Quantitative Methods in Finance

Tutorial, Part 9:
Distributed–lag models.

Example 1: Write down the infinite distributed-lag model and explain the terms impact, intermediate, cumulative, and long-term multiplier.

Infinite distributed-lag model and types of multipliers

In empirical estimation we often encounter lagged variables. These are very important, since they allow not only static, but also dynamic evaluation of the relations between economic and financial categories. Thus, we can e.g. distinguish between the short-term and the long-term propensity to consume, between the expected (desired) and the actual stock of capital, or between the expected and the actual amount of money in circulation.

A model that includes not only current, but also lagged values of explanatory variables (for the sake of simplicity, we will take into account only one explanatory variable, denoted by *x*) is called the distributed–lag model. This means that the effect of changes in the explanatory variable on the dependent variable is distributed over a longer period of time. If we do not determine its length, meaning that we do not define how far in the past we want to go, we get the infinite distributed-lag model, which can in its general form be written as follows:

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \ldots + u_t \, .$$

We distinguish the following types of effects:

a) Regression coefficient β_0 – *short-term or impact multiplier*: If the value of the explanatory variable in a given unit of time increases by one unit of measurement, the dependent variable changes in the same time unit on average by β_0 units of measurement.

b) Regression coefficients β_1, β_2, … – *intermediate multipliers*: If the value of the explanatory variable in a given unit of time increases by one unit of measurement, the dependent variable changes e.g. for example in two time units on average by β_2 units of measurement.

c) Subtotals of regression coefficients – *cumulative multipliers* (let us take a period of three consecutive time units): If the value of the explanatory variable in a given unit of time increases by one unit of measurement, the dependent variable changes in all three time units together on average by $\beta_0 + \beta_1 + \beta_2$ units of measurement (by β_0 in a given time unit, by β_1 in the next time unit, and by β_2 in the last time unit).

d) Sum of all regression coefficients – *long-term or total multiplier*: If the value of the explanatory variable in a given unit of time increases by one unit of measurement, the value of the dependent variable changes in the long-run on average by $\sum_{k=0}^{\infty} \beta_k$ units of measurement.

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<u>Example 2:</u> Based on data for 20 years on imports (*I*) and gross domestic product (*GDP*), we estimated the following (autoregressive) function of imports for a given country:

$$\hat{I}_t = 2.45 + 0.70 GDP_t + 0.25 I_{t-1}$$
$$t: \quad (17.0) \quad (3.2) \qquad\quad (2.5)$$

while taking into account the so-called Koyck transformation (the values are in billion EUR).

a) Write down and explain the Koyck assumption.
b) Based on the above estimates of the import function, calculate and interpret the impact multiplier, the second intermediate multiplier, and the equilibrium (total) multiplier. How is the latter parameter called in this case from the economics point of view?
c) Calculate and interpret the median lag.

Koyck model (geometric scheme of distributed lags)

The infinite distributed-lag model can in its general form be written as:

$$y_t = \alpha + \beta_0 x_t + \beta_1 x_{t-1} + \beta_2 x_{t-2} + \ldots + u_t \,.$$

When estimating such a model, we can help ourselves with a variety of assumptions about the distribution of the effects of the explanatory variable on the dependent variable. Koyck assumed that all the effects of the lagged explanatory variable have the same sign and are in time decreasing in a geometric sequence. This means that the values of the regression coefficients of the lagged explanatory variable are geometrically decreasing by moving into the past (i.e. by increasing the lag length). This assumption can be written as follows:

$$\beta_k = \beta_0 \lambda^k; \qquad k = 0, 1, 2, \ldots,$$

where the parameter λ, also referred to as the rate of decline (decay), takes the values between 0 and 1. The values of the regression coefficients for lags are dependent on the regression coefficient β_0, which represents the effect of the explanatory variable on the dependent variable contemporaneously, and the rate of decline λ. The closer the value of λ is to one (zero), the slower (faster) is the decline of the effects of a lagged explanatory variable on the dependent variable.

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Example 3: The data file `moneydemand2.dta` contains the following data for the U.S. for the period 1960–1993: the quantity of money in circulation (*M1*; in billion USD at 1987 prices), gross domestic product (*GDP*; in billion USD at 1987 prices), and the reference interest rate (*IR*; in percent). Estimate the following regression model:

$$M1_t^* = \beta_1 + \beta_2 GDP_t + \beta_3 IR_t + u_t ,$$

where $M1_t^*$ represents the desired or long-term amount of money in circulation. Explain the model that you have just taken into account and interpret the results. Check for the presence of first- and higher-order autocorrelation and, if necessary, correct for it.

Partial adjustment (stock adjustment) model

Based on the above regression model we can observe that the equilibrium (desired) amount of money in circulation is a linear function of gross domestic product and the interest rate. As the long-term quantity of money in circulation $M1^*$ is not directly observed (known), we can help ourselves with the partial adjustment process:

$$M1_t - M1_{t-1} = \delta(M1_t^* - M1_{t-1}) ; \qquad 0 < \delta \le 1 .$$

Actual change in the quantity of money in circulation in time unit *t* equals a share δ (called the adjustment coefficient) of the desired change in the quantity of money during this time unit. If we insert the initial regression model into the previous expression, we obtain:

$$M1_t - M1_{t-1} = \delta(\beta_1 + \beta_2 GDP_t + \beta_3 IR_t + u_t - M1_{t-1}) ,$$

which we can properly rearrange as:

$$M1_t = \delta\beta_1 + \delta\beta_2 GDP_t + \delta\beta_3 IR_t + (1-\delta) M1_{t-1} + \delta u_t .$$

In a completely general form, we can write the previous equation as:

$$M1_t = \gamma_1 + \gamma_2 GDP_t + \gamma_3 IR_t + \gamma_4 M1_{t-1} + v_t ,$$

where the following holds:

$$\gamma_1 = \delta\beta_1 ; \qquad \gamma_2 = \delta\beta_2 ; \qquad \gamma_3 = \delta\beta_3 ; \qquad \gamma_4 = (1-\delta) ; \qquad v_t = \delta u_t .$$

The initial model represents the long-term or equilibrium money-demand function, while the final autoregressive model represents the short-term money-demand function. The regression coefficient β_2 represents the long-term effect of the gross domestic product on the amount of money in circulation, whereas the regression coefficient γ_2 represents the short-term effect. The same reasoning applies for the regression coefficient β_3, which represents the long-term effect of the interest rate on the amount of money in circulation, whereas the regression coefficient γ_3 represents the interest rates short-term effect.

It also follows from the restrictions based on our autoregressive model that the coefficients of the long-term function are calculated by dividing the regression coefficients γ_1, γ_2 and γ_3 from the autoregressive function by the adjustment coefficient δ. The latter tells us what share of the gap between the desired and the actual amount of money in circulation is eliminated in each time unit.

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Computer printout of the results in Stata:

```
. tsset year
        time variable:  year, 1960 to 1993
                delta:  1 unit

. regress m1 gdp ir

      Source |       SS       df       MS              Number of obs =      34
-------------+------------------------------           F(  2,    31) =  141.67
       Model |   240672.24     2   120336.12           Prob > F      =  0.0000
    Residual |  26331.0384    31  849.388335           R-squared     =  0.9014
-------------+------------------------------           Adj R-squared =  0.8950
       Total |  267003.278    33  8091.00842           Root MSE      =  29.144

------------------------------------------------------------------------------
          m1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gdp |   .1192877   .0071285    16.73   0.000      .104749    .1338264
          ir |  -23.30547   2.438946    -9.56   0.000    -28.27973   -18.33121
       _cons |    425.344   19.92984    21.34   0.000     384.6968    465.9912
------------------------------------------------------------------------------

. regress m1 gdp ir l.m1

      Source |       SS       df       MS              Number of obs =      33
-------------+------------------------------           F(  3,    29) =  142.25
       Model |  240881.011     3  80293.6704           Prob > F      =  0.0000
    Residual |    16369.56    29  564.467588           R-squared     =  0.9364
-------------+------------------------------           Adj R-squared =  0.9298
       Total |  257250.571    32  8039.08035           Root MSE      =  23.759

------------------------------------------------------------------------------
          m1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gdp |   .0566796   .0160087     3.54   0.001      .023938    .0894211
          ir |  -11.64861   3.421357    -3.40   0.002    -18.64607   -4.651148
             |
          m1 |
         L1. |   .6254733   .1489465     4.20   0.000     .3208434    .9301032
             |
       _cons |   148.0334   68.39056     2.16   0.039     8.159004    287.9078
------------------------------------------------------------------------------

. predict em1, resid
(1 missing value generated)
```

```
. regress em1 l.em1, nocons

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  1,    31) =   12.65
       Model |  4721.54612     1  4721.54612           Prob > F      =  0.0012
    Residual |  11569.7725    31  373.218469           R-squared     =  0.2898
-------------+------------------------------           Adj R-squared =  0.2669
       Total |  16291.3187    32  509.103708           Root MSE      =  19.319

------------------------------------------------------------------------------
         em1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         em1 |
         L1. |   .5563226   .1564106     3.56   0.001     .2373211    .8753241
------------------------------------------------------------------------------

. scalar rho=_b[l.em1]

. regress em1 gdp ir l.m1 l.em1

      Source |       SS       df       MS              Number of obs =      32
-------------+------------------------------           F(  4,    27) =    3.41
       Model |  5470.19205     4  1367.54801           Prob > F      =  0.0221
    Residual |  10818.6816    27   400.69191           R-squared     =  0.3358
-------------+------------------------------           Adj R-squared =  0.2374
       Total |  16288.8736    31  525.447536           Root MSE      =  20.017

------------------------------------------------------------------------------
         em1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gdp |   .0174963   .0142944     1.22   0.232    -.0118334    .0468261
          ir |  -2.603332   2.996615    -0.87   0.393    -8.751878    3.545215
             |
          m1 |
         L1. |  -.1725952    .134589    -1.28   0.211    -.4487489    .1035585
             |
         em1 |
         L1. |   .6440759   .1744669     3.69   0.001     .2860995    1.002052
             |
       _cons |   71.01483   61.81335     1.15   0.261     -55.8157    197.8454
------------------------------------------------------------------------------

. scalar lm=e(N)*e(r2)

. display lm, invchi2tail(1,0.05), chi2tail(1,lm)
10.746363 3.8414588 .00104484

. qui regress m1 gdp ir l.m1
. estat bgodfrey, nomiss0

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |         10.746               1                   0.0010
---------------------------------------------------------------------------
                        H0: no serial correlation
```

```
. estat bgodfrey

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |         11.030               1                   0.0009
---------------------------------------------------------------------------
                        H0: no serial correlation

. estat bgodfrey, lag(1/28)

Breusch-Godfrey LM test for autocorrelation
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |         11.030               1                   0.0009
       2     |         11.994               2                   0.0025
       3     |         13.825               3                   0.0032
       4     |         13.888               4                   0.0077
       5     |         14.466               5                   0.0129
       6     |         14.555               6                   0.0240
       7     |         19.916               7                   0.0058
       8     |         20.909               8                   0.0074
       9     |         20.933               9                   0.0130
      10     |         22.895              10                   0.0111
      11     |         24.361              11                   0.0113
      12     |         24.745              12                   0.0161
      13     |         24.751              13                   0.0249
      14     |         24.751              14                   0.0371
      15     |         25.032              15                   0.0495
      16     |         25.033              16                   0.0693
      17     |         25.549              17                   0.0831
      18     |         25.552              18                   0.1104

      .
      .
      .

      26     |         32.713              26                   0.1706
      27     |         32.713              27                   0.2067
      28     |         32.714              28                   0.2464
---------------------------------------------------------------------------
                        H0: no serial correlation

. newey m1 gdp ir l.m1, lag(17)

Regression with Newey-West standard errors          Number of obs  =        33
maximum lag: 17                                     F(  3,    29)  =    376.56
                                                    Prob > F       =    0.0000

------------------------------------------------------------------------------
             |             Newey-West
          m1 |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         gdp |   .0566796   .0153896     3.68   0.001     .0252044    .0881547
          ir |  -11.64861   2.656746    -4.38   0.000    -17.08226   -6.214952
             |
          m1 |
         L1. |   .6254733   .1481754     4.22   0.000     .3224206     .928526
             |
       _cons |   148.0334   63.55881     2.33   0.027     18.04103    278.0258
------------------------------------------------------------------------------
```

■