第二章关系作业答案

1. 解：$A \times B = \{(1,a),(1,b),(1,c),(1,d),(2,a),(2,b),(2,c),(2,d)\}$;

$B \times A = \{(a,1),(b,1),(c,1),(d,1),(a,2),(b,2),(c,2),(d,2)\}$;

$$\begin{aligned}A^2 \times B = \{&((1,1),a),((1,2),a),((2,1),a),((2,2),a),\\ &((1,1),b),((1,2),b),((2,1),b),((2,2),b),\\ &((1,1),c),((1,2),c),((2,1),c),((2,2),c),\\ &((1,1),d),((1,2),d),((2,1),d),((2,2),d)\};\end{aligned}$$

$A \times (B-C) = \{1,a),(1,b),(2,a),(2,b)\}$.

4. 解：$\mathscr{P}(A) = \{\emptyset,\{a,\{b,c\}\}\}$;

$\mathscr{P}(B) = \{\emptyset,\{\emptyset\},\{\{\emptyset\}\},\{\emptyset,\{\emptyset\}\}\}$;

$A \times B = \{(\{a,\{b,c\}\},\emptyset),(\{a,\{b,c\}\},\{\emptyset\})\}$.

5.(1) 证明：令$(x,y) \in A \times (B \cup C)$

$\Rightarrow x \in A \wedge y \in B \cup C$

$\Rightarrow x \in A \wedge (y \in B \vee y \in C)$

$\Rightarrow (x \in A \wedge y \in B) \vee (x \in A \wedge y \in C)$

$\Rightarrow ((x,y) \in A \times B) \vee ((x,y) \in A \times C)$

$\therefore A \times (B \cup C) \subseteq (A \times B) \cup (A \times C)$;

令$(x,y) \in (A \times B) \cup (A \times C)$

$\Rightarrow (x \in A \wedge y \in B) \vee (x \in A \wedge y \in C)$

$\Rightarrow x \in A \wedge (y \in B \vee y \in C)$

$\Rightarrow x \in A \wedge y \in B \cup C$

$\Rightarrow (x,y) \in A \times (B \cup C)$

$\therefore (A \times B) \cup (A \times C) \subseteq A \times (B \cup C)$;

$\therefore A \times (B \cup C) = (A \times B) \cup (A \times C)$ Q.E.D.

6. 解：不具备自反性($(b,b) \notin R$);

不具备反自反性($(a,a) \in R$);

不具备对称性($(a,b) \in R, (b,a) \notin R$);

具备反对称性($(b,a) \notin R, (c,a) \notin R$);

具备传递性.

8. 解: $R \circ S = \{(a,e),(c,b),(b,e)\}$;

$S \circ R = \{(d,b),(c,b)\}$;

$R \circ R = \{(a,b),(b,b)\}$.

12. 解: $r(R) = \{(1,2),(2,1),(1,3),(1,1),(2,2),(3,3)\}$;

$s(R) = \{(1,2),(2,1),(1,3),(1,1),(3,1)\}$;

$t(R) = \{(1,2),(2,1),(1,3),(1,1),(2,3)\}$.

18. 证明：$\because R$是自反的, $\therefore I \subseteq R$

$\because R \subseteq s(R), \therefore I \subseteq s(R)$, 故$s(R)$是自反的;

同理, $I \subseteq t(R)$, 故$t(R)$是自反的 Q.E.D.

22. 证明：可知R是自反、对称且传递的.

$\because R$是自反的, $\therefore I \subseteq R$

$\because I^{-1} = I, \therefore I \subseteq R^{-1}$

故R^{-1}是自反的;

$\forall (x,y) \in R^{-1}$, 有$(y,x) \in R$

$\because R$是自反的, $\therefore \forall (a,b) \in R, (b,a) \in R$

$\therefore (x,y) \in R \Rightarrow (y,x) \in R^{-1}$

故R^{-1}是对称的;

$\forall (x,y) \in R^{-1} \wedge (y,z) \in R^{-1}$

有$(y,x) \in R \wedge (z,y) \in R$

$\because R$是传递的

$\therefore (z,x) \in R$

$\therefore (x,z) \in R^{-1}$

故R^{-1}是传递的;

综上所证，R^{-1}是等价关系.　　Q.E.D.

24. 解：$[1]_R = \{1,5\}, [2]_R = \{2,3,6\}, [3]_R = \{2,3,6\},$

$[4]_R = \{4\}, [5]_R = \{1,5\}, [6]_R = \{2,3,6\}.$

33. 解：最大元为a, 无最大小元, 极大元为a, 极小元为d, e.

35. 解: 哈斯图见图1.

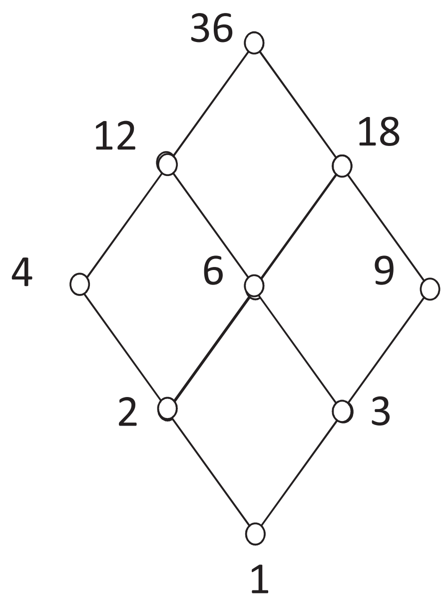

Figure 1: 35对应哈斯图