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**Boolean Identities** **Date:** _Feb 7 2023___________

**Learning Goal** Simplify boolean expressions using boolean identities.

Recall that we use boolean algebra in order to evaluate complex digit circuits. Our goal is to simplify digital circuits (boolean expressions) as much as possible and for that we need boolean identities.

**Boolean Identities**

Boolean identities represent equivalent boolean expressions. Two boolean expressions are equal if they have the same truth table.

| Identity Name | AND Form | OR Form |
|---|---|---|
| Identity Law | $1x = x$ | $0 + x = x$ |
| Null Law | $0x = 0$ | $1 + x = 1$ |
| Idempotent Law | $xx = x$ | $x + x = x$ |
| Inverse Law | $x\bar{x} = 0$ | $x + \bar{x} = 1$ |
| Commutative Law | $xy = yx$ | $x + y = y + x$ |
| Associative Law | $(xy)z = x(yz)$ | $(x + y) + z = x + (y + z)$ |
| Distributive Law | $x + yz = (x + y)(x + z)$ | $x(y + z) = xy + xz$ |
| Absorption Law | $x(x + y) = x$ | $x + xy = x$ |
| DeMorgan's Law | $\overline{(xy)} = \bar{x} + \bar{y}$ | $\overline{(x + y)} = \bar{x}\bar{y}$ |
| Double Complement Law | $\bar{\bar{x}} = x$ | |

**Proofs**

$x(x + y) = x$

x(x + y)

= xx + xy (distributive)

= x + xy (idempotent)

= x(1 + y) (distributive)

= x1 (null)

= x (identity)

$x + xy = x$

x + xy

= x(1 + y) (distributive)

= x (null)

$x + yz = (x + y)(x + z)$

(x + y)(x + z)

= (x + y)x + (x + y)x (distributive)

= x(x + y) + z(x + y) (commutative)

= xx + xy + xz + zy (distributive)

= x + xy + xz + zy (idenpotent)

= x(1 + y + z) + zy (distributive)

= x + yz (null)

**DeMorgan's Laws**

DeMorgan's Laws are used to simplify a complement (opposite) of a boolean expression. It says:

**not (A and B)** → **not A or not B**

**not (A or B)** → **not A and not B**

However,

**not (A and B)** $\neq$ **not A and not B**

**Example** Simplify $\overline{(xy + \bar{x}\bar{y})}$

= !(xy) * !(xy)

= (!x + !y)(!!x + !!y)

= (!x + !y)(x + y)

= (!x)x + (!x)y + (!y)x + (!y)y

= (!x)y + (!y)x

**Practice** Simplify $\overline{(x(\bar{y} + z))}$

= !x + !(!y + z)

= !x + (!!y)(!z)

= !x + y(!z)

A boolean expression can have an infinite number of equivalent boolean expressions. The goal of using boolean identities is to reduce an expression to its simplest terms. Simplest usually means with as few terms, no brackets and no extraneous negatives.

**Practice** Simplify the following.

1. $x(\bar{x} + y)$
2. $xy + \bar{x}y$
3. $(\bar{x} + \bar{y})(\bar{x} + y)$
4. $\bar{x} + y\bar{x}$
5. $(w + \bar{x} + y + \bar{z})y$
6. $x(x + (xy))$
7. $\overline{(x + \bar{x})}$
8. $\overline{(\bar{x} + \bar{x})}$
9. $w + (w\bar{x}yz)$
10. $\bar{w}\overline{(wxyz)}$