

Autocorrelation

$$\text{Var} - \text{Cov}(u) = \underline{W} \neq \sigma^2 \cdot I \quad \text{Slides 44-45.}$$

Example, AR(1):

$$W = \frac{1}{1 - \rho^2} \cdot \begin{bmatrix} \sigma^2 & \rho \sigma^2 & \dots & \dots & \rho^{T-1} \sigma^2 \\ \rho \sigma^2 & \sigma^2 & \dots & \dots & \rho^{T-2} \sigma^2 \\ \rho^2 \sigma^2 & \rho \sigma^2 & \dots & \dots & \rho^{T-3} \sigma^2 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ \rho^{T-1} \sigma^2 & \rho^{T-2} \sigma^2 & \dots & \dots & \sigma^2 \end{bmatrix}$$

ρ - coefficient of first-order autocorrelation

Slide 45:

No autocorrelation, $W = \sigma^2 \cdot I$:

$$\begin{aligned} \text{Var} - \text{Cov}(b) &= (X^T X)^{-1} X^T \sigma^2 I X (X^T X)^{-1} = \\ &= \sigma^2 \cdot (X^T X)^{-1} X^T X (X^T X)^{-1} \\ &= \sigma^2 \cdot (X^T X)^{-1} \underbrace{I}_{\text{}} \end{aligned}$$

Autocorrelation, $W \neq \sigma^2 \cdot I$:

$$\text{Var} - \text{Cov}(b) = (X^T X)^{-1} X^T W X (X^T X)^{-1}$$

"sandwich" estimator

Slides 45-46:

Least squares estimator:

① Estimator of the regression coefficients β_j :

$$b = (X^T X)^{-1} X^T y \quad \checkmark$$

② Estimator of the variance of stoch. var. u :

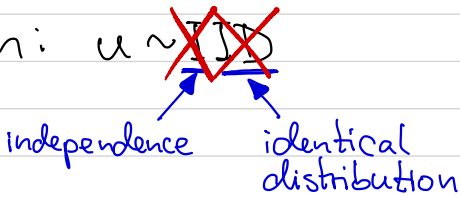
$$s_e^2 = \frac{RSS}{n-k} \quad \times$$

③ Estimator of the variance-covariance matrix of regression coefficient estimates:

$$\text{var-cov}(b) = s_e^2 \cdot (X^T X)^{-1} \quad \times$$

Slide 52:

Our assumption: $u \sim \text{IID}$



The diagram shows the text "Our assumption: $u \sim \text{IID}$ ". The "IID" is crossed out with a large red 'X'. Two blue arrows point from the 'I' and 'D' of the crossed-out "IID" to the words "independence" and "identical distribution" respectively, which are written in blue below the main text.

HAC: heteroscedasticity and autocorrelation consistent (estimator of variance).

Slides 53-55:

Example, variables:

- hm1: harmonized money aggregate M1;
- ppr: income of households;
- r_vp: interest rate on demand deposits;
- r_vv: interest rate on short-term deposits;
- c_zp: consumer price index.

Regression models with dummy explanatory variables

Dummy trap:

gender	D_1	D_2
M	1	0
F	0	1

Slide 2.

$$D_{1i} + D_{2i} = 1, \forall i$$

One can write: $D_{1i} = 1 - D_{2i}$ or $D_{2i} = 1 - D_{1i}$, resulting in perfect multicollinearity.

Types of regression models with dummy explanatory variables

- (A)
- y - gross wage
 - gender (male, female): $D = \begin{cases} 0; \text{female} \\ 1; \text{male} \end{cases}$

- (B)
- y - gross wage
 - education (elementary school, college, university):

$$D_1 = \begin{cases} 0; \text{other} \\ 1; \text{college} \end{cases}$$

$$D_2 = \begin{cases} 0; \text{other} \\ 1; \text{university} \end{cases}$$

C

- y - savings
- gender (male, female):

$$D_1 = \begin{cases} 0; & \text{female} \\ 1; & \text{male} \end{cases}$$

- labour force participation (employed, unemployed):

$$D_2 = \begin{cases} 0; & \text{unemployed} \\ 1; & \text{employed} \end{cases}$$

D

- y - gross wage
- gender (male, female): $D = \begin{cases} 0; & \text{female} \\ 1; & \text{male} \end{cases}$
- x_3 - years of employment

E

- y - gross wage
- education (elementary school, college, university):

$$D_1 = \begin{cases} 0; & \text{other} \\ 1; & \text{college} \end{cases}$$

$$D_2 = \begin{cases} 0; & \text{other} \\ 1; & \text{university} \end{cases}$$

- x_4 - years of employment

(F)

- y - savings
- gender (male, female):

$$D_1 = \begin{cases} 0; & \text{female} \\ 1; & \text{male} \end{cases}$$

- marital status (single, married):

$$D_2 = \begin{cases} 0; & \text{single} \\ 1; & \text{married} \end{cases}$$

- x_4 - years of employment