Part I

Assignment two

1 Question 1

The dual problem can be formulated as follows:

$$max \sum_{S\epsilon\mathcal{F}} \mu_s - (n-1)\pi_1 + \sum_{i=2}^{n} \pi_i$$

$$\sum_{(i,j)\epsilon A,\, i\epsilon S,\, j\notin S} \mu_s - kz_{ij} \leq c_{ij} \quad {\scriptstyle (i,j)\in A}$$

$$z_{ij} - \pi_1 - \pi_i \leq 0 \quad (i,\, j) \in A'$$

$$z_{ij} + \pi_1 + \pi_i \leq 0$$

$$\mu_s \geq 0 \quad s \in \mathcal{F}$$

$$z_{ij} \leq 0 \quad (i,\, j) \in A$$

$$\pi_1 \gtreqless 0,\; \pi_i \gtreqless 0 \quad i = 1 \ldots N$$

2 Question 2

To check that it's the optimal solution i should write the dual problem and check that

The dual problem is the following:

$$min\ -3y_a + 2y_b + 6y_c + 29y_d$$

$$-2y_a - y_b + y_c + 2y_d \geq 1$$

$$-y_a + 3y_b - y_c + y_d \geq 2$$

$$y_i \geq 0 \quad i = a, b, c, d$$

And these are complementary slachness conditions:

$$x_1\left(-2y_a - y_b + y_c + 2y_d - 1\right) = 0$$

$$x_2\left(-y_a + 3y_b - y_c + y_d - 2\right) = 0$$

$$y_a\left(-2x_1 - x_2 + 3\right) = 0$$

follows that $y_a = 0$

$$y_b\left(-x_1 + 3x_2 - 24\right) = 0$$

follows that we don't know the value of y_b

$$y_c\left(x_1 - x_2 - 6\right) = 0$$

follows that $y_c = 0$

$$y_d\left(2x_1 + x_2 - 29\right) = 0$$

follows that we don't know the value of y_d
We can calculate the missing $y_b$and $y_d$using the first two equations:

$$\begin{cases} -2y_a - y_b + y_c + 2y_d - 1 = 0 & y_a = 0 \\ -y_a + 3y_b - y_c + y_d - 2 = 0 & y_c = 0 \end{cases}$$

and we end up with:

$$\begin{cases} y_b = \frac{3}{7} \\ y_d = \frac{5}{7} \end{cases}$$

So $y_{bar} = \left[0, \frac{3}{7}, 0, \frac{5}{7}\right]$
Since y_{bar} does not contain any negative value it's a feasible solution. This proves that the given values are optimal.

3 Question 3

Let $P = \{x \epsilon R^n : \ Ax \leq b\}$be a convex polyhedron. A vector x is a basic solution if among all the m vectors there are n being linearly indipendent.
Teorem 3.6 about *Strong Duality*

4 Question 4

No time yet