



# Bottlenecks Identification in “Very Large” Multiclass Queueing Models

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G.Casale – G.Serazzi  
serazzi@elet.polimi.it



- ❑ motivations
- ❑ multiclass pitfalls
- ❑ networks with multiple bottlenecks
- ❑ “very large” models
- ❑ conclusions



- ❑ complexity of modelling actual computer infrastructures
  - ❑ large installations comprising thousand of servers
  - ❑ strongly multiclass workload
  - “very large” models (VLM)
- ❑ emerging distributed technologies and applications
  - ❑ peer-to-peer technologies
    - ❑ collaboration, middlewares, file sharing, games, messaging, ...
  - ❑ grid computing
  - ❑ web services
    - ❑ interoperability
  - ❑ wireless and ubiquitous computing
  - ❑ ...

# “Very large” models (VLM)



- ❑ Intel (2001)
  - ❑ 100000 clients
  - ❑ 3000 servers
- ❑ Vodaphone Italy (2004)
  - ❑ 500 server Sun, 400 server HP, 2000 server NT
  - ❑ 40 Millions/day of SMS, 20 Millions customers
  - ❑ 500 update/sec on the customer care DB
- ❑ Unicredit bank (2004)
  - ❑ 10 large mainframes
  - ❑ 1000-1500 servers
  - ❑ Transactions: 36 Millions/day

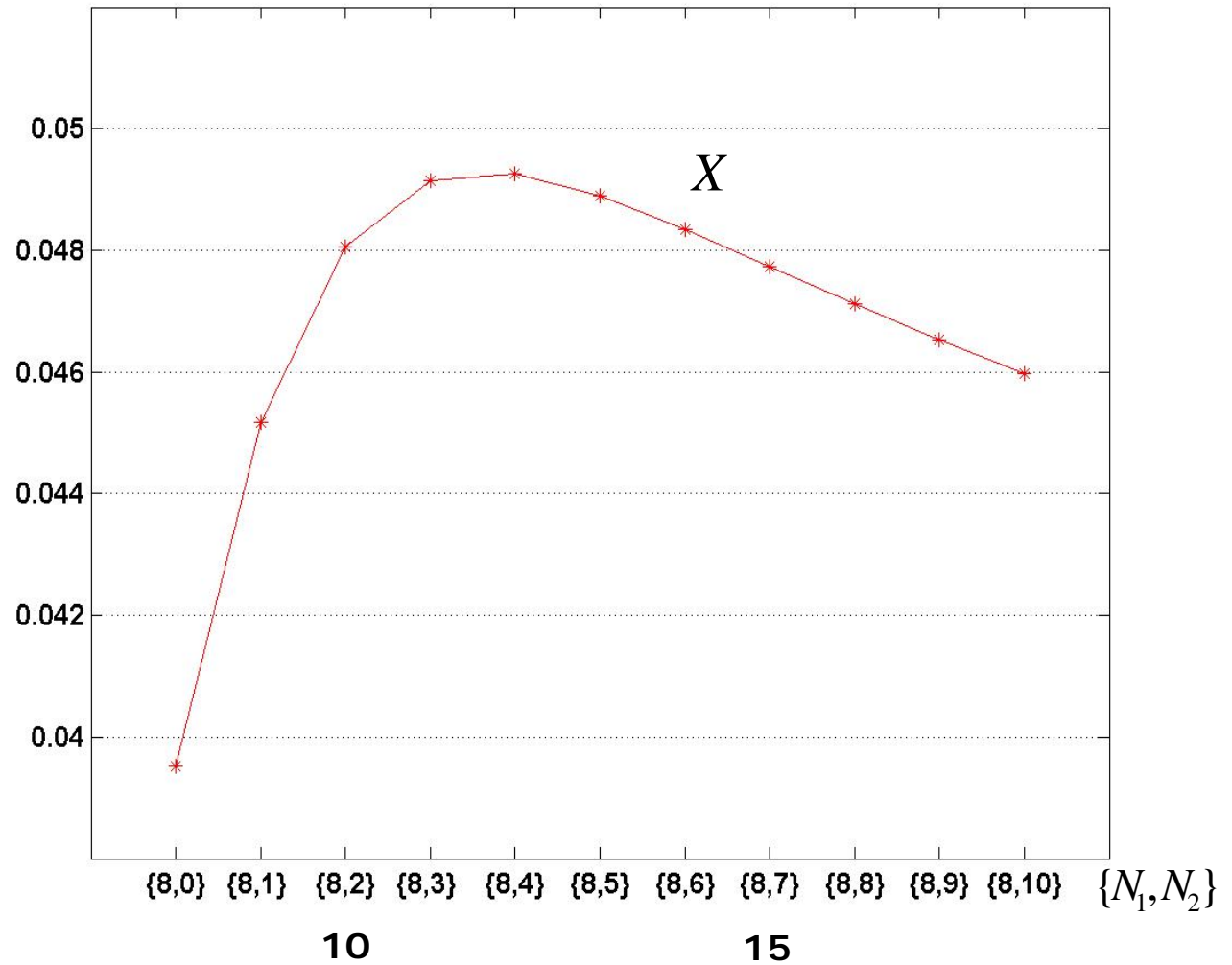


- ❑ “the throughput  $X$  of the network increases as the total number of customers  $N$  increases”
- ❑ “the sum of the utilizations  $U_{TOT} = \sum_{i=1}^M U_i$  of the stations increases with  $N$ ”
- ❑ “for a given population, only one bottleneck station exists in the network”

# X increases as N



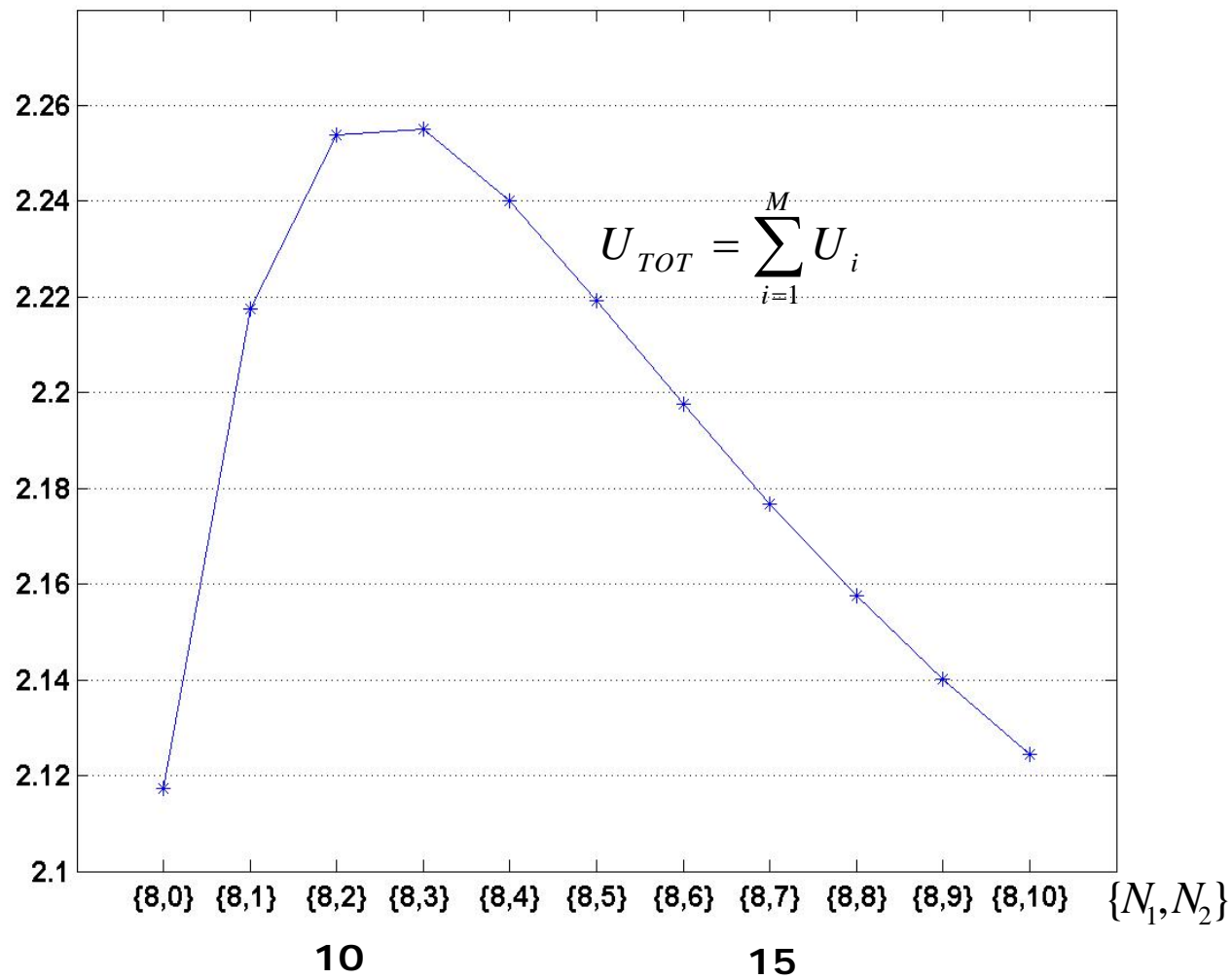
Stations	Classes	
	25	5
	15	30
	10	15



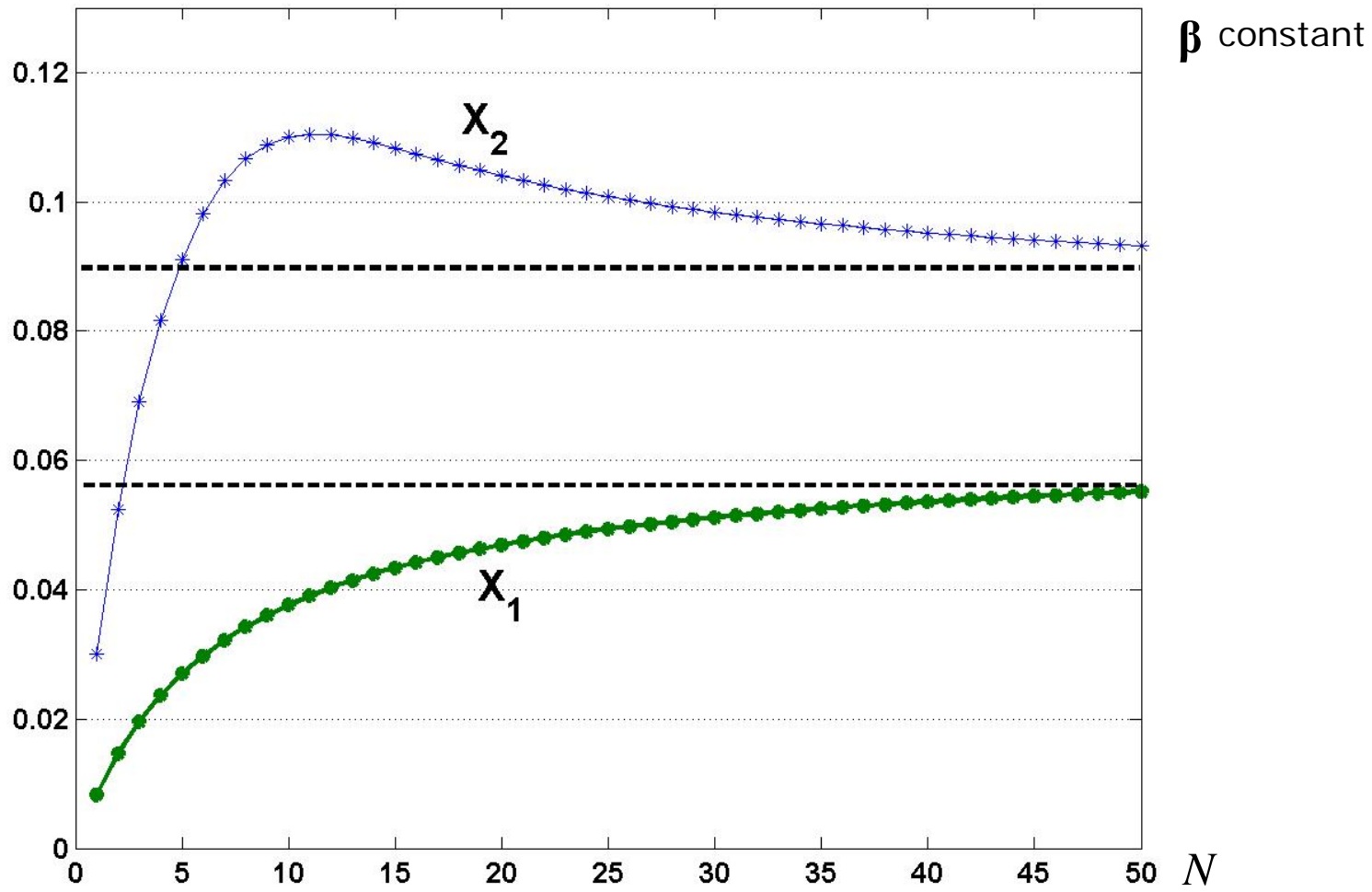
# Global utilization increases as N



Stations	Classes	
	60	25
	55	90
	24	50



# Asymptotic Performance Indices





- loading matrix

$$\mathbf{L} = \{ L_{mr} = V_{mr} S_{mr} \}$$

		<i>Customer Classes</i>			
		$L_{11}$	$L_{12}$	$\dots$	$L_{1R}$
		$L_{21}$	$L_{22}$	$\dots$	$L_{2R}$
		$\dots$	$\dots$	$\dots$	$\dots$
		$L_{M1}$	$\dots$	$\dots$	$L_{MR}$
Queueing Stations		$\begin{bmatrix} L_{11} & L_{12} & \dots & L_{1R} \\ L_{21} & L_{22} & \dots & L_{2R} \\ \dots & \dots & \dots & \dots \\ L_{M1} & \dots & \dots & L_{MR} \end{bmatrix}$			

- $\beta$  (% of jobs per-class)

$$\beta_r = \frac{N_r}{\sum_{r=1}^R N_r} = \frac{N_r}{N}$$

- population mix

$$\boldsymbol{\beta} = \{\beta_1, \beta_2, \dots, \beta_R\}, \quad \sum_{r=1}^R \beta_r = 1$$

- population vector

$$\mathbf{N} = \{N_1, N_2, \dots, N_R\} = N \cdot \boldsymbol{\beta}$$

# Taxonomy of stations



*Classes*

$\underline{\mathbf{L}} =$

60	5
15	70
40	50
20	10
20	55

*Stations*

## □ Natural bottlenecks

$$\beta_1 = 100\% \quad \beta_2 = 0\% \quad \rightarrow \quad U_{\max} = U_1$$

$$\beta_1 = 0\% \quad \beta_2 = 100\% \quad \rightarrow \quad U_{\max} = U_2$$

## □ Network bottlenecks

$$\beta_1 = 50\%, \quad \beta_2 = 50\% \quad \rightarrow \quad U_{\max} = U_3$$

## □ Potential bottlenecks set (network + natural) bottlenecks

$$\mathbf{\Pi} = \{1, 2, 3\}$$

## □ Dominated stations

4 has all components less than those of 3

## □ Masked-off stations

5 not dominated, but never saturates

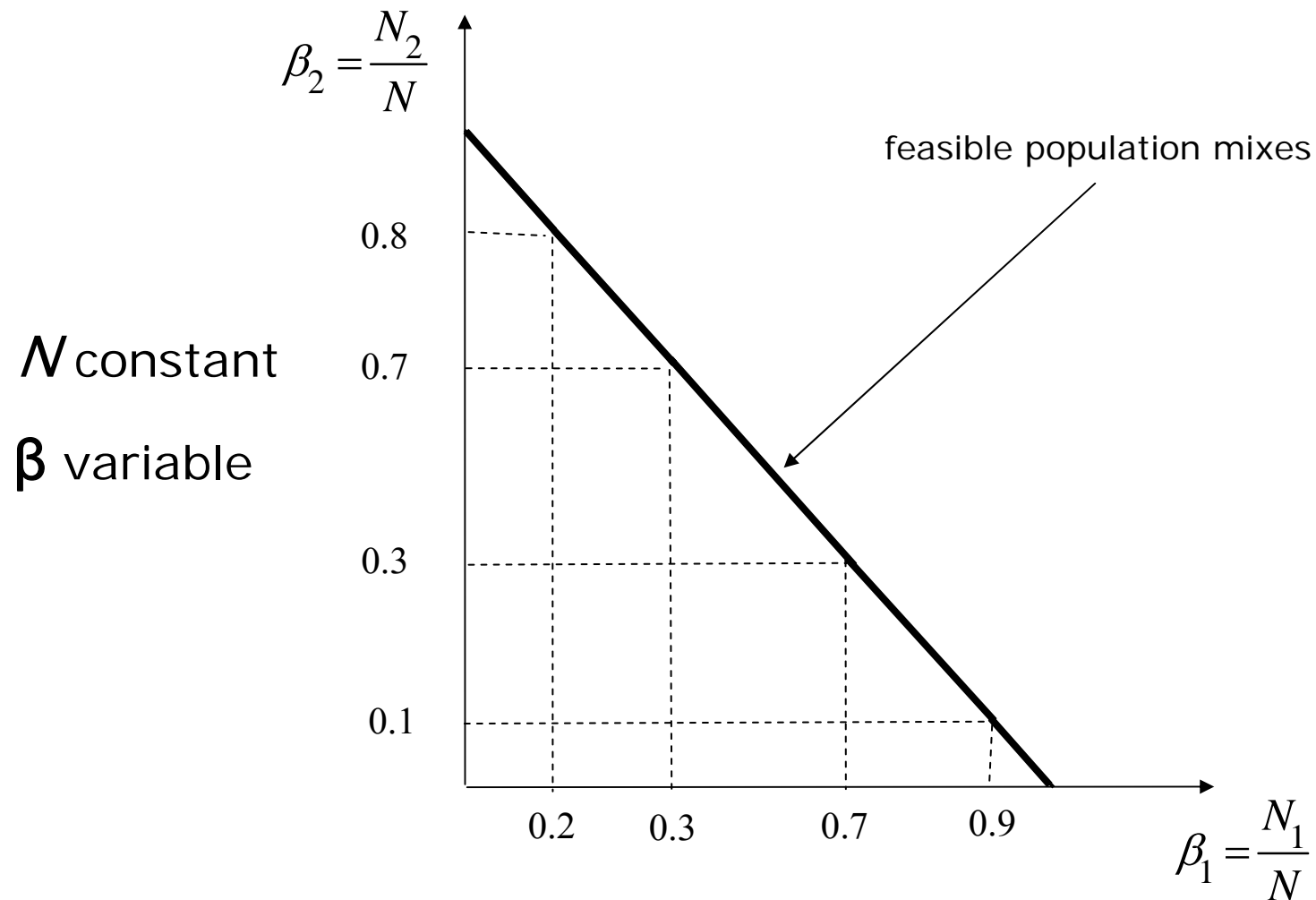
# Types of population growth



- ❑ **unbalanced** population growth
  - ❑ only the customers of one class increases to infinity (degenerate to a **single class** case)
  
- ❑ **proportional** population growth
  - ❑ the customers grow to infinity keeping constant the population mix  $\beta$  (or the arrival rate mix  $\lambda$ )

e.g.,  $\beta_1 = 50\%$ ,  $\beta_2 = 50\% \rightarrow (10,10), (30,30), (100,100)$

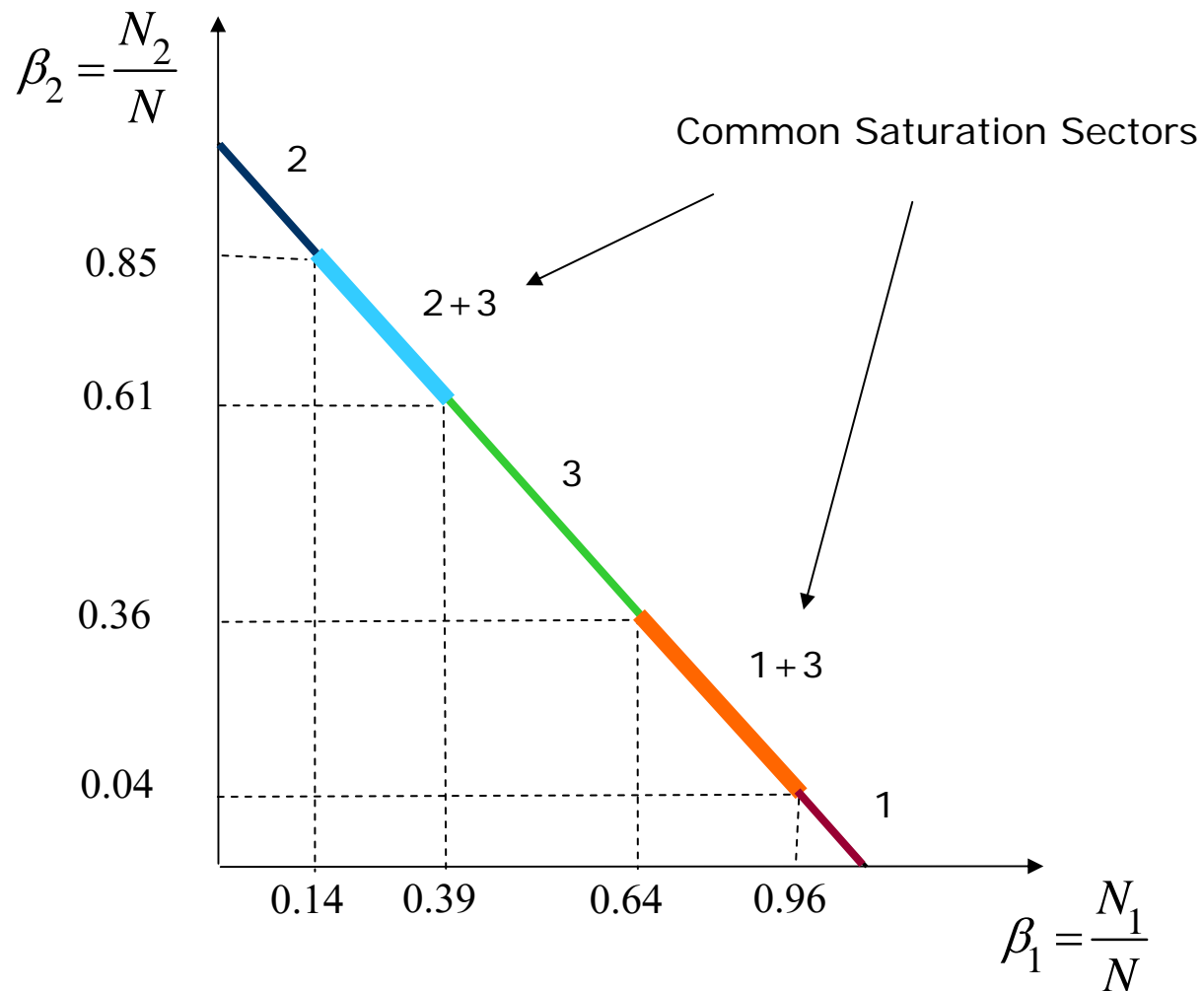
# proportional pop. growth (closed model)



# Bottlenecks in closed models (2 classes)



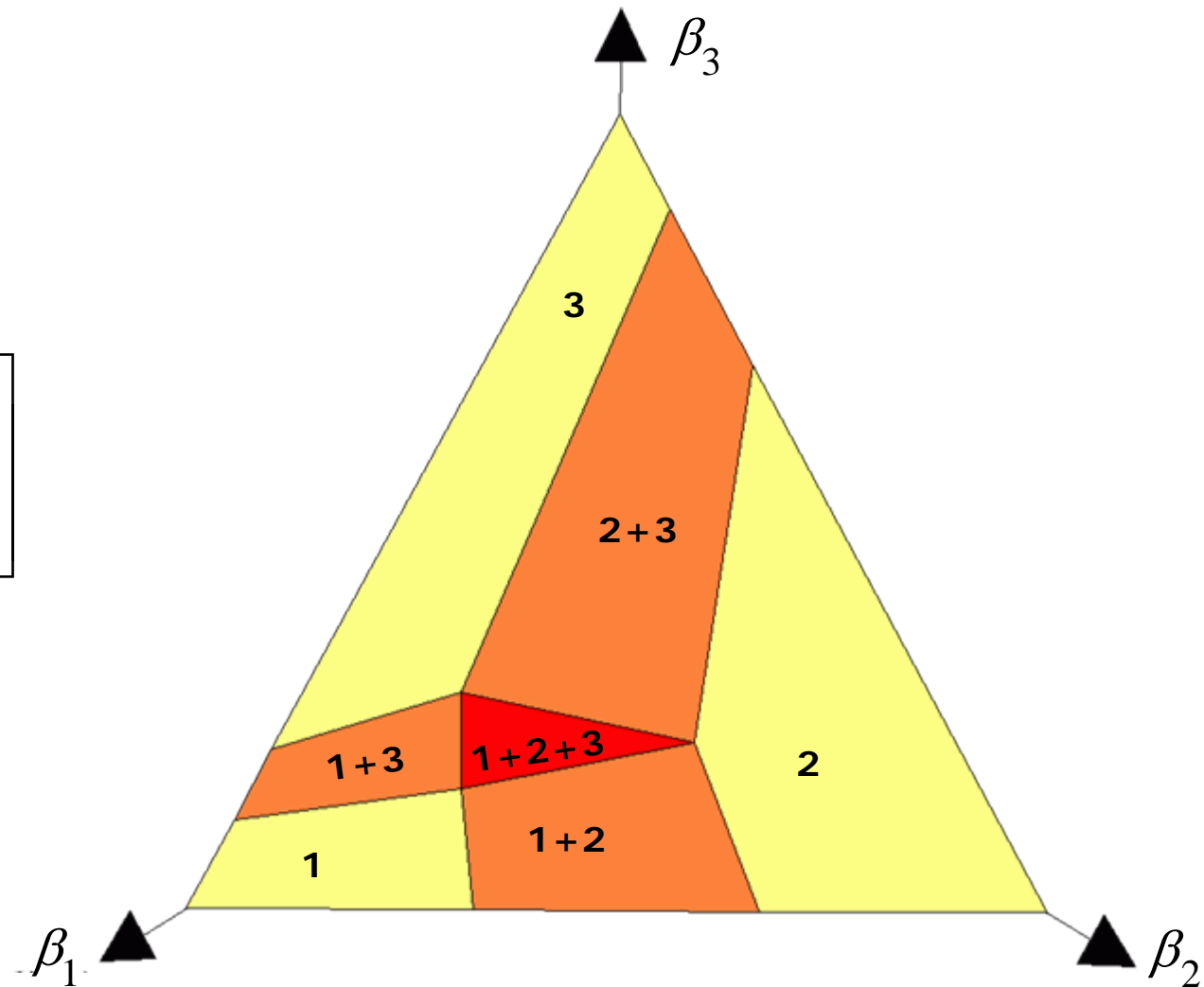
	Classes	
Stations	60	5
	15	70
	40	50



# Bottlenecks in closed models (3 classes)



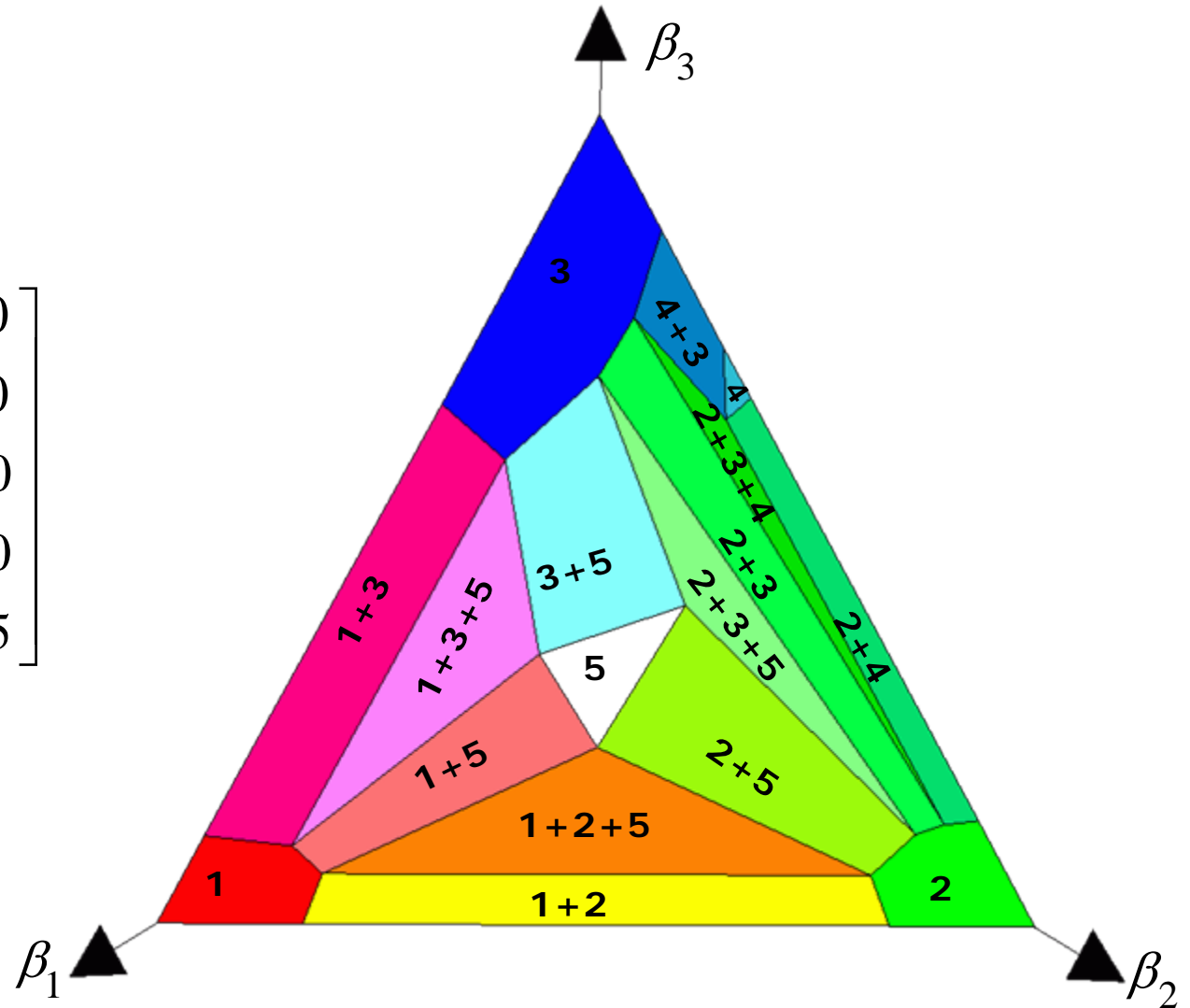
	Classes		
Stations	100	40	50
	50	80	70
	90	30	90



# Complex saturation behavior



Stations	Classes		
	50	10	10
	10	50	10
	20	10	60
	10	20	50
	25	25	35

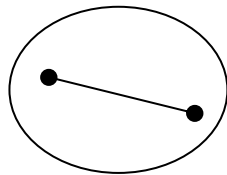


# The convex hull problem

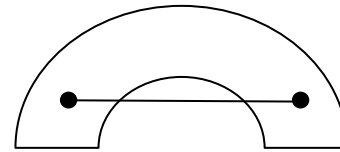


## □ Convex set

- Every line segment joining any pair of points lies entirely in the set



Convex set

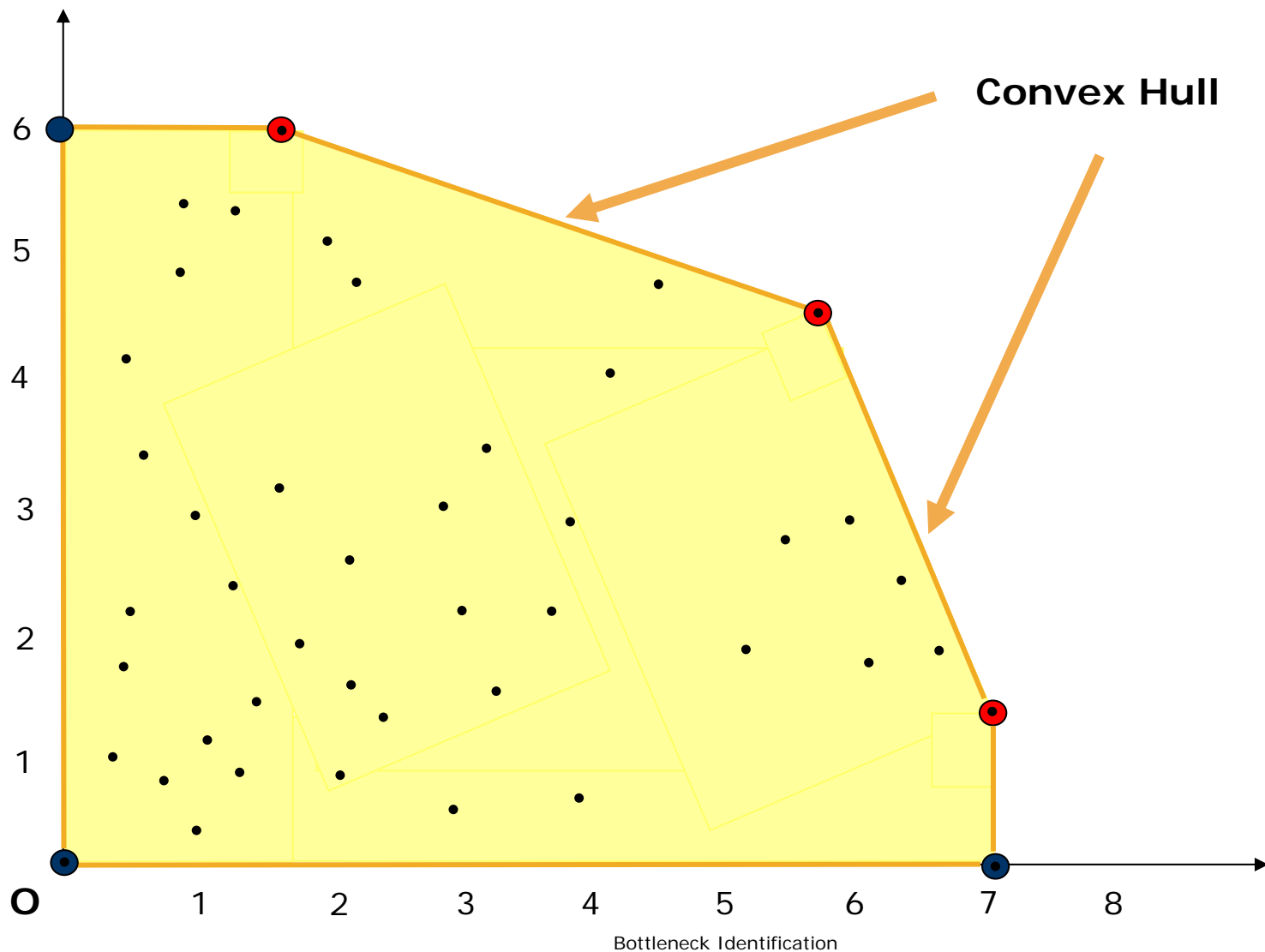


Non-convex set

- **Convex hull problem:** find the smallest convex set containing a given set of points
- Fast algorithms in 2D [ $O(M \log M)$ ] and in 3D [ $O(M^2)$ ] exist (M number of points)



# Convex hull in 2 dimensions





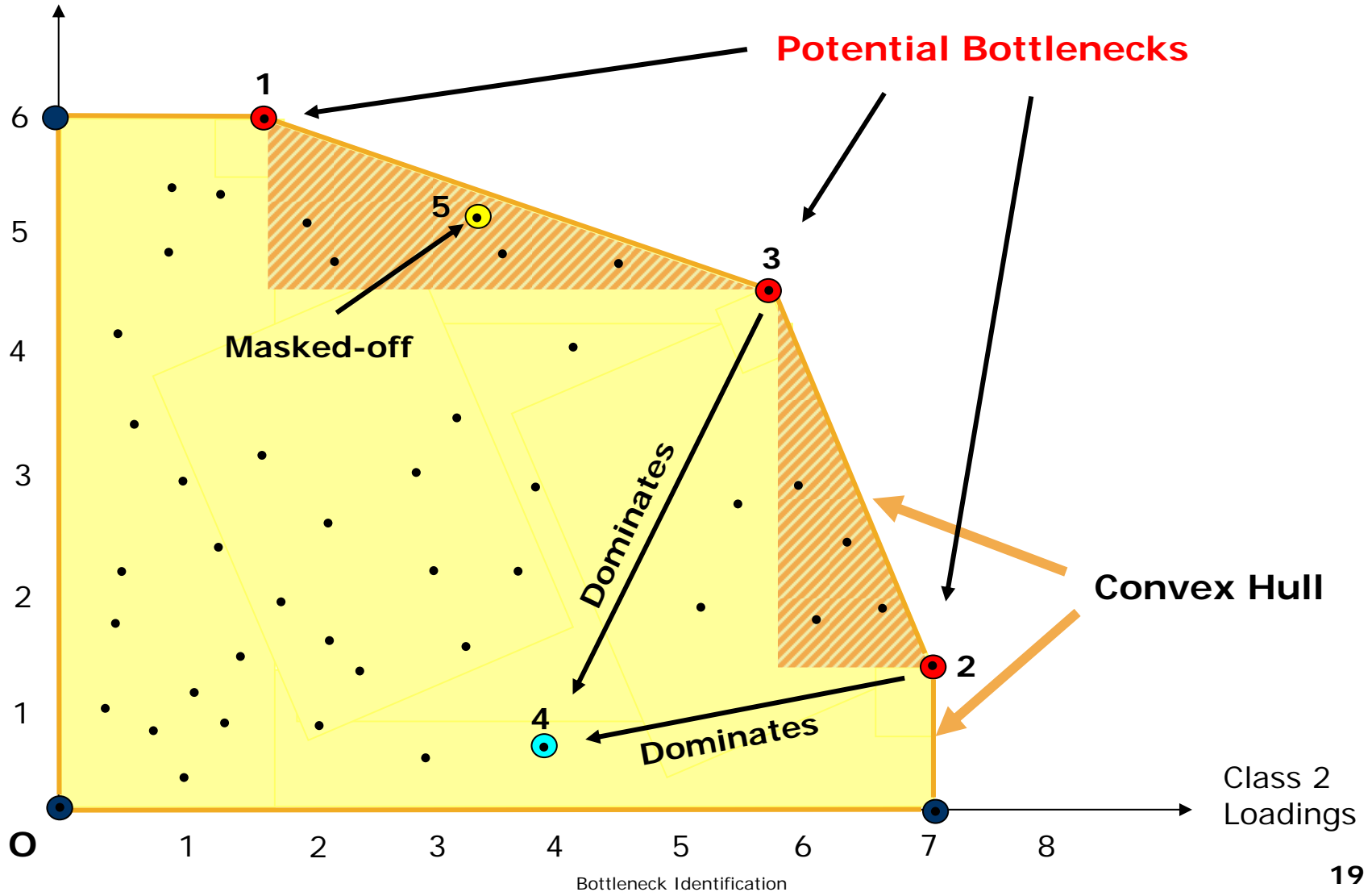
- ❑ Use of computational geometry techniques for the queueing network analysis
- ❑ Define the loading matrix  $L$
- ❑ Solve the **convex hull problem**
  - ❑ Gives knowledge on the sets of stations that can saturate together (common saturation sectors)
  - ❑ Allows fast computation of the bottlenecks set as a function of the population mix



# Potential Bottleneck Identification

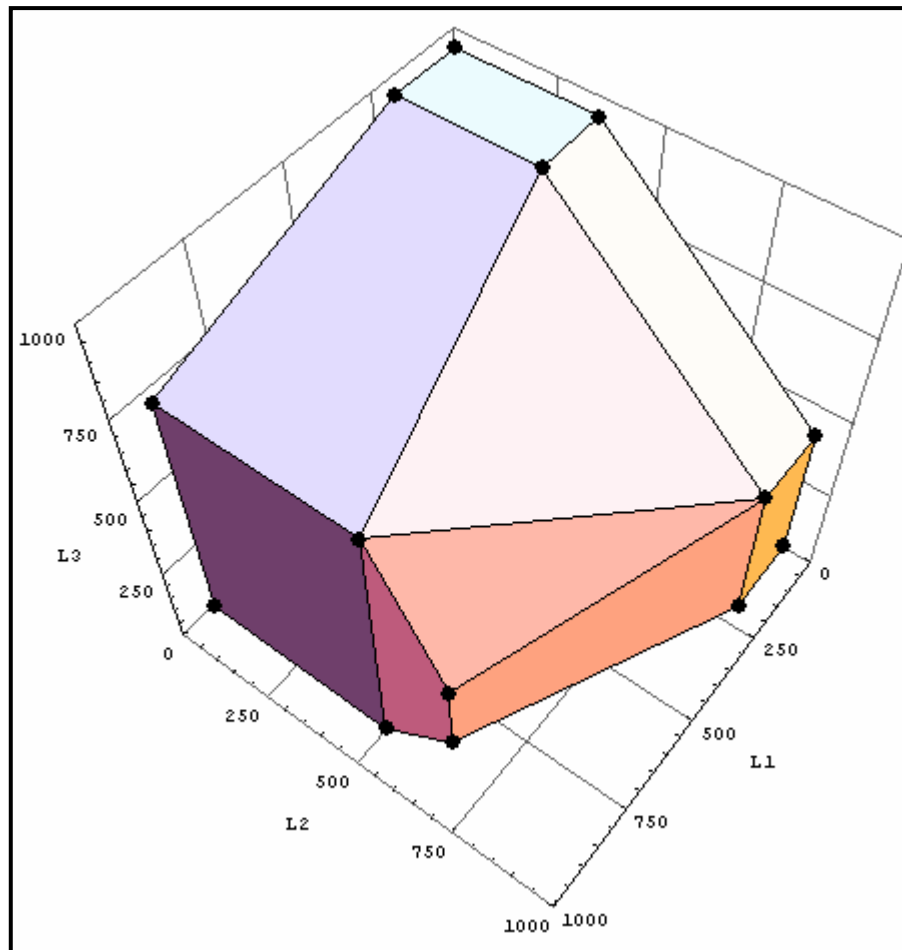
## Convex hull of the loading matrix

Class 1 Loadings



# Potential Bottleneck Identification

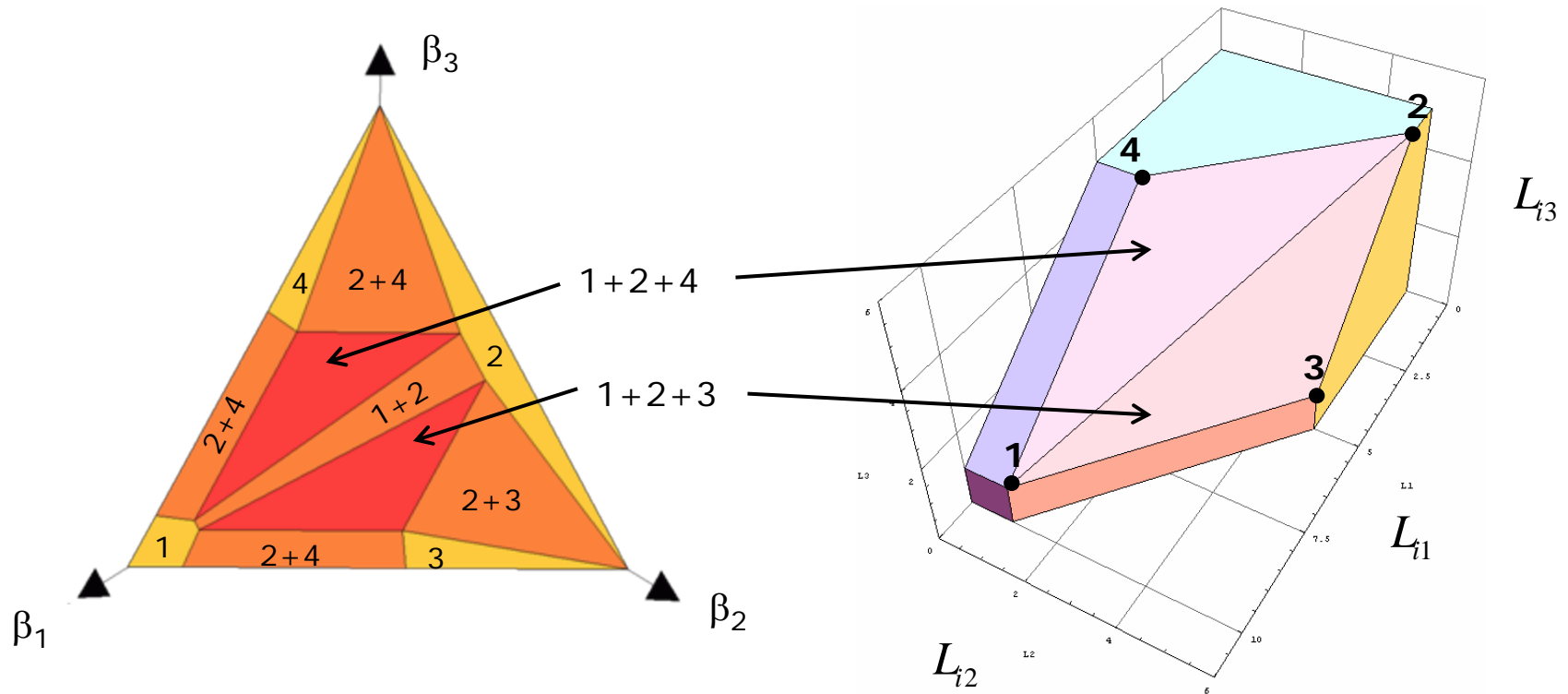
## Convex hull of a 3-class model



# Saturation sectors and convex hulls



- The convex hull of the loading matrix and the beta space are closely related



- **3+4** and **1+3+4** are not saturation sectors because there is no edge between **3** and **4** on the convex hull
- allows fast computation of the saturation sectors

# Redundancy elimination

- ❑ The time complexity of the convex hull is exponential in the number of classes

CONVEX HULL CPU TIME	R=3	R=6	R=7	R=8	R=9
M=1000	<0.1 s	1 s	32 s	161 s	>1day
M=10000	<0.1 s	21 s	200 s	>1day	>1day
M=100000	0.12 s	100 s	>1day	>1day	>1day
M=1000000	72 s	463 s	>1day	>1day	>1day

Tested on a AMD Athlon 2800XP+ - 256KB CACHE – 768Mb RAM

- ❑ Redundancy elimination techniques instead of convex hulls
  - Polynomial time complexity in the number of stations  $M$  and in the number of classes  $R$
  - Loose information on the set of stations saturating together (identifies only the potential bottleneck set)

# Potential bottlenecks Identification

## Experimental results



Redundancy Elimination CPU TIME	R=5	R=10	R=25	R=50	R=100
M=1000	4 secs	6 secs	15 secs	48 secs	80 secs
M=10000	2 minutes	4 minutes	10 minutes	31 minutes	66 minutes
M=100000	5 hours	7 hours	9 hours	16 hours	34 hours

Tested on a Intel Xeon Dual Processor 2.80 Ghz – 512KB CACHE – 1Gb RAM

- ❑ Redundancy Elimination is formulated as a set of independent problems
  - parallelization: grid computing, clusters, ...
- ❑ Heuristic strategies for quick identification of dominated and masked-off stations are available

# Fast computation of the saturation sectors



- ❑ “Old” computation scheme: brute force approach, explore all possible sets of saturating stations. Requires the solution of a large number of linear systems to identify the  $\delta$ s of the saturation sectors edges
- ❑ “New” computation scheme: use convex hull informations to solve only a minimal set of linear systems

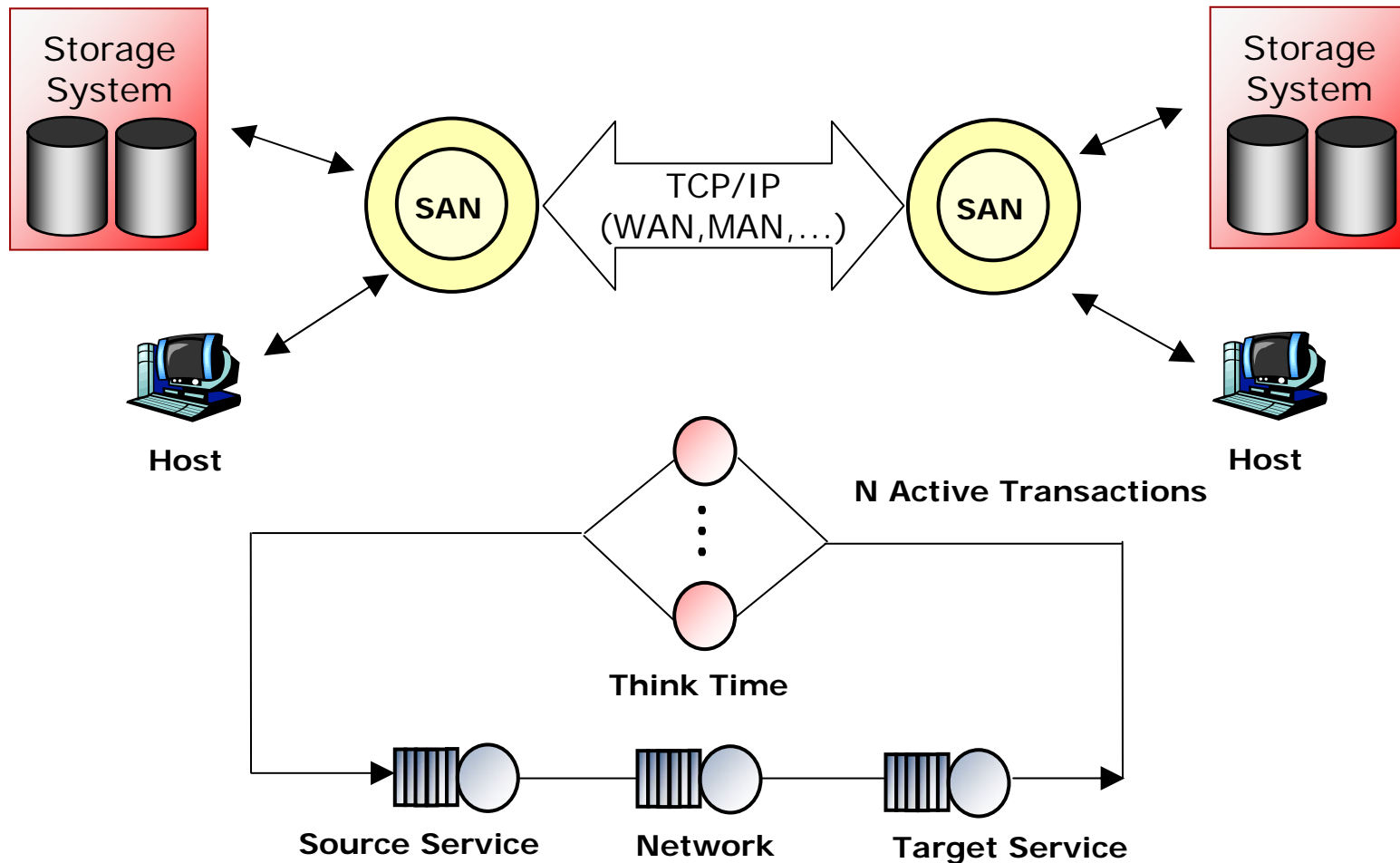
	3 customer classes			4 customer classes		
	Old	New	↓	Old	New	↓
<b>M=10 stations</b>	108.72	42.50	<b>-60.56%</b>	297.78	114.82	<b>-61.44%</b>
<b>M=100</b>	1914.88	173.06	<b>-90.96%</b>	33299.83	1123.02	<b>-99.66%</b>
<b>M=1000</b>	9712.82	390.56	<b>-95.98%</b>	384068.5	3954.01	<b>-98.97%</b>

Mean number of linear systems to be solved to identify the saturation sectors (tested on 100 random models)



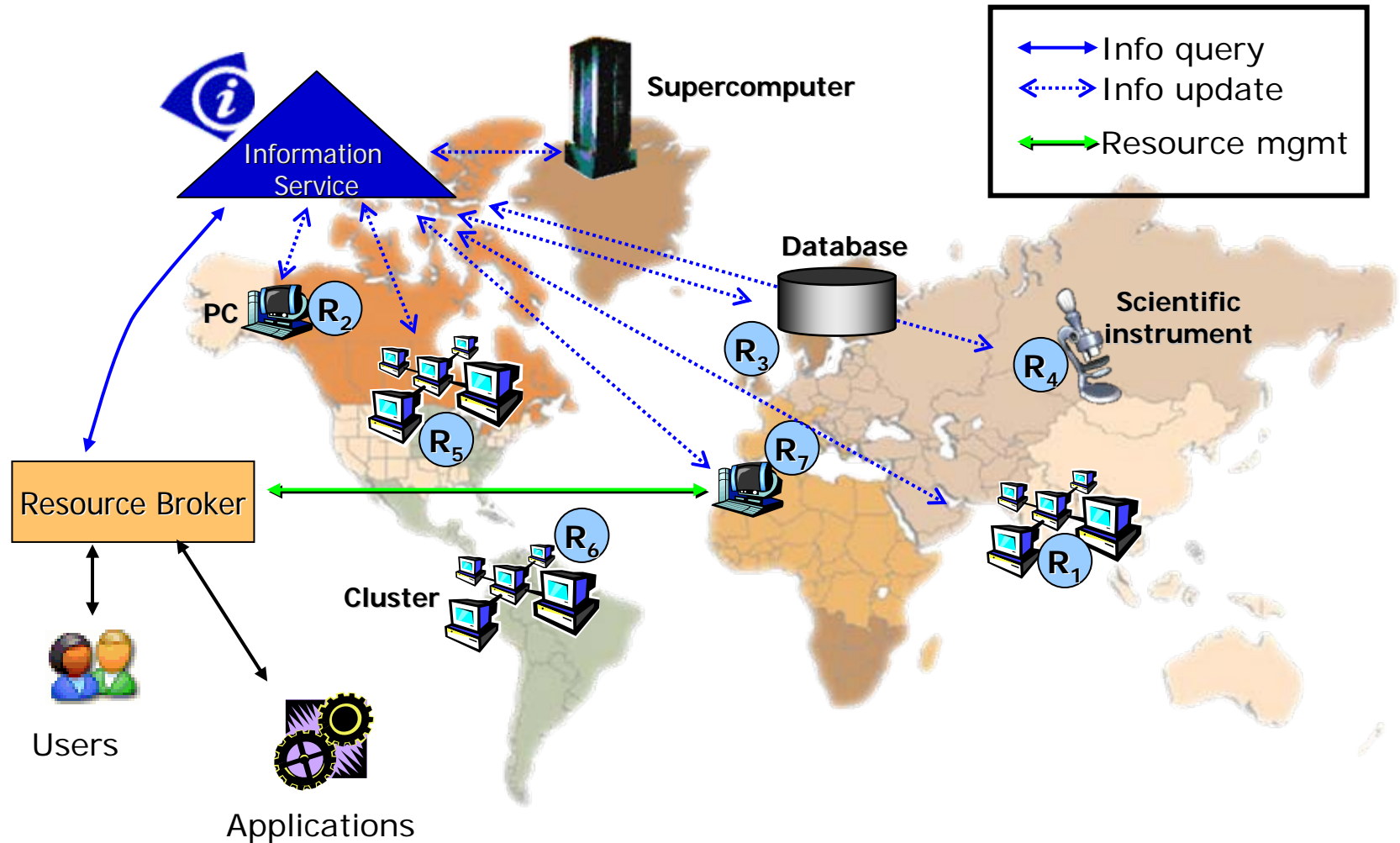


- ❑ Experimental studies report accurate performance modelling of remote SAN environments

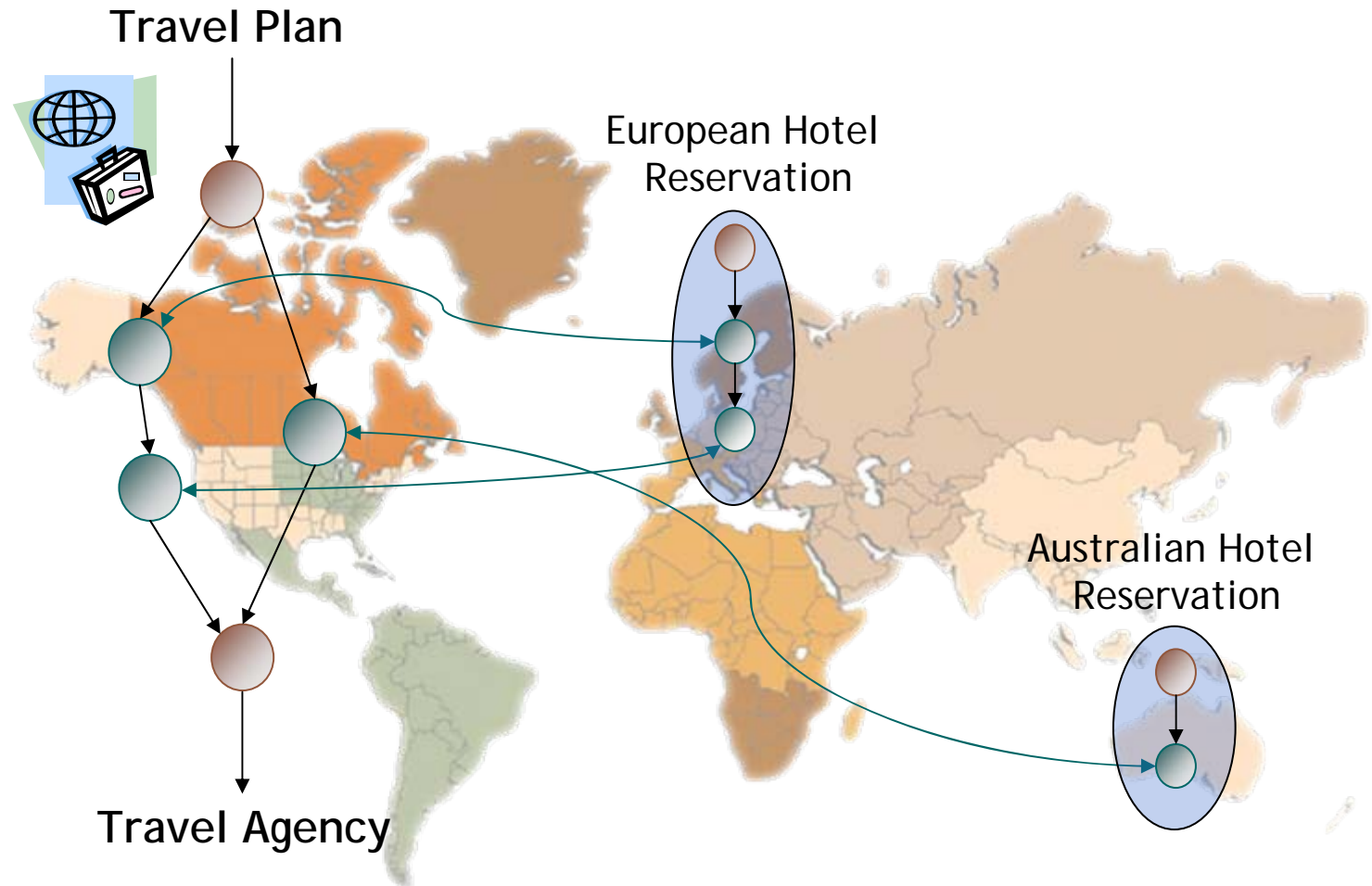


# Applications

## Grid computing



❑ Several issues: optimal scheduling, performance forecasting, ...



- ❑ Each workflow requires cooperation with several different services
- ❑ Several queues can be experienced
- ❑ Strongly multiclass workload (different types of services)



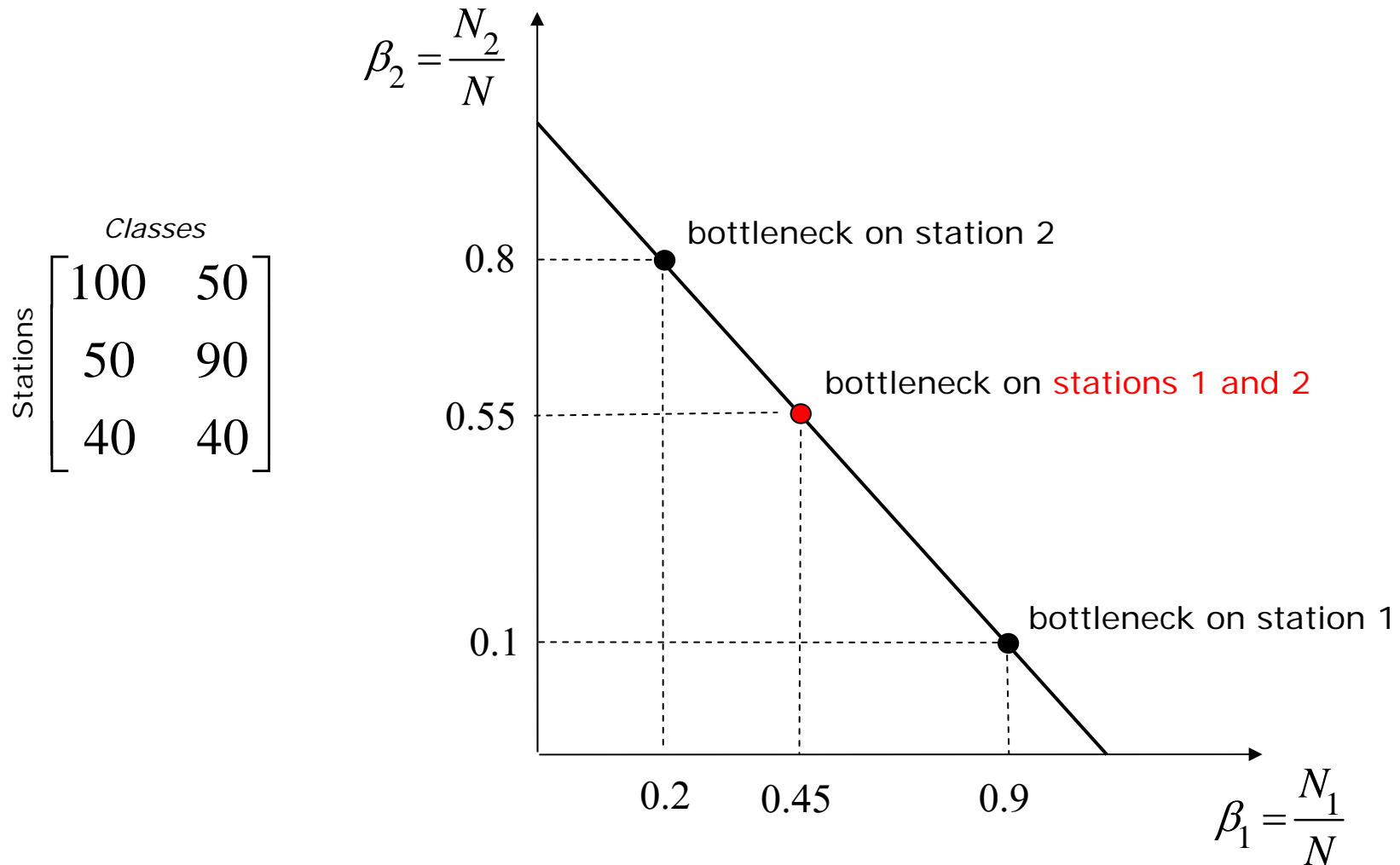
- ❑ Web caching and Web replication
  - ❑ proxy TCP connection caching models
  - ❑ content-aware load balancing
  - ❑ content delivery networks (CDN)
- ❑ Massively Multiplayer Games
  - ❑ scalability issues and load balancing
- ❑ P2P File sharing
- ❑ ...



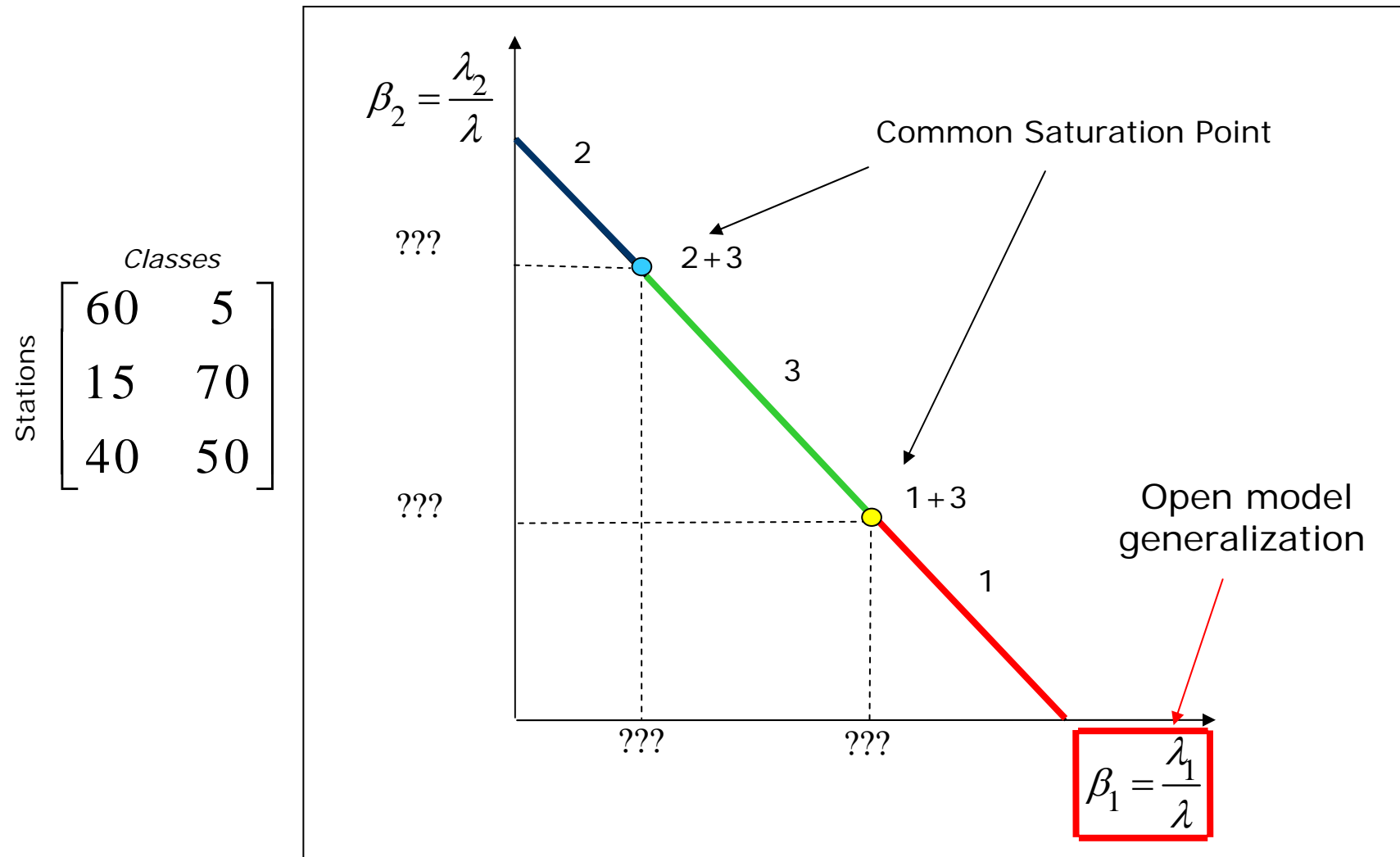
- ❑ Balbo G., Serazzi G., *Asymptotic analysis of multiclass queueing networks: common bottlenecks*. Performance Evaluation, 26: pp. 51-72 (1996)
- ❑ Balbo G., Serazzi G., *Asymptotic analysis of multiclass queueing networks: multiple bottlenecks*. Performance Evaluation, 30: pp. 115-152 (1997)
- ❑ Litoiu M., Rolia J. A., Serazzi G.: *Designing Process Replication and Activation: A Quantitative Approach*. IEEE Trans. Software Eng. 26(12): 1168-1178 (2000)
- ❑ Rosti E., Schiavoni F., Serazzi G., *Queueing network models with two classes of customers*, Proc. of Mascots 97, pp 229-234, Israel (1997)
- ❑ Casale G., Serazzi G., *Estimating Bottlenecks of Very Large Models*, In Performance Evaluation: Stories and Perspectives, pp. 89-104. Austrian Computing Group (OGC) Press (2003)



# Only one bottleneck in the network

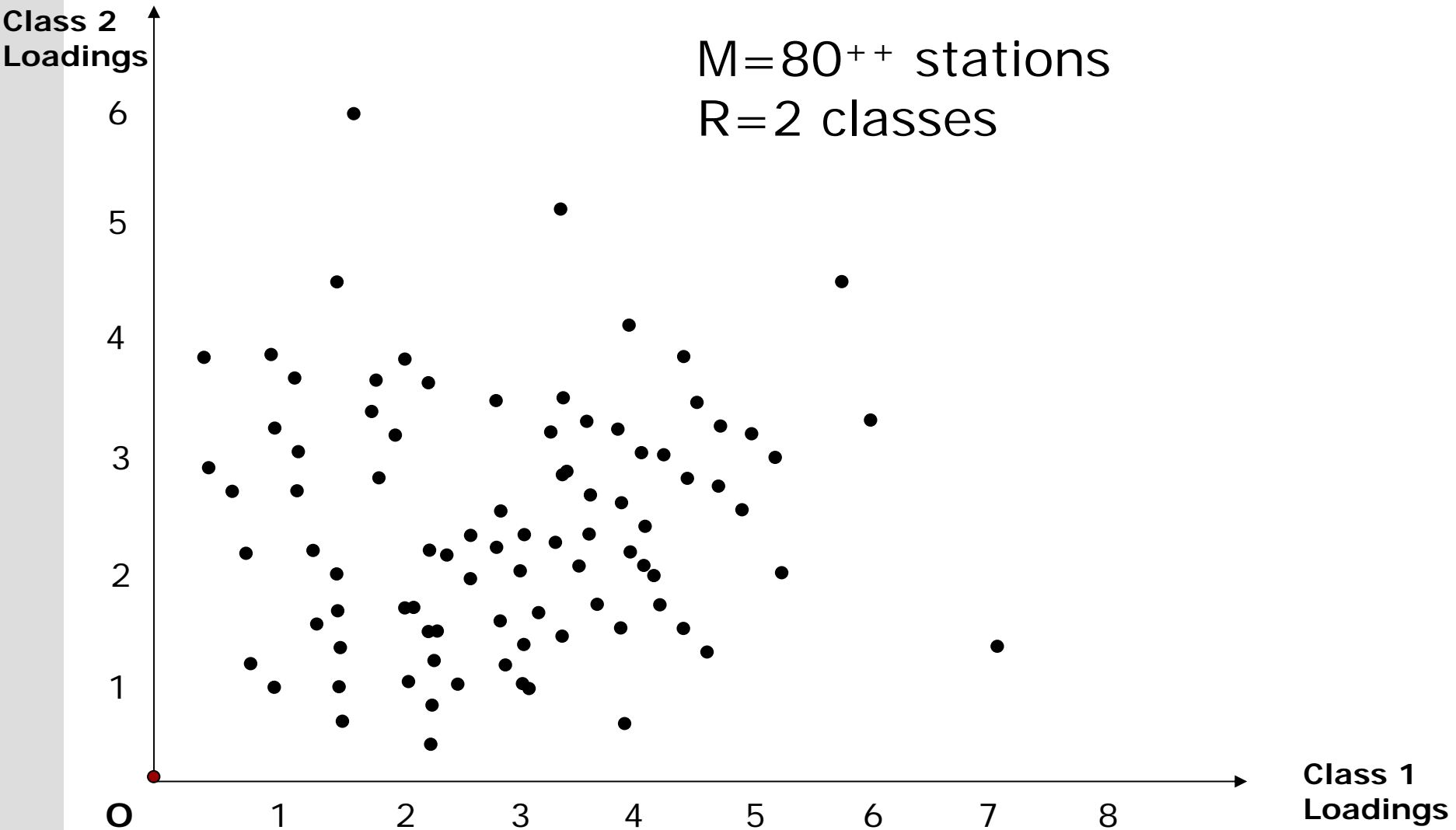


# Bottleneck migration in open models

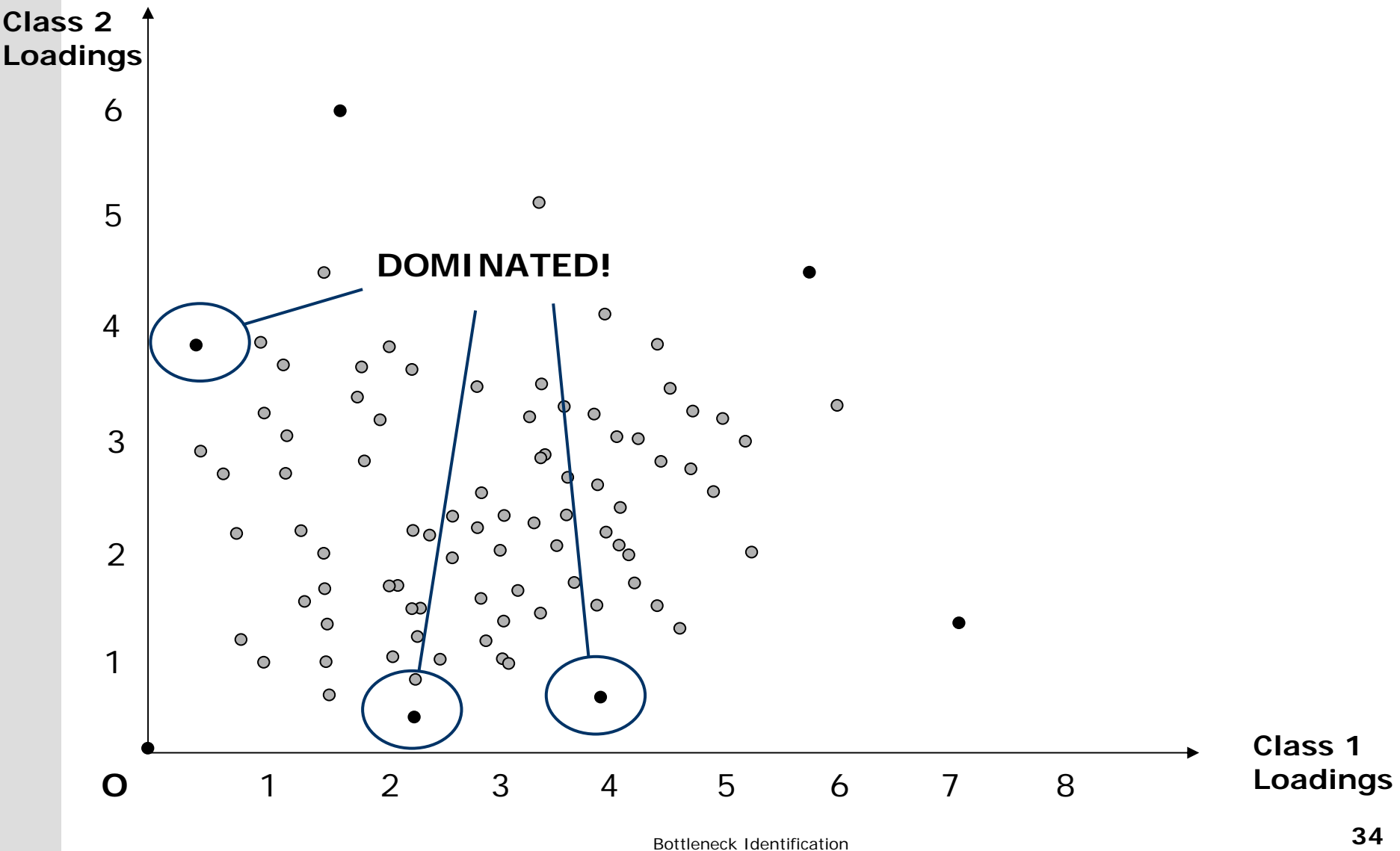


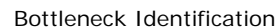


# A simple case study



# A superset of the potential bottlenecks set

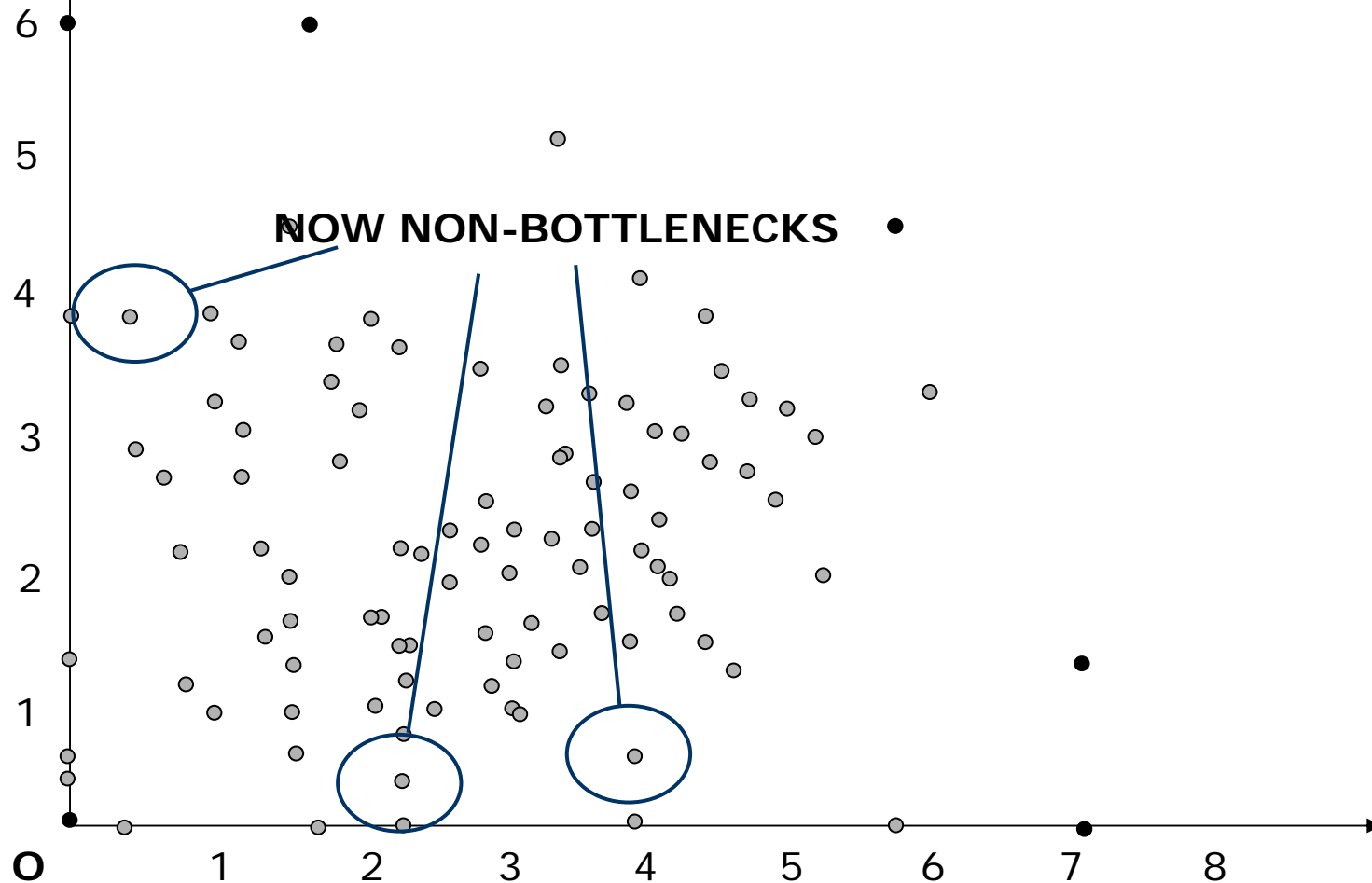




# A new REP gives the pot. bottleneck set



Class 2  
Loadings

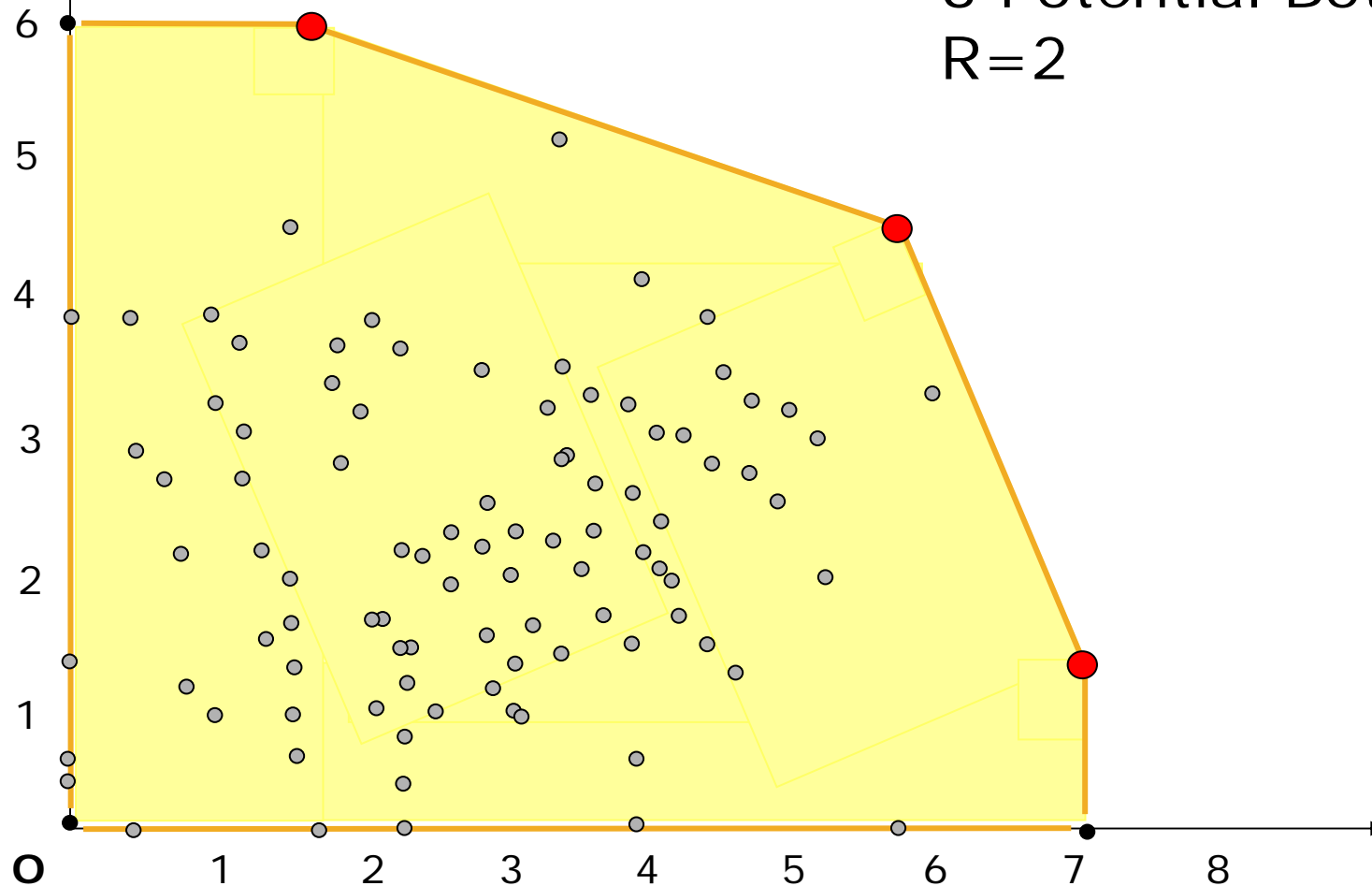


Class 1  
Loadings

# The convex hull of the model



Class 2  
Loadings



3 Potential Bottlenecks  
 $R=2$

Class 1  
Loadings

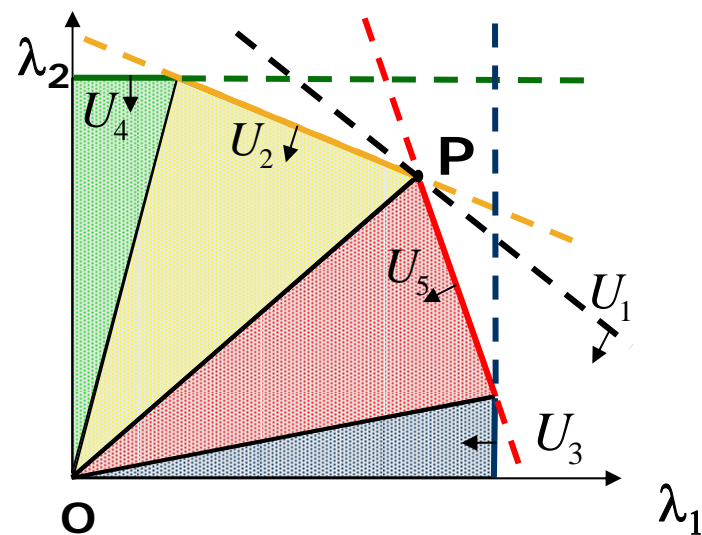
# Optimal operational points

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# Multiclass open networks

- ❑ Saturation occurs when one or more constraints  $U_k \leq 1$  are active ( $U_k = 1$ )
- ❑ Any set of active constraints defines a face of the convex hull of the feasible region



- ⇒
1. The convex hull identifies the actual bottlenecks
  2. Bottleneck migration occurs at the interface of two or more faces
  3. The common saturation sector in the arrival rate space is at most  $(R-1)$ -dimensional