Politecnico di Milano
STATISTICS (079086), 2009/2010, Prof. A.Barchielli
Problem set n. 2

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Exercise 1 Let $X_1, \ldots, X_n$ be a random sample from a Gaussian distribution $\mathcal{N}(4.2, 4)$.

1. Calculate $P(X_1 > 3)$; [0.726]
2. Find x such that $P(X_1 \leq x) = 0.7$; [0.524]
3. Calculate $P(\bar{X} \leq 3)$, with $n = 3$; [0.149]
4. Calculate $P(|\bar{X} - 4.2| \leq 0.3)$, with $n = 3$ and $n = 25$; [0.206,0.546]
5. What is the minimum n such that $P(|\bar{X} - 4.2| \leq 0.3) \geq 0.8$? [74]

Exercise 2 Let X_1 and X_2 be two independent random variables with Exponential distribution and mean 1/2. Set $Y = X_1 + X_2$. Moreover, let Z be a random variable with Uniform distribution over the interval $(0, 1)$, and set $Cov(Y, Z) = 0.05$.

1. Compute the correlation coefficient of Y and Z. *[The correlation coefficient of Y and Z is $corr(Y, Z) = Cov(Y, Z)/\sqrt{Var(Y) \times Var(Z)}$. We know that X_1 and X_2 are independent and thus $Var(X_1 + X_2) = Var(X_1) + Var(X_2) = 2\,\frac{1}{4} = \frac{1}{2}$. Moreover, $Var(Z) = \frac{1}{12}$. Hence, $corr(Y, Z) = \frac{0.05}{\sqrt{\frac{1}{2}\frac{1}{12}}} = 0.245$]*
2. Compute the mean and variance of $W = Y - 2Z$. *[We know that $E(Y) = E(X_1 + X_2) = E(X_1) + E(X_2) = 2\,\frac{1}{2} = 1$ and $E(Z) = \frac{1}{2}$. Hence, $E(W) = E(Y - 2Z) = E(Y) - 2E(Z) = 1 - 1 = 0$. Moreover, $Var(W) = Var(Y - 2Z) = Var(Y) + 4Var(Z) - 4Cov(Y, Z) = \frac{1}{2} + 4\,\frac{1}{12} - 4 \times 0.05 = 0.633$]*

Exercise 3 The quantity of coffee powder utilized by an automatic coffee machine for a cup of coffee is a Gaussian random variable with mean 30 grams. Find the standard deviation such that 98% of the cups contains al least 27 grams of coffee powder. [$\sigma = 1.46$]

Exercise 4 The glycemic level in the blood of an adult woman is a Gaussian random variable with mean $\mu = 80$ mg%ml and standard deviation $\sigma = 20$ mg%ml. If this level is 1.5σ greater than the mean value, the patient is considered hill (hyperglycemic). Find the probability that an adult woman is hyperglycemic. [0.0668]

Exercise 5 A test for students consists in 100 questions each with two possible answers.

a) Suppose the student chooses the answer at random. Find the approximate probability that the student correctly answers to at least 55 questions. [0.184, normal approximation]

b) The teacher decides to set a minimum score y to avoid that student answering at random pass the test. The minimum score y is then set as that value such that the probability to obtain at least y at random is 0.05. Find y. [59]

Exercise 6** An avicultural farm rears two types of chickens A and B. Let X be the random variable that indicates the weight of the eggs produced by type A chickens, and let Y be the random variable that indicates the weight of the eggs produced by type B chickens. If $X \sim \mathbb{N}(60, 4)$ and $Y \sim \mathbb{N}(55, 2.5)$, and X and Y are independent, find the probability the variable $(X + Y)/2$ takes values in the interval $(55, 58)$. [0.6267]

Exercise 7** Let $X \sim f(x, \theta)$ be the random variable indicating the daily journey time (in minutes) from Peter's house to the Politecnico:

$$f(x,\theta) = \begin{cases} \frac{\theta}{30^\theta}(x-30)^{\theta-1} & \text{if } 30 < x < 60 \\ 0 & \text{otherwise} \end{cases}$$

with θ unknown positive parameter.

1. Express in terms of θ the probability that Peter reaches the Politecnico in less than 35 minutes. $[1/6^\theta]$
2. Indicate the appropriate probabilistic model describing the number of days out of 100 days in which Peter reaches the Politecnico in less than 35 minutes. $[Bin(100, 1/6^\theta)]$
3. Express in terms of θ the approximate probability that Peter reaches the Politecnico in less than 35 minutes at least 40 days out of 100 days. $[1 - \Phi\left((3.95 \times 6^\theta - 10)/\sqrt{6^\theta - 1}\right)]$
4. Calculate the probability found at point 3. when $\theta = 0.6$ $[\simeq 1 - \Phi(1.13) \simeq 0.129]$