Introduction to AMPL

ORLAB - Operations Research Laboratory

Stefano Gualandi

Politecnico di Milano, Italy

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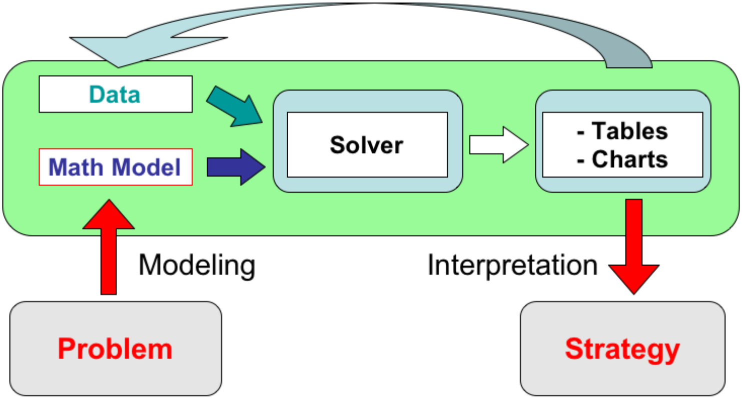
What you do during lab sessions



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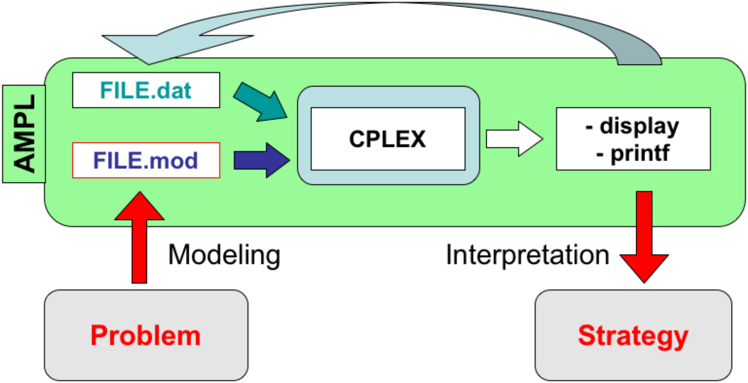

WARNINGS

- **Warning 1**: Lab sessions complement exercise sessions
- **Warning 2**: read carefully the error messages!
- Common pitfalls:
  - unbounded variables
  - missing constraints
  - missunderstood constraints
  - wrong quantification in the summation of constraints
  - wrong objective function

A short introduction to AMPL

- You write two separate text (ASCII) files:
  - *FILENAME*.**mod**: it contains the model of the problem, i.e., variables, constraints, and objective function;
  - *FILENAME*.**dat**: it contains the data specifing an instance of the problem, i.e., the coefficients of the constraints and objective function
- Useful AMPL keywords (syntax sugar):
  - `set`: defines set of elements;
  - `param`: defines constant parameters like constants, vectors, or matrices; `default` is used to give default values to parameters;
  - `var`: defines continuous variables. They can be defined also as `integer` and `binary`;
  - `sum`: defines a linear sum;
  - `subject to`: defines a family of constraints;
  - `minimize` or `maximize`: defines the objective function;

Using AMPL

1. select the solver you want to use (CPLEX):
   `% option solver cplex;`
   or: `% option solver gurobi;`
2. write/load the model (.mod) and data (.dat) files
3. solve the problem
4. if you get (syntax) errors, check your files and try again
5. if you get a solution, look it:
   `% display x;`
6. in any case, inspect how AMPL interpret your .mod file:
   `% expand Cost;`
   `% expand MatrixConstraints;`

An example: math model

Math model:

$$\begin{aligned} \min \quad & \sum_{i \in I} c_i x_i \\ \text{s.t.} \quad & Ax \leq b \\ & x \geq 0 \quad x \in R^n \end{aligned}$$

where

- A matrix $m \times n$
- $I = \{1, \ldots, n\}$ set indexes of columns of A
- $J = \{1, \ldots, m\}$ set indexes of rows of A

An example: math model (expanded)

Math model (expanded):

$$\begin{aligned} \min \quad & \sum_{i \in I} c_i x_i \\ \text{s.t.} \quad & \sum_{i \in I} a_{ij} x_i \leq b_j, \quad \forall j \in J, \\ & x_i \in R_+, \quad \forall i \in I. \end{aligned}$$

where

- A matrix $m \times n$
- $I = \{1, \dots, n\}$ set indexes of columns of A
- $J = \{1, \dots, m\}$ set indexes of rows of A

An example: FILE.mod

```
# YOU CAN WRITE COMMENTS
set I;
set J;

param A{J,I};
param b{J};
param c{I};

var x{I}, >= 0;

minimize Cost:
    sum {i in I} c[i]*x[i];

subject to MatrixConstraint {j in J}:
    sum {i in I} A[j,i]*x[i] <= b[j];
```

Exercise 4: Oil refinery

An oil refinery mixes 4 types of raw oil in order to produce 3 kinds of fuel: *normal*, *super*, *green*. The following table shows the maximum quantity available for each type of raw oil and the corresponding cost per barrel:

Type of raw oil	Barrels available	Cost (euro/barrel)
1	5000	9
2	2400	7
3	4000	12
4	1500	6

Exercise 4: Oil refinery

The technical specification of each type of fuel constrains the quantity of raw oil used to produce each type of fuel. The following table shows this *blending* constraint along with the cost per barrel of the fuels:

Type of fuel	Required raw oil	Price (euro/barrel)
normal	at least 20% of type 2 at most 30% of type 3	12
super	at least 40% of type 3	18
green	at most 50% of type 2	10

The oil refinery wants to mix the different types of raw oil in such a way to maximize its profit.

Exercise 4: Sets, parameters and data

- Sets:
  - I = set of oils
  - J = set of fuels
- Parameters:
  - c_i = cost/barrel of oil i
  - d_i = availability of oil i
  - r_j = price of fuel j
- Variables:
  - $x_{i,j}$ = quantity of oil i in fuel j
  - y_j = quantity of fuel j sold

Exercise 4: Math model

Objective function:

$$\max \quad \sum_{j \in J} r_j y_j - \sum_{i \in I, j \in J} c_i x_{i,j}$$

subject to constraints:

$$\sum_{j \in J} x_{i,j} \leq d_i \qquad \forall i \in I \qquad \rightarrow \text{oil availability}$$

$$y_j = \sum_{i \in I} x_{i,j} \qquad \forall j \in J \qquad \rightarrow \text{fuels produced}$$

$$x_{i,j} \leq q_{i,j}^{max} y_j \qquad \forall i \in I, \forall j \in J \qquad \rightarrow \textit{blending} \text{ constraints}$$

$$x_{i,j} \geq q_{i,j}^{min} y_j \qquad \forall i \in I, \forall j \in J \qquad \rightarrow \textit{blending} \text{ constraints}$$

$$x_{i,j} \geq 0 \qquad \forall i \in I, \forall j \in J$$

$$y_j \geq 0 \qquad \forall j \in J$$

Exercise 4: Additional question

Suppose you are working at the Eurepan Parlament, and you want to decrease the consumption of *normal* and *super* fuels in order to favorite the *green* fuel.

What decision do you take? Why?

Exercise 5: Exam - September 2006

An italian manager has to plan the production of shoes for the Della Calli' company. The company can produce a given set of shoes $M = \{1, \ldots, m\}$ using the set of employees $O = \{1, \ldots, m\}$. A worker i takes $p_{i,j}$ minutes to produce a pair of shoes of model j, and he can work at maximum for q_i minutes. For each model j is given the selling price g_j, and the maximum amount r_j which can be sold on the market.

Formulate the problem of maximizing the overall money gain.

Exercise 5: Exam - September 2006

- Sets:
  - M = set of shoe models
  - O = set of employees
- Parameters:
  - $p_{i,j}$ = time for worker i to produce a pair of model j
  - q_i = maximum working time for worker i
  - g_j = selling price of model j
  - r_j = maximum production for model j
- Variables:
  - $x_{i,j}$ = quantity of shoes of model j produced by worker i

Challenge 1: N-Queens

Express an ILP model to solve the N-Queens problem, that is to locate N queens on a chess board in such a way that no queen threatsany other queen. Further, solve with AMPL the N-Queens problem for N = 3, 5, 10, 20, 100, 1000.

Links and email

- Lab sessions web site:
  http://home.dei.polimi.it/gualandi/FRO-D
- E-mail:
  gualandi_at_elet.polimi.it
- Italian guide to AMPL:
  http://.../gualandi/FRO-D/guida_ampl.pdf
- Set of examples of translating math model into AMPL
  http://.../gualandi/FRO-D/ampl_examples.pdf