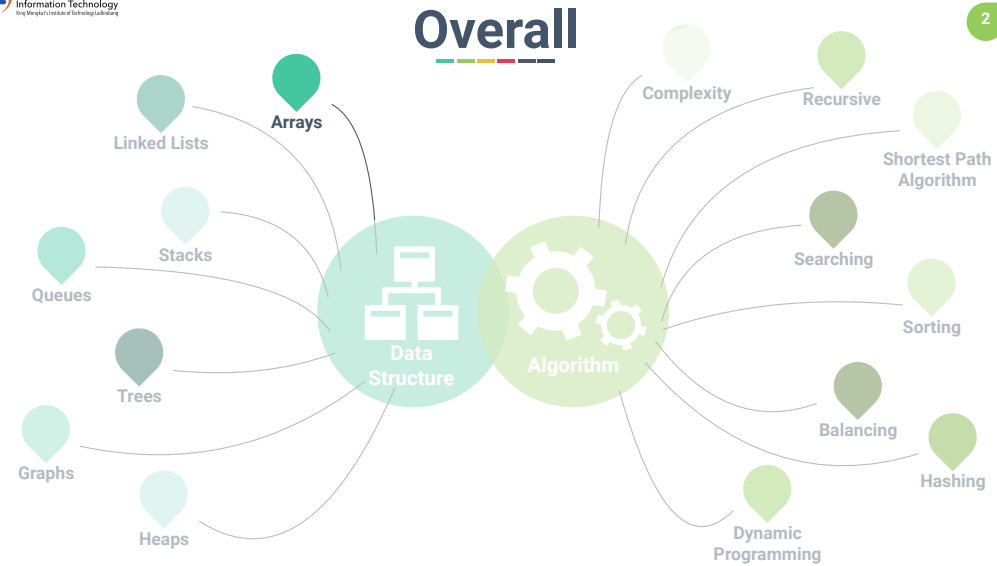


Chapter 2: Arrays

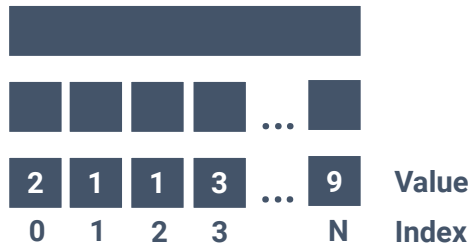
Dr. Sirasit Lochanachit

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Arrays



An array is a chunk of memory, consisting of equal-size elements. Each of those elements have an integer index, which uniquely refers to the value stored. The values are all of the same type (integer, character, etc.).

Physical Level Arrays



- A computer will have a large number of bytes of memory.
- It has a memory address to keep track of where a data is stored.
- Each byte has a unique number as its address.
- Although the number is sequential, any byte/element in a RAM can be accessed to read or write with a constant time $O(1)$.

Array of Characters

Address/byte#	Value	Index
6000	C	0
6001	A	1
6002	R	2

- In Python, it represents a unicode character with 16 bits (i.e. 2 bytes).
- Each element/cell in array is index with an integer starting with 0, 1, 2, and so on.
- Since each cell has an equal-size bytes, any element can be accessed constantly with this formula:
 - $\text{start_address} + \text{elem_size} * \text{index}$

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Exercise

Given an array:

Start Address is 6000,
element size is 8,

What is the address of the element at index 6?

- $\text{start_address} + \text{elem_size} * \text{index}$

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Array of Characters

Address/byte#	Value	Index
6000	C	0
6001	A	1
6002	R	2

- Luckily, a programming language calculate memory addresses of an array automatically, so we can focus on values and indexes.

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Python List



- Python list is a referential-type array that stores the memory addresses (references) of a value instead of the value itself.
- Strings can be in any length, but memory addresses are fixed-size.

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Arrays

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2-Dimensional Arrays

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'R'
'e'
'f'
'r'

2	1	16	30
12	10	12	3
20	81	13	23

2	1	16	30
12	10	12	3
20	81	13	23

(a) One dimension

(b) Two dimensions

(c) Three dimensions

(0,0)	(0,1)	(0,2)	(0,3)
(1,0)	(1,1)	(1,2)	(1,3)
(2,0)	(2,1)	(2,2)	(2,3)

2-Dimensional Arrays

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2-D Arrays

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6000	6001	6002	6003
(0,0)	(0,1)	(0,2)	(0,3)
6004	6005	6006	6007
(1,0)	(1,1)	(1,2)	(1,3)
6008	6009	6010	6011
(2,0)	(2,1)	(2,2)	(2,3)

2	1	16	30
12	10	12	3
20	81	13	23

(a) Two dimensions

Size/length			
(0)	(1)	(2)	(3)
Rank			
(0)	2	1	16
(1)	12	10	12
(2)	20	81	13

(b) Nested-one dimension

Example

Python Code: 2-D Arrays

```
c = np.array([[['h','e','l','l','o'], ['w','o','r','l','d']]])
```

Rank	6000	6001	6002	6003	6004	6005	6006	6007	6008	6009
(0)	h	e	l	l	o					
	(0,0)	(0,1)	(0,2)	(0,3)	(0,4)					
	6010	6011	6012	6013	6014	6015	6016	6017	6018	6019
(1)	w	o	r	l	d					
	(1,0)	(1,1)	(1,2)	(1,3)	(1,4)					

Exercise

Suppose start address is 6000, find the address of index (1,4)

- $\text{start_address} + \text{elem_size} * \text{index}$
- Where $\text{index} = (\text{rank} * \text{array_length}) + \text{target_index}$

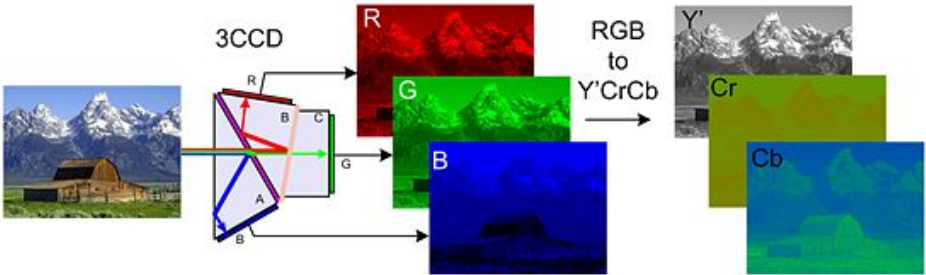
3-D Arrays

Python Code: 3-D Arrays

```
a = np.array([[[[1,2,3],[4,5,6]],  
               [[7,8,9],[10,11,12]],  
               [[13,14,15],[16,17,18]]]])
```

Dimension
#1
#2
#3

3-D Arrays



Asymptotic Performance

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Asymptotic Performance

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	Add Operation	Running Time	Remove Operation	Running Time
Beginning				
End				
Between				

Data	10	5	8	7	1			
------	----	---	---	---	---	--	--	--

	Add Operation	Running Time	Remove Operation	Running Time
Beginning				
End	data.append(val)	$O(1)$		
Between				

Data	10	5	8	7	1	20		
------	----	---	---	---	---	----	--	--

Asymptotic Performance

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Asymptotic Performance

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	Add Operation	Running Time	Remove Operation	Running Time
Beginning				
End	data.append(val)	$O(1)$	data.pop()	$O(1)$
Between				

Data	10	5	8	7	1			
------	----	---	---	---	---	--	--	--

	Add Operation	Running Time	Remove Operation	Running Time
Beginning			data.pop(0) Del data[0]	$O(n)$
End	data.append(val)	$O(1)$	data.pop()	$O(1)$
Between				

Data		5	8	7	1			
------	--	---	---	---	---	--	--	--

Asymptotic Performance

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Asymptotic Performance

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	Add Operation	Running Time	Remove Operation	Running Time
Beginning			data.pop(0) Del data[0]	$O(n)$
End	data.append(val)	$O(1)$	data.pop()	$O(1)$
Between				

Data

5	8	7	1				
---	---	---	---	--	--	--	--

	Add Operation	Running Time	Remove Operation	Running Time
Beginning	data.insert(0, val)	$O(n)$	data.pop(0) Del data[0]	$O(n)$
End	data.append(val)	$O(1)$	data.pop()	$O(1)$
Between				

Data

9	5	8	7	1			
---	---	---	---	---	--	--	--

Asymptotic Performance

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Asymptotic Performance

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	Add Operation	Running Time	Remove Operation	Running Time
Beginning	data.insert(0, val)	$O(n)$	data.pop(0) Del data[0]	$O(n)$
End	data.append(val)	$O(1)$	data.pop()	$O(1)$
Between			data.remove(val)	$O(n)$

Data

9	5		7	1			
---	---	--	---	---	--	--	--

	Add Operation	Running Time	Remove Operation	Running Time
Beginning	data.insert(0, val)	$O(n)$	data.pop(0) Del data[0]	$O(n)$
End	data.append(val)	$O(1)$	data.pop()	$O(1)$
Between			data.remove(val)	$O(n)$

Data

9	5	7	1				
---	---	---	---	--	--	--	--

Asymptotic Performance

	Add Operation	Running Time	Remove Operation	Running Time
Beginning	data.insert(0, val)	$O(n)$	data.pop(0) Del data[0]	$O(n)$
End	data.append(val)	$O(1)$	data.pop()	$O(1)$
Between	data.insert(index, val)	$O(n)$	data.remove(val)	$O(n)$

Data

9	0	5	7	1			
---	---	---	---	---	--	--	--

Asymptotic Performance

Operation	Running Time
len(data)	$O(1)$
data[i]	$O(1)$
Data[i] = val	$O(1)$
c * data	$O(n)$
data.reverse()	$O(n)$
data.sort()	$O(n \log n)$

Searching in Array

Sequence

1. Search (element by element)

1	20	81	23	90	13	91
1	20	81	23	90	13	91
1	20	81	23	90	13	91
1	20	81	23	90	13	91
1	20	81	23	90	13	91

Binary

1. Sort

1	13	20	23	81	90	91
---	----	----	----	----	----	----

2. Search

1	13	20	23	81	90	91
1	13	20	23	81	90	91

Searching in Array

Sequence

1. Search (element by element)

1	20	81	23	90	13	91
1	20	81	23	90	13	91
1	20	81	23	90	13	91
1	20	81	23	90	13	91
1	20	81	23	90	13	91

Pseudocode: Sequential/linear search

```

linear_search(list, target_value)
  for each item in the list
    if item value == target_value
      return the item's location
    end if
  end for
  return 'no match'
END

```

Binary Search

- Binary Search
 - Locate a target value in a sequence of n elements that are sorted.
 - $\text{mid} = (\text{low} + \text{high}) / 2$
 - Initially, $\text{low} = 0$, $\text{high} = n-1$
- For instance, find number 5.

Data	1	5	7	9	10	11	20
Index	0	1	2	3	4	5	6

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Binary Search

- Binary Search
 - If target value $< \text{data}[\text{mid}]$, next interval is from low to $\text{mid}-1$.
 - If target value $> \text{data}[\text{mid}]$, next interval is from $\text{mid} + 1$ to high .

	low		mid		high		
Data	1	5	7	9	10	11	20
Index	0	1	2	3	4	5	6
							$\text{mid} = (0 + 6) / 2 = 3$
	low	mid	high				
Data	1	5	7	9	10	11	20
Index	0	1	2	3	4	5	6
							$\text{mid} = (0 + 2) / 2 = 1$

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