Tutorial, Part 4

Example 1

$\hat{C} = a_1 + \underset{(a_2)}{0.92}\, DI$ total effect of DI on C

$\widehat{CPI} = c_1 + \underset{(c_2)}{0.78}\, DI$ total effect of DI on CPI

$\hat{C} = b_1 + b_2 DI + b_3 CPI$ $\quad var(b_3) = 0.09$

$t: \quad (4.64) \quad (1.95)$

$b_2 = ?$

Effect of DI on C decomposition:

DIRECT : DI on C $\longrightarrow b_2$

+ INDIRECT : DI on C via CPI $\longrightarrow c_2 \cdot b_3 = 0.78\, b_3$

TOTAL : DI on C $\longrightarrow a_2 = 0.92$

$b_2 + 0.78\, b_3 = 0.92$

$b_2 = 0.92 - 0.78\, b_3$

$$t_{b_3} = \frac{b_3 - \overset{=0}{\beta_3}}{\sqrt{var(b_3)}} = \frac{b_3}{\sqrt{0.09}} = 1.95$$

$b_3 = 1.95 \cdot \sqrt{0.09} = 0.585$

$\underline{b_2} = 0.92 - 0.78 \cdot 0.585 = \underline{0.4637}$

Example 2

Model 1: linear

PRM $\quad MRT_{(i)} = \beta_1 + \beta_2 \, POP65_{(i)} + \beta_3 \, TOBACCO_{(i)} + u_{(i)}$

SRM $\quad \widehat{MRT}_{(i)} = b_1 + b_2 \, POP65_{(i)} + b_3 \, TOBACCO_{(i)}$

$\widehat{MRT} = 54.765 + 52.793 \, POP65 + 1.441 \, TOBACCO$

Interpretation (e.g. $b_2 = 52.793$): If the percentage of the population above age 65 increases by 1 percentage point the number of deaths per 100 000 inhabitants on average increases by 52.8 deaths, ceteris paribus.

Model 2: log-linear

$MRT = \beta_1 \, POP65^{\beta_2} \, TOBACCO^{\beta_3} \cdot u$

$\widehat{\ln MRT} = b_1 + b_2 \ln POP65 + b_3 \ln TOBACCO$

$\widehat{\ln MRT} = 3.449 + 0.786 \ln POP65 + 0.283 \ln TOBACCO$

Let's consider: $\ln Y_i = \beta_1 + \beta_2 \ln X_i + u_i$

$i=1 \longrightarrow \ln Y_1 = \beta_1 + \beta_2 \ln X_1$

$i=2 \longrightarrow \ln Y_2 = \beta_1 + \beta_2 \ln X_2$

subtract (assume: u_1 and u_2 cancel out)

$\ln Y_2 - \ln Y_1 = \beta_1 - \beta_1 + \beta_2 \ln X_2 - \beta_2 \ln X_1$

$\Delta \ln Y = \beta_2 \Delta \ln X \qquad *$

$$\Delta \ln Y = \ln \frac{Y_2}{Y_1} = \ln \frac{Y_1 + (Y_2 - Y_1)}{Y_1} = \ln\left(1 + \frac{\Delta Y}{Y_1}\right) \approx \frac{\Delta Y}{Y_1}$$

(approximation: $\ln(1+z) \approx z$ if $z \approx 0$ (really small))

$\Delta \ln Y \approx \frac{\Delta Y}{Y_1}$ relative change

$\Delta \ln X \approx \frac{\Delta X}{X_1}$

$$* \quad \frac{\Delta Y}{Y_1} \approx \beta_2 \frac{\Delta X}{X_1} \quad / \cdot 100$$

$$\frac{100\,\Delta Y}{Y_1} \approx \beta_2 \frac{100\,\Delta X}{X_1}$$

$$\%\Delta Y \approx \beta_2 \%\Delta X \quad \rightarrow \text{when } \%\Delta X = 1\%$$

$$\%\Delta Y \approx \beta_2 \%$$

ELASTICITY

Interpretation (e.g. $b_2 = 0.786$): If the percentage of the population above age 65 increases by 1 % the number of deaths per 100 000 inhabitants on average increases by (approximately) 0.786 %, ceteris paribus.

Model 3: log-lin

$$MRT = e^{\beta_1 + \beta_2 POP65 + \beta_3 TOBACCO + u}$$

$$\widehat{\ln MRT} = b_1 + b_2\, POP65 + b_3\, TOBACCO$$

$$\widehat{\ln MRT} = 5.746 + 0.063\, POP65 + 0.002\, TOBACCO$$

Let's consider: $\ln Y_i = \beta_1 + \beta_2 X_i + u_i$

$$i=1 \rightarrow \ln Y_1 = \beta_1 + \beta_2 X_1$$

$$i=2 \rightarrow \ln Y_2 = \beta_1 + \beta_2 X_2$$

subtract

$$\ln Y_2 - \ln Y_1 = \beta_2 (X_2 - X_1)$$

$$\Delta \ln Y = \beta_2 \Delta X$$

$$\frac{\Delta Y}{Y_1} \approx \beta_2 \Delta X \quad / \cdot 100$$

$$\frac{100\,\Delta Y}{Y_1} \approx 100 \beta_2 \Delta X$$

$$\%\Delta Y \approx 100 \beta_2 \Delta X \quad \rightarrow \text{when } \Delta X = 1 \text{ u.m.}$$

$$\%\Delta Y \approx 100 \beta_2$$

SEMI-ELASTICITY

Interpretation (e.g. $b_2 = 0.063$): If the percentage of the population above 65 increases by 1 percentage point the number of deaths per 100 000 inhabitants on average increases by (approximately) 6.3%, ceteris paribus.

Model 4: lin – log

$$\widehat{MRT} = b_1 + b_2 \ln POP65 + b_3 \ln TOBACCO$$

$$\widehat{MRT} = -1718.884 + 652.256 \ln POP65 + 202.187 \ln TOBACCO$$

Let's consider: $Y_i = \beta_1 + \beta_2 \ln X_i + u_i$

$i=1 \rightarrow \quad Y_1 = \beta_1 + \beta_2 \ln X_1$ } subtract

$i=2 \rightarrow \quad Y_2 = \beta_1 + \beta_2 \ln X_2$

$$Y_2 - Y_1 = \beta_2 (\ln X_2 - \ln X_1)$$

$$\Delta Y = \beta_2 \Delta \ln X$$

$$\Delta Y \approx \beta_2 \frac{\Delta X}{X_1} \quad / \cdot 100$$

$$100 \Delta Y \approx \beta_2 \frac{100 \Delta X}{X_1}$$

$$\Delta Y \approx \frac{\beta_2}{100} \% \Delta X \quad \rightarrow \text{when } \% \Delta X = 1\%$$

$$\Delta Y \approx \frac{\beta_2}{100} \text{ u.m.}$$

SEMI-ELASTICITY

Interpretation (e.g. $b_2 = 652.256$): If the percentage of the population above age 65 increases by 1% the number of deaths per 100 000 inhabitants on average increases by 6.52 deaths, ceteris paribus.

Model 5: "mixed"

$MRT = e^{\beta_1 + \beta_2 POP65} \cdot TOBACCO^{\beta_3} \cdot u$

$$\widehat{\ln MRT} = b_1 + b_2 \, POP65 + b_3 \ln TOBACCO$$

$$\widehat{\ln MRT} = 4.584 + 0.061 \, POP65 + 0.299 \ln TOBACCO$$

Interpretation: in the same manner as in the previous models

Example 3

Let's consider a quadratic regression model:

$$Y_i = \beta_1 + \beta_2 X_i + \beta_3 X_i^2 + u_i$$

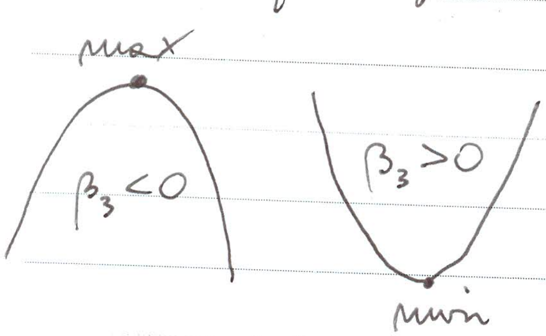
$$\frac{dY}{dX} = \beta_2 + 2\beta_3 X \qquad \frac{dY}{dX} \approx \frac{\Delta Y}{\Delta X} \Rightarrow$$

$$\Delta Y \approx (\beta_2 + 2\beta_3 X)\Delta X \qquad \Delta Y \approx \frac{dY}{dX}\Delta X$$

when $\Delta X = 1$ m.u. $\Rightarrow \Delta Y \approx \beta_2 + 2\beta_3 X$

$\Rightarrow$ With non-linear models the change in Y depends on the exact value of X at which we observe the change.

In the vertex ("turning point") of the parabola the effect of X on Y changes direction (sign):



(concave) (convex)

$$\frac{dY}{dX} = \beta_2 + 2\beta_3 X = 0$$

$$X = -\frac{\beta_2}{2\beta_3}$$

a) $\Delta SALES \approx \frac{\partial SALES}{\partial ADV} \cdot \Delta ADV$

$\Delta SALES \approx (0.46 - 2 \cdot 0.002 \cdot ADV)\, \Delta ADV$

$\Delta ADV = 1$ u.m. $\longrightarrow \Delta SALES \approx 0.46 - 0.004\, ADV$

Interpretation: If advertising costs increase by 1 unit of measurement (=1000 monetary units) the sales on average change by (0.46 − 0.004 ADV) units of measurement (= mio monetary units), ceteris paribus.

At 4000 MU (=4 u.m.) level: $\Delta SALES \approx 0{,}46 - 0{,}004 \cdot 4 = 0{,}444$

Interpretation: If advertising costs increase from 4000 to 5000 MU the sales on average increase by 444000 MU, ceteris paribus

b) Interpretation of $b_4 = 1.122$: If R&D costs increase by 1% the sales on average increase by (approximately) 1,122 %, ceteris paribus

Interpretation of $b_5 = 0.008$: If percentage of employees with higher education increases by 1 percentage point the sales on average increase by (approximately) 0,8 %, ceteris paribus.

c) Not true. The two models are non-linear in variables. We can linearize them and estimate the transformed models using LSE method.

Example 2

M_B (BASE model):

$$C_t = \beta_1 + \beta_2 Y_t + \beta_3 ST_t + \beta_4 TR_t + \beta_5 FT_t + \mu_t$$

$H_0: \beta_j = 0 \quad ; \quad \forall j = 4,5$ (exclude TR and FT)

$H_1: \beta_j \neq 0 \quad ; \quad \exists j = 4,5$ (cannot exclude TR or FT)

M_R (RESTRICTED model):

$$C_t = \beta_1 + \beta_2 Y_t + \beta_3 ST_t + \mu_t$$

I. F-test

$$F = \frac{(RSS_R - RSS_B)/(k_B - k_R)}{RSS_B/(n-k_B)} = \frac{(0.1594 - 0.1413)/(5-3)}{0.1413/(25-5)} = 1.283$$

$F_C(m_1 = 2, m_2 = 20, \alpha = 0.05) = 3.49$

$F < F_C \Rightarrow$ cannot reject H_0 (at $\alpha = 0.05$)

II. LM (Lagrange Multiplier) - test

$M_R: \quad C_t = b_1 + b_2 Y_t + b_3 ST_t + e_t$

M_{AR} (AUXILIARY regression model):

$$e_t = g_1 + g_2 Y_t + g_3 ST_t + g_4 TR_t + g_5 FT_t$$

$R^2 = 0.1137$

$LM = n \cdot R^2_{AR} = 25 \cdot 0.1137 = 2.8425 \quad (\sim \chi^2_{(\#\,restrictions)})$

$\chi^2_{c\,(m=2,\ \alpha=0.05)} = 5.997$

$LM < \chi^2_c \quad \Rightarrow$ same conclusion

III. LR (Likelihood Ratio) - test

(Stata printout for M_B estimation)

$$\ln L_B = -\frac{n}{2}\left(\ln(2\pi) + \ln\left(\frac{RSS_B}{n}\right) + 1\right) =$$
$$= -\frac{25}{2}\left(\ln(2\pi) + \ln\left(\frac{0.1413}{25}\right) + 1\right) = 29.224$$

(Stata printout for M_R estimation)

$$\ln L_R = -\frac{n}{2}\left(\ln(2\pi) + \ln\left(\frac{RSS_R}{n}\right) + 1\right) =$$
$$= -\frac{25}{2}\left(\ln(2\pi) + \ln\left(\frac{0.1594}{25}\right) + 1\right) = 27.715$$

$$LR = -2(\ln L_R - \ln L_B) = -2(27.715 - 29.224) = 3.0181 \quad (\sim \chi^2)$$

$\chi^2_{c\,(m=2,\ \alpha=0.05)} = 5.997$

$LR < \chi^2_c \quad \Rightarrow$ same conclusion