Due date: 11/11/2009 at 10.30

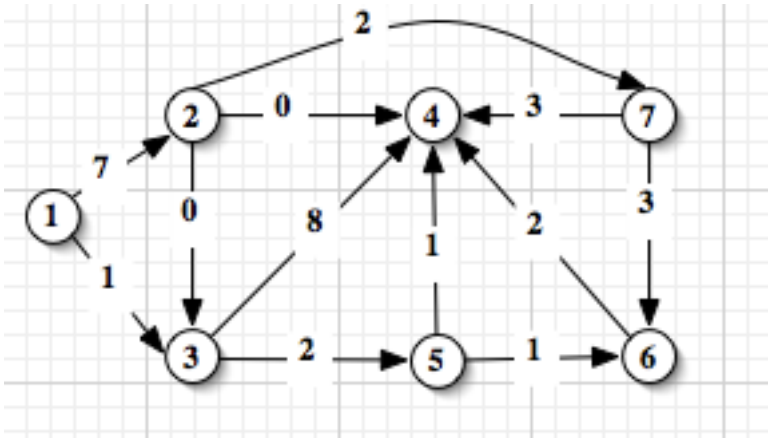
Return the homework personally to the teacher or send it in PDF format by e-mail from the official Politecnico account.

1. A factory produces three goods (1, 2, 3) over a time horizon of T days ($\{1, \ldots, T\}$). For each unit of product i the profit is $p_i, i = 1, \ldots, 3$. Two resources are used to produce the goods: workforce (w) and raw material (r) whose availability is W_t and R_t, respectively, for $t = 1, \ldots, T$. The workforce has a cost of c_{wt} per unit and the raw material has a cost of c_{rt} per unit, depending on the day. In order to produce one unit of good i, A_{wi}, and A_{ri} units of resources are needed, $i = 1, \ldots, 3$. Unused workforce cannot be cumulated for the next day, while raw material can, with the limit of the inventory capacity B. This means that in planning the production we may think of buying more raw material than that required by the production of the day, storing it in the inventory. Let us assume that the inventory at the beginning is empty. The presence of the inventory allows us to exploit profitable costs, or tackle possible raw material shortages. Plan the production so that the total profit (given by the difference of the gain due to the production and the costs due to the resources) is maximized.

   Formulate the problem as a Linear Programming model (specify the meaning of variables, the objective function and the constraints).

   Variant Suppose that when the production of a good (1, 2 or 3) is started, a fixed amount of raw material F has to be discarded to start up the machines, and this quantity does not depend on the actual production. How does the model has to be modified?

2. Determine the shortest path tree having root in node 1 for the graph below. What is the most efficient algorithm for the instance in the picture? Illustrate the intermediate steps and the possible pre-processing phase.



3. To friends are lost in the desert and they have 8 liters of water. The water is initially contained in a bottle with acapacity of 8 liters. In addition the two friends have two other bottles with capacity 5 and 3 liters respectively. In order to find the way out of the desert, they decide to split and to walk along different directions so as to maximize the probability of being rescued. The problem they face is to divide equally the water (4 liters each) without spilling it. Therefore the only operations they can do is to pour the water from a bottle to another so that either the first one is emptied or the second is filled in. The friends have to find a sequence of movements so that they end up with the bottle of 8 liters and the bottle of 5 liters containing 4 liters each. Help them in finding the shortest sequence using the search of a path on a suitable graph. Describe the graph and what you should look for.

   Variant Suppose that the two fiends want to minimize the energies spent in moving the water. The energy is proportional to the weight of the container (including the weight of the water) that has to be lifted to pour the water into the other container. Suppose that the empty container of 8 liters has weight 2, that of 5 liters has weight 5, and that of 3 liters has weight 1, and that the weight of the water is 1 per liter. How can we consider this fact in the graph model? Does the solution change? Is there any weight combination that induce a change in the solution?