Quantitative Methods in Finance

Tutorial, Part 13:
Univariate modelling of volatility.

Example: We will examine and model volatility based on univariate volatility models, where we focus on the return to the euro exchange rate against the US dollar. The data are daily and run from 7 July 2002 to 6 June 2013, giving a total of 3,988 observations. The data are contained in Stata data file `currencies.dta`. The programming code is given in Stata Do file `currencies-commands-t13.do`.

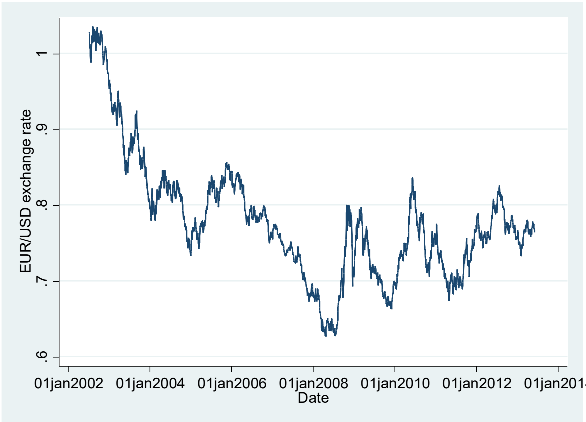
a) Inspect the euro exchange rate time series visually. Does it look stationary? Construct continuously compounded percentage returns (percentage changes). Check for stationarity of the newly generated time series. What do you find?
b) Fit a constant-only model of the percentage return to the euro by the ordinary least squares estimator and test for ARCH effects by using Engle's Lagrange multiplier test. What do you find?
c) Identify the mean equation as an ARMA model (or components thereof) by inspecting the appropriate information criterion. Also, identify the variance equation as a GARCH(1,1) model. Estimate the two equations together by the maximum likelihood estimator. Test jointly the ARCH(1) and GARCH(1) coefficients for statistical significance.
d) Let us assume that the financial markets respond differently to unanticipated increases in the euro rate than they do to unanticipated decreases. We can see if the data support this supposition by specifying a GARCH model that allows an asymmetric effect of "news" (disturbances or unanticipated changes). Estimate two such models; the GJR model and the exponential GARCH (EGARCH) model. For the former, also generate the news-response curve. What do you find?
e) Let us assume that there exists a feedback from the conditional variance to the conditional mean, i.e. that the eure rate is partly determined by its volatility (risk). We can see if the data support this supposition by specifying a GARCH-in-mean (GARCH–M) model with a feedback term in the mean equation. What do you find?
f) Suppose that the GJR model was estimated using the first 2,890 observations (from 8 July 2002 till 5 June 2010), leaving 1,097 remaining observations for constructing mean and volatility forecasts (for the period from 6 June 2010 till 6 June 2013). Generate static and dynamic mean and volatility forecasts and present them graphically. What do you find?

Computer printout of the results in Stata:

Data exploration

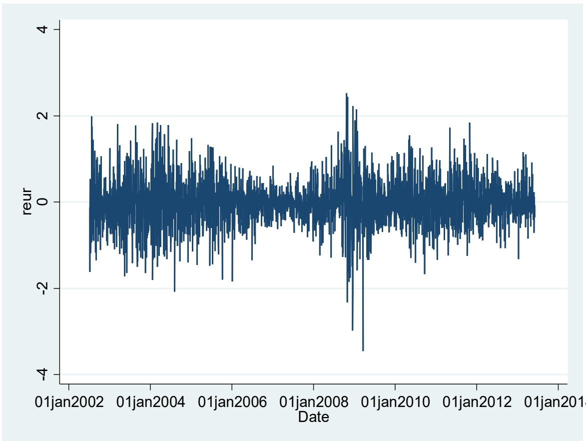
```
. tsset Date
        time variable:  Date, 07jul2002 to 06jun2013
                delta:  1 day
```

```
. twoway line EUR Date
```



Generating continuously compounded percentage returns (percentage changes) and performing the ADF test

```
. generate reur=100*(ln(EUR/L.EUR))
(1 missing value generated)

. twoway line reur Date
```




```
. dfgls reur, notrend maxlag(7)

DF-GLS test for unit root                  Number of obs = 3,979
Variable: reur
Lag selection: User specified              Maximum lag   =     7

                              -------- Critical value ---------
 [lags]        DF-GLS mu           1%            5%           10%
----------------------------------------------------------------
       7         -18.102       -2.580        -1.951        -1.628
       6         -19.396       -2.580        -1.951        -1.628
       5         -21.739       -2.580        -1.951        -1.628
       4         -24.261       -2.580        -1.952        -1.628
       3         -28.166       -2.580        -1.952        -1.628
       2         -32.168       -2.580        -1.952        -1.629
       1         -38.850       -2.580        -1.952        -1.629
----------------------------------------------------------------
Opt lag (Ng-Perron seq t) = 6 with RMSE = .4781864
Min SIC  = -1.461914 at lag 1 with RMSE = .4804462
Min MAIC = -.6670497 at lag 7 with RMSE = .4780518
```

Fitting a constant-only model and testing for ARCH effects

```
. regress reur

      Source |       SS           df       MS      Number of obs   =     3,987
-------------+----------------------------------   F(0, 3986)      =      0.00
       Model |           0         0           .   Prob > F        =         .
    Residual |   905.48945     3,986  .227167449   R-squared       =    0.0000
-------------+----------------------------------   Adj R-squared   =    0.0000
       Total |   905.48945     3,986  .227167449   Root MSE        =    .47662

------------------------------------------------------------------------------
        reur | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
       _cons |  -.0074132   .0075483    -0.98   0.326    -.0222121    .0073857
------------------------------------------------------------------------------

. estat archlm, lags(1/10)
LM test for autoregressive conditional heteroskedasticity (ARCH)
---------------------------------------------------------------------------
    lags(p)  |          chi2               df                 Prob > chi2
-------------+-------------------------------------------------------------
       1     |        191.261               1                   0.0000
       2     |        205.141               2                   0.0000
       3     |        207.426               3                   0.0000
       4     |        207.624               4                   0.0000
       5     |        216.005               5                   0.0000
       6     |        235.139               6                   0.0000
       7     |        313.283               7                   0.0000
       8     |        317.933               8                   0.0000
       9     |        331.409               9                   0.0000
      10     |        331.086              10                   0.0000
---------------------------------------------------------------------------
         H0: no ARCH effects      vs.  H1: ARCH(p) disturbance
```

Identifying the ARMA components and estimating a GARCH(1,1) model

```
. generate variable=reur
(1 missing value generated)

. scalar pset=4
. scalar qset=4

. forvalues p=0/`=pset' {
  2. forvalues q=0/`=qset' {
  3. qui arima variable, arima(`p',0,`q')
  4. scalar bic_`p'_`q'=-2*e(ll)+e(rank)*ln(e(N))
  5. }
  6. }

. forvalues p=0/`=pset' {
  2. forvalues q=0/`=qset' {
  3. display "SBIC(`p',`q') = "bic_`p'_`q'
  4. }
  5. }
SBIC(0,0) = 5421.1927
SBIC(0,1) = 5334.993
SBIC(0,2) = 5342.9514
SBIC(0,3) = 5351.1197
SBIC(0,4) = 5355.7096
SBIC(1,0) = 5335.735
SBIC(1,1) = 5342.9826
SBIC(1,2) = 5351.2263
SBIC(1,3) = 5358.6979
```

```
SBIC(1,4) = 5363.4543
SBIC(2,0) = 5342.8287
SBIC(2,1) = 5350.2025
SBIC(2,2) = 5357.9651
SBIC(2,3) = 5356.9569
SBIC(2,4) = 5371.6321
SBIC(3,0) = 5350.4465
SBIC(3,1) = 5358.2112
SBIC(3,2) = 5364.3527
SBIC(3,3) = 5364.6364
SBIC(3,4) = 5371.7929
SBIC(4,0) = 5356.5335
SBIC(4,1) = 5363.1554
SBIC(4,2) = 5371.4462
SBIC(4,3) = 5371.9948
SBIC(4,4) = 5380.7008
```

. arch reur, ma(1) arch(1) garch(1) nolog

```
ARCH family regression -- MA disturbances


Sample: 08jul2002 - 06jun2013                     Number of obs   =      3,987
Distribution: Gaussian                            Wald chi2(1)    =     183.13
Log likelihood = -2320.973                        Prob > chi2     =     0.0000

------------------------------------------------------------------------------
             |                 OPG
        reur | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------

reur         |
       _cons |  -.0145532   .0074799    -1.95   0.052    -.0292136    .0001071
-------------+----------------------------------------------------------------
ARMA         |
          ma |
         L1. |   .2011728   .0148657    13.53   0.000     .1720366     .230309
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .0253159   .0021344    11.86   0.000     .0211326    .0294992
             |
       garch |
         L1. |   .9732146   .0019799   491.55   0.000     .9693341    .9770951
             |
       _cons |   .0003358   .0001139     2.95   0.003     .0001125    .0005591
------------------------------------------------------------------------------
```

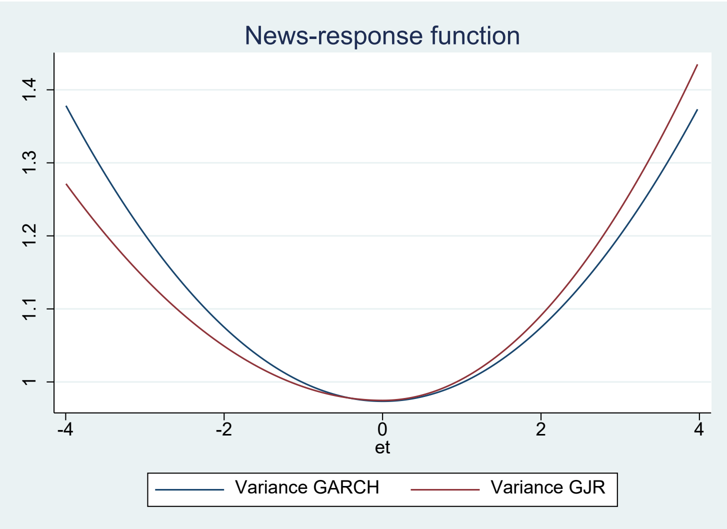
. test [ARCH]L1.arch [ARCH]L1.garch

```
 ( 1)  [ARCH]L.arch = 0
 ( 2)  [ARCH]L.garch = 0

           chi2(  2) = 1.7e+06
         Prob > chi2 =    0.0000
```

. generate et=(_n-1994)/500
. predict var_GARCH, variance at(et 1)
. label variable var_GARCH "Variance GARCH"

Estimating asymmetric GARCH(1,1) models

```
. arch reur, ma(1) arch(1) tarch(1) garch(1) nolog     // GJR model //

ARCH family regression -- MA disturbances

Sample: 08jul2002 - 06jun2013                     Number of obs     =      3,987
Distribution: Gaussian                            Wald chi2(1)      =     182.62
Log likelihood = -2318.698                        Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |                 OPG
        reur | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
reur         |
       _cons |  -.0124398    .007545    -1.65   0.099    -.0272278    .0023482
-------------+----------------------------------------------------------------
ARMA         |
          ma |
         L1. |   .2004768   .0148349    13.51   0.000     .1714008    .2295527
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .0185563   .0029701     6.25   0.000      .012735    .0243776
             |

       tarch |
         L1. |   .0105838   .0038379     2.76   0.006     .0030617    .0181058
             |
       garch |
         L1. |   .9743547   .0019509   499.45   0.000      .970531    .9781783
             |
       _cons |   .0003619   .0001098     3.30   0.001     .0001467     .000577
------------------------------------------------------------------------------

. predict var_GJR, variance at(et 1)
. label variable var_GJR "Variance GJR"
. line var_GARCH var_GJR et in 2/l, m(i) c(l) title(News-response function)
```




```
. arch reur, ma(1) earch(1) egarch(1) nolog       // EGARCH model //

ARCH family regression -- MA disturbances

Sample: 08jul2002 - 06jun2013                     Number of obs     =      3,987
Distribution: Gaussian                            Wald chi2(1)      =     189.82
```

```
Log likelihood = -2325.341                        Prob > chi2     =     0.0000

------------------------------------------------------------------------------
             |                 OPG
        reur | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
reur         |
       _cons |  -.0138688   .0075768    -1.83   0.067    -.0287191    .0009815
-------------+----------------------------------------------------------------
ARMA         |
          ma |
         L1. |   .1983254    .014395    13.78   0.000     .1701119     .226539
-------------+----------------------------------------------------------------
ARCH         |
       earch |
         L1. |   .0069072   .0036314     1.90   0.057    -.0002101    .0140246
             |
     earch_a |
         L1. |   .0600615    .004954    12.12   0.000     .0503518    .0697712
             |
      egarch |
         L1. |   .9975515   .0006838  1458.73   0.000     .9962111    .9988918
             |
       _cons |  -.0001273   .0012745    -0.10   0.920    -.0026252    .0023706
------------------------------------------------------------------------------
```

Estimating a GARCH-M model

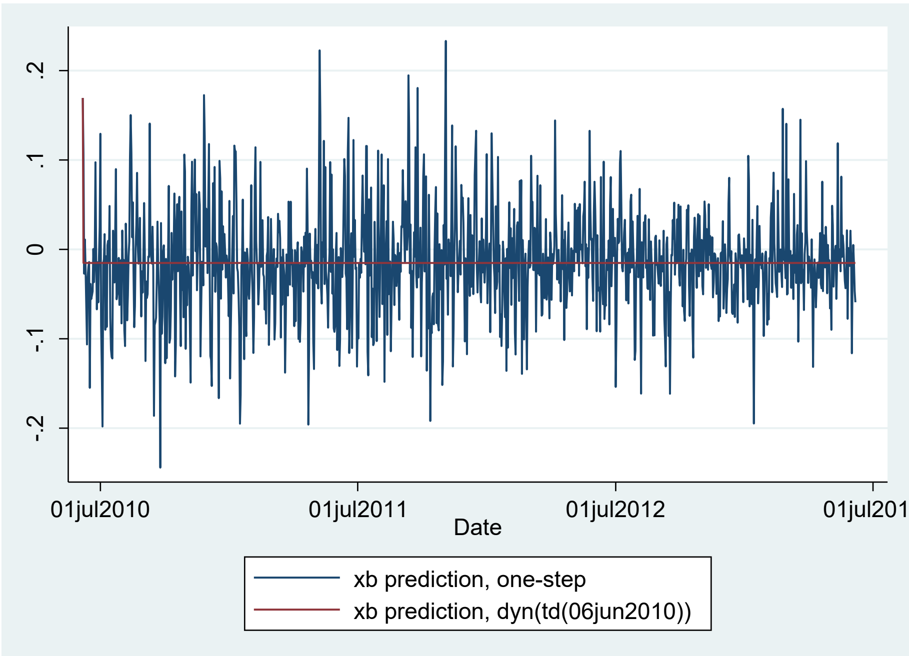
. arch reur, ma(1) arch(1) garch(1) archm nolog

```
ARCH family regression -- MA disturbances

Sample: 08jul2002 - 06jun2013                     Number of obs   =      3,987
Distribution: Gaussian                            Wald chi2(2)    =     183.18
Log likelihood =  -2320.24                        Prob > chi2     =     0.0000

------------------------------------------------------------------------------
             |                 OPG
        reur | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
reur         |
       _cons |  -.0280391   .0136946    -2.05   0.041      -.05488   -.0011982
-------------+----------------------------------------------------------------
ARCHM        |
      sigma2 |   .0865367   .0734325     1.18   0.239    -.0573884    .2304617
-------------+----------------------------------------------------------------
ARMA         |
          ma |
         L1. |   .2006999   .0148769    13.49   0.000     .1715417     .229858
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .0255826   .0021656    11.81   0.000     .0213382    .0298271
             |
       garch |
         L1. |   .9729244   .0020038   485.55   0.000     .9689971    .9768517
             |
       _cons |   .0003427   .0001146     2.99   0.003     .0001181    .0005673
------------------------------------------------------------------------------
```

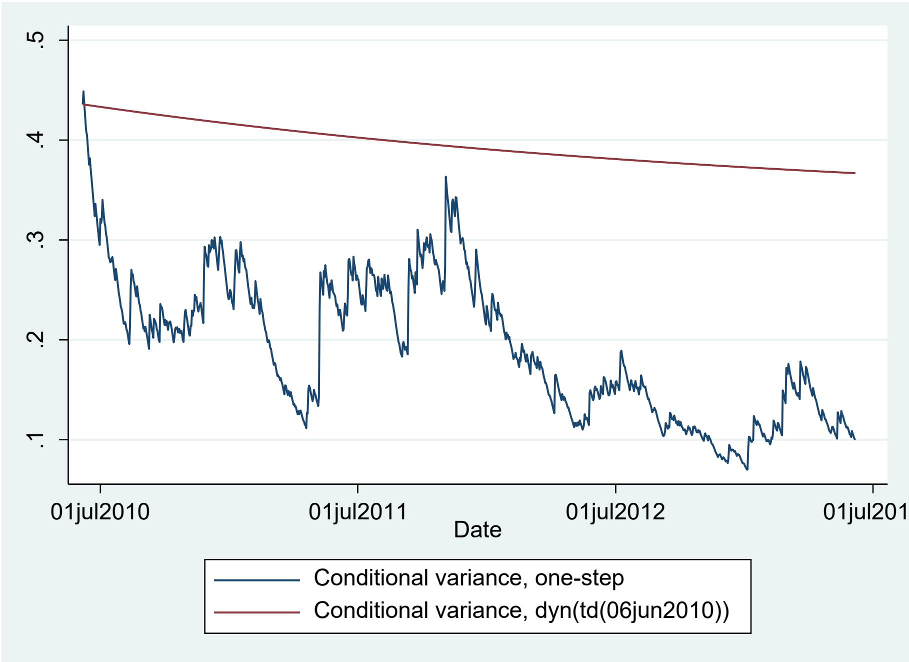
Mean and volatility forecasting based on the GJR model

. arch reur if Date<=td(05jun2010), ma(1) arch(1) tarch(1) garch(1) nolog

```
ARCH family regression -- MA disturbances

Sample: 08jul2002 - 05jun2010                    Number of obs     =      2,890
Distribution: Gaussian                           Wald chi2(1)      =      66.82
Log likelihood = -1753.218                       Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |                 OPG
        reur | Coefficient  std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
reur         |
       _cons |  -.0151996   .0084702    -1.79   0.073     -.031801    .0014017
-------------+----------------------------------------------------------------
ARMA         |
          ma |
         L1. |   .1437661   .0175869     8.17   0.000     .1092965    .1782357
-------------+----------------------------------------------------------------
ARCH         |
        arch |
         L1. |   .0169227   .0032851     5.15   0.000      .010484    .0233614
             |
       tarch |
         L1. |   .0161222   .0045458     3.55   0.000     .0072126    .0250319
             |
       garch |
         L1. |   .9740383   .0020868   466.75   0.000     .9699481    .9781284
             |
       _cons |   .0003237   .0001169     2.77   0.006     .0000947    .0005527
------------------------------------------------------------------------------

. predict mean_stat_reur, xb
. predict mean_dyn_reur, xb dynamic(td(06jun2010))

. twoway (tsline mean_stat_reur) (tsline mean_dyn_reur) if Date>=td(06jun2010),
  legend(rows(2))
```



```
. predict var_stat_reur, variance
. predict var_dyn_reur, variance dynamic(td(06jun2010))

. twoway (tsline var_stat_reur) (tsline var_dyn_reur) if Date>=td(06jun2010),
  legend(rows(2))
```



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