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Introduction to Data Structures and Algorithms

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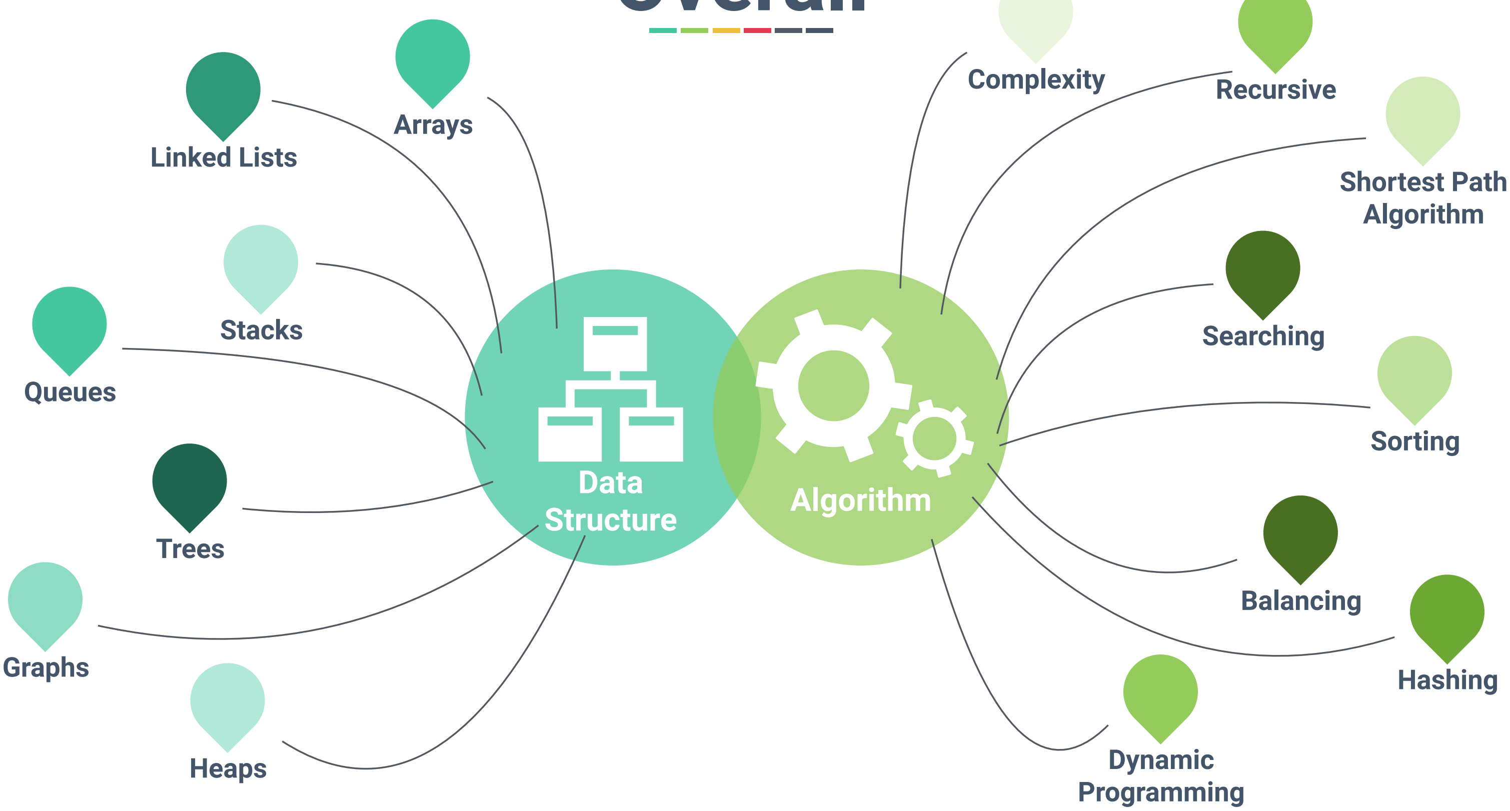
Overall



Overall

- Data Structures
- Algorithms

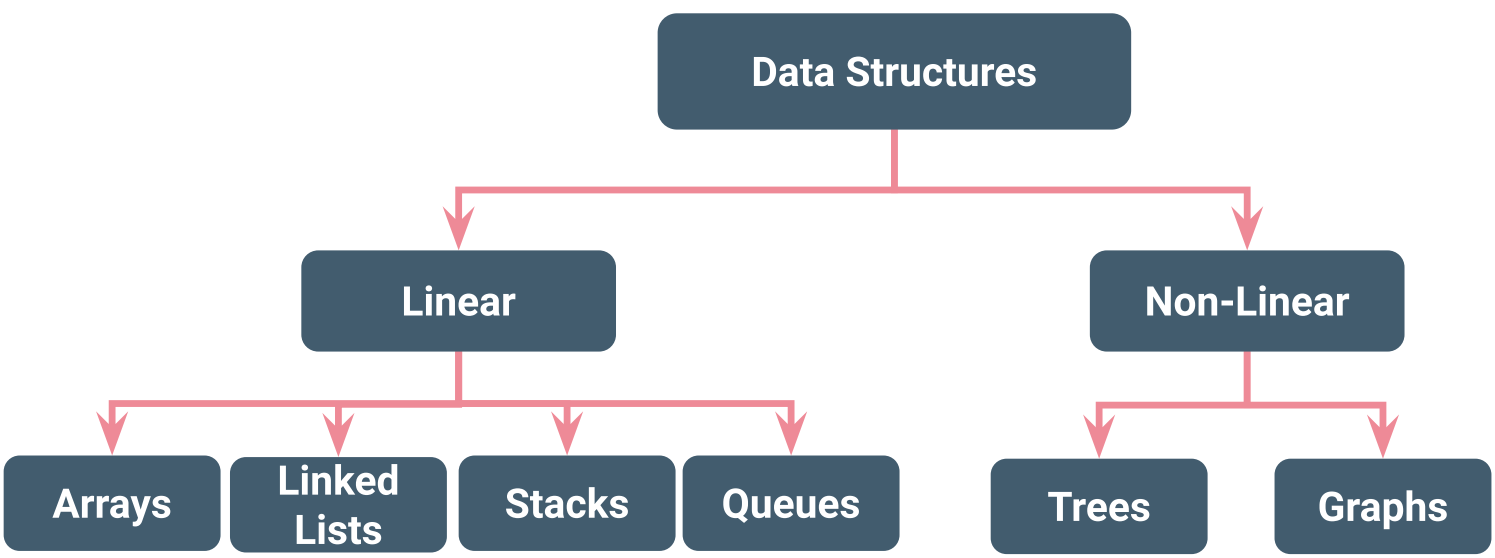
What is “Data Structure” ?

- Way to store and organise data
- Enable efficient access and modification of data
- Designed for a specific algorithm
  - Strengths and limitations
  - Time and space complexity

Abstract Data Type

- A data type, where only behavior is defined but not implementation.
- Similar to a black box where input and output are known, but not how.
- Examples: Array, List, Map, Queue, Set, and etc.

Type of Data Structure



Check out for a comprehensive list of data structures at
https://en.wikipedia.org/wiki/List_of_data_structures

What is “Algorithm” ?

- Well-defined procedure or set of instructions to accomplish a task
- Sequence of steps that transform input into output
- Tool to solve a well-specified computational problem

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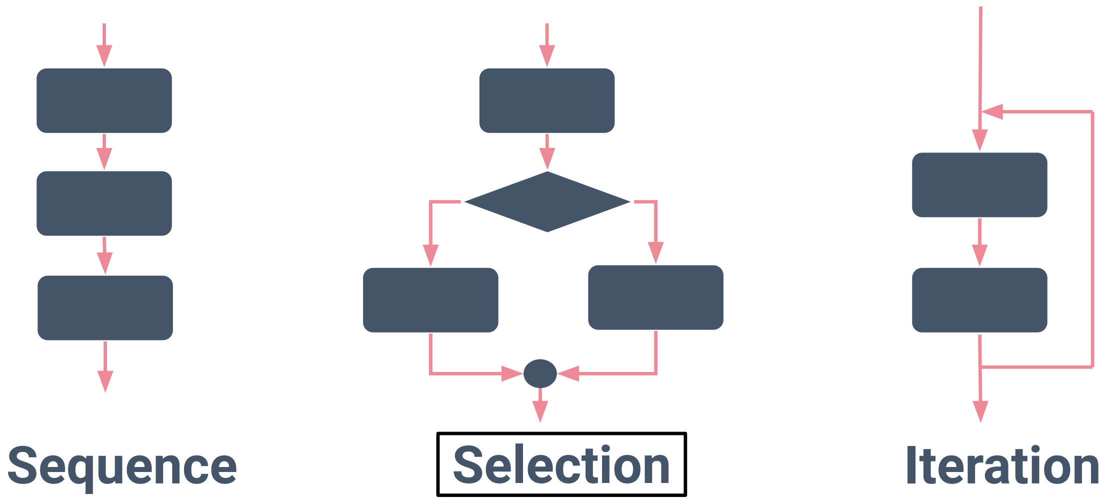
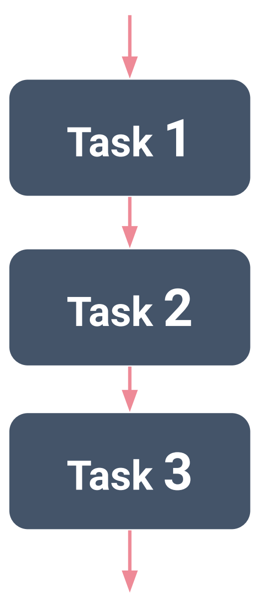
Example: Sorting numbers

1. Input:
    - A sequence of n numbers: *<a1, a2, …, an>*
    - <31, 41, 59, 26, 41>
2. Sorting Algorithms
3. Output:
    - A permutation (reordering) *<a’1, a’2, …, a’n>* of input sequence such that *a’1 <= a’2 <= … <= a’n*
    - <26, 31, 41, 41, 59>

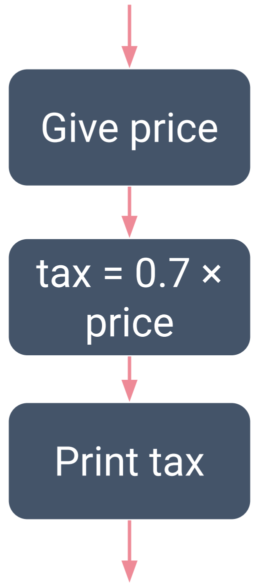
Control Structure



Sequence



Sequence



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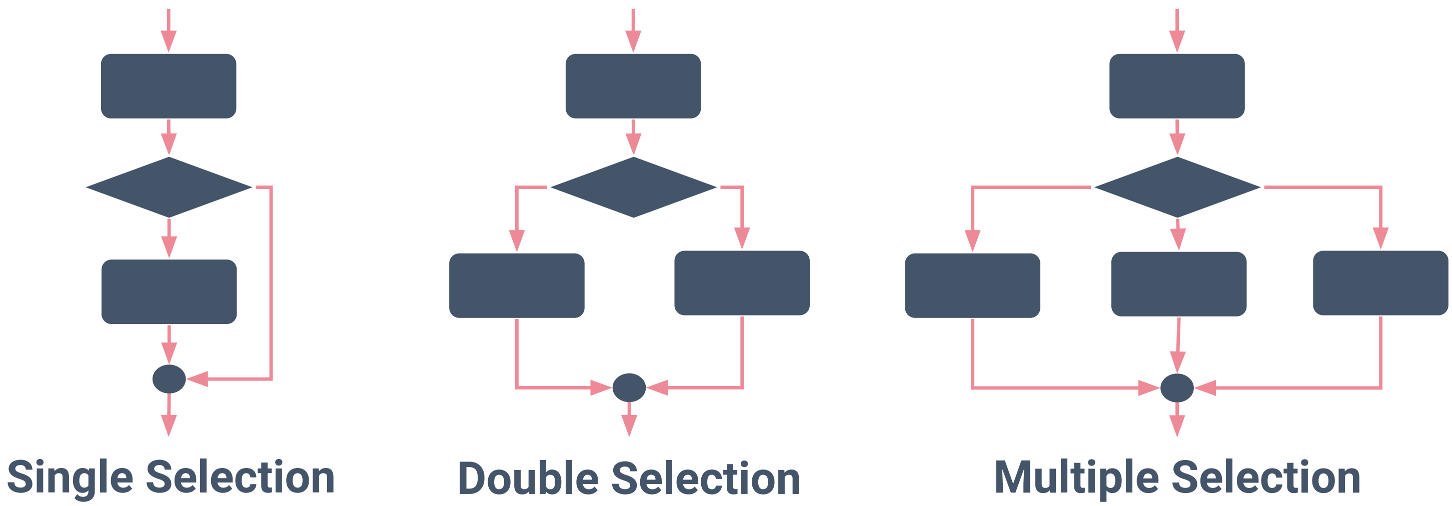
Exercise: Sequence

Write a flowchart that converts USD currency to Thai baht. At the end of the flowchart, print the Thai baht. (Exchange rate USD to Thai baht is 32.00)

Selection



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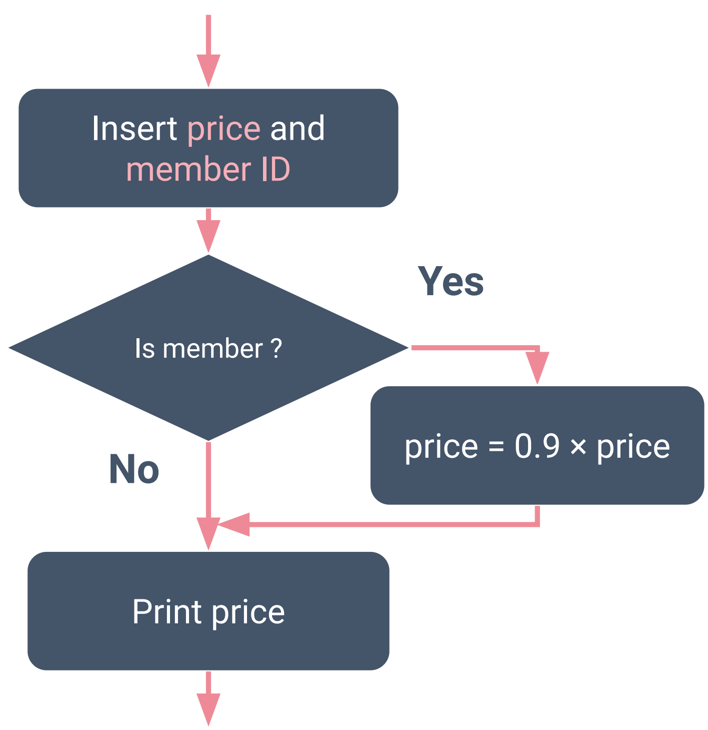
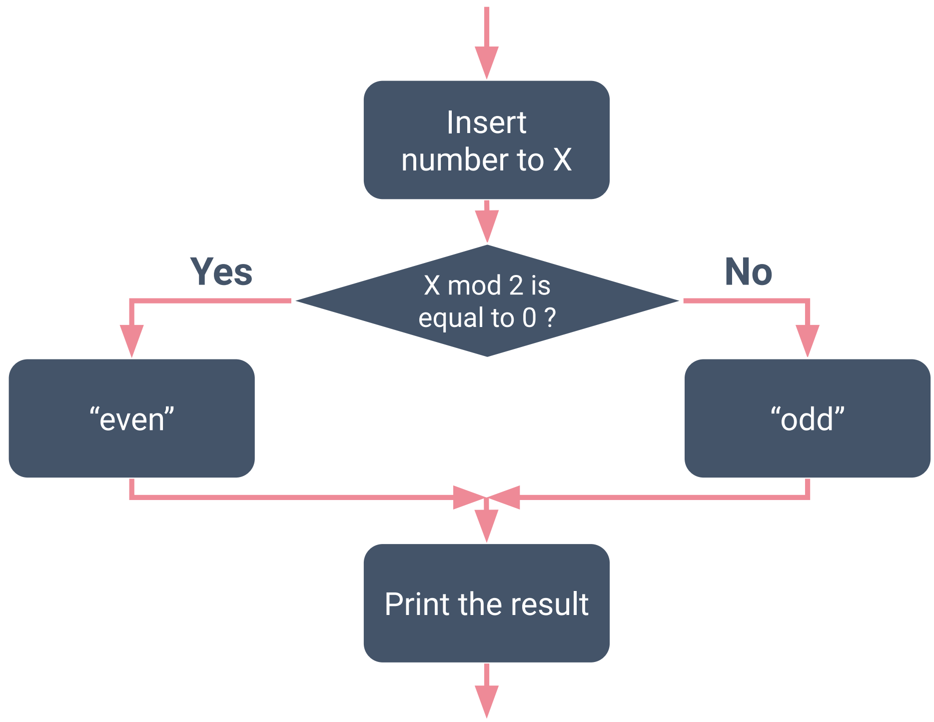
Selection



Selection



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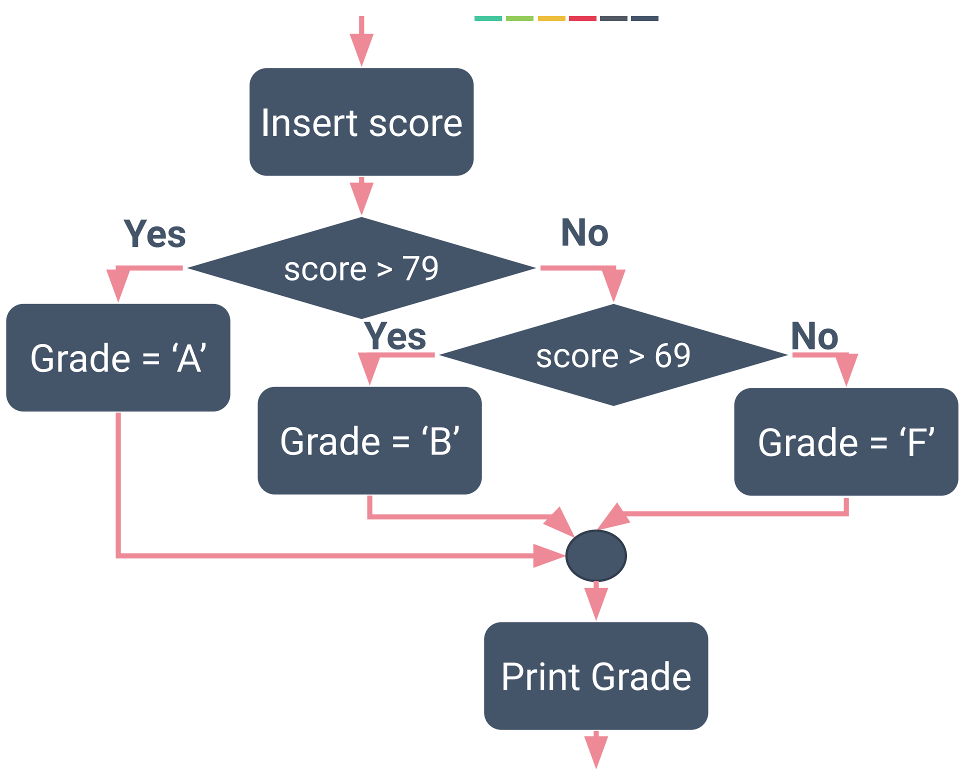
Selection



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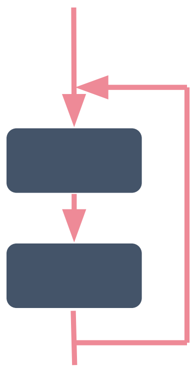
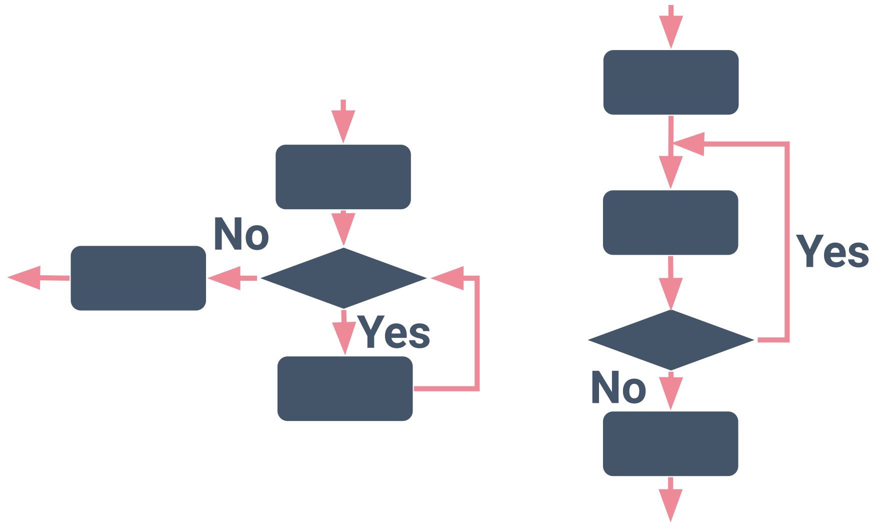
Iteration



Infinite Loop



Finite Loop

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Exercise: Control Flow

Write a flowchart describing the logic of factorial function.

Hint: The factorial n!, is the product of all positive integers less than or equal to n.

For example: 5! = 5 * 4 * 3 * 2 * 1 = 120

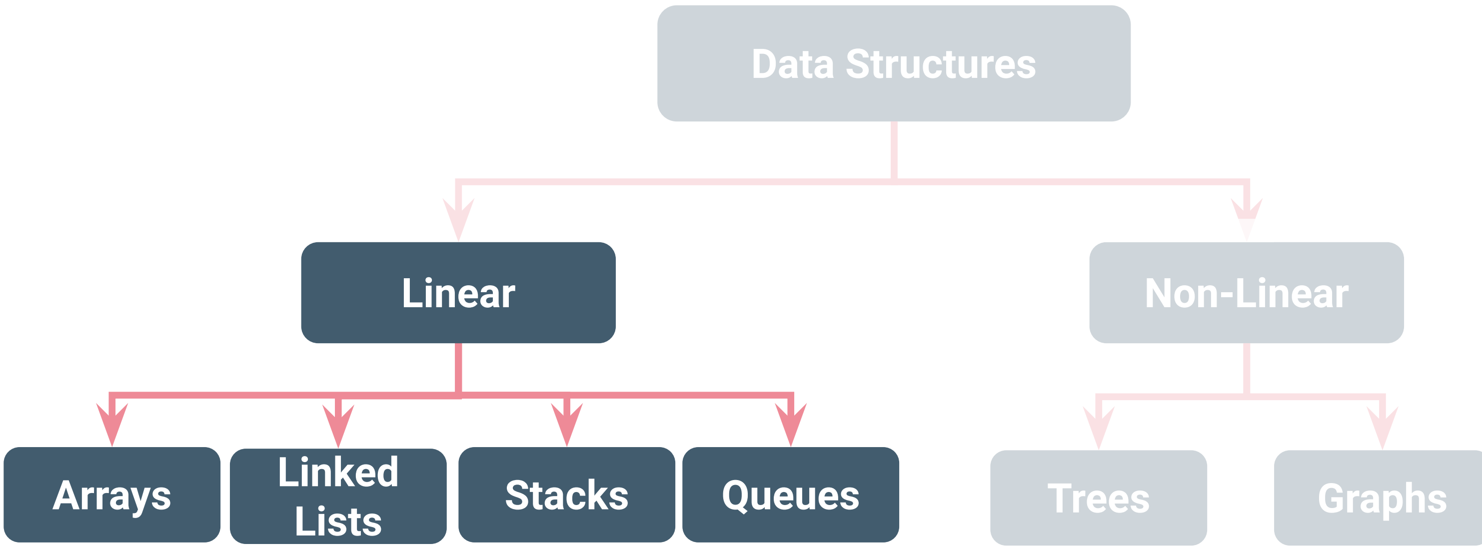
What kind of problems are solved by algorithms?

- Human Genome Project
  - identifying all genes of human beings
- Internet: Routing, searches, and security
  - Shortest path, search engines, encrypted communication
- E-commerce
  - Ads, recommendations, authentications
- Commercial enterprises
  - Resource allocation:
    - crew assignment on flights, package delivery route

Data Structures and Algorithms

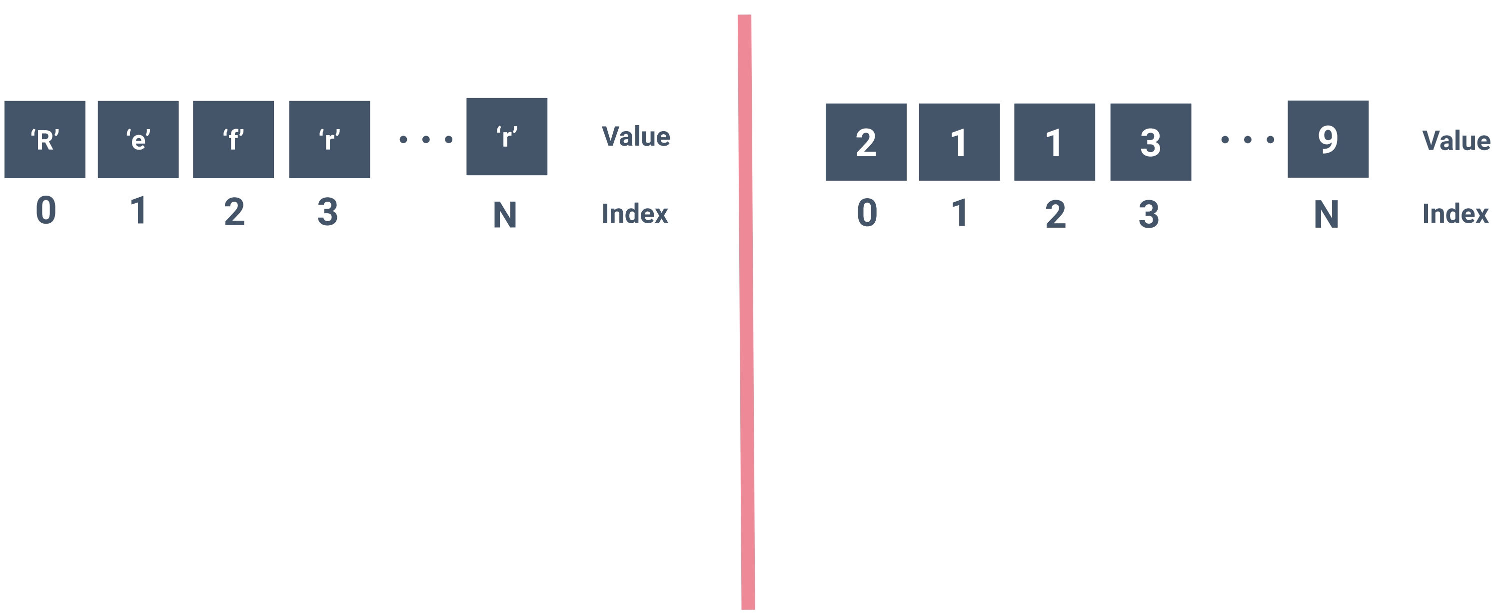
- Data Structures
  - Way to store and organise data
- Algorithms
  - Sequence of steps performed on data structures to perform a task

Type of Data Structure

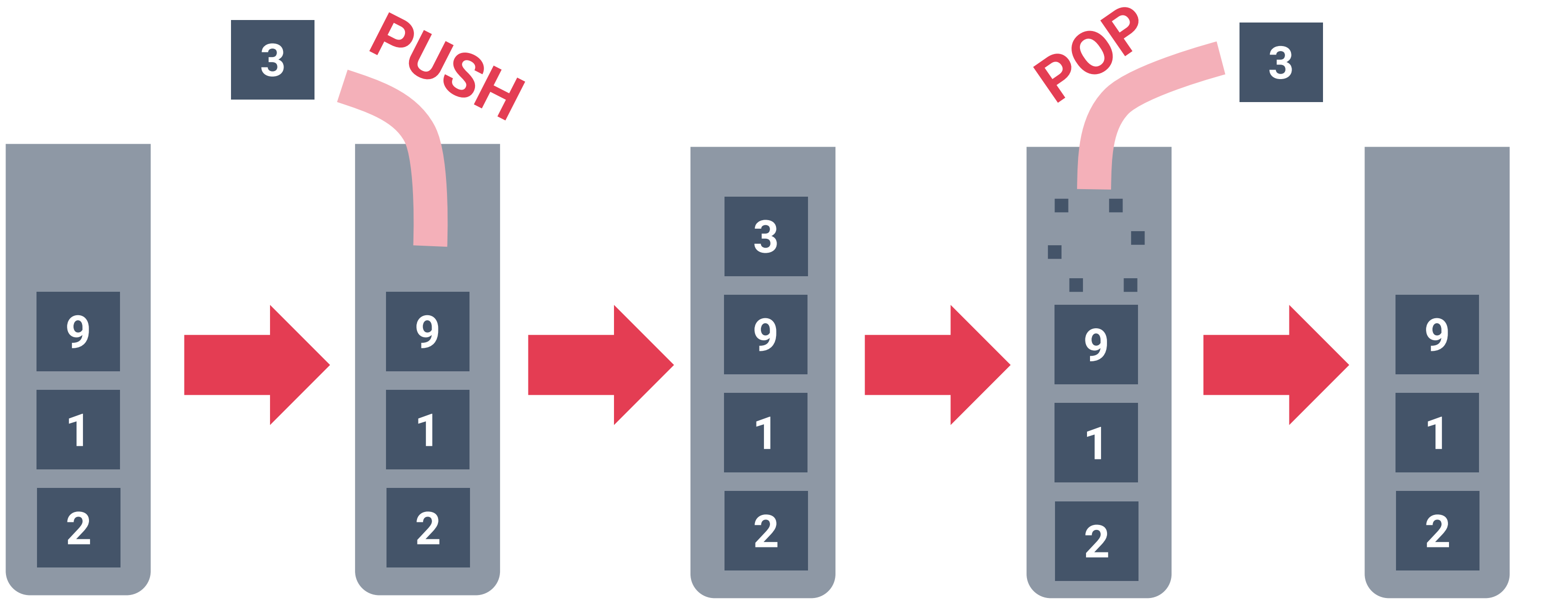


Check out for a comprehensive list of data structures at https://en.wikipedia.org/wiki/List_of_data_structures

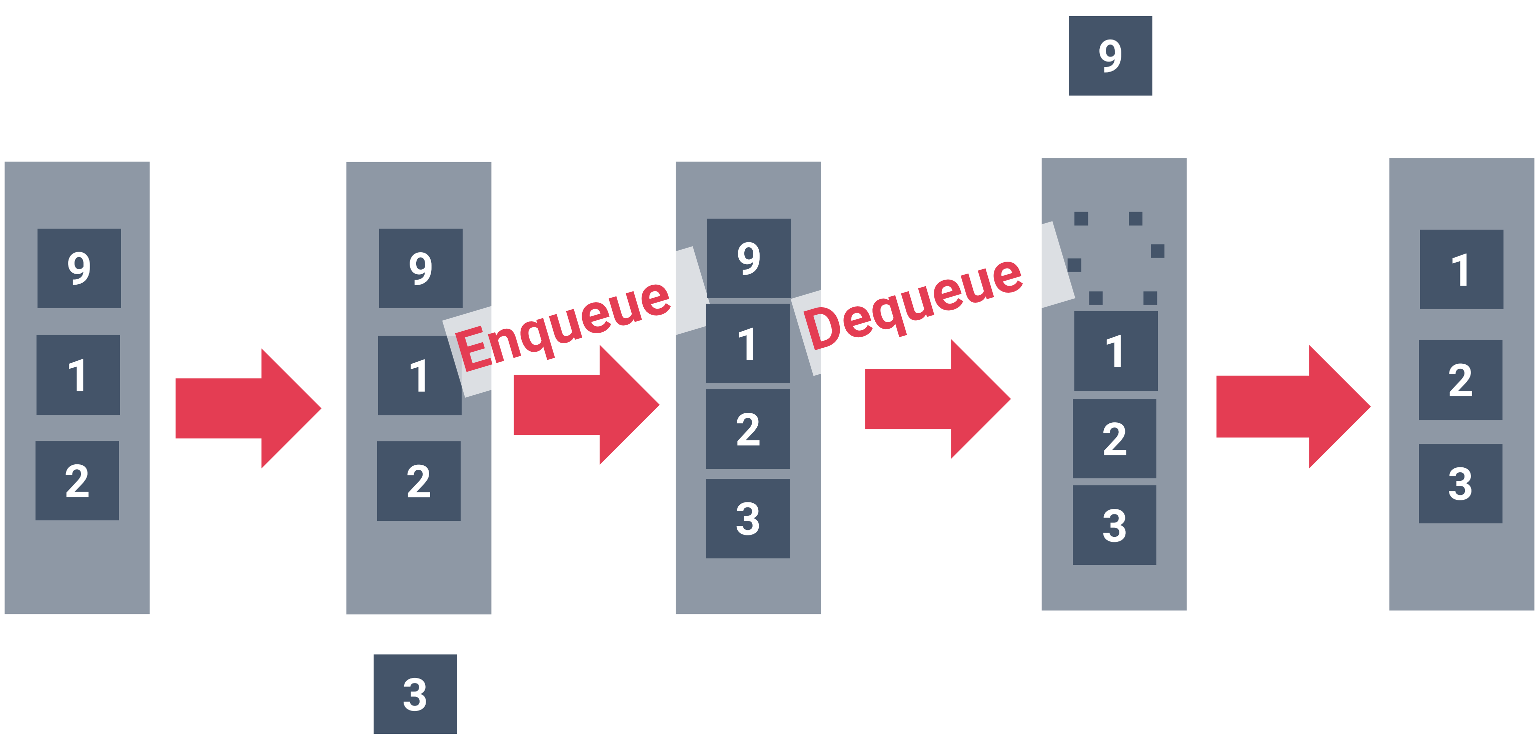
Arrays



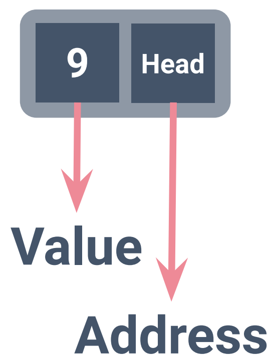
Stacks



Queues

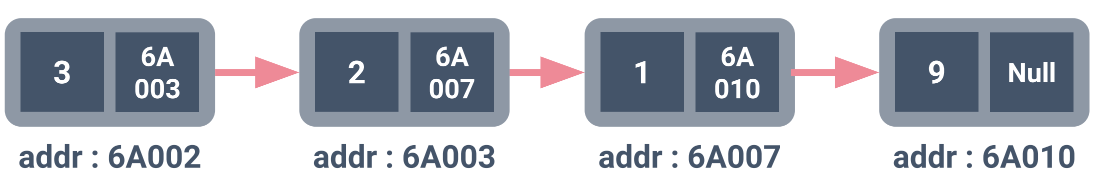


Linked Lists



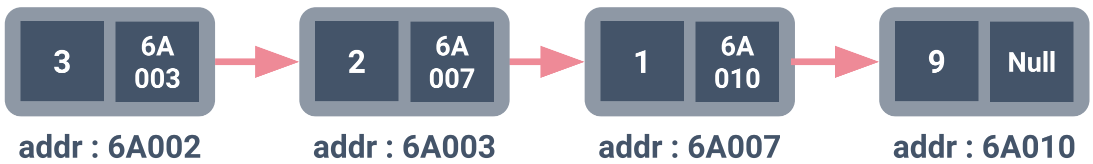
Linked Lists

- **Linked Lists**



Linked Lists VS Arrays

- **Linked Lists**



- **Arrays**

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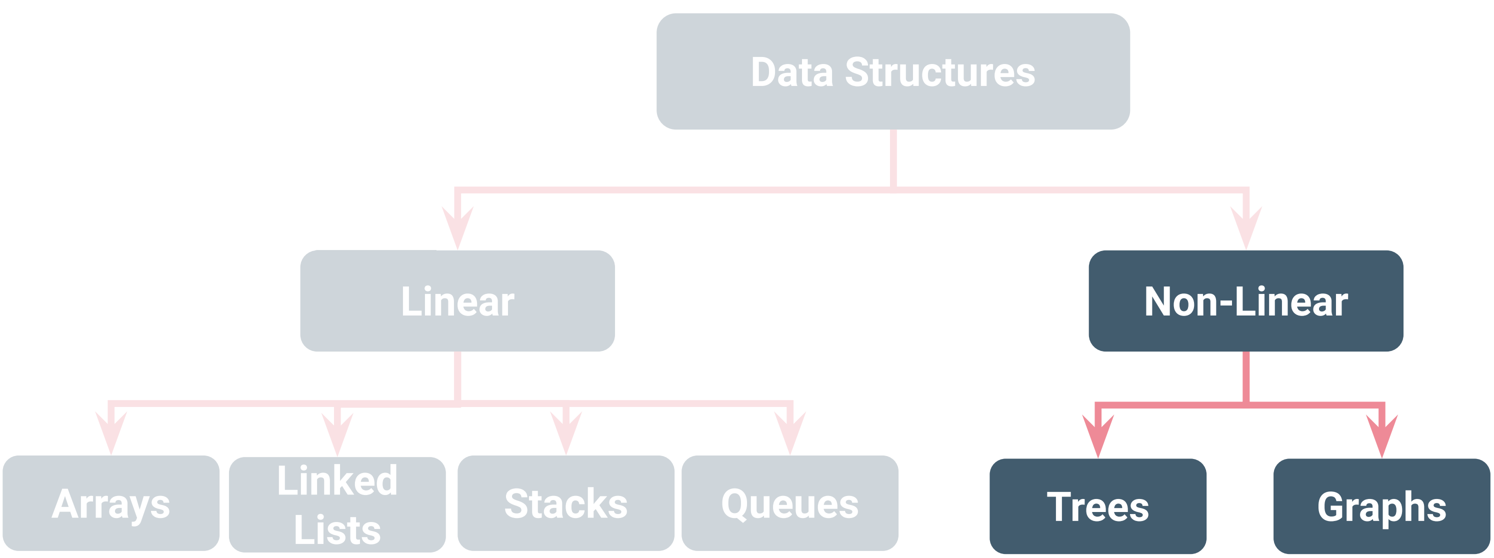
Type of Data Structure



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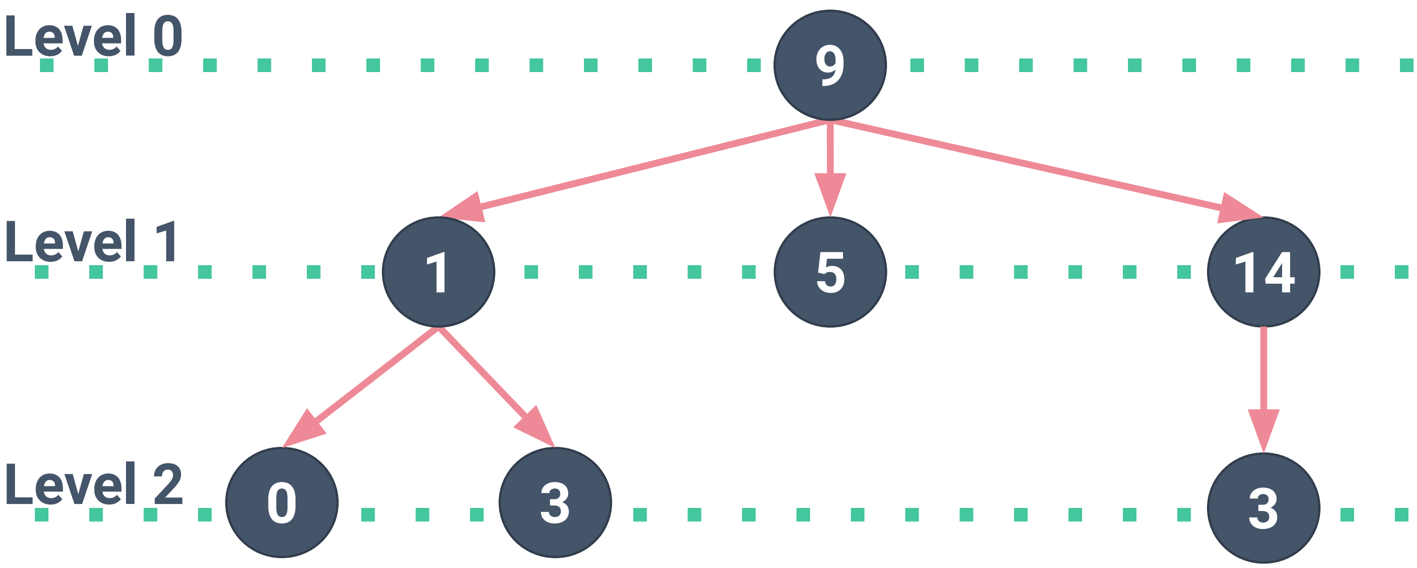
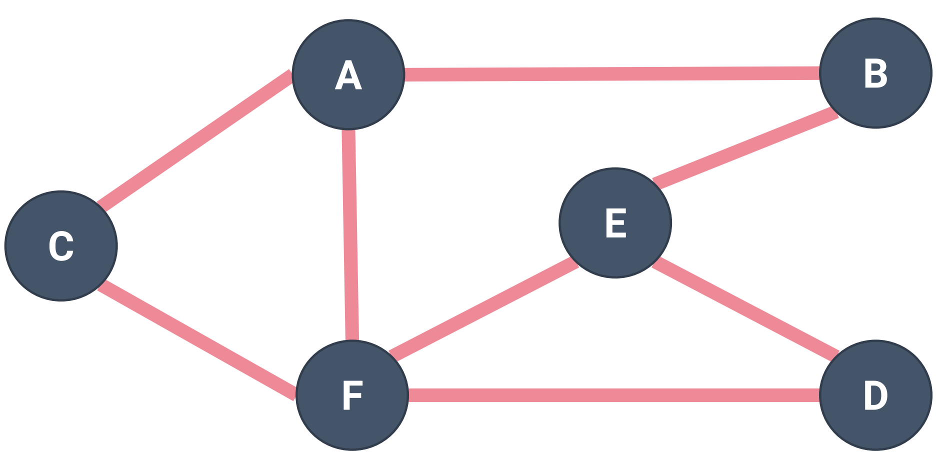
Trees



Graphs



Algorithm Analysis

- If computers were infinitely fast, **any correct method** for solving a problem would do.
- Computing time and space in memory are a limited resource.
- The solution should run as fast as possible.
- For the same problem, it can be solved by different algorithms but they are often differ in their efficiency.
- **Algorithms that are efficient in terms of time or space are preferred.**

Algorithm Analysis

- How do we measure algorithm **efficiency** or **performance**?
  - Use running time as an indicator.

Benchmark Analysis

- Example: Summation of *n* integers
  - Time required for the iterative solution seems to increase as we increase the value of *n*.
- Running time depends on many factors
  - Hardware (CPU, RAM, etc.)
  - Software (OS, Programming language, etc.)
- Need an alternative way for analysing algorithms with respect to running time

Algorithm Analysis

- How do we measure algorithm **efficiency** or **performance**?
  - Use running time as an indicator.

- Running time can be expressed as the number of operations or steps executed.
  - theSum = 0 -> 1 step/time
  - for i in range(1,n+1):
    theSum = theSum + i -> n steps/times

Time complexity
T(n) = 1+n

Algorithm Analysis

- The parameter ***n*** is often referred as "**size of the problem**" or **input size**
- The running time of an algorithm or data structure operation **increases** with the input size.
- Running time as a function of the input size - f(n)
- As *n* gets larger, the constant will become less significant.
- Order of magnitude/growth describes the running time that is most important.

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Asymptotic Notation

- Example:
  - Suppose an algorithm runs on an input size n.
  - Times required to execute is $T(n) = 2n^2 + n + 1$
  - The $\mathbf{n^2}$ terms become larger when **n** gets larger.
  - The running time of this algorithm grows as $\mathbf{n^2}$.
- Asymptotic notation represents algorithm's complexity
  - Ignores constant factors and slower growing terms.
  - Focus on the main components that affect the growth.
  - Big-O notation

Big-O notation

- **Big-*O* notation (Order)**
  - Example:
    - Given T(n) = 1+n, then T(n) = O(n)
    - Given T(n) = $2n^2$ + n + 1, then T(n) = O(n^2)

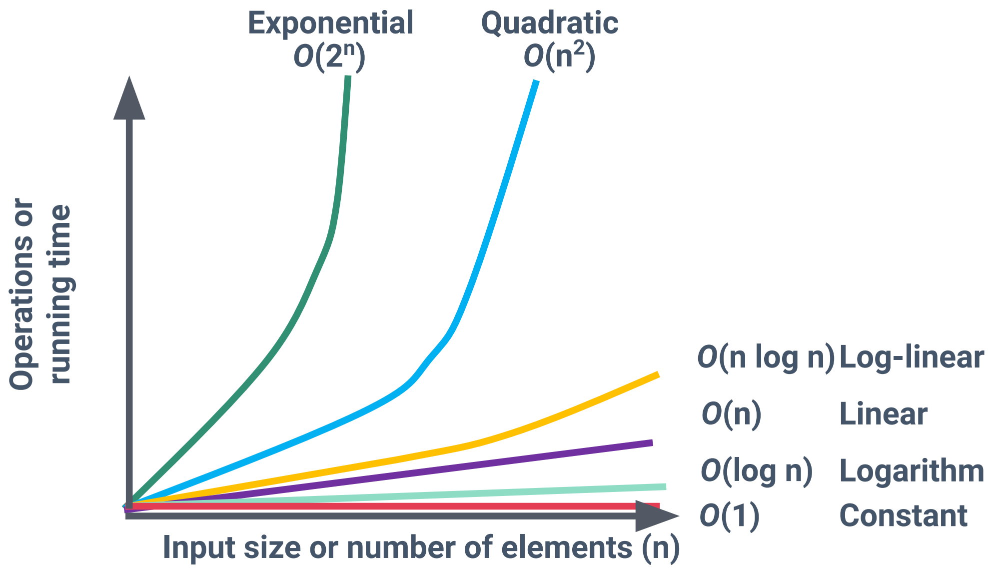
- **O(n) means time complexity will never exceed n.**

Big-O notation

- **Big-*O* notation (Order)**
  - Common f(n):
    - O(1): constant
    - O(log n): logarithm
    - O(n): Linear
    - O(n log n): Log linear
    - O(n^2): Quadratic
    - O(2^n): Exponential
    - O(n!): Factorial

Time Complexity

In general, the standard functions of input size n are shown in figure.

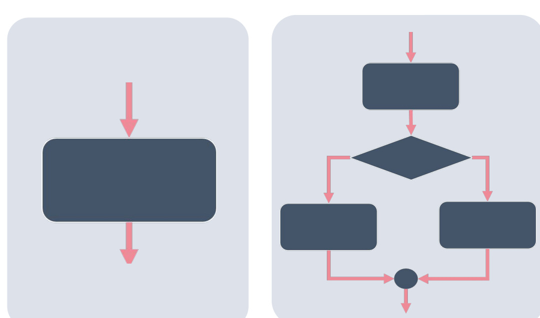


Calculating Time Complexity

Mathematic

$f(n) = 3$

Computer



- Simple statements (read, write, assign)
- Simple operations (+ - * / == > >= < <=)

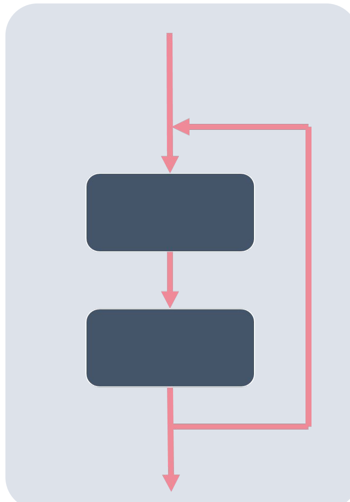
Big-O

$O(1)$

Constant

Calculating Time Complexity

Mathematic	Computer	Big-O
$f(n) = n-3$	Linear loop	$O(n)$ → Linear



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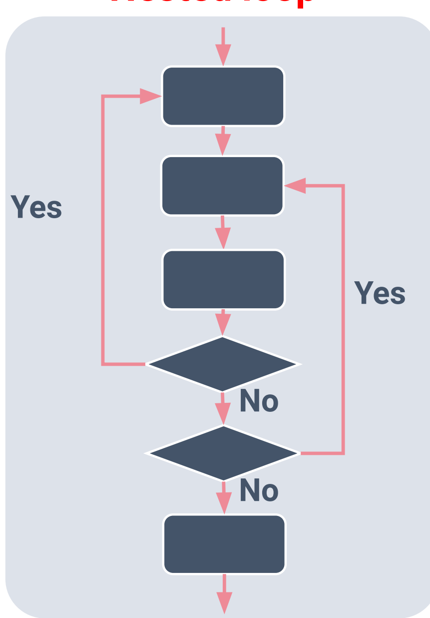
Calculating Time Complexity

Mathematic	Computer	Big-O
$f(n) = n^2 + n + 7$	Nested loop	$O(n^2)$ → Quadratic



Loops

	Python Code	Time complexity
Linear	for i in range (0,n): do something	T(n) = n, O(n)
Linear	for i in range (0,n,2): do something	T(n) = n/2, O(n)

Nested	for i in range (0,n):
	for j in range (0,n):
	do something
		T(n) = n^2, O(n^2)

Loops

	Python Code	n=1000	Time complexity
Logarithmic	i = 1 while i < n: do something i *= 2	1 2 4 8 16 32 64 128 256 512	T(n) = log n, O(log n)
Logarithmic	i = n while i >= 1: do something i //= 2	1000 500 250 125 62 31 15 7 3 1	T(n) = log n, O(log n)

Loops

Python Code

Time complexity

Linear logarithmic

```
for i in range (0,n):
        j = 1
        while j < n:
                do something
                j *= 2
```

$T(n) = n\log n, O(n\log n)$

Loops

Python Code

Time complexity

Dependent Nested

```
for i in range (0,n):
        for j in range(0, i+1)
                do something
```

$T(n) = n(n+1)/2, O(n^2)$

Number of iterations of the inner loop depends on the outer loop

For the inner loop, the number of iterations is (n+1)/2

Exercise

Time complexity

- $n^2 + 100n$
- $n^2 * n + 100n^2 \log n$
- $123 + \log 657$
- $(n + \log n)^3$
- $n(2 + \log n)$
- $1 + 2 + 3 + ... + n$

O(?)
O(?)
O(?)
O(?)
O(?)
O(?)

Exercise

Time complexity

- $n^2 + 100n$
- $n^2 * n + 100n^2 \log n$
- $123 + \log 657$
- $(n + \log n)^3$
- $n(2 + \log n)$
- $1 + 2 + 3 + ... + n$

$O(n^2)$
$O(n^3)$
$O(1)$
$O(n^3)$
$O(n \log n)$
$O(n)$

Calculating Time Complexity

Python Code: Factorial

```
def factorial1(n):
  if n <= 1:
    return 1
  else:
    fact = 1
    for i in range(2,n+1):
      fact *= i
    return fact
```

→ $T(1)$

→ $T(1)$

→ $T(n)$

→ $T(1)$

$T(n) = 2 + n + 1 = n + 3$

Complexity: $O(n)$

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Calculating Time Complexity

Python Code: Simple nested loops

```
def simple(n):
  for i in range(n):
    for j in range(n):
      print("i: {0}, j: {1}".format(i,j))
```

→ $T(n)$

→ $T(n)$

$T(n) = n*n$

Complexity: $O(n^2)$

Calculating Time Complexity

Python Code: Element uniqueness v1

```
def unique1(s):
  for i in range(len(s)):
    for j in range(i+1, len(s)):
      if s[i] == s[j]:
        return False  # Found duplicate pair
  return True       # All elements are unique
```

→ *T*(n)

→ *T*(n)

T(n) = n(n-1)/2

Complexity: *O*(n^2)

Calculating Time Complexity

Python Code: Element uniqueness v2

```
def unique2(s):
  temp = sorted(s)  # create a sorted copy of s
  for i in range(1, len(temp)):
    if temp[i-1] == temp[i]:
      return False   # Found duplicate pair
  return True        # All elements are unique
```

→ *T*(n log n)

→ *T*(n)

T(n) = n log n + n

Complexity: *O*(n log n)

Calculating Time Complexity

Python Code: Prefix averages v1

```
def prefix_average1(s):
    n = len(s)
    a = [0] * n                  # create list of n zeros
    for i in range(n):
        total = 0                # compute each element
        for j in range(i+1):
            total += s[j]
        a[i] = total / (i+1)     # record the average
    return a
```

→ $T(1)$

→ $T(n)$

→ $T(n)$

→ $T(n)$

$T(n) = 1 + n + 2n + n(n+1)/2$

Complexity: $O(n^2)$

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Calculating Time Complexity

Python Code: Prefix averages v2

```
def prefix_average2(s):
    n = len(s)
    a = [0] * n                  # create list of n zeros
    for i in range(n):
        a[i] = sum(s[0:i+1])/(i+1) # record the average
    return a
```

→ $T(1)$

→ $T(n)$

→ $T(n)$

→ $T(n)$

$T(n) = 1 + n + n(n+1)/2$

Complexity: $O(n^2)$

Calculating Time Complexity

Python Code: Prefix averages v3

```
def prefix_average3(s):
  n = len(s)
  a = [0] * n                # create list of n zeros
  total = 0
  for i in range(n):
    total += s[i]      # update total sum to include s[i]
    a[i] = total / (i+1) # compute average based on
current sum
  return a
```

$T(1)$
$T(n)$
$T(1)$

$T(n)$

$T(n) = 2 + n + 2n$

v3 is asymptotically better than v2 and v1

Complexity: $O(n)$

Exercise 1

Python Code

```
def example1(s):
  n = len(s)
  total = 0
  for i in range(n):     # loop from 0 to n-1
    total += s[i]
  return total
```

Complexity: $O(?)$

Exercise 2

Python Code

```
def example2(s):
  n = len(s)
  total = 0
  for i in range(0,n,2):   # Increment of 2
    total += s[i]
  return total
```

Complexity: O(?)

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Exercise 3

Python Code

```
def example3(s):
  n = len(s)
  total = 0
  for i in range(n):       # loop from 0 to n-1
    for k in range(1+i):   # loop from 0 to i
      total += s[k]
  return total
```

Complexity: O(?)

Exercise 4

Python Code

```
def example4(s):
  n = len(s)
  prefix = 0
  total = 0
  for i in range(n):
    prefix += s[i]
    total += prefix
  return total
```

Complexity: *O*(?)

Exercise 5

Python Code

```
# Assume that A and B have equal length of n
def example5(A,B):
  n = len(A)
  count = 0
  for i in range(n):          # loop from 0 to n-1
    total = 0
    for j in range(n):        # loop from 0 to n-1
      for k in range(1+j):    # loop from 0 to j
        total += A[k]
    if B[i] == total:
      count += 1
  return count
```

Complexity: *O*(?)