates of the three stock exchange indices for the following 8 days.
- By choosing appropriate matrices **A** and **B** in the structural VAR (SVAR) model, reproduce the Cholesky decomposition that is regularly employed in the VAR framework to orthogonalise the (correlated) disturbance terms. In short, calculate the estimated orthogonalization matrix  $\mathbf{A}^{-1}\mathbf{B}$  and show that you obtain the same values by employing the Cholesky decomposition on the variance-covariance matrix of the VAR model.
- The SVAR model in g) was exactly (just) identified. Estimate an overidentified short-run SVAR model by imposing the restriction that the growth rate of FTSE100 does not affect the growth rate of S&P500 contemporaneously (i.e.  $\mathbf{A}_{3,2} = 0$ ), only with lags.
- Following the recursive logic from point g), estimate an overidentified long-run SVAR model by imposing the restriction that the long-run response of the growth rate of SP500 to a shock to (the equation for) the growth rate of FTSE100 is zero (i.e.  $\mathbf{C}_{3,2} = 0$ ).

## Computer printout of the results in Stata:

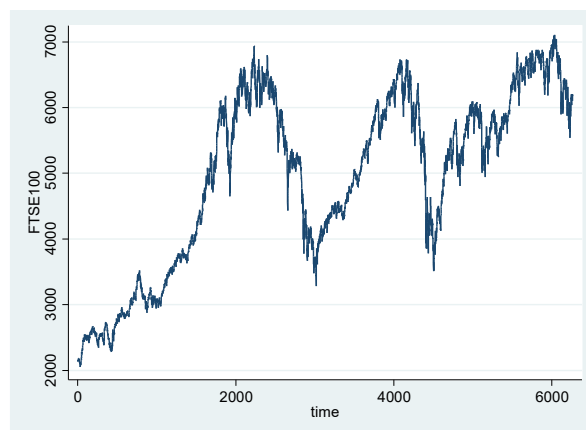
Testing for stationarity and generating stationary variables

```
. generate time=_n
. tsset time
      time variable:  time, 1 to 6262
                delta:  1 unit
```

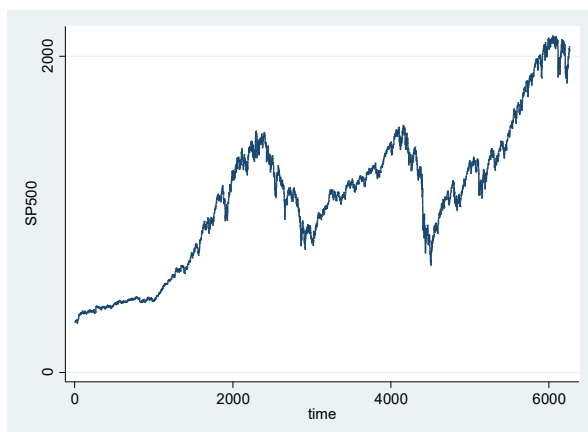
```
. twoway line DAX30 time
```



```
. twoway line FTSE100 time
```



```
. twoway line SP500 time
```



```
. dfuller DAX30
```

Dickey-Fuller test for unit root Number of obs = 6261

	Test	----- Interpolated Dickey-Fuller -----		
	Statistic	1% Critical	5% Critical	10% Critical
		Value	Value	Value
Z(t)	-0.947	-3.430	-2.860	-2.570

MacKinnon approximate p-value for Z(t) = 0.7720

```
. dfuller FTSE100
```

Dickey-Fuller test for unit root Number of obs = 6261



# Estimating a VAR model

```
. varsoc rDAX30 rFTSE100 rSP500, maxlag(10)
```

Selection-order criteria						Number of obs = 6251		
Sample: 12 - 6262								
lag	LL	LR	df	p	FPE	AIC	HQIC	SBIC
0	59679.5				1.0e-12	-19.0934	-19.0923	-19.0902
1	60341	1322.9	9	0.000	8.3e-13	-19.3022	-19.2977	-19.2892
2	60392.6	103.29	9	0.000	8.2e-13	-19.3158	-19.308	-19.2932*
3	60422.7	60.082	9	0.000	8.1e-13	-19.3226	-19.3114*	-19.2902
4	60428.4	11.352	9	0.252	8.2e-13	-19.3215	-19.3069	-19.2794
5	60453.4	50.187	9	0.000	8.1e-13	-19.3267	-19.3087	-19.2749
6	60463	19.003	9	0.025	8.1e-13	-19.3268	-19.3055	-19.2653
7	60476.5	27.051	9	0.001	8.1e-13*	-19.3283*	-19.3036	-19.2571
8	60482.5	12.017	9	0.212	8.1e-13	-19.3273	-19.2993	-19.2464
9	60486.9	8.9106	9	0.446	8.1e-13	-19.3258	-19.2945	-19.2353
10	60496.5	19.039*	9	0.025	8.1e-13	-19.326	-19.2913	-19.2257
Endogenous: rDAX30 rFTSE100 rSP500								
Exogenous: cons								

```
. var rDAX30 rFTSE100 rSP500, lags(1/3)
```

## Vector autoregression

Sample: 5 - 6262	No. of obs	=	6258
Log likelihood = 60493.06	AIC	=	-19.32344
FPE = 8.14e-13	HQIC	=	-19.31224
Det(Sigma_ml) = 8.06e-13	SBIC	=	-19.29113

Equation	Parms	RMSE	R-sq	chi2	P>chi2
rDAX30	10	.014051	0.0728	491.5445	0.0000
rFTSE100	10	.010649	0.1224	872.4073	0.0000
rSP500	10	.011476	0.0064	40.60215	0.0000

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
rDAX30					
rDAX30					
L1.	-.1464566	.0197743	-7.41	0.000	-.1852136 -.1076996
L2.	-.0475436	.0199284	-2.39	0.017	-.0866026 -.0084846
L3.	-.0437496	.0197713	-2.21	0.027	-.0825007 -.0049986
rFTSE100					
L1.	-.1095166	.0255905	-4.28	0.000	-.1596731 -.0593602
L2.	-.0533342	.0257767	-2.07	0.039	-.1038556 -.0028128
L3.	-.0210292	.0247012	-0.85	0.395	-.0694426 .0273843
rSP500					
L1.	.4294045	.0197804	21.71	0.000	.3906357 .4681733
L2.	.1090047	.0219026	4.98	0.000	.0660764 .151933
L3.	.0897897	.0209804	4.28	0.000	.0486689 .1309106
_cons	.7937616	.0312191	25.43	0.000	.7325733 .8549499
rFTSE100					
rDAX30					
L1.	-.0738281	.0149872	-4.93	0.000	-.1032025 -.0444537
L2.	-.0497505	.015104	-3.29	0.001	-.0793538 -.0201472
L3.	-.0173996	.0149849	-1.16	0.246	-.0467695 .0119703

rFTSE100						
L1.	-.1937018	.0193953	-9.99	0.000	-.231716	-.1556877
L2.	-.0675766	.0195365	-3.46	0.001	-.1058674	-.0292858
L3.	-.0647442	.0187213	-3.46	0.001	-.1014373	-.028051
rSP500						
L1.	.4247581	.0149918	28.33	0.000	.3953748	.4541414
L2.	.1064824	.0166003	6.41	0.000	.0739465	.1390183
L3.	.0748367	.0159013	4.71	0.000	.0436707	.1060027
_cons	.8610715	.0236613	36.39	0.000	.8146962	.9074469
-----						
rSP500						
rDAX30						
L1.	.0257694	.0161512	1.60	0.111	-.0058864	.0574252
L2.	.004467	.016277	0.27	0.784	-.0274354	.0363694
L3.	-.0077052	.0161487	-0.48	0.633	-.0393561	.0239457
rFTSE100						
L1.	-.0161221	.0209017	-0.77	0.441	-.0570886	.0248445
L2.	-.0176055	.0210538	-0.84	0.403	-.0588701	.0236592
L3.	.0057493	.0201753	0.28	0.776	-.0337937	.0452922
rSP500						
L1.	-.071862	.0161561	-4.45	0.000	-.1035274	-.0401966
L2.	-.0457346	.0178895	-2.56	0.011	-.0807974	-.0106718
L3.	.0061645	.0171363	0.36	0.719	-.027422	.0397511
_cons	1.117282	.025499	43.82	0.000	1.067305	1.16726
-----						

VAR model diagnostics and Granger causality tests

. varwle

Equation: rDAX30

lag	chi2	df	Prob > chi2
1	472.3495	3	0.000
2	29.83905	3	0.000
3	20.44017	3	0.000

Equation: rFTSE100

lag	chi2	df	Prob > chi2
1	818.0678	3	0.000
2	60.34163	3	0.000
3	35.28578	3	0.000

Equation: rSP500

lag	chi2	df	Prob > chi2
1	26.47035	3	0.000
2	15.82732	3	0.001
3	.2906968	3	0.962

Equation: All

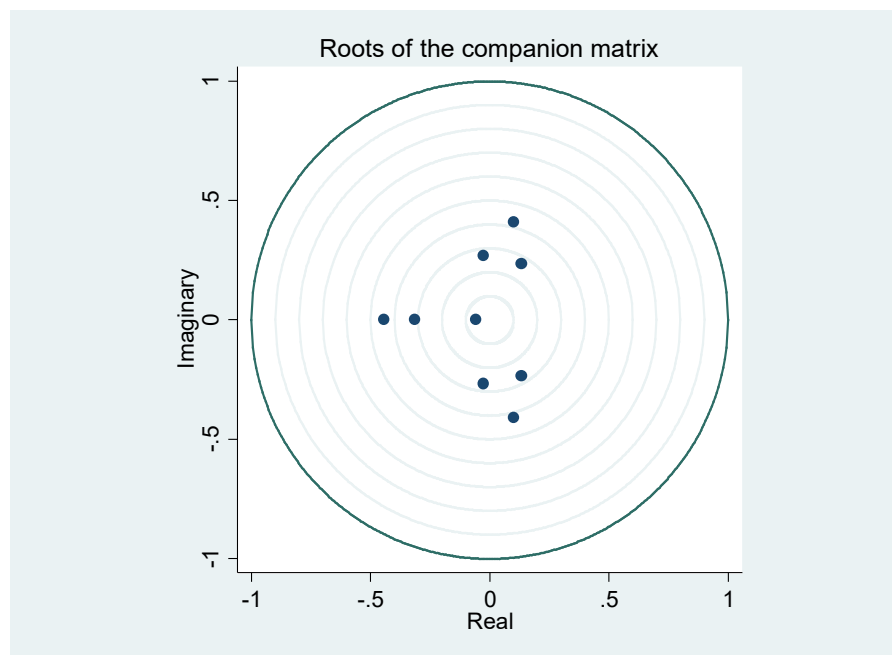
lag	chi2	df	Prob > chi2
1	1579.707	9	0.000
2	129.4776	9	0.000
3	60.68804	9	0.000

**. varstable, graph**

Eigenvalue stability condition

Eigenvalue	Modulus
-.4451482	.445148
.09963105 + .4097661i	.421704
.09963105 - .4097661i	.421704
-.3150297	.31503
-.02736248 + .26788i	.269274
-.02736248 - .26788i	.269274
.1309877 + .234633i	.26872
.1309877 - .234633i	.26872
-.05835511	.058355

All the eigenvalues lie inside the unit circle.  
VAR satisfies stability condition.



**. varlmar**

Lagrange-multiplier test

lag	chi2	df	Prob > chi2
1	6.9372	9	0.64366
2	16.6934	9	0.05374

H0: no autocorrelation at lag order

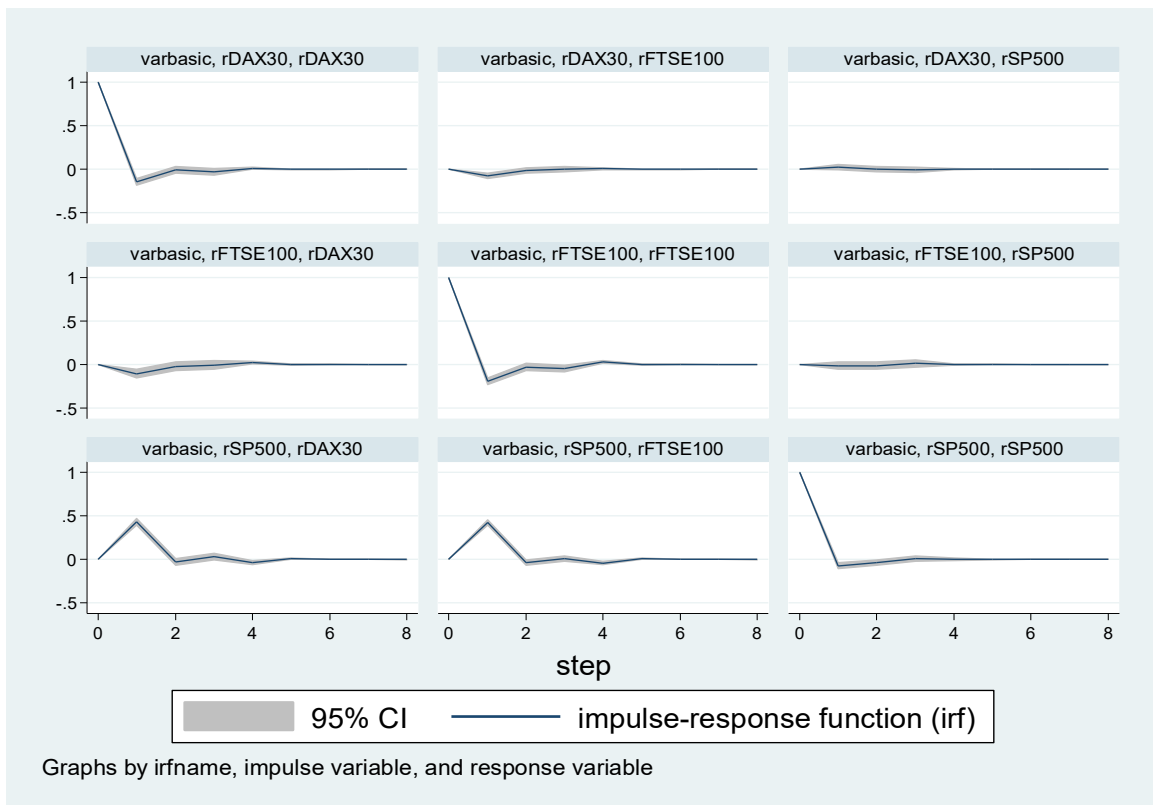
**. vargranger**

Granger causality Wald tests

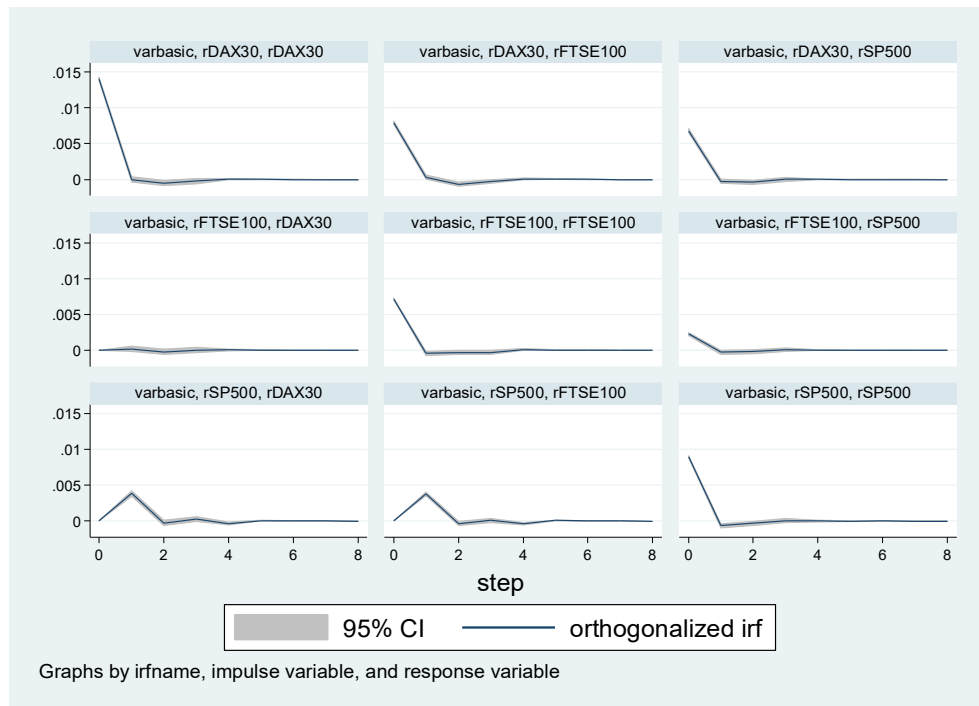
Equation	Excluded	chi2	df	Prob > chi2
rDAX30	rFTSE100	20.911	3	0.000
rDAX30	rSP500	482.09	3	0.000
rDAX30	ALL	483.18	6	0.000
rFTSE100	rDAX30	33.114	3	0.000
rFTSE100	rSP500	815.65	3	0.000
rFTSE100	ALL	833.48	6	0.000
rSP500	rDAX30	2.8103	3	0.422
rSP500	rFTSE100	1.3015	3	0.729
rSP500	ALL	3.7582	6	0.709

Model interpretation (IRFs, OIRFs and FEVDs)

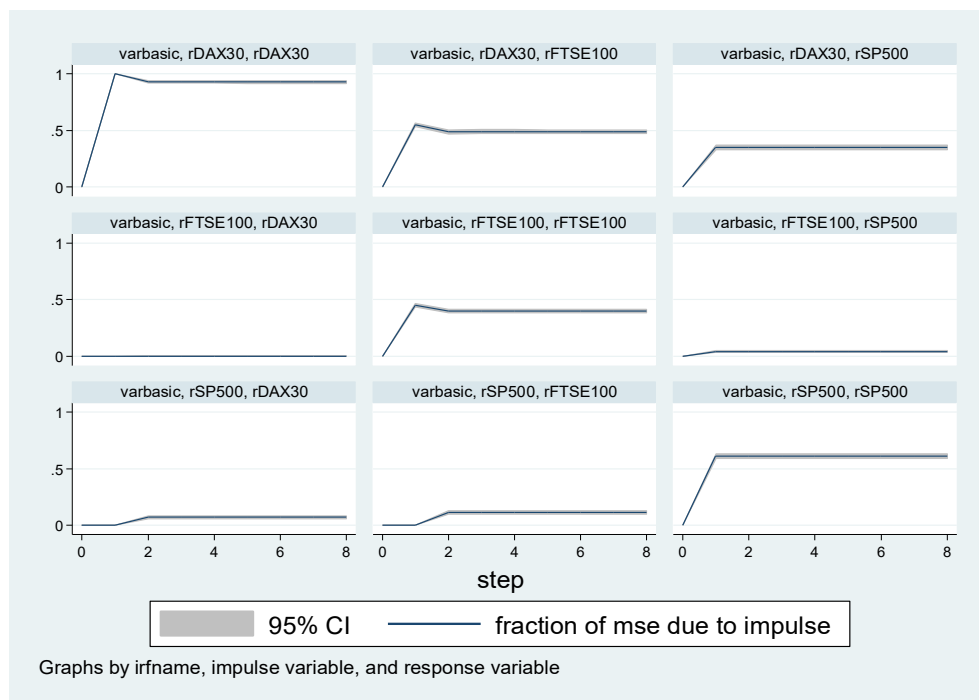
**. varbasic rDAX30 rFTSE100 rSP500, lags(1/3) irf**



```
. varbasic rDAX30 rFTSE100 rSP500, lags(1/3)
```



```
. varbasic rDAX30 rFTSE100 rSP500, lags(1/3) fevd
```

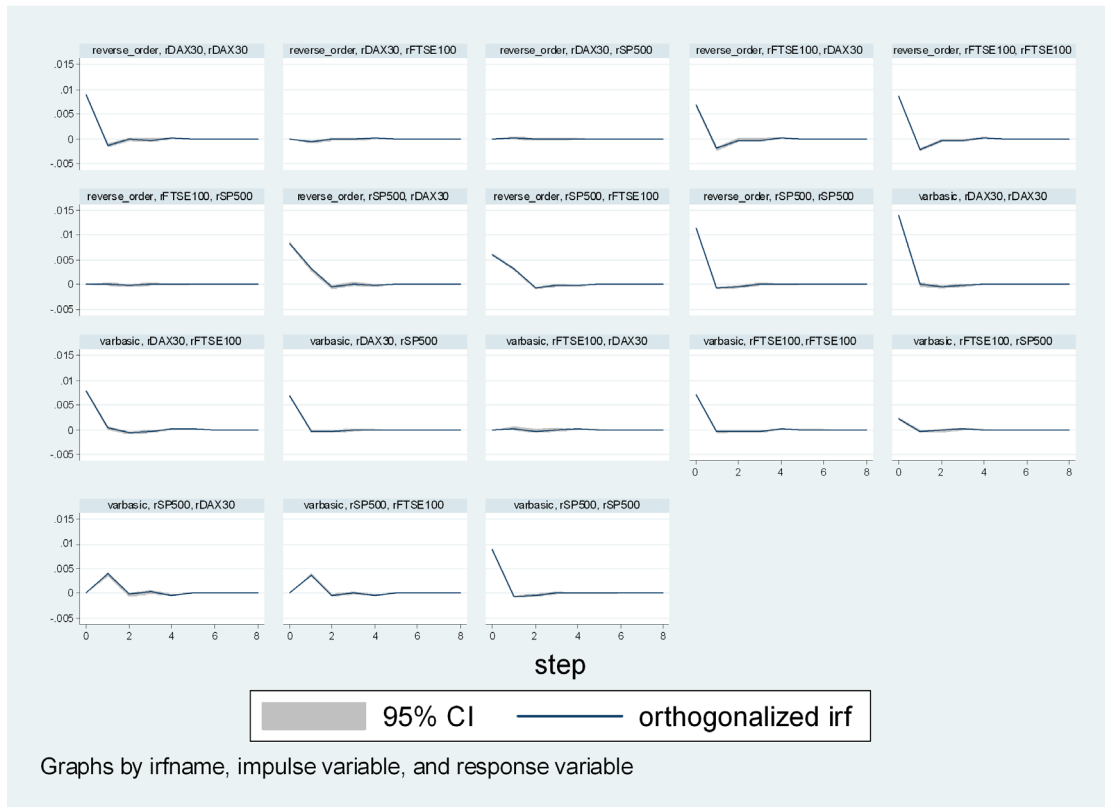


*Sensitivity analysis: Changing the ordering of variables*

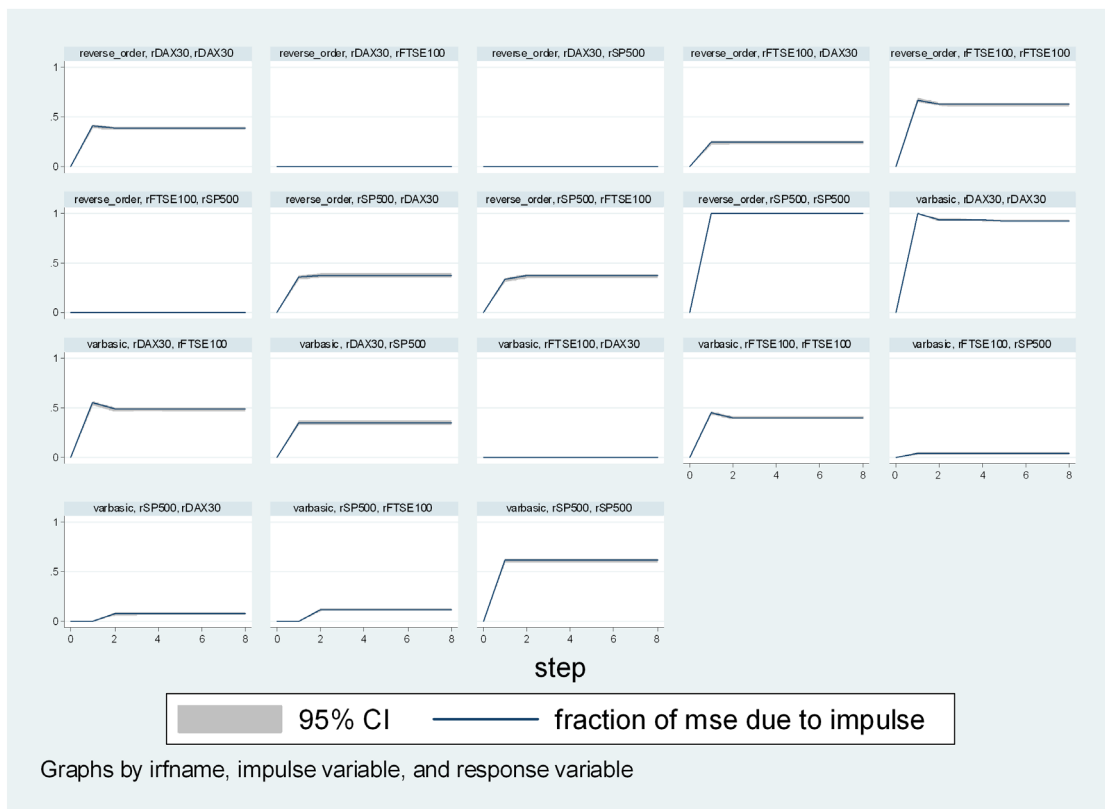
```
. irf create reverse_order, replace order(rSP500 rFTSE100 rDAX30)
irfname reverse_order not found in _varbasic.irf
(file _varbasic.irf updated)
```



. irf graph oirf



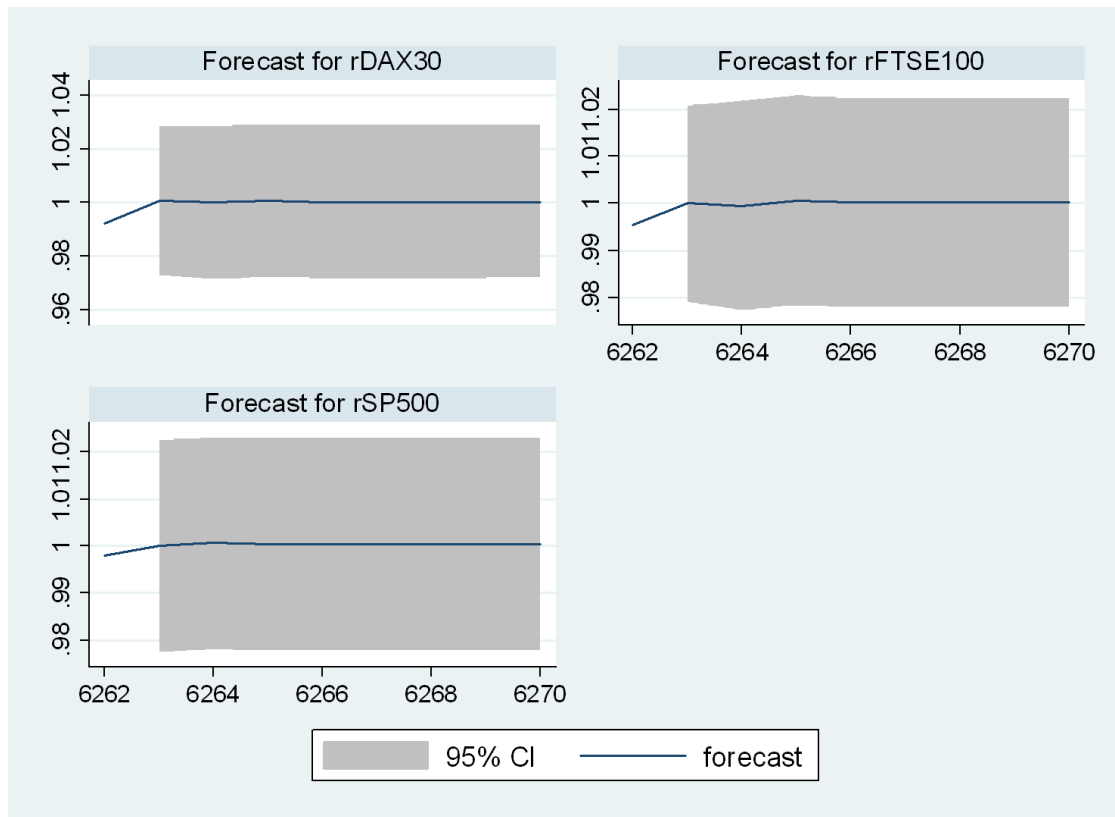
. irf graph fevd



Forecasting based on VAR(3) for the next 8 time periods (days)

```
. qui var rDAX30 rFTSE100 rSP500, lags(1/3)

. fcast compute f_, step(8)
. fcast graph f_rDAX30 f_rFTSE100 f_rSP500
```



Reproducing the Cholesky decomposition in a short-run SVAR model

```
. matrix A=(1,0,0\.,1,0\.,.,1)
. matrix B=(.,0,0\0,.,0\0,0,.)

. matrix list A

A[3,3]
      c1  c2  c3
r1    1   0   0
r2    .   1   0
r3    .   .   1

. matrix list B

symmetric B[3,3]
      c1  c2  c3
r1    .
r2    0   .
r3    0   0   .

. svar rDAX30 rFTSE100 rSP500, lags(1/3) aeq(A) beq(B)
Estimating short-run parameters

Iteration 0:   log likelihood = -19708.556
Iteration 1:   log likelihood =  31170.466
```

```

Iteration 2:   log likelihood = 44871.694
Iteration 3:   log likelihood = 51596.374
Iteration 4:   log likelihood = 56993.194
Iteration 5:   log likelihood = 60066.687
Iteration 6:   log likelihood = 60474.684
Iteration 7:   log likelihood = 60493.043
Iteration 8:   log likelihood = 60493.057
Iteration 9:   log likelihood = 60493.057

```

#### Structural vector autoregression

```

( 1)  [a_1_1]_cons = 1
( 2)  [a_1_2]_cons = 0
( 3)  [a_1_3]_cons = 0
( 4)  [a_2_2]_cons = 1
( 5)  [a_2_3]_cons = 0
( 6)  [a_3_3]_cons = 1
( 7)  [b_1_2]_cons = 0
( 8)  [b_1_3]_cons = 0
( 9)  [b_2_1]_cons = 0
(10)  [b_2_3]_cons = 0
(11)  [b_3_1]_cons = 0
(12)  [b_3_2]_cons = 0

```

```

Sample:   5 - 6262
Exactly identified model
Number of obs   =      6,258
Log likelihood  =    60493.06

```

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
/a_1_1	1	(constrained)				
/a_2_1	-.5622149	.0064251	-87.50	0.000	-.574808	-.5496219
/a_3_1	-.3043617	.0120314	-25.30	0.000	-.3279428	-.2807806
/a_1_2	0	(constrained)				
/a_2_2	1	(constrained)				
/a_3_2	-.3182253	.0158744	-20.05	0.000	-.3493385	-.2871121
/a_1_3	0	(constrained)				
/a_2_3	0	(constrained)				
/a_3_3	1	(constrained)				
/b_1_1	.0140395	.0001255	111.87	0.000	.0137936	.0142855
/b_2_1	0	(constrained)				
/b_3_1	0	(constrained)				
/b_1_2	0	(constrained)				
/b_2_2	.007136	.0000638	111.87	0.000	.007011	.007261
/b_3_2	0	(constrained)				
/b_1_3	0	(constrained)				
/b_2_3	0	(constrained)				
/b_3_3	.0089612	.0000801	111.87	0.000	.0088042	.0091182

```

. matrix Aest=e(A)
. matrix Best=e(B)

. matrix list Aest

```

```

Aest[3,3]
      rDAX30    rFTSE100    rSP500
rDAX30         1          0          0
rFTSE100    -.56221492         1          0
rSP500     -.30436171    -.31822526         1

```

```

. matrix list Best

symmetric Best[3,3]
      rDAX30    rFTSE100    rSP500
rDAX30  .01403954
rFTSE100 0    .00713597
rSP500  0    0    .00896123

. matrix ortmat_est=inv(Aest)*Best
. matrix list ortmat_est

ortmat_est[3,3]
      rDAX30    rFTSE100    rSP500
rDAX30  .01403954    0    0
rFTSE100 .00789324    .00713597    0
rSP500  .00678493    .00227085    .00896123

. qui var rDAX30 rFTSE100 rSP500, lags(1/3)
. matrix sig_var=e(Sigma)

. matrix ortmat_var=cholesky(sig_var)
. matrix list ortmat_var

ortmat_var[3,3]
      rDAX30    rFTSE100    rSP500
rDAX30  .01403954    0    0
rFTSE100 .00789324    .00713597    0
rSP500  .00678493    .00227085    .00896123

Short-run overidentified SVAR model

. matrix A=(1,0,0\.,1,0\.,0,1)
. matrix B=(.,0,0\0,.,0\0,0,.)

. matrix list A

A[3,3]
      c1  c2  c3
r1    1   0   0
r2    .   1   0
r3    .   0   1

. matrix list B

symmetric B[3,3]
      c1  c2  c3
r1    .
r2    0   .
r3    0   0   .

. svar rDAX30 rFTSE100 rSP500, lags(1/3) aeq(A) beq(B)
Estimating short-run parameters

Iteration 0:  log likelihood = -19707.394
Iteration 1:  log likelihood = 31491.505
Iteration 2:  log likelihood = 45977.397
Iteration 3:  log likelihood = 52920.253
Iteration 4:  log likelihood = 55493.852
Iteration 5:  log likelihood = 58854.138
Iteration 6:  log likelihood = 60284.729
Iteration 7:  log likelihood = 60298.307
Iteration 8:  log likelihood = 60298.314
Iteration 9:  log likelihood = 60298.314

```

# Structural vector autoregression

```
( 1) [a_1_1]_cons = 1
( 2) [a_1_2]_cons = 0
( 3) [a_1_3]_cons = 0
( 4) [a_2_2]_cons = 1
( 5) [a_2_3]_cons = 0
( 6) [a_3_2]_cons = 0
( 7) [a_3_3]_cons = 1
( 8) [b_1_2]_cons = 0
( 9) [b_1_3]_cons = 0
(10) [b_2_1]_cons = 0
(11) [b_2_3]_cons = 0
(12) [b_3_1]_cons = 0
(13) [b_3_2]_cons = 0
```

Sample: 5 - 6262

Overidentified model

Number of obs = 6,258

Log likelihood = 60298.31

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
/a_1_1	1	(constrained)				
/a_2_1	-.5622149	.0064251	-87.50	0.000	-.574808	-.5496219
/a_3_1	-.4832727	.0083236	-58.06	0.000	-.4995867	-.4669587
/a_1_2	0	(constrained)				
/a_2_2	1	(constrained)				
/a_3_2	0	(constrained)				
/a_1_3	0	(constrained)				
/a_2_3	0	(constrained)				
/a_3_3	1	(constrained)				
/b_1_1	.0140395	.0001255	111.87	0.000	.0137936	.0142855
/b_2_1	0	(constrained)				
/b_3_1	0	(constrained)				
/b_1_2	0	(constrained)				
/b_2_2	.007136	.0000638	111.87	0.000	.007011	.007261
/b_3_2	0	(constrained)				
/b_1_3	0	(constrained)				
/b_2_3	0	(constrained)				
/b_3_3	.0092445	.0000826	111.87	0.000	.0090825	.0094064

LR test of identifying restrictions: chi2(1) = 389.5

Prob > chi2 = 0.000

## Long-run overidentified SVAR model

```
. matrix C=(.,0,0\.,.,0\.,.,.)
. matrix list C
```

```
C[3,3]
      c1  c2  c3
r1     .   0   0
r2     .   .   0
r3     .   .   .
```

```
. svar rDAX30 rFTSE100 rSP500, lags(1/3) lreq(C)
Estimating long-run parameters
```

```
Iteration 0: log likelihood = -21830.051
Iteration 1: log likelihood = 29961.237
Iteration 2: log likelihood = 53616.74
Iteration 3: log likelihood = 60046.934
Iteration 4: log likelihood = 60491.992
Iteration 5: log likelihood = 60493.057
Iteration 6: log likelihood = 60493.057
```

# Structural vector autoregression

```
( 1) [c_1_2]_cons = 0
( 2) [c_1_3]_cons = 0
( 3) [c_2_3]_cons = 0
```

Sample: 5 - 6262                      Number of obs        =        6,258  
Exactly identified model              Log likelihood        =        60493.06

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
/c_1_1	.0138664	.0001239	111.87	0.000	.0136235	.0141093
/c_2_1	.0079758	.0001074	74.27	0.000	.0077653	.0081863
/c_3_1	.008072	.0001089	74.15	0.000	.0078587	.0082854
/c_1_2	0	(constrained)				
/c_2_2	.0063526	.0000568	111.87	0.000	.0062414	.0064639
/c_3_2	.0030708	.0000767	40.01	0.000	.0029204	.0032212
/c_1_3	0	(constrained)				
/c_2_3	0	(constrained)				
/c_3_3	.0056697	.0000507	111.87	0.000	.0055703	.005769

```
. matrix C=(.,0,0\.,.,0\.,0,.)
. matrix list C
```

```
C[3,3]
      c1  c2  c3
r1     .   0   0
r2     .   .   0
r3     .   0   .
```

```
. svar rDAX30 rFTSE100 rSP500, lags(1/3) lreq(C)
Estimating long-run parameters
```

```
Iteration 0: log likelihood = -21830.143
Iteration 1: log likelihood = 29935.143
Iteration 2: log likelihood = 53512.185
Iteration 3: log likelihood = 59327.247
Iteration 4: log likelihood = 59687.476
Iteration 5: log likelihood = 59688.163
Iteration 6: log likelihood = 59688.163
```

# Structural vector autoregression

```
( 1) [c_1_2]_cons = 0
( 2) [c_1_3]_cons = 0
( 3) [c_2_3]_cons = 0
( 4) [c_3_2]_cons = 0
```

Sample: 5 - 6262                      Number of obs        =        6,258  
Overidentified model                  Log likelihood        =        59688.16

	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]	
/c_1_1	.0138664	.0001239	111.87	0.000	.0136235	.0141093
/c_2_1	.0079758	.0001074	74.27	0.000	.0077653	.0081863
/c_3_1	.008072	.0001089	74.15	0.000	.0078587	.0082854
/c_1_2	0	(constrained)				
/c_2_2	.0063526	.0000568	111.87	0.000	.0062414	.0064639
/c_3_2	0	(constrained)				
/c_1_3	0	(constrained)				
/c_2_3	0	(constrained)				
/c_3_3	.0064479	.0000576	111.87	0.000	.0063349	.0065608

LR test of identifying restrictions: chi2(1) = 1610                      Prob > chi2 = 0.000

