



***Politecnico di Milano***  
*Facoltà di Ingegneria dell'Informazione*

## **5 – Multiple Access**

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**Reti Mobili Distribuite**

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# Packet access

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- ❑ At logical layer multiple access can be managed in a dynamic and distributed way using multiple access protocols
- ❑ First multiple access protocols have been designed for LANs
- ❑ Nowadays multiple access protocols are mainly used in wireless networks (no more shared medium wired LANs)



# Packet access: Classification

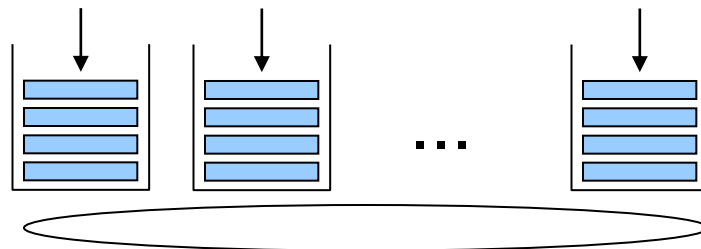
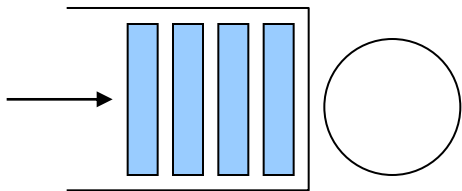
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- Scheduled access
  - Transmissions on the channel are sequential with no conflicts
  - Polling schemes
  - Centralized scheduling schemes
- Random access
  - Transmission are partially uncoordinated and can overlap (collision)
  - Conflicts are resolved using distributed procedures based on random retransmission delay



# Packet access: Local and global queues

- ❑ In the case of multiplexing (single station) we have a single queue that is managed according to a scheduling algorithm
- ❑ In case of multiple access with  $M$  stations with local queues we still have the opportunity to use a single “virtual” queue at a central decision point that schedule access to the channel

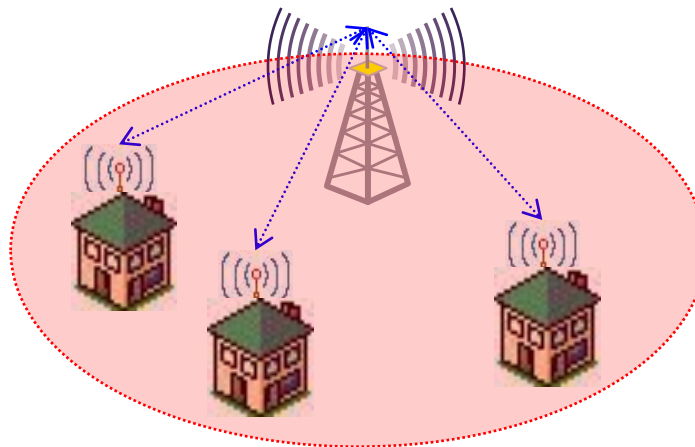




# Packet access: Local and global queues

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- However, to inform the scheduler of the status of the local queues and provide access grants we have to use the channel (coordination signaling)
- The centralized scheduling approach is quite flexible but complex
- It is adopted in several wireless technologies like e.g. WiMax





# Assumptions and notation

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- ❑ In the following we drop the assumption of global coordination and analyze distributed mechanisms
- ❑ Let us assume that arrival times in the  $M$  local queues are described by a Poisson process with rate  $\lambda/M$  ( $\lambda$  global rate)
- ❑ The system status is described by vector

$$\mathbf{n} = (n_1, n_2, \dots, n_M)$$

- ❑ Where  $n_i$  is the number of packets in queue  $i$
- ❑ The system evolution is described by the process  $N(t)$



# Polling

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- ❑ Polling schemes are scheduled access schemes where stations access the channel according to a cyclic order
- ❑ The polling message, or *token*, is the grant for access the channel
- ❑ The token can be distributed by a central station (roll-call polling) or passed from station to station (hub polling or token system)
- ❑ Let us assume that packet transmission time is  $T$  and that token passing time is  $h$ , both constant
- ❑ Polling schemes differentiate based on the service policy (exhaustive, gated, limited)



# Exhaustive Polling

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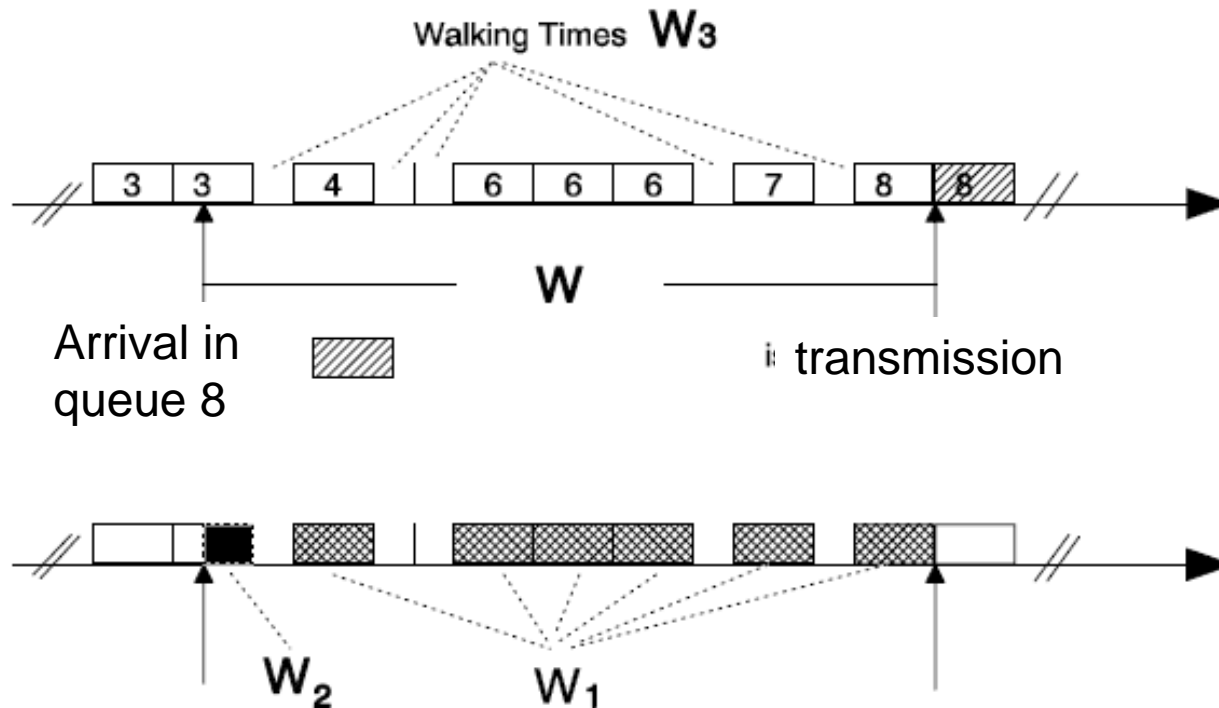
- With exhaustive polling, stations when receive the token transmit all packets in the queue before releasing it
- Let us analyze the behavior of this system
- The probability that the channel is transmitting a packet at a random time  $t$  is give by

$$\rho = \lambda T$$



# Exhaustive Polling

- The average waiting time  $E[W]$  in the queue can be calculated considering three components  $E[W] = W_1 + W_2 + W_3$





# Exhaustive Polling

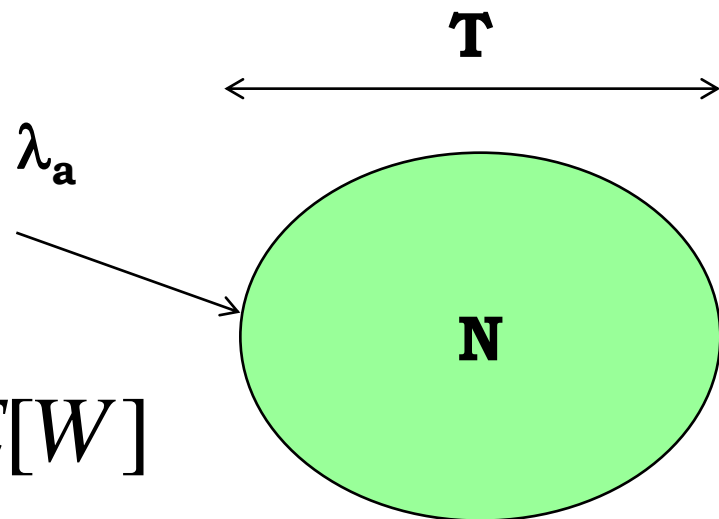
$$W_1 = E[N_c]T$$

- $E[N_c]$  is the average number of packets transmitted before considered packet
- Using Little's result is can be expressed as:

$$E[N_c] = \lambda E[W]$$

- Therefore:

$$W_1 = \lambda T E[W] = \rho E[W]$$





# Exhaustive Polling

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$$W_2 = \rho \frac{T}{2} + (1 - \rho) \frac{h}{2}$$

$$W_3 = \frac{M - 1}{2} h$$

□ The total average waiting time is given by:

$$E[W] = \rho E[W] + \rho \frac{T}{2} + (1 - \rho) \frac{h}{2} + \frac{(M - 1)}{2} h$$



# Exhaustive Polling

- Solving by  $E[W]$  we get:

$$E[W] = \underbrace{\frac{\rho}{2(1-\rho)} T}_{\text{Waiting time of a single queue (M/D/1)}} + \underbrace{\frac{M-\rho}{2(1-\rho)} h}_{\text{Additional waiting time due to token passing time}}$$

Waiting time of a single queue (M/D/1)

Additional waiting time due to token passing time

- Note that:

$$\rho_{\max} = 1$$



# Exhaustive Polling

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- The average token cycle time is given by the transmission time of all packets that arrive during a cycle plus the token passing time

$$E[C] = \lambda E[C]T + Mh$$

$$E[C] = \frac{Mh}{1 - \rho}$$



# Gated Polling

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- With gated polling, stations when receive the token can transmit all packets that are in queue at the time when the token arrives
- The expression of the average waiting time is similar to previous case with an additional term
- This is the additional cycle the packet has to wait when it arrives when the token is already at the station

$$W_4 = \frac{\rho}{M} Mh = h\rho$$



# Gated Polling

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□ Therefore we get:

$$E[W] = \frac{\rho}{2(1-\rho)} T + \frac{M + \rho}{2(1-\rho)} h$$

□ Again  $\rho_{\max} = 1$



# Limited Polling

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- With limited polling, stations when receive the token can transmit only up to  $k$  packets
- The special case of  $k=1$  is called *Round-Robin*
- Here we have one more additional term which are the additional cycles the packet has to wait, one per each packet in the queue at the arrival moment

$$W_5 = \frac{E[N_c]}{M} Mh = \lambda E[W] h$$





# Limited Polling

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□ Therefore we get:

$$E[W] = \frac{\rho}{2(1 - \rho \frac{h+T}{T})} T + \frac{M + \rho}{2(1 - \rho \frac{h+T}{T})} h$$

□ Now we have:

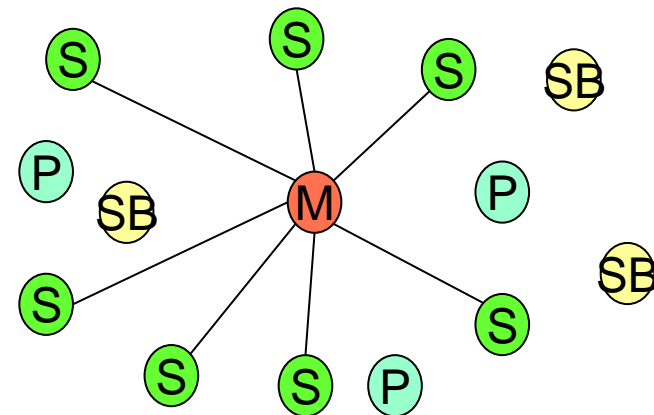
$$\rho_{\max} = \frac{T}{T + h}$$



# Polling in real networks

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- There are several examples where polling is used for regulating access to a channel in wireless technologies
  - WiFi (Point Coordination Function – PCF or HCF – Hybrid Coordination Function)
  - Bluetooth
- The main difference with simple schemes we considered so far is that the station sequence can be dynamically changed





# Polling in Bluetooth

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- Some key characteristics of Bluetooth multiple access mechanism make the direct application of previously derived formulas not possible:
  - Queues are not visited in a sequential order (master queue is always visited in odd slots)
  - Token passing time is always one slot, but the slot is used for data transmission if the queue is not empty
  - Exhaustive service makes no sense for Bluetooth since after each packet transmission by the master/slave at least a slot is used by the slave/master
  - Bluetooth makes use of packets with different lengths (1, 3, or 5 slots)
- We derive expressions for the waiting time in two special cases



# Polling in Bluetooth

- Let us assume the master has one separate queue per slave and all queues are visited according to a fixed sequence
- Arrival in the queues are independent Poisson processes

$m$ : number of BT devices

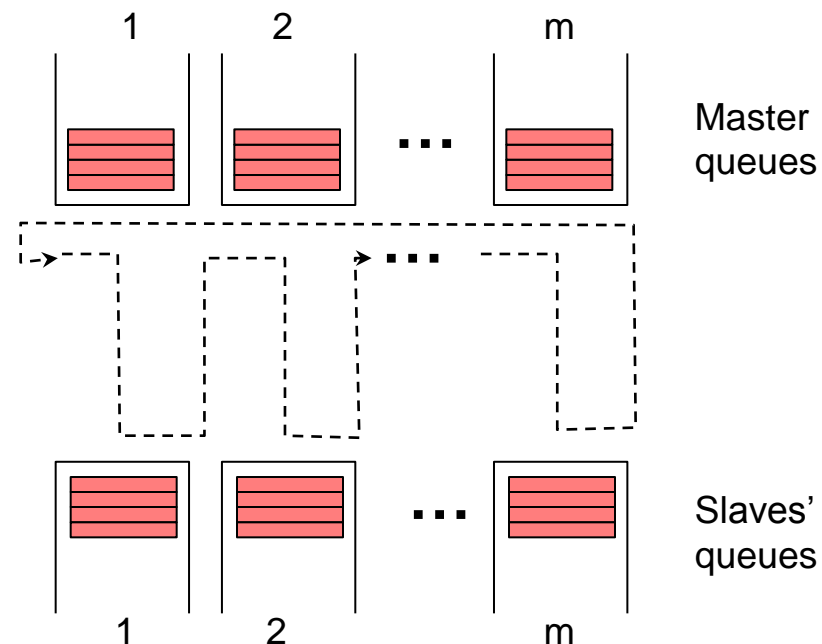
$m-1$ : slaves

$M=2(m-1)$ : total queues

$\gamma$ : arrival rate in each queue

$\lambda$ : total arrival rate

$T$ : slot duration





# Polling in Bluetooth

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## □ Case 1)

- 1-limited service (round-robin)
- 1-slot packets only

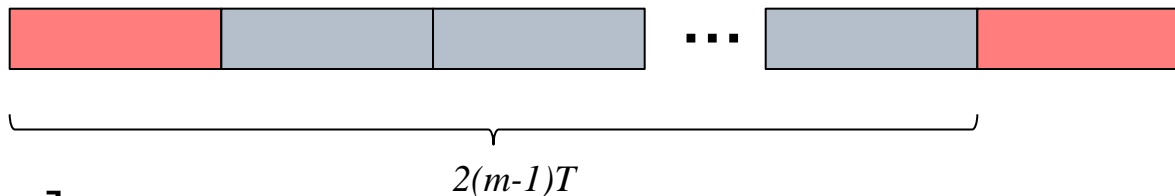
## □ We observe that

- Cycle length is fixed and equal to  $2(m-1)$  slots
- System is equivalent to a TDMA with  $2(m-1)$  slots per frame
- There are several equivalent ways of calculating the waiting time
- We use the same approach adopted for the general polling schemes



# Polling in Bluetooth

## □ Case 1)



$$W_1 = \gamma TE[W]$$

$$W_2 = \frac{2(m-1)T}{2} = (m-1)T$$

$$W_3 = 0$$

$$W_4 = 0$$

$$W_5 = \gamma TE[W] [2(m-1) - 1]$$

$$E[W] = \frac{(m-1)T}{1 - \gamma T - 2(m-1)\gamma T + \gamma T} = \frac{(m-1)T}{1 - 2(m-1)\gamma T}$$



# Polling in Bluetooth

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- Case 2)
  - 1-limited service (round-robin)
  - 1, 3, and 5-slots packets
- We observe that:
  - The system is equivalent to a polling system with:
    - Token passing time equal to 1 slot
    - Service time equal to packet length minus one slot

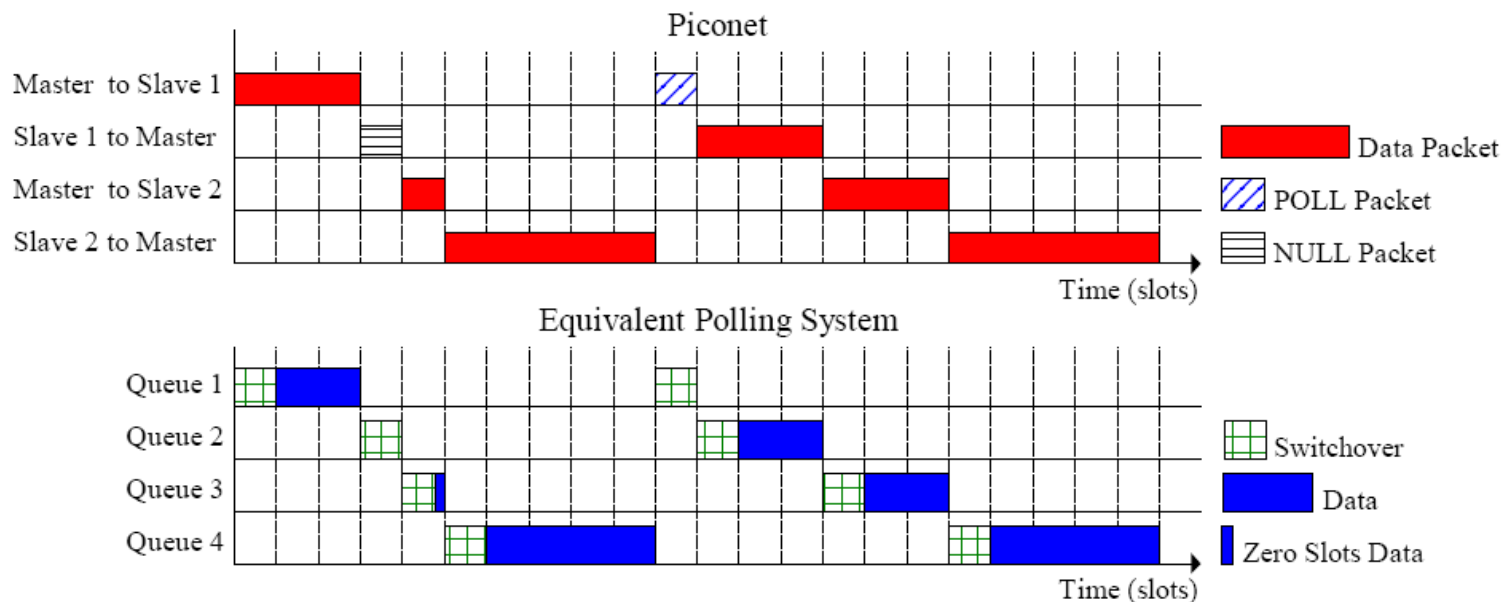




# Polling in Bluetooth

## □ Case 2)

- 1-limited service (round-robin)
- 1, 3, and 5-slots packets





# Polling in Bluetooth

## □ Case 2)

■ Notation:  $p_1$  : prob. of 1 - slot packets

$p_3$  : prob. of 3 - slot packets

$p_5$  : prob. of 5 - slot packets

$L$  : packet length

$$\bar{L} = p_1 + 3p_3 + 5p_5$$

$X$  : service duration in the equiv. system

$$\bar{X} = 2p_3 + 4p_5$$

$$\bar{X}^2 = 4p_3 + 16p_5$$

$$E[z] = \frac{\bar{X}^2}{2\bar{X}}$$



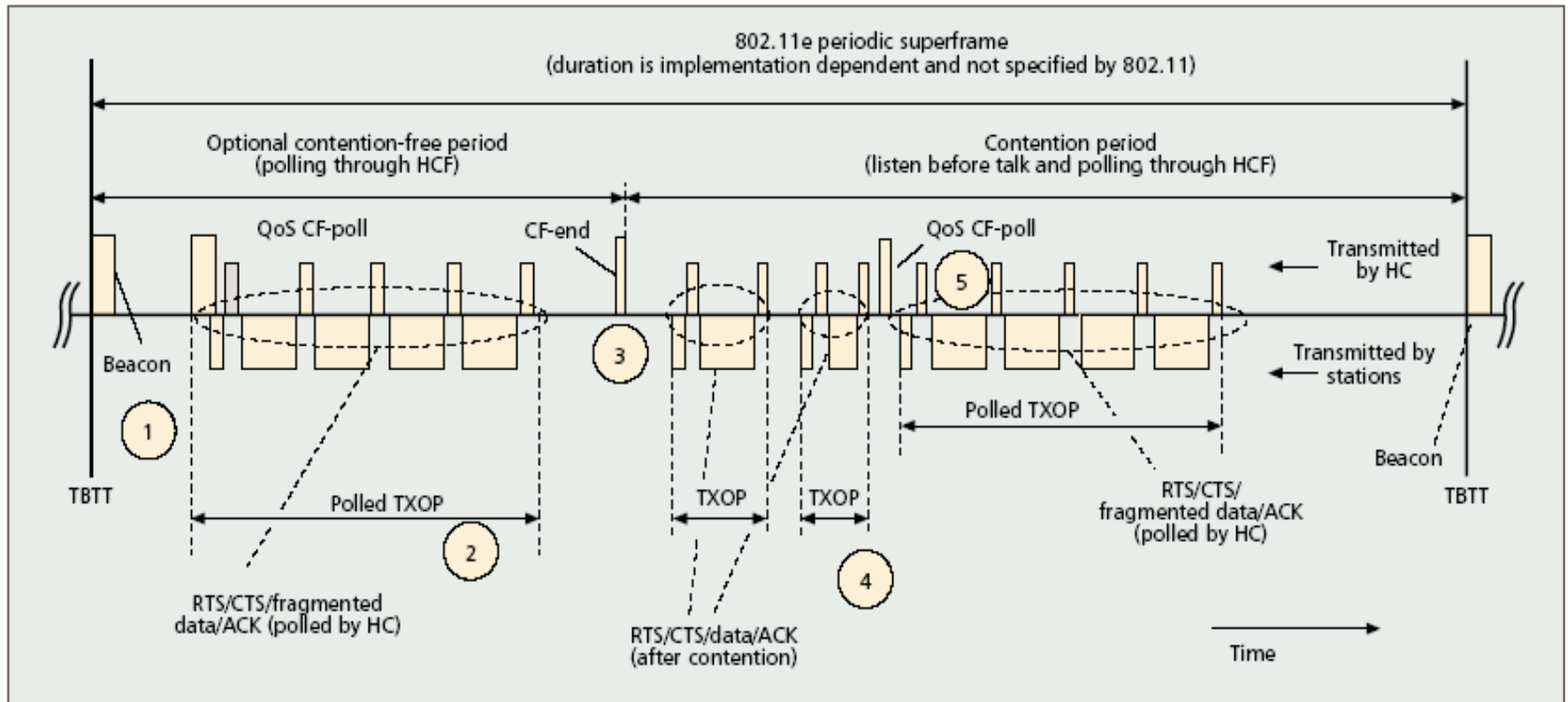
# Polling in Bluetooth

## □ Case 2)

■ Waiting time:

$$\begin{aligned} E[W] &= \frac{\rho}{1 - \frac{(L-1+1)T}{LT} \rho} E[z] + \frac{M + \rho}{2 \left( 1 - \frac{(L-1+1)T}{LT} \rho \right)} T = \\ &= \frac{\rho}{1 - \frac{L}{L-1} \rho} E[z] + \frac{2(m-1) + \rho}{2 \left( 1 - \frac{L}{L-1} \rho \right)} T = \\ &= \frac{2(m-1)\gamma(L-1)T}{1 - 2(m-1)\gamma LT} \frac{\bar{X}^2}{2(L-1)} + \frac{2(m-1) + 2(m-1)\gamma(L-1)T}{2(1 - 2(m-1)\gamma LT)} T = \\ &= \frac{(m-1)\gamma}{1 - 2(m-1)\gamma L} \bar{X}^2 + \frac{(m-1) + (m-1)\gamma(L-1)T}{1 - 2(m-1)\gamma LT} T = \end{aligned}$$

# Polling in WiFi (IEEE 802.11e)





# Random access

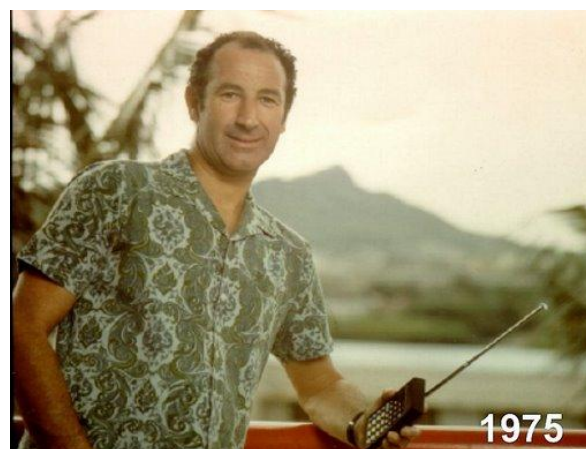
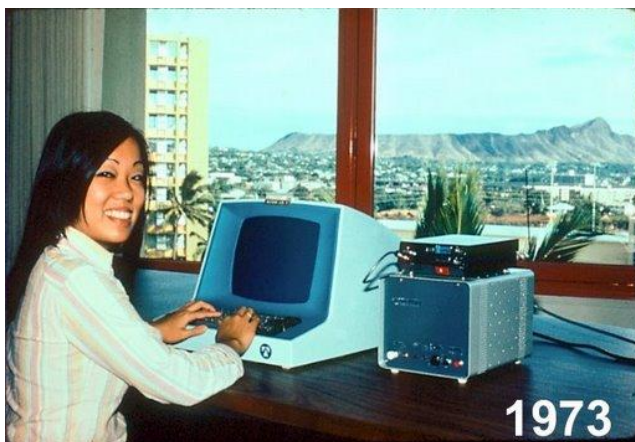
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- ❑ With random access there are possible conflicts on the channel (collisions)
- ❑ Conflicts are resolved using the channel feedback and some procedure to select a random waiting time
- ❑ The minimum channel feedback a station need to have is the information if its transmission was sccessful or not
- ❑ The first and simplest random access protocol is **Aloha** which uses just this minimum feedback

# AlohaNet

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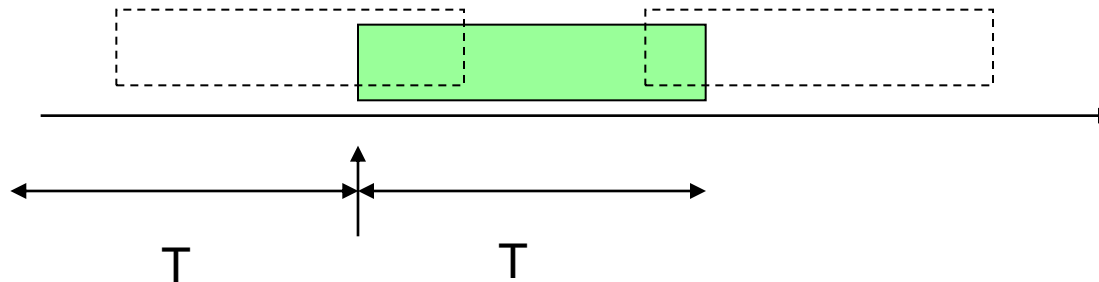
- The ALOHA network was created at the University of Hawaii in 1970 under the leadership of Prof. Norman Abramson
- It was the first wireless network!





# Aloha

- The access mechanism is very simple:
  - When there is a packet to be transmitted, just transmit it.
  - If transmission fails, wait for a random time and retransmit





# Aloha

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- Let us assume the transmission starting times on channel are a Poisson process with rate  $\lambda$
- Let us consider the normalized rate  $G = \lambda T$
- The success probability is given by the probability that there is no other transmission in a  $2T$  interval

$$P_s = e^{-2G}$$

- The normalized throughput  $S$  is therefore given by:

$$S = Ge^{-2G}$$

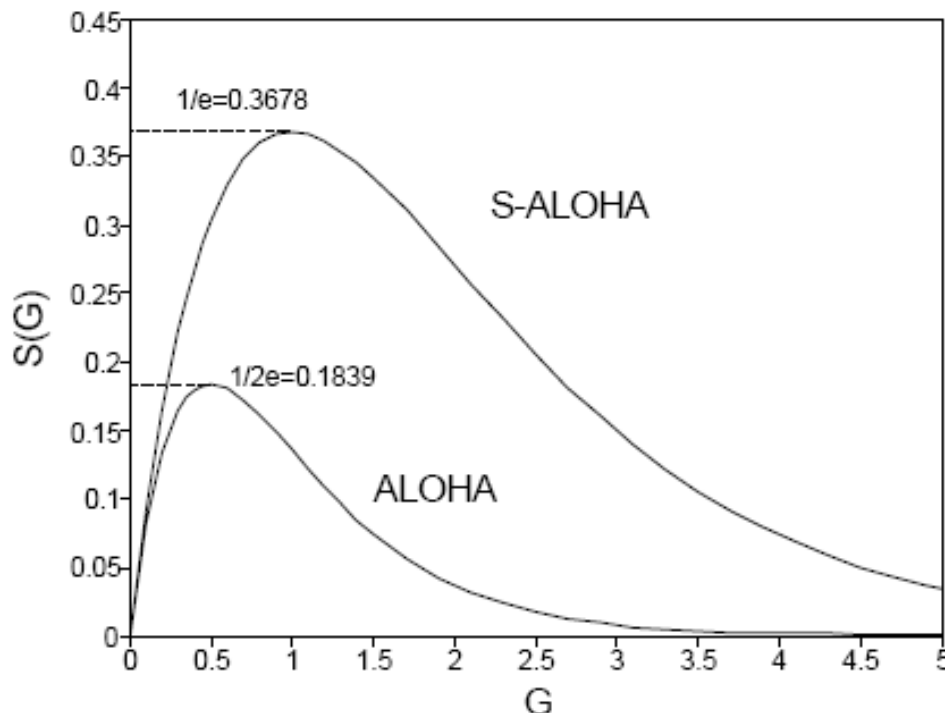




# Aloha

- If transmissions are somehow synchronized (slotted Aloha) the vulnerability period reduces to  $T$  and therefore

$$S = Ge^{-G}$$



Infinite  
population  
model



# Aloha: Single buffer

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- ❑ Unfortunately, the traffic on the channel is the combination of new transmissions and retransmissions and it can increase if throughput reduces
- ❑ To evaluate the dynamic behavior of Aloha let us consider an enhanced model
- ❑ Let us assume we have  $M$  stations with a single transmission buffer (max 1 packet)
- ❑ Channel is slotted
- ❑ If buffer is empty a new packet arrive and is immediately transmitted in the slot with probability  $\alpha$
- ❑ If buffer is full, packet is retransmitted with probability  $\beta$



# Aloha: Single buffer

- The system status is given by  $n(t)$ , the number of full buffers at time  $t$
- $n(t)$  is a discrete Markov chain
- The probability that in a slot there are  $i$  arrivals given  $n$  buffers are full is:

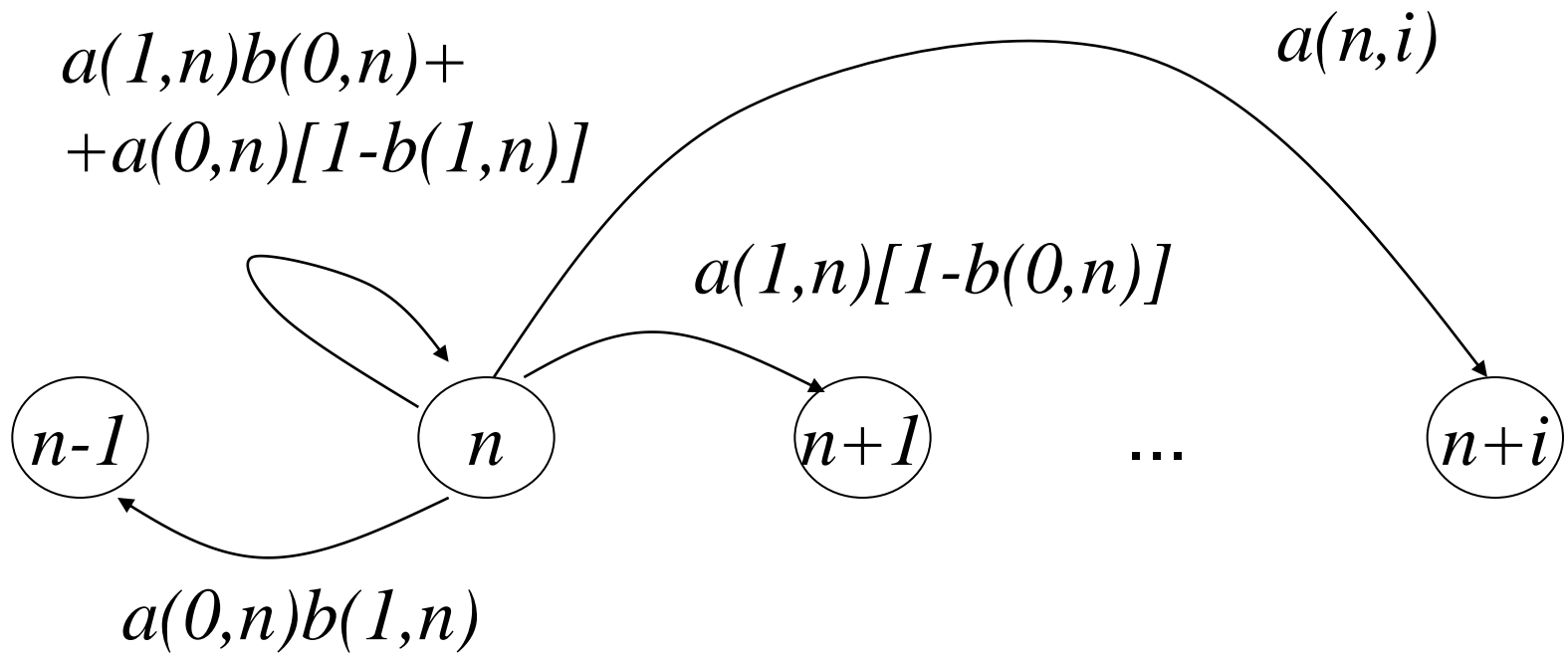
$$a(i, n) = \binom{M-n}{i} \alpha^i (1-\alpha)^{M-n-i}$$

- While the probability that there are  $i$  retransmission is:

$$b(i, n) = \binom{n}{i} \beta^i (1-\beta)^{n-i}$$



# Aloha: Single buffer





# Aloha: Single buffer

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- We can solve (numerically) the chain and get the stationary state probability  $\pi_n$
- The throughput  $S$  is given by

$$S = E[s(n)] = \sum_{n=0}^M \pi_n s(n)$$

$$s(n) = a(1, n)b(0, n) + a(0, n)b(1, n)$$



# Aloha: Single buffer

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- The traffic on the channel is:

$$g(n) = (M - n)\alpha + n\beta$$

- and the new arrivals:

$$a(n) = (M - n)\alpha$$

- We can express  $n$  as a function of  $g$ :

$$n = \frac{g - M\alpha}{\beta - \alpha}$$

- And calculate  $a(g) = M\alpha - (g - M\alpha)\frac{\alpha}{\beta - \alpha}$

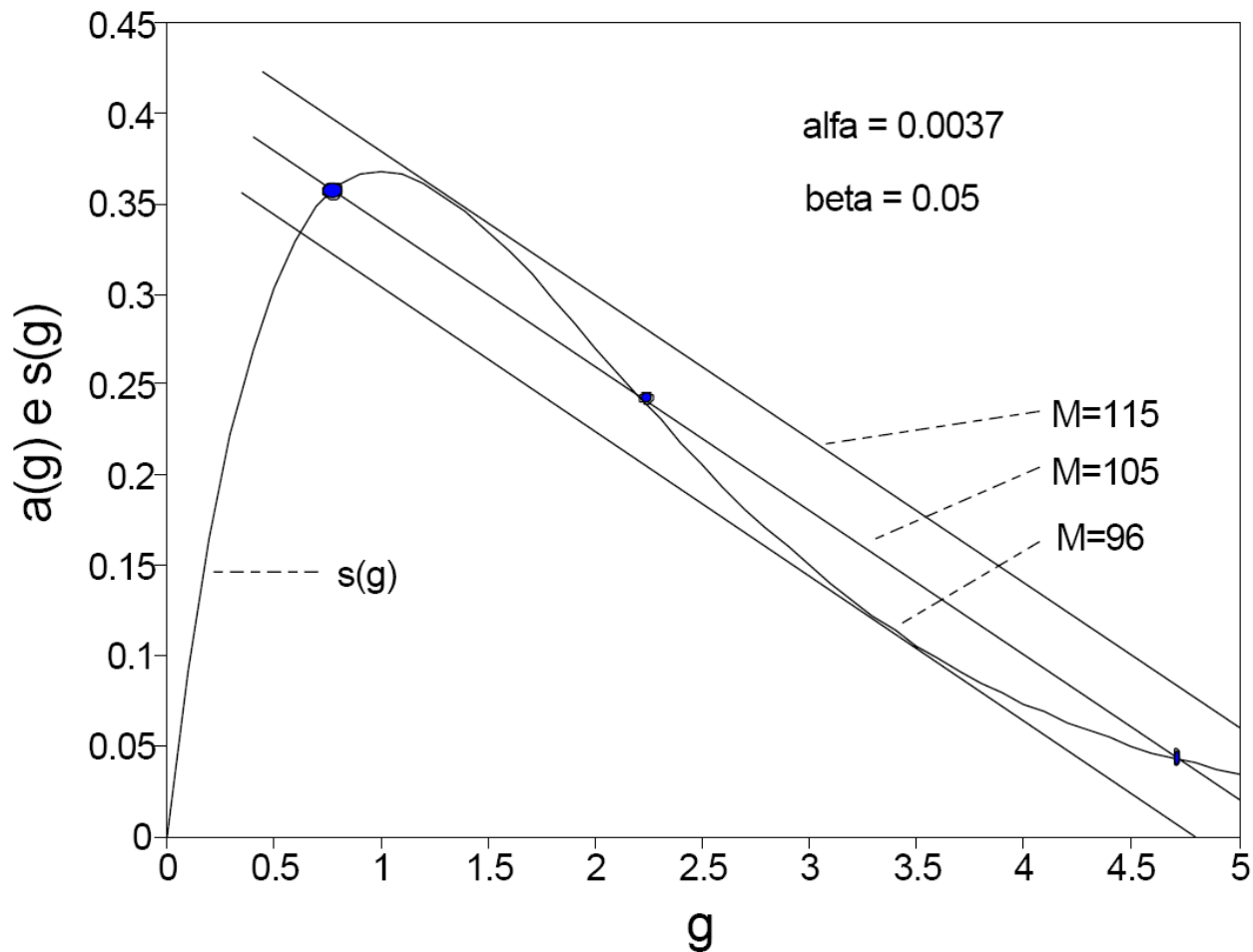


# Aloha: Single buffer

- Similarly, we can also get  $s(g)$  using stationary probabilities (the curve is very close to that of the infinite population model)
- We can consider points where  $s(g)=a(g)$  as equilibrium points of the process
- Equilibrium points can be “stable” or “unstable”



# Aloha: Single buffer

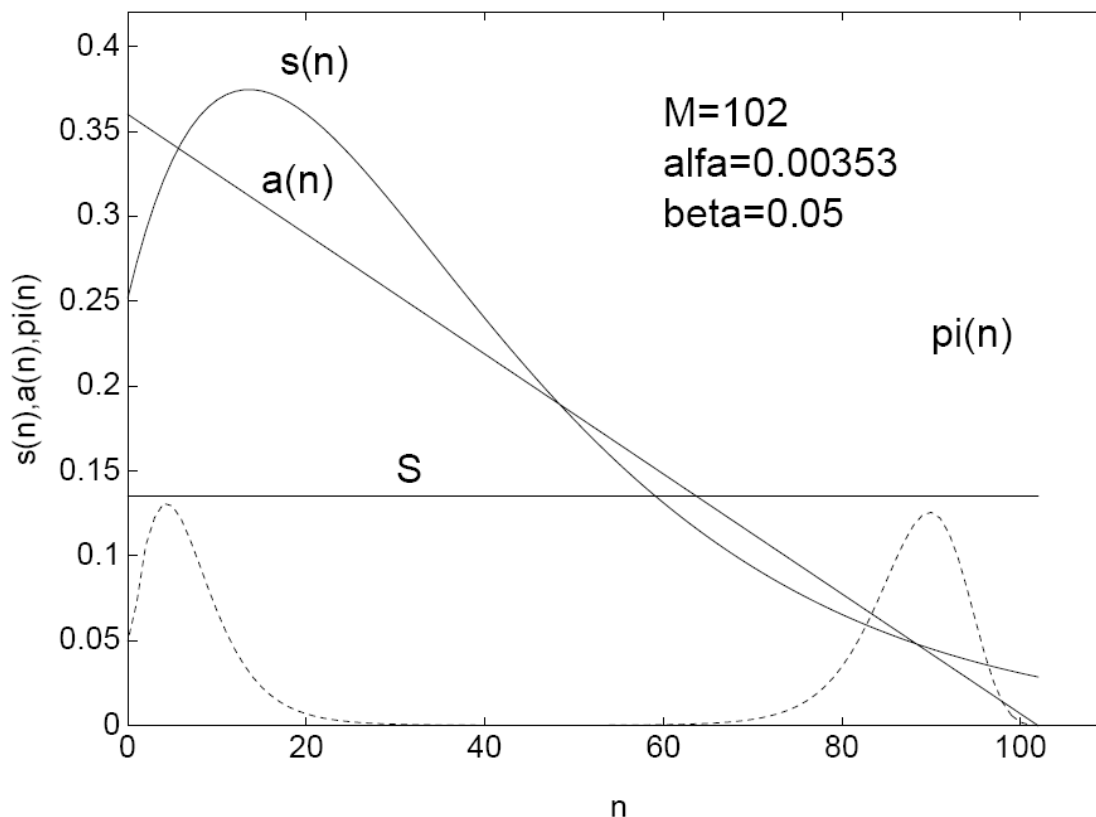






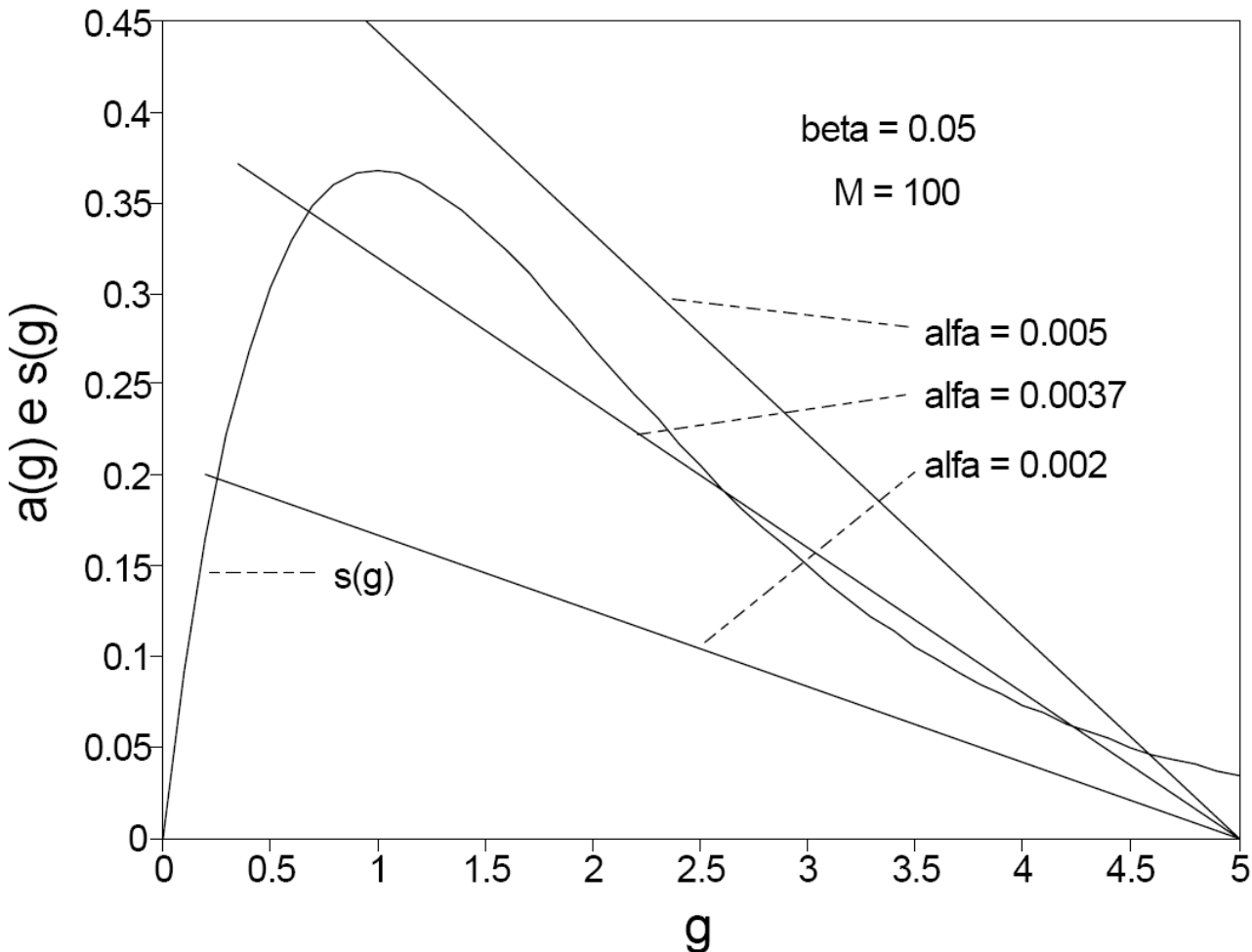
# Aloha: Single buffer

□ What are really equilibrium points?



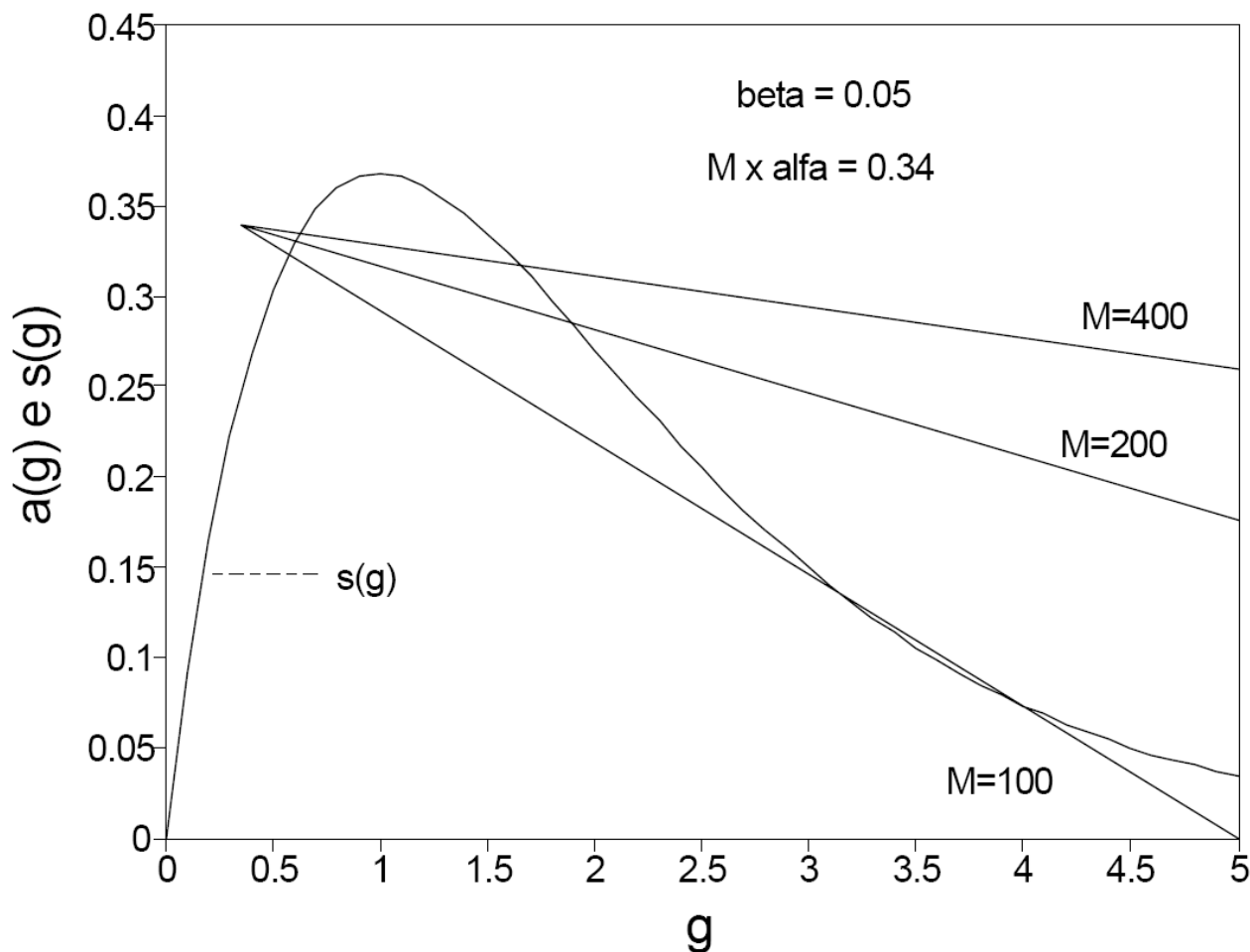


# Aloha: Single buffer





# Aloha: Single buffer





# Aloha stability

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- If more than one equilibrium point exists, we cannot guarantee the system has a stable throughput
- If traffic is low (number of stations) throughput is stable
- To stabilize Aloha the only way is to limit traffic on the channel (keeping it out of the system if necessary)
- With the minimum feedback considered so far, stabilizing the system is not possible



# Stabilized Aloha

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- Let's assume a richer feedback from the channel: stations know at the end of each transmission interval if it was empty (E), if there was a successful transmission (S), or a collision (C)
- Let's also assume the slotted Aloha case for simplicity and the single buffer model
- In order to control traffic on the channel we need to use transmission probability
- Let's assume the same probability  $\beta$  for new transmissions and retransmissions



# Stabilized Aloha

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- The traffic on the channel is given by  $G = n \beta$
- Assuming we know the number  $n$  of busy buffers, the best strategy to limit traffic is to set dynamically  $\beta = 1/n$  so that  $G = 1$  and throughput is maximized
- However, the number  $n$  of busy buffers cannot be known directly by stations and needs to be estimated at each slot  $k$
- Let's denote with  $\hat{n}_k$  the estimated value and with  $\beta_k = 1 / \hat{n}_k$  the probability value at slot  $k$



# Stabilized Aloha

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- Let's assume  $n_k$  is distributed according to a Poisson distribution and let's set its average value to  $\hat{n}_k$

$$P(n_k = i) = \frac{\hat{n}_k^i}{i!} e^{-\hat{n}_k}$$

- This is the 'a priori' distribution of  $n_k$
- Due to channel feedback, after a slot, the distribution 'a posteriori' is modified
- This is obviously different depending on the observed feedback



# Stabilized Aloha

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- If we observe a successful transmission ( $S$ ) the obvious update rule is:

$$\hat{n}_{k-1} = \hat{n}_k - 1 + S$$

- Since 1 packet leaved the system and, on average,  $S$  new arrivals increased the number of busy buffers





# Stabilized Aloha

- If we observe an empty slot ( $E$ ), we have:

$$P(n_k = i / E) = \frac{P(n_k = i; E)}{P(E)} = \frac{(1 - \beta)^i \frac{\hat{n}_k^i}{i!} e^{-\hat{n}_k}}{e^{-G}}$$

- Since the mechanism keeps  $G=1$  with  $\beta_k = 1 / \hat{n}_k$

$$P(n_k = i / E) = \frac{(\hat{n}_k - 1)^i}{i!} e^{-(\hat{n}_k - 1)}$$

- which is equal to the 'a priori' distribution with an average value of  $\hat{n}_k - 1$
- So the update rule is again:  $\hat{n}_{k-1} = \hat{n}_k - 1 + S$



# Stabilized Aloha

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- Since the update rule is the same we can consider a single event type, we can call no-collision (NC)

- Therefore the feedback we have is just

$$[C; NC]$$

- This is usually called binary channel feedback



# Stabilized Aloha

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- If we observe a collision (C), we could calculate the 'a posteriori' distribution as in previous case.

- However, since the system need to be in equilibrium we can simply write that:

$$P(E)x_e + P(S)x_s + P(C)x_c = -1/e$$

- Where  $x$  are the variation in the estimated  $n_k$  (without including new arrivals)



# Stabilized Aloha

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□ We have

$$x_e = x_s = -1$$

$$P(E) = e^{-G} \Big|_{G=1} = e^{-1}$$

$$P(S) = Ge^{-G} \Big|_{G=1} = e^{-1}$$

$$P(C) = 1 - e^{-G} - Ge^{-G} \Big|_{G=1} = 1 - 2e^{-1}$$

$$\text{therefore: } x_c = \frac{1}{e-2}$$

□ and the updated rule is

$$\hat{n}_{k-1} = \hat{n}_k + \frac{1}{e-2} + S$$



# Stabilized Aloha

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- It is possible to show that the system is stable and that the most convenient value for  $S$  is

$$S = e^{-1}$$



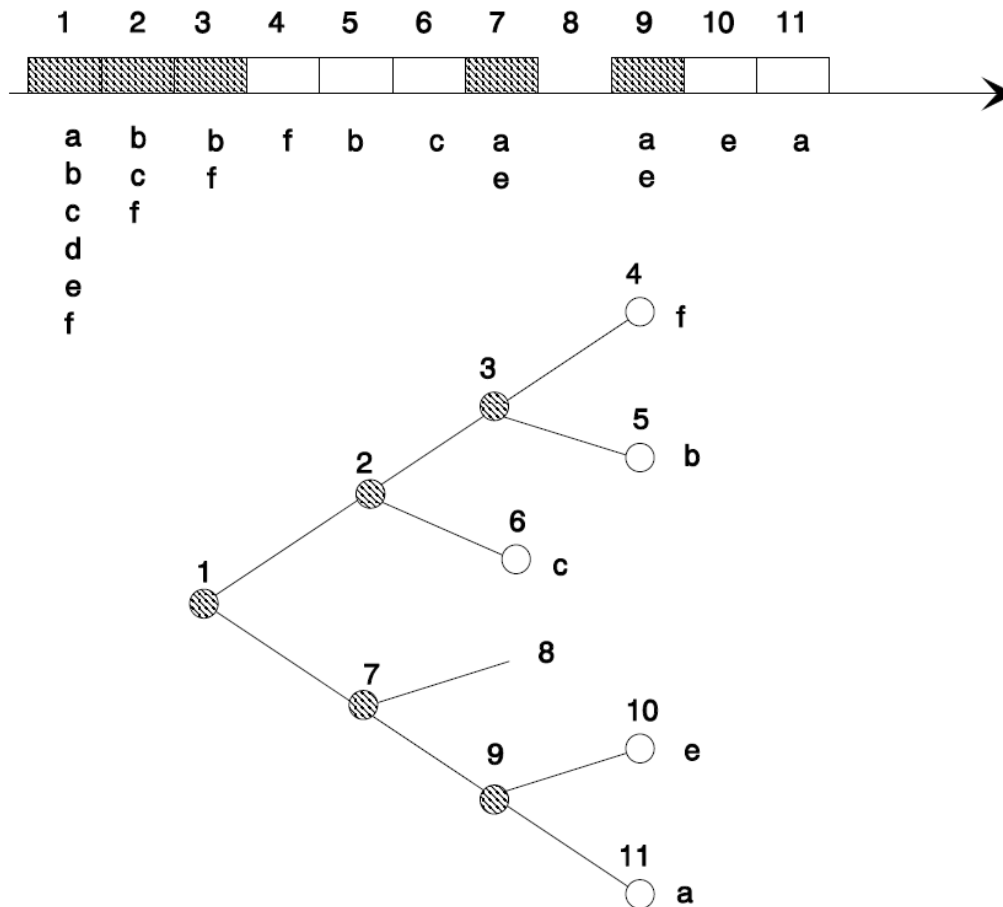
# Collision Resolution Algorithms

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- ❑ Another very popular way of stabilizing Aloha that provides a performance (throughput) much higher than in previous case is based on the so called stack algorithms
- ❑ The idea is to solve the collision among a group of stations (all packets successfully transmitted) before moving on with the access mechanism
- ❑ The simplest version of CRA is based on binary trees
- ❑ After a collision stations are divided in two groups randomly
- ❑ The first group retransmits while the other waits until all the packets of the first group are successfully transmitted



# Collision Resolution Algorithms





# Aloha in real networks

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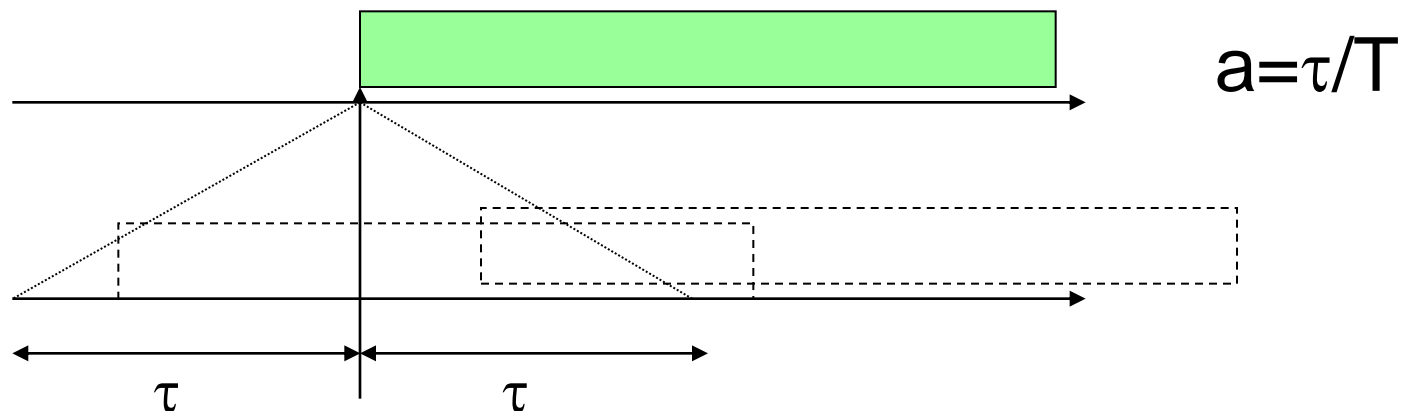
- There are several technologies where Aloha is still adopted including:
  - Random access signaling channel of cellular systems (like e.g. GSM)
  - Reservation channel of WiMax
  - RFID





# Carrier Sense Multiple Access (CSMA)

- ❑ If the channel feedback is richer, more efficient random access mechanisms can be adopted
- ❑ If the propagation time is short wrt to transmission time we can sense the channel status
- ❑ CSMA: like Aloha but transmit only when you sense the channel free





# Carrier Sense Multiple Access (CSMA)

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- On the channel we have cycles of Busy (at least one station sense the channel as busy) and Idle (all stations sense the channel free) periods
- The throughput  $S$  can be given by:

$$S = \frac{\alpha}{B + I}$$

- where  $B$  and  $I$  are the average busy and idle periods and  $\alpha$  is the probability that there is a success transmission in a busy period



# Carrier Sense Multiple Access (CSMA)

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- Making the same assumptions of the aloha infinite population model we have:  $\alpha = e^{-aG}$

$$I = \frac{1}{G}$$

$$B = e^{-aG} (1 + a) + (1 - e^{-aG}) (1 + a + Z)$$

- where  $Z$  is the time when colliding transmission partially overlap



# Carrier Sense Multiple Access (CSMA)

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□ It can be shown that:

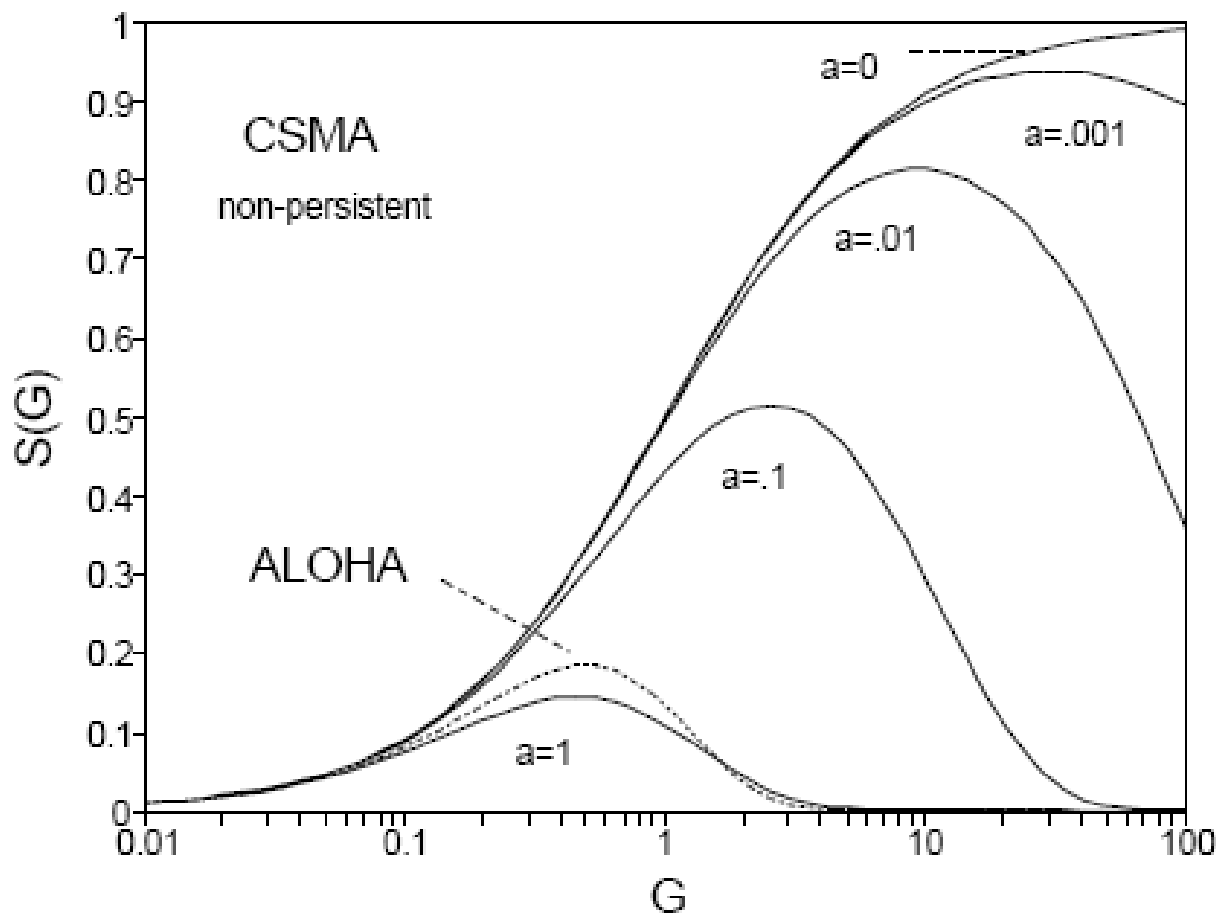
$$Z = a + \frac{ae^{-aG}}{1 - e^{-aG}} - \frac{1}{G}$$

□ Therefore we get:

$$S = \frac{Ge^{-aG}}{G(1 + 2a) + e^{-aG}}$$

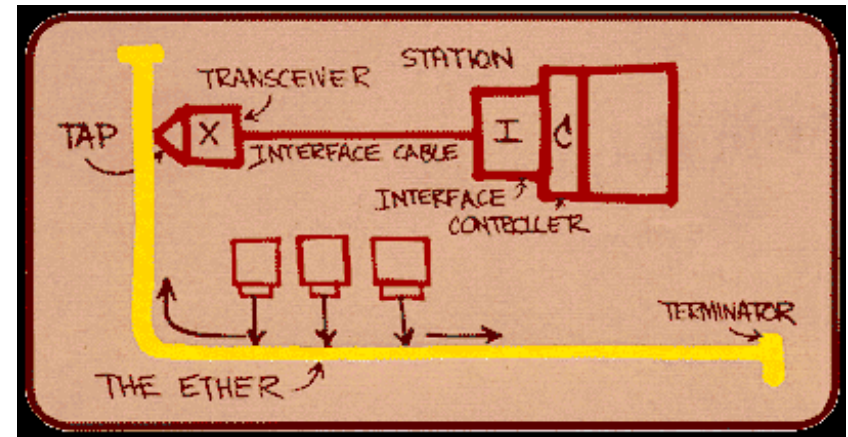


# Carrier Sense Multiple Access (CSMA)



# CSMA in real networks

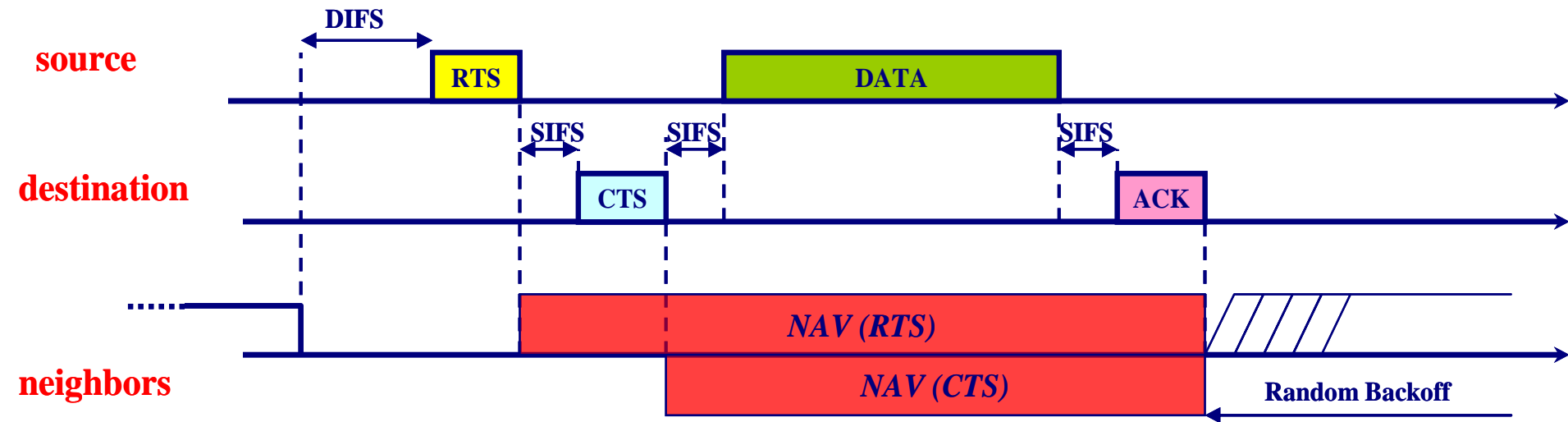
- There are several technologies that are based on variants of the CSMA protocol
- Including Ethernet



- Today the most famous and widely used one is WiFi



# IEEE 802.11 random access





# IEEE 802.11 random access

- Similarly to the general model we can derive a model for WiFi

$$\alpha = e^{-aG}$$

a = interframe space

b = duration of RTS and CTS

$$I = \frac{1}{G}$$

$$B = e^{-aG} (1 + 3a + 2b) + (1 - e^{-aG})(b + a + Z)$$

$$S = \frac{Ge^{-aG}}{G(1 + 2a) + e^{-aG} - G(1 - b)(1 - e^{-aG}) + (2a + 2b)Ge^{-aG}}$$