Progreee report: A domain decomposition method for a 2D elliptic equation

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Domain decomposition methods are widely used techniques for solving partial differential equations(PDEs) in scientific computing. It partitions the spatial region of interest into multiple sub-domains, and solve them independently. As sub-domains can be processed in parallel, domain decomposition methods are well suited for multi-core machine.

In this project, we will focus on the following elliptic equation in 2D:

$$\begin{cases} \Delta u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \\ u(0,y) = \sin 2\pi y, u(1,y) = \cos 2\pi y \\ \frac{\partial}{\partial y} u(x,0) = 0, \frac{\partial}{\partial y} u(x,1) = 0 \end{cases}$$

, where $x \in [0,3]$ and $y \in [0,1]$.

We will divide the region of problem into 3 overlapped sub-domains along x-axis, as shown in Fig.1. The matching square grid will be used for simplicity. The Gauss-Seidel iteration scheme will be used in the sub-domain, and Additive Schwarz method(ASM) will be used to connect sub-domains.

Objective:
1. Solve a 100 x 3000 grid with initial conditions and plot the final result.
2. Study the effect of grid size on the rate of convergence.
3. Study the effect of overlapped region on the rate of convergence.
4. Compare the rate of convergence with and without domain decomposition.

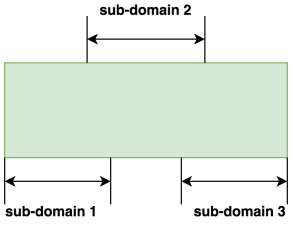

Figure 1: Schematic drawing of sub-domains