University of Ljubljana

SEB SCHOOL OF ECONOMICS AND BUSINESS

3. Hypotheses Testing

Prof. Dr. Miroslav Verbič

miroslav.verbic@ef.uni-lj.si

www.miroslav-verbic.si

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Motivation

The basic task of econometrics is to confront the existing economic theories with the economic reality by using the methods of statistical inference.

P. Samuelson, *Econometrica*, 1954 (22)

Statistical Inference

Null hypothesis → **Alternative hypothesis**

$$H_0 : \beta_j = \beta_0 \quad ; \quad H_1 : \beta_j \neq \beta_0$$

$$H_0 : \beta_j \leq \beta_0 \quad ; \quad H_1 : \beta_j > \beta_0$$

$$H_0 : \beta_j \geq \beta_0 \quad ; \quad H_1 : \beta_j < \beta_0$$

Simple hypothesis ↔ **Composite hypothesis**

One-tailed test ↔ **Two-tailed test**

Statistical Inference

Goal: REJECT H_0

What to put into the null hypothesis?

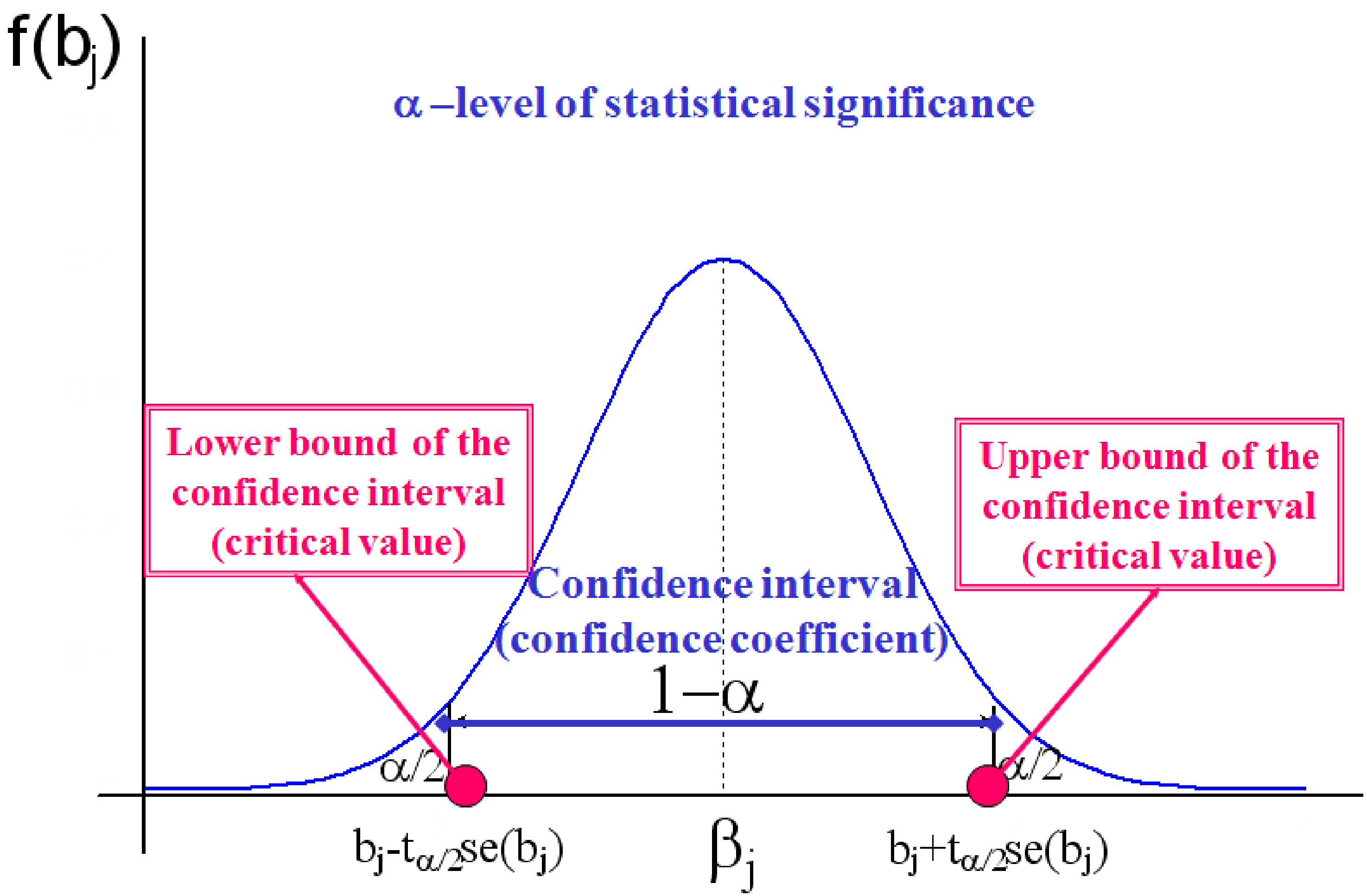
H_o is not rejected! → **H_o is rejected!**

Computer software includes (direct) testing of the following hypothesis:

$$H_0 : \beta_j = \beta_0 \quad ; \quad H_1 : \beta_j \neq \beta_0$$

Simple hypothesis! **Two-tailed test!**

Defining the Confidence Intervals



Hypotheses about Individual Values of β_j

Essential elements:

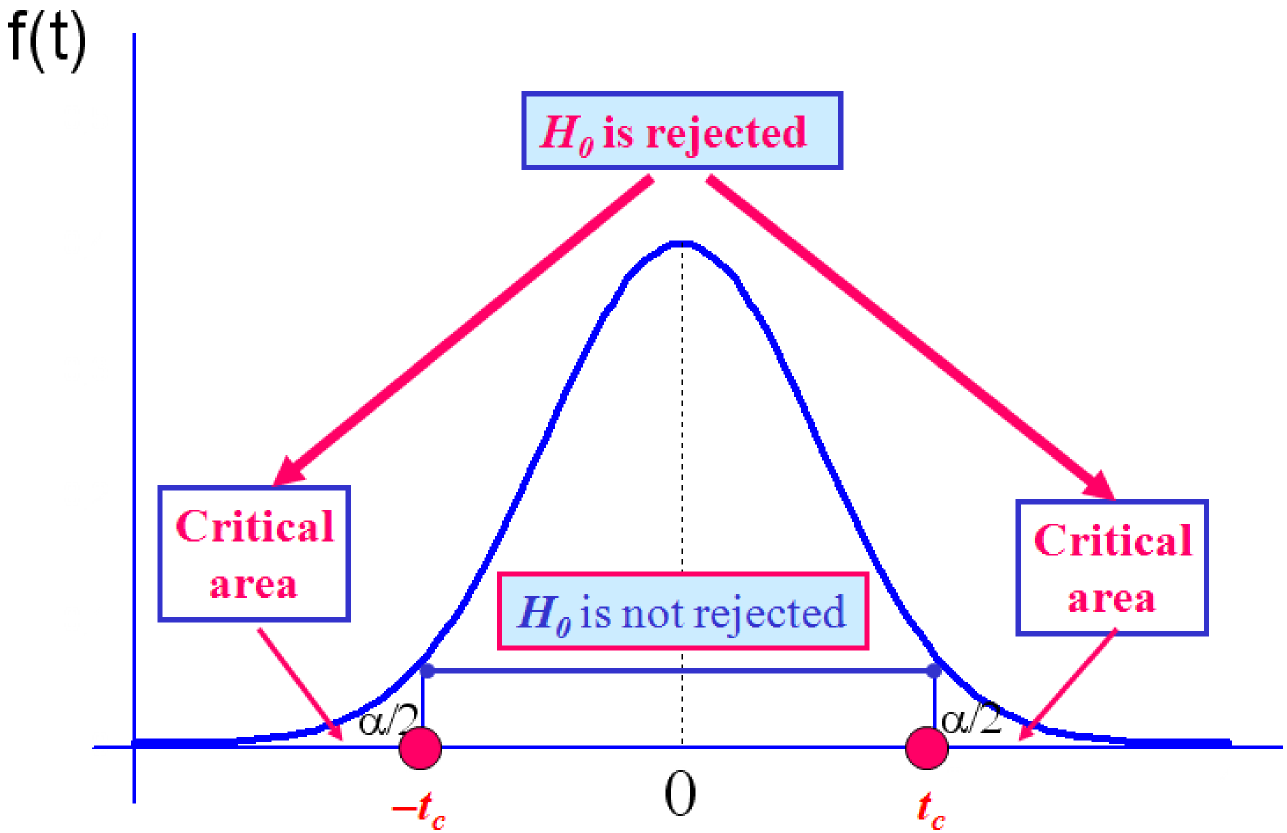
① test statistic

② sample distribution of the test statistic taking into account H_o

$$t = \frac{b_j - \beta_0}{\sqrt{\mathrm{var}(b_j)}} = \frac{b_j - \beta_0}{se(b_j)} \underset{H_0}{\sim} t_{(n-k)}$$

$$\Pr(-t_{\alpha/2} \leq \frac{b_j - \beta_0}{se(b_j)} \leq t_{\alpha/2}) = 1 - \alpha$$

Hypotheses about Individual Values of β_j



If the value of t–statistic is in the critical area,
H_0 is rejected at the level of statistical significance α!

Hypotheses about Individual Values of β_j

Hypothesis testing scheme about the values of regression coefficients based on *t*–statistic

Hypothesis type	H_0	H_1	H_0 is rejected, if
Two-tailed test	$\beta_j = \beta_{j0}$	$\beta_j \neq \beta_{j0}$	$\lvert t \rvert \geq t_{\alpha/2}$
One-tailed test	$\beta_j \leq \beta_{j0}$	$\beta_j > \beta_{j0}$	$t \geq t_{\alpha}$
One-tailed test	$\beta_j \geq \beta_{j0}$	$\beta_j < \beta_{j0}$	$t \leq -t_{\alpha}$

Type I Error ! ⟷ **Type II Error !**

Rule of thumb:

***H_0* is rejected at $\alpha = 0.05$**

- **$t = 2$ or above; two-tailed test**
- **$t = 1.65$ or above; one-tailed test**

Exact Level of Statistical Significance

***p*–value** or **exact level of statistical significance** is the *smallest* statistical significance level that enables *rejection of the null hypothesis* based on a *given data sample*.

E.g. $p = \Pr\left(|t| \geq t_{\alpha/2}\right)$ for the two-tailed test.

The calculated *p*–value is *different* for *every data sample*.

Most often used rule of thumb: *H_o* is rejected, if it holds:

$$p \leq (\alpha = 0.05)$$

In general: *p*–value is the probability of obtaining test statistic value at least as extreme as the one actually observed, under the assumption that the *null hypothesis is true*.

Meaning of Hypotheses Testing

Rejection of the null hypothesis means:

- The ***difference*** between the regression coefficient estimate and the value β_0, tested in the null hypothesis, is ***"statistically significant"*** (statistically significantly different from 0).
- It is unlikely that the difference was established (purely) by chance. This probability is equal to the level of statistical significance of the test α, or $\alpha/2$ for the two-tailed test.
- If the value of regression coefficient, tested in the null hypothesis, is equal to zero ($\beta_0 = 0$), then rejection of the null hypothesis means that the ***regression coefficient estimate*** is statistically significantly different from 0 (the estimated regression coefficient is ***"statistically significant"***).

Meaning of Hypotheses Testing

Statistical significance ⟷ **Economic / theoretical acceptability**

- **A statistically significant effect of an explanatory variable on the dependent variable, measured by the value of the *t*–statistic, does not yet mean the theoretical acceptability of the given explanatory variable.**
- **High value of the *t*–statistic does not mean that this explanatory variable is more important than another explanatory variable with a lower value of *t*–statistic. We can not make conclusions about this based on the value b_j either. For this purpose we use the standardised regression coefficients.**
- **We have to be aware that the value of *t*–statistic depends on several already mentioned determinants. This has to be taken into account when doing statistical inference.**

Statistical Inference

Conclusion based on sample data	Actual state of nature (on the population)	
	H_0 is true	H_0 is false
H_0 is rejected	False conclusion $Pr = \alpha$ (Type I Error)	Correct conclusion $Pr = 1 - \beta$ (Power of the test)
H_0 is not rejected	Correct conclusion $Pr = 1 - \alpha$	False conclusion $Pr = \beta$ (Type II Error)

Due to the presence of Type II Error, the null hypothesis is either rejected or not rejected, but not accepted. If the null hypothesis is rejected, a conclusion is made from the alternative hypothesis. The alternative hypothesis is neither accepted nor rejected, except on those occasions, when its distribution is explicitly known.

Statistical Inference in Econometrics

Goal: Application of the *linear regression model* for determining the validity of theory based on a sample of data.

The model is used for testing hypotheses about a given data generating process (DGP), where we consider only linear restrictions (for now).

The hypothesis has to be testable:

– we are able to write it down in terms of mathematical notation;

– we are able to test it within our model.

Statistical Inference in Econometrics

Nested and non-nested models: models are nested, if one can be obtained by applying a restriction on the other.

$$
\begin{aligned}
M_1 &= (\beta_1, \beta_2, \beta_3, \beta_4, \beta_5) \\
M_2 &= (\beta_1, 0\ , \beta_3, \beta_4, \beta_5) \\
M_3 &= (\beta_1, \beta_2, 0\ , \beta_4, \beta_5)
\end{aligned}
$$

Restricted models and non-restricted models:

– the restricted model (“null” model) is the model that has the null hypothesis already incorporated;

– the non-restricted model (“alternative” model) is the base model, i.e. the model from the alternative hypothesis.

Testing Linear Combinations of b

$$H_0 : \mathbf{c}^{\mathrm{T}}\boldsymbol{\beta} = r \quad ; \quad H_1 : \mathbf{c}^{\mathrm{T}}\boldsymbol{\beta} \neq r$$

$$t = \frac{\mathbf{c}^{\mathrm{T}}\mathbf{b} - r}{se(\mathbf{c}^{\mathrm{T}}\mathbf{b})} = \frac{\mathbf{c}^{\mathrm{T}}\mathbf{b} - r}{\sqrt{\operatorname{var}(\mathbf{c}^{\mathrm{T}}\mathbf{b})}}$$

$$\operatorname{var}(\mathbf{c}^{\mathrm{T}}\mathbf{b}) = \mathbf{c}^{\mathrm{T}}((\operatorname{var}-\operatorname{cov}(\mathbf{b}))\mathbf{c}$$

Testing Linear Combinations of **b**

$$\text{var}-\text{cov}(\mathbf{b}) = s_e^2\left(\mathbf{X}^{\mathrm{T}}\mathbf{X}\right)^{-1}$$

$$var\left(ab_j + cb_l\right) = a^2\,var\left(b_j\right) + c^2\,var\left(b_l\right) + 2ac\,cov\left(b_j, b_l\right)$$

$$var\left(ab_j + cb_l + db_m\right) = a^2\,var\left(b_j\right) + c^2\,var\left(b_l\right) + d^2\,var\left(b_m\right) +$$
$$+2ac\,cov\left(b_j, b_l\right) + 2ad\,cov\left(b_j, b_m\right) + 2cd\,cov\left(b_l, b_m\right)$$

Trinity of Tests in Econometrics

1. **Wald Test (*F*–test)**
2. **Lagrange Multiplier Test (*LM*–test)**
3. **Likelihood Ratio Test (*LR*–test)**

Fundaments of *F*–test and *LM*–test

BASE MODEL

RSS_B

$$RSS_B = \mathbf{e}_B^T \mathbf{e}_B$$

RESTRICTED MODEL

RSS_R

$$RSS_R = \mathbf{e}_R^T \mathbf{e}_R$$

$$RSS_R \geq RSS_B$$

$$R_R^2 \leq R_B^2$$

?

Are the differences in *RSS* statistically significant?

Wald Test (*F*–test)

$$F = \frac{(RSS_R - RSS_B)/m}{RSS_B/(n-k_B)} \sim F_{(m,\, n-k_B)}$$

Degrees of freedom:

$m = (n - k_R) - (n - k_B) = k_B - k_R$

$$F = \frac{((1-R_R^2)-(1-R_B^2))/m}{(1-R_B^2)/(n-k_B)} = \frac{(R_B^2-R_R^2)/m}{(1-R_B^2)/(n-k_B)} \sim F_{(m,\, n-k_B)}$$

I. $F < F_c$ **at given statistical significance level;**
H_0 **is not rejected**

II. $F \geq F_c$ **at given statistical significance level;**
H_0 **is rejected**

Wald Test (*F*–test)

Comparable multiple determination coefficient of the restricted model, when the dependent variable in the restricted model is defined differently than in the base (unrestricted) model:

$$R_R^2 \quad \Rightarrow \quad R_R^{*2} = 1 - \frac{RSS_R}{TSS_B}$$

Wald Test (*F*–test)

APPLICATION OF THE *F*–STATISTIC (WALD TEST)

❶ **Testing hypotheses on particular values of regression coefficients or linear combinations of regression coefficients (equivalent to the *t*–statistic)**

$$F = \frac{(RSS_R - RSS_B)/m}{RSS_B/(n-k_B)} \sim F_{(m,\, n-k_B)}$$

Wald Test (*F*–test)

❷ Simultaneous testing of multiple hypotheses on regression coefficients

A Testing statistical significance of model reduction "General to Simple"

$$H_o: \beta_l = 0 \\ \beta_{l+1} = 0 \\ \vdots \\ \beta_{l+(m-1)} = 0$$

H_1 : At least one value is not equal to 0

$$F = \frac{(RSS_R - RSS_B)/m}{RSS_B/(n-k_B)} \sim F_{(m,n-k_B)}$$

Wald Test (*F*–test)

B Testing statistical significance of model expansion "Simple to General"

$$H_o: \beta_{k+1} = 0$$
$$\beta_{k+2} = 0$$
$$\vdots$$
$$\beta_{k+m} = 0$$

H_1 : At least one value is not equal to 0

$$F = \frac{(RSS_B - RSS_N)/m}{RSS_N/(n-k_N)} \sim F_{(m,\, n-k_N)}$$

Lagrange Multiplier (LM) Test

Example of model reduction:

$$y_i = \beta_1 + \beta_2 x_{2i} + \cdots + \beta_k x_{ki} + \beta_{k+1} x_{k+1,i} + \cdots + \beta_{k+m} x_{k+m,i} + u_i$$

1

$$H_o : \beta_{k+1} = 0; \quad \beta_{k+2} = 0; \quad \ldots; \quad \beta_{k+m} = 0$$

H_1 : At least one value is not equal to 0

2

Estimate the restricted regression model and calculate e_i

$$y_i = g_1 + g_2 x_{2i} + \cdots + g_k x_{ki} + e_i \quad \Rightarrow \quad e_i$$

Lagrange Multiplier (LM) Test

3 **Estimate the auxiliary regression:**

$$\hat{e}_i = g_1 + g_2 x_{2i} + \cdots + g_k x_{ki} + g_{k+1} x_{k+1,i} + \cdots + g_{k+m} x_{k+m,i} \quad \Rightarrow \quad R^2$$

4 $$LM = nR^2 \sim \chi_m^2$$

5 $$LM \geq \chi_m^2$$

H_o **is rejected**

Likelihood Ratio (LR) Test

BASE MODEL

$\ln L_B$

RESTRICTED MODEL

$\ln L_R$

$$\ln L = -\frac{n}{2}\left[\ln(2\pi) + \ln\left(\frac{RSS}{n}\right) + 1\right]$$

?

Are the differences in ln L statistically significant?

$$LR = -2\left(\ln L_R - \ln L_B\right) \sim \chi^2_m$$

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