Politecnico di Milano
STATISTICS (079086), 2009/2010, Prof. A.Barchielli
Problem set n. 1

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Exercise 1 [M.Maravalle et al., *Esercizi di statistica*, McGraw-Hill, 1996]

1. Let X be a binomial random variable with parameters $n = 25$ and $p = 0.2$. Calculate $P(X < \mu_X - 2\sigma_X)$, where μ_X denotes the mean of X and σ_X denotes the standard deviation of X; [0.004]
2. If X is a Poisson random variable such that $P(X = 0) = P(X = 1)$, find the mean value of X; [1]
3. If X is a Poisson random variable such that $P(X = 0) = 1/2$, find the mean value of X; [$ln2$]
4. Let X be a binomial random variable with parameters n and p. Assume that $E(X) = 5$ and $Var(X) = 4$. Find n and p; [$n = 25$ and $p = 1/5$]

Exercise 2 Let X be a discrete random variable with $P(X = 1) = P(X = -1) = \frac{\theta}{2(1+\theta)}$ and $P(X = 0) = \frac{1}{1+\theta}$, $\theta > 0$.

a) Compute the mean and variance of X. [$E(X) = 0$, $Var(X) = \frac{\theta}{1+\theta}$]

b) Determine the density of $Y = X^2$ and compute its mean and variance. [$P(Y = 1) = \frac{\theta}{1+\theta}$ and $P(Y = 0) = \frac{1}{1+\theta}$, $\theta > 0$; $E(Y) = \frac{\theta}{1+\theta}$, $Var(Y) = \frac{\theta}{(1+\theta)^2}$]

Exercise 3

1. If X is a uniform random variable on $(1, 2)$, find z such that $P(X > z + \mu_X) = 1/4$; [1/4]
2. If X is a uniform random variable on (a, b). Assume that $E(X) = 1$ and $Var(X) = 4/3$. Find a and b and deduce $P(X < 0)$; [0.25]

Exercise 4 Let X be uniformly distributed over the interval $[0, 1]$. Let $Y = -\frac{1}{3}lnX$.

a) Compute $P(Y \leq 0)$ [0]

b) Determine the cumulative distribution function and density of Y. [$Y \sim Esp(1/3)$]

Exercise 5 Let X be exponentially distributed with parameter $1/\lambda$.

a) Compute the density of $Y = X^2$. [$f_Y(t) = \frac{\lambda}{2}\frac{e^{-\lambda\sqrt{t}}}{\sqrt{t}}\,\mathbf{1}_{(0,\infty)}(t)$]

b) Compute the density of $Y = X^{1/\alpha}$, where $\alpha > 0$. [$f_Y(t) = \lambda\alpha t^{\alpha-1}e^{-\lambda t^\alpha}\,\mathbf{1}_{(0,\infty)}(t)$]