STRENGTH OF MATERIALS

[Mechanics of Solids]

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Chapter

1

Simple Stresses and Strains

1.1. CLASSIFICATION OF LOADS

A load may be defined as the combined effect of external forces acting on a body. The loads may be classified as : (*i*) dead loads, (*ii*) live or fluctuating loads, (*iii*) inertia loads or forces, and (*iv*) centrifugal loads or forces.

The other way of classification is (*i*) tensile loads, (*ii*) compressive loads, (*iii*) torsional or twisting loads, (*iv*) bending loads, and (*v*) shearing loads.

The load may also be a 'point' (or concentrated) or 'distributed.'

Point load. A point load or concentrated load is one which is *considered to act at a point.* In actual practice, the load has to be distributed over a small area, because, such small knife-edge contacts are generally neither possible, nor desirable.

Distributed load. A distributed load is one which is *distributed or spread in some manner over the length of the beam.* If the spread is uniform, (*i.e.*, at the uniform rate, say w kN or N/metre run) it is said to be uniformly distributed load and is abbreviated as u.d.l. If the spread is not at uniform rate, it is said to be non-uniformly distributed load. *Triangulary* and *trapezoidally* distributed loads fall under this category.

1.2. STRESS

Fig. 1.1. Tensile stress

Fig. 1.2. Compressive stress

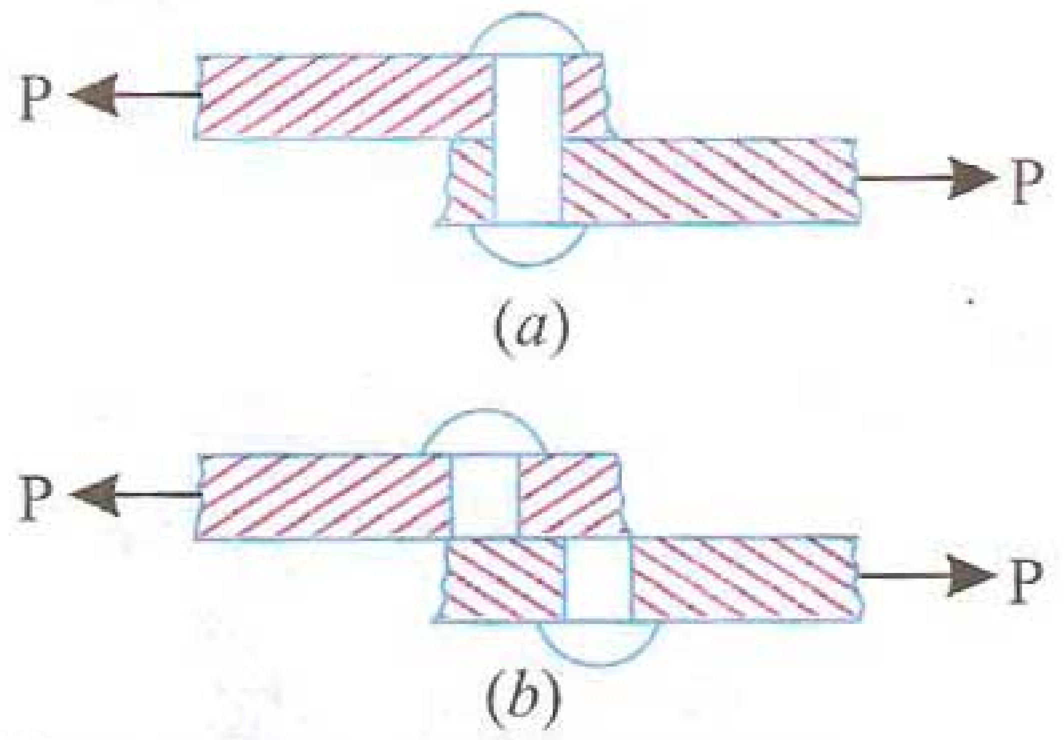


Fig. 1.3. (*a*) Rivet resisting shear.
(*b*) Rivet failure due to shear.

When a body is acted upon by some load or external force, it undergoes deformation (*i.e.,* change in shape or dimensions) which increases gradually. During deformation, the material of the body resists the tendency of the load to deform the body, and when the load influence is taken over by the internal resistance of the material of the body, it becomes stable. This *internal resistance which the body offers to meet with the load is called* **stress.**

Stress can be considered either as total stress or unit stress. Total stress represents the total resistance to an external effect and is expressed in N, kN or MN. Unit stress represents the resistance developed by a unit area of cross-section, and is expressed in kN/m^2 or MN/m^2 or N/mm^2. For the remainder of this text, the word stress will be used to signify unit stress.

The various types of stresses may be classified as :

1. **Simple or direct stress**

   (*i*) Tension (*ii*) Compression (*iii*) Shear.

2. **Indirect stress**

   (*i*) Bending (*ii*) Torsion.

3. **Combined stress.** Any possible combination of types 1 and 2.

This chapter deals with simple stresses only.

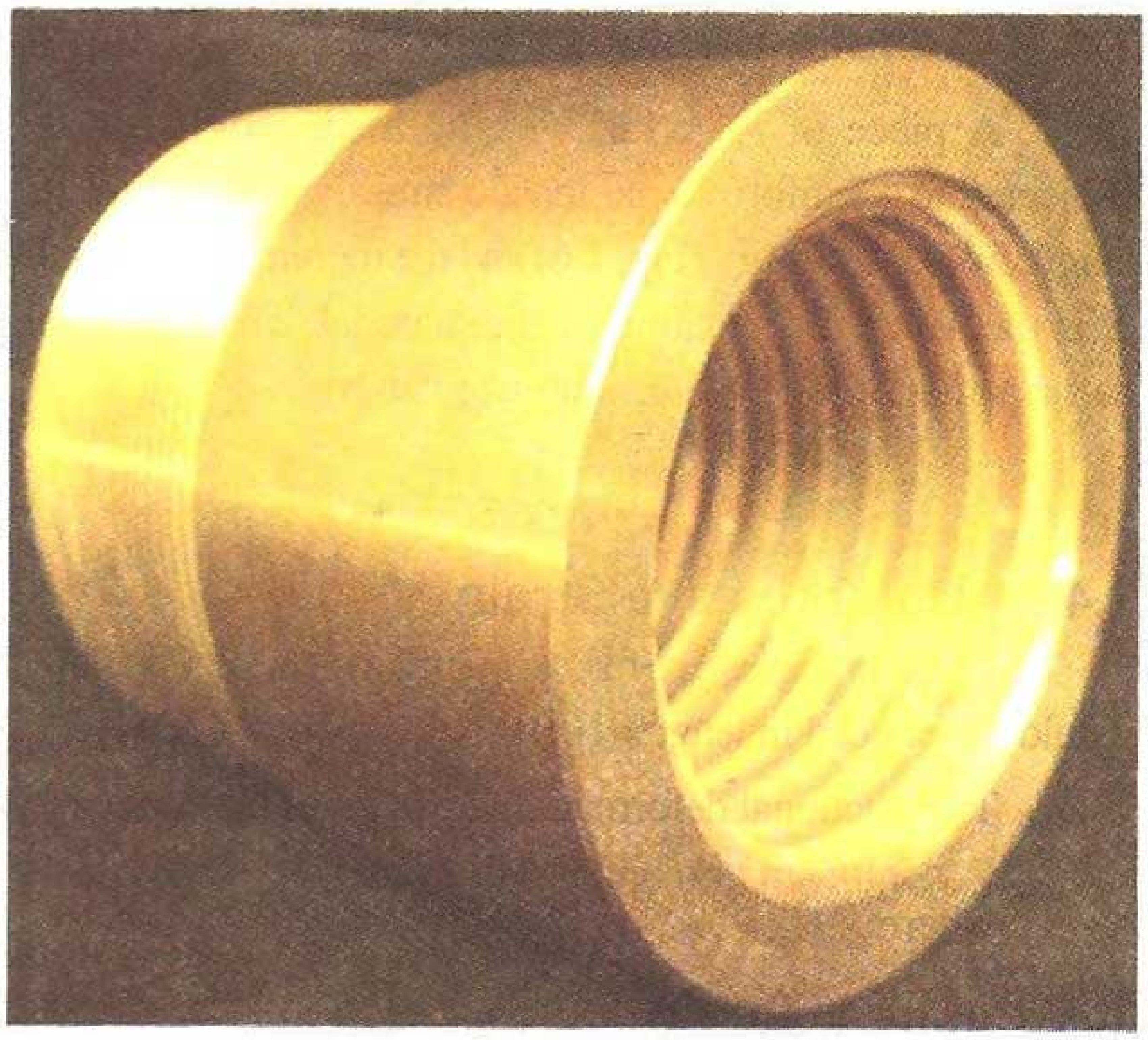
1.3. SIMPLE STRESS

Simple stress is often called *direct stress* because it develops under direct loading conditions. That is, simple tension and simple compression occur when the applied force, called load, is in line with the axis of the member (axial loading) (Figs. 1.1 and 1.2), and simple shear occurs, when equal,

The chain link at the bottom of the bulldozer spreads the weight and provides a strong grip.

parallel, and opposite forces tend to cause a surface to slide relative to the adjacent surface (Fig. 1.3).

In certain loading situations, the stresses that develop are not simple stresses. For example, referring to Fig. 1.4, the member is subjected to a load which is perpendicular to the axis of the member (transverse loading) (Fig. 1.5). This will cause the member to bend, resulting in deformation of the material and stresses being developed internally to resist the deformation. All three types of stresses—tension, compression and shear—will develop, but they will not be simple stresses, since they were not caused by direct loading.



Bronze nut.

When any type of simple stress σ (sigma) develops, we can calculate the magnitude of the stress by,

$$\sigma = \frac{P}{A} \quad \text{... (1.1)}$$

where,

σ = Stress, kN/m^2 or N/mm^2,

P = Load $\left[\begin{array}{c}\text{external force causing} \\ \text{stress to develop}\end{array}\right]$, kN or N, and

A = Area over which stress develops, m^2 or mm^2.

It may be noted that in cases of either simple tension or simple compression, the areas which resist the load are perpendicular to the direction of forces. When a member is subjected to simple shear, the resisting area is parallel to the direction of the force. Common situations causing shear stresses are shown in Figs. 1.3 and 1.4.

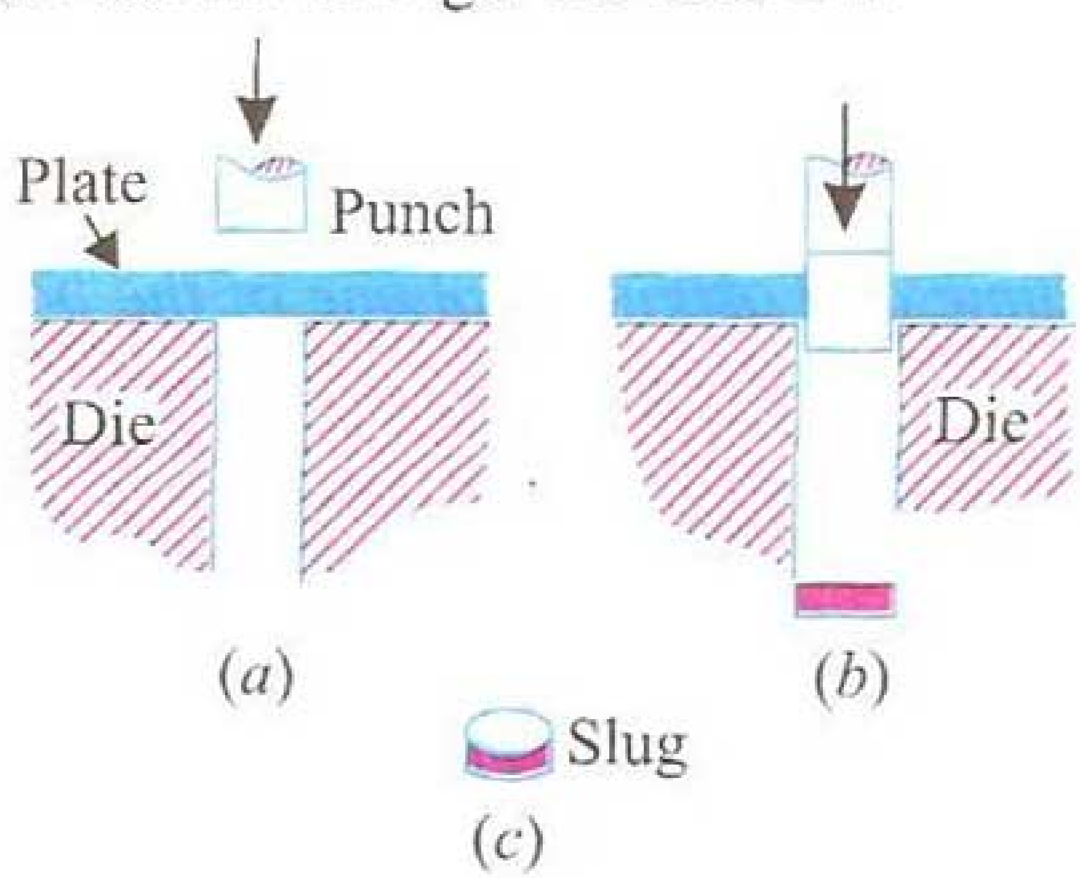


Fig. 1.4. (a) Punch approaching plate;
(b) Punch shearing plate;
(c) Slug showing sheared area.

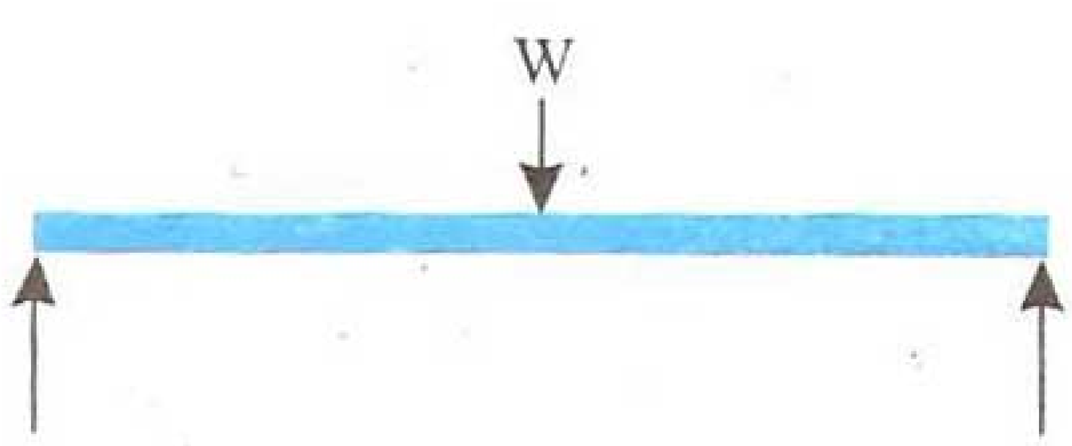


Fig. 1.5. Simply supported beam. (Transverse loading)

1.4. STRAIN

Any element in a material subjected to stress is said to be strained. *The* **strain** *(e) is the deformation produced by stress.* The various types of strains are explained below :

1.4.1. Tensile Strain

A piece of material, with uniform cross-section, subjected to a uniform axial tensile stress, will increase its length from l to $(l + \delta l)$, (Fig. 1.6) and the increment of length δl is the actual deformation of the material. The fractional deformation or the tensile strain is given by

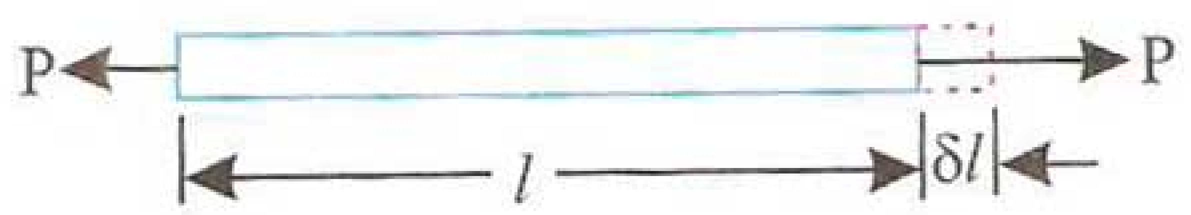


Fig. 1.6

$$e_t = \frac{\delta l}{l} \qquad \text{...(1.2)}$$

1.4.2. Compressive Strain

Under compressive forces, a similar piece of material would be reduced in length (Fig. 1.7) from l to $(l - \delta l)$.

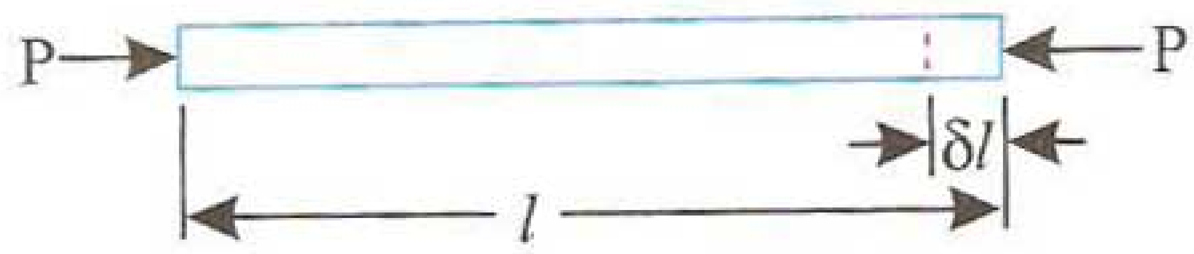


Fig. 1.7

The fractional deformation again gives the strain e_c,

where, $$e_c = \frac{\delta l}{l} \qquad \text{...(1.2a)}$$

1.4.3. Shear Strain

In case of a shearing load, a shear strain will be produced which is *measured by the angle through which the body distorts.*

In Fig. 1.8 is shown a rectangular block *LMNP* fixed at one face and subjected to force *F*. After application of force, it distorts through an angle ϕ and occupies new position *LM'N'P*. The shear strain (e_s) is given by

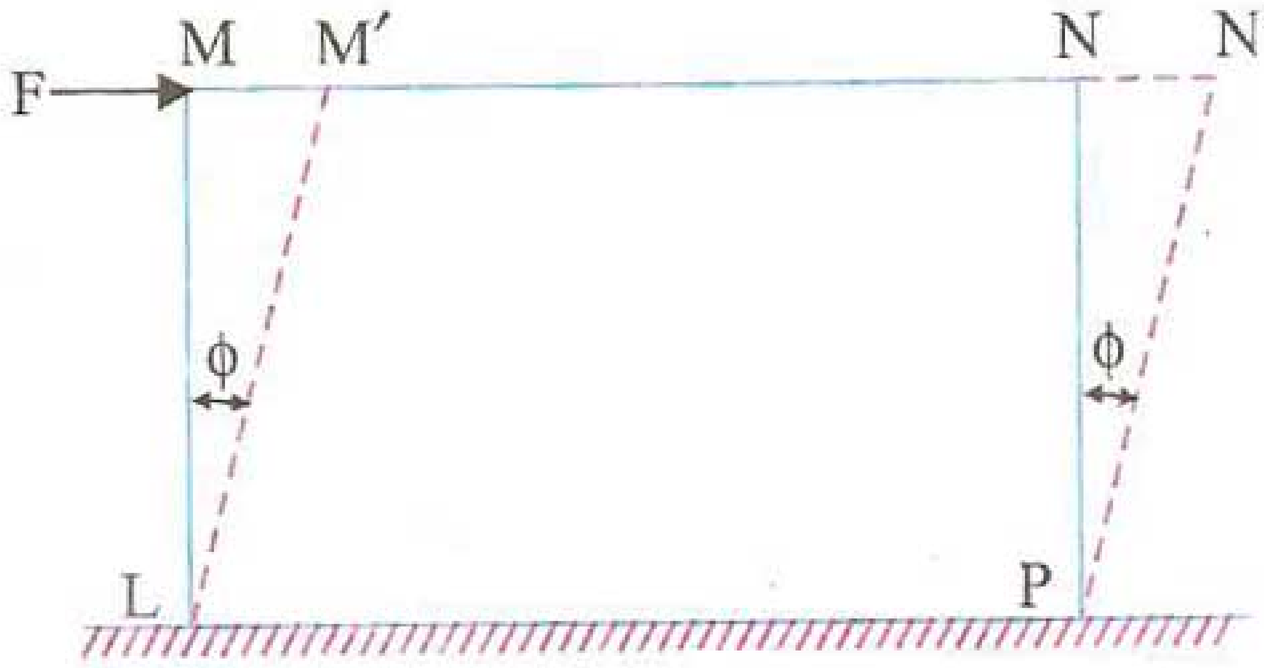


Fig. 1.8

$$e_s = \frac{NN'}{NP} = \tan\phi$$
$$= \phi \text{ (radians) .....since}$$

ϕ is very small.

The above result has been obtained by assuming NN' equal to arc (as NN' is small) drawn with centre P and radius PN.

1.4.4. Volumetric Strain

It is defined as the *ratio between change in volume and original volume of the body,* and is denoted by e_v.

$$\therefore \qquad e_v = \frac{\text{Change in volume}}{\text{Original volume}} = \frac{\delta V}{V} \qquad \text{...(1.3)}$$

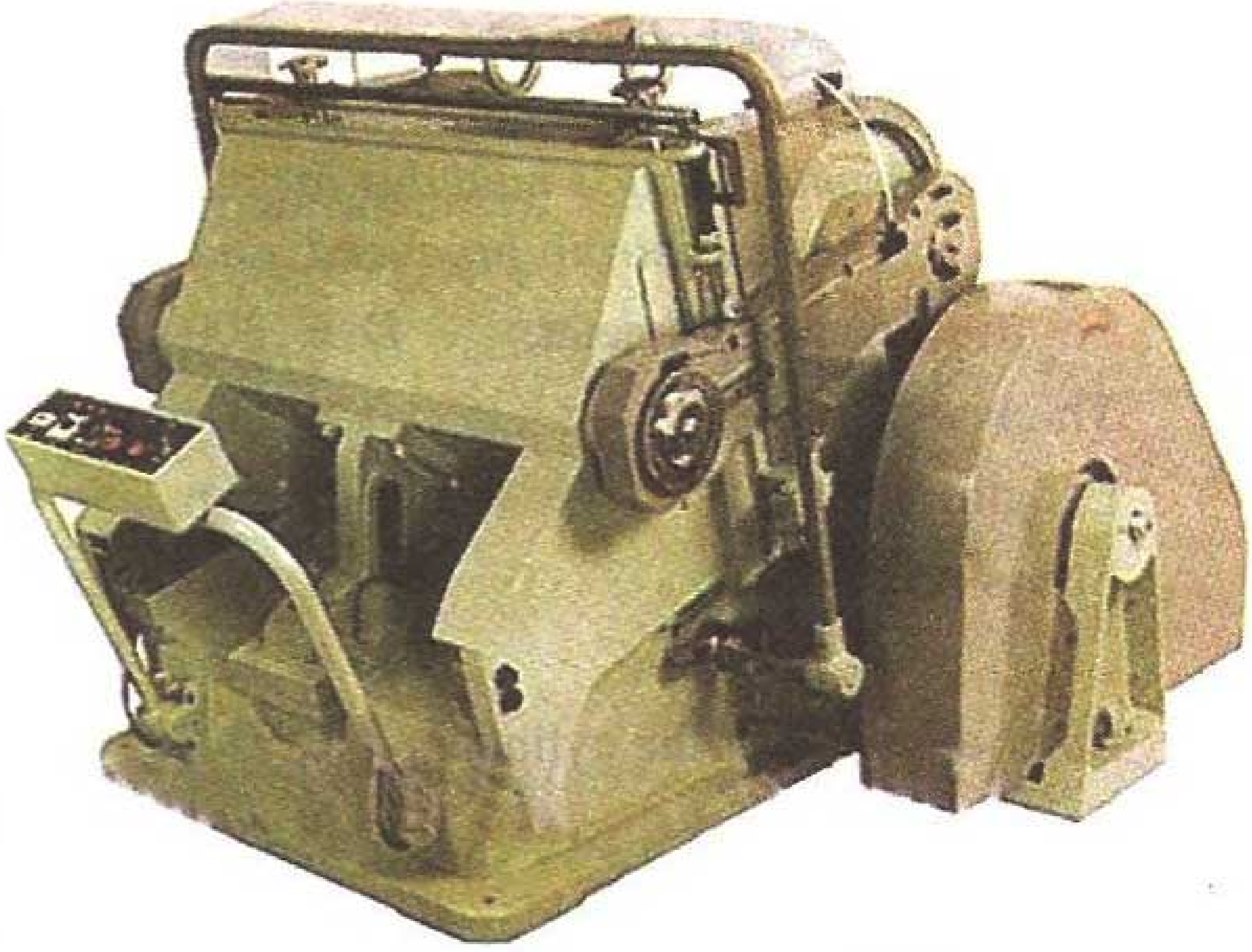
The strains which disappear with the removal of load are termed as *elastic strains* and the body which regains its original position on the removal of force is called an *elastic body.* The body is said to be *plastic* if the *strains exist even after the removal of external force.* There is always a limiting value of load up to which the strain totally disappears on the removal of load—the stress corresponding to this load is called *elastic limit.*

Robert Hooke discovered experimentally that within elastic limit, stress varies *directly* as strain

i.e., $\qquad \text{Stress} \propto \text{Strain}$

or, $$\frac{\text{Stress}}{\text{Strain}} = \text{a constant}$$

This constant is termed as *Modulus of elasticity.*


Heavy duty punching machine.

(i) Young's modulus:

It is the *ratio between tensile stress and tensile strain or compressive stress and compressive strain.* It is denoted by E. It is the same as modulus of elasticity.

or, $$E = \frac{\sigma}{e}\left[=\frac{\sigma_t}{e_t} \text{ or } \frac{\sigma_c}{e_c}\right] \quad ...(1.4)$$

(ii) Modulus of rigidity:

It is defined as the *ratio of shear stress* τ *(tau) to shear strain* and is denoted by C, N or G. It is also called *shear modulus of elasticity.*

or, $$\frac{\tau}{e_s} = C, N \text{ or } G \quad ...(1.5)$$

(iii) Bulk or volume modulus of elasticity:

It may be defined as the *ratio of normal stress* (on each face of a solid cube) *to volumetric strain* and is denoted by the letter K.

or, $$\frac{\sigma_n}{e_v} = K \quad ...(1.6)$$

Example 1.1. *A square steel rod 20 mm × 20 mm in section is to carry an axial load (compressive) of 100 kN. Calculate the shortening in a length of 50 mm.* $E = 2.14 \times 10^8$ *kN/m²*.

Solution.

Area, $A = 0.02 \times 0.02 = 0.0004 \text{ m}^2$

Length, $l = 50 \text{ mm or } 0.05 \text{ m}$

Load, $P = 100 \text{ kN}$

$E = 2.14 \times 10^8 \text{ kN/m}^2$

Shortening of the rod δl:

Stress, $$\sigma = \frac{P}{A}$$

$\therefore$ $$\sigma = \frac{100}{0.0004} = 250000 \text{ kN/m}^2$$

Also, $$E = \frac{\text{Stress}}{\text{Strain}}$$

or, $$\text{Strain} = \frac{\text{Stress}}{E} = \frac{250000}{2.14 \times 10^8}$$

or, $$\frac{\delta l}{l} = \frac{250000}{2.14 \times 10^8}$$

$\therefore$ $$\delta l = \frac{250000}{2.14 \times 10^8} \times l = \frac{250000}{2.14 \times 10^8} \times 0.05$$
$$= 0.0000584 \text{ m or } 0.0584 \text{ mm}$$

Hence, the shortening of the rod **= 0.0584 mm (Ans.)**

Example 1.2. *A hollow cast-iron cylinder 4 m long, 300 mm outer diameter, and thickness of metal 50 mm is subjected to a central load on the top when standing straight. The stress produced is 75000 kN/m². Assume Young's modulus for cast-iron as 1.5 × 10⁸ kN/m² and find (i) magnitude of the load, (ii) longitudinal strain produced, and (iii) total decrease in length.*

Solution. Outer diameter, $D = 300 \text{ mm} = 0.3 \text{ m}$

Thickness, $t = 50 \text{ mm} = 0.05 \text{ m}$

Length, $l = 4 \text{ m}$

Stress produced, $\sigma = 75000 \text{ kN/m}^2$

$E = 1.5 \times 10^8 \text{ kN/m}^2$

Inner diameter of the cylinder, $d = D - 2t = 0.3 - 2 \times 0.05 = 0.2 \text{ m}$

(*i*) Magnitude of the load *P* :

Using the relation,
$$\sigma = \frac{P}{A}$$

or,
$$P = \sigma \times A = 75000 \times \frac{\pi}{4}(D^2 - d^2) = 75000 \times \frac{\pi}{4}(0.3^2 - 0.2^2)$$

or,
$$P = \mathbf{2945.2 \text{ kN}} \quad \textbf{(Ans.)}$$

(*ii*) Longitudinal strain produced, *e* :

Using the relation,

Strain,
$$e = \frac{\text{Stress}}{E} = \frac{75000}{1.5 \times 10^8}$$
$$= \mathbf{0.0005} \quad \textbf{(Ans.)}$$

(*iii*) Total decrease in length, δ*l* :

Using the relation,
$$\text{Strain} = \frac{\text{Change in length}}{\text{Original length}} = \frac{\delta l}{l}$$
$$0.0005 = \frac{\delta l}{4}$$
$$\delta l = 0.0005 \times 4 \text{ m} = 0.002 \text{ m} = 2 \text{ mm}$$

Hence, decrease in length $= \mathbf{2 \text{ mm}}$ **(Ans.)**

Depending on the purpose and function, machine parts are made of different alloys and materials.

Example 1.3. *The following observations were made during a tensile test on a mild steel specimen 40 mm in diameter and 200 mm long.*

Elongation with 40 kN load (within limit of proportionality),

$$\delta l = 0.0304 \text{ mm}$$

Yield load $= 161$ *kN*

Maximum load $= 242$ *kN*

Length of specimen at fracture

$= 249$ *mm*

Determine :

(*i*) *Young's modulus of elasticity,* (*ii*) *Yield point stress,*

(*iii*) *Ultimate stress, and* (*iv*) *Percentage elongation.*

Solution.

(*i*) Young's modulus of elasticity *E* :

Stress,
$$\sigma = \frac{P}{A} = \frac{40}{\frac{\pi}{4} \times (0.04)^2} = 3.18 \times 10^4 \text{ kN/m}^2$$

Strain,
$$e = \frac{\delta l}{l} = \frac{0.0304}{200} = 0.000152$$

$\therefore$
$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{3.18 \times 10^4}{0.000152}$$
$$= \mathbf{2.09 \times 10^8 \ kN/m^2} \quad \textbf{(Ans.)}$$

(*ii*) Yield point stress :

$$\text{Yield point stress} = \frac{\text{Yield point load}}{\text{Area}}$$

To bear high compressive and variable stresses, automobile wheels are made of alloys.

$$= \frac{161}{\frac{\pi}{4} \times (0.04)^2} = \mathbf{12.8 \times 10^4\ kN/m^2} \quad \textbf{(Ans.)}$$

(*iii*) **Ultimate stress :**

$$\text{Ultimate stress} = \frac{\text{Maximum load}}{\text{Area}}$$

$$= \frac{242}{\frac{\pi}{4} \times (0.04)^2} = \mathbf{19.2 \times 10^4\ kN/m^2} \quad \textbf{(Ans.)}$$

(*iv*) **Percentage elongation :**

$$\text{Percentage elongation} = \frac{\text{Length of specimen at fracture} - \text{original length}}{\text{Original length}}$$

$$= \frac{249 - 200}{200} = 0.245 = \mathbf{24.5\%} \quad \textbf{(Ans.)}$$

Example 1.4. *A steel wire 2 m long and 3 mm in diameter is extended by 0.75 mm when a weight W is suspended from the wire. If the same weight is suspended from a brass wire, 2.5 m long and 2 mm in diameter, it is elongated by 4.64 mm. Determine the modulus of elasticity of brass if that of steel be 2.0 × 10⁵ N/mm².* **(AMIE Summer, 2000)**

Solution. *Given* : $l_s = 2$ m, $d_s = 3$ mm, $\delta l_s = 0.75$ mm, $E_s = 2.0 \times 10^5$ N/mm²; $l_b = 2.5$ m, $d_b = 2$ mm, $\delta l_b = 4.64$ mm.

Modulus of elasticity of brass, E_b :

From Hooke's law, we know that

$$\delta l = \frac{Pl}{AE}$$

where, δl = Extension, l = Length, A = Cross-sectional area, and E = Modulus of elasticity.

Parts made from sand casting using grey cast iron, ductile iron and copper alloys.

Case I : For steel wire :

$$\delta l_s = \frac{P l_s}{A_s E_s}$$

or, $$0.75 = \frac{P \times (2 \times 1000)}{\left(\frac{\pi}{4} \times 3^2\right) \times 2.0 \times 10^5}$$

or, $$P = 0.75 \times \left(\frac{\pi}{4} \times 3^2\right) \times 2.0 \times 10^5 \times \frac{1}{2000} \quad ...(i)$$

Case II : For brass wire:

$$\delta l_b = \frac{P l_b}{A_b E_b}$$

$$4.64 = \frac{P \times (2.5 \times 1000)}{\left(\frac{\pi}{4} \times 2^2\right) \times E_b}$$

or, $$P = 4.64 \times \left(\frac{\pi}{4} \times 2^2\right) \times E_b \times \frac{1}{2500} \quad ...(ii)$$

Equating eqns. (*i*) and (*ii*), we get

$$0.75 \times \left(\frac{\pi}{4} \times 3^2\right) \times 2.0 \times 10^5 \times \frac{1}{2000} = 4.64 \times \left(\frac{\pi}{4} \times 2^2\right) \times E_b \times \frac{1}{2500}$$

or, $$\mathbf{E_b = 0.909 \times 10^5\ N/mm^2}\ \textbf{(Ans.)}$$

Example 1.5. *A steel bar is 900 mm long; its two ends are 40 mm and 30 mm in diameter and the length of each rod is 200 mm. The middle portion of the bar is 15 mm in diameter and 500 mm long. If the bar is subjected to an axial tensile load of 15 kN, find its total extension.*

Take $E = 200\ GN/m^2$ (G stands for giga and $1G = 10^9$).

Solution. Refer to Fig. 1.9.

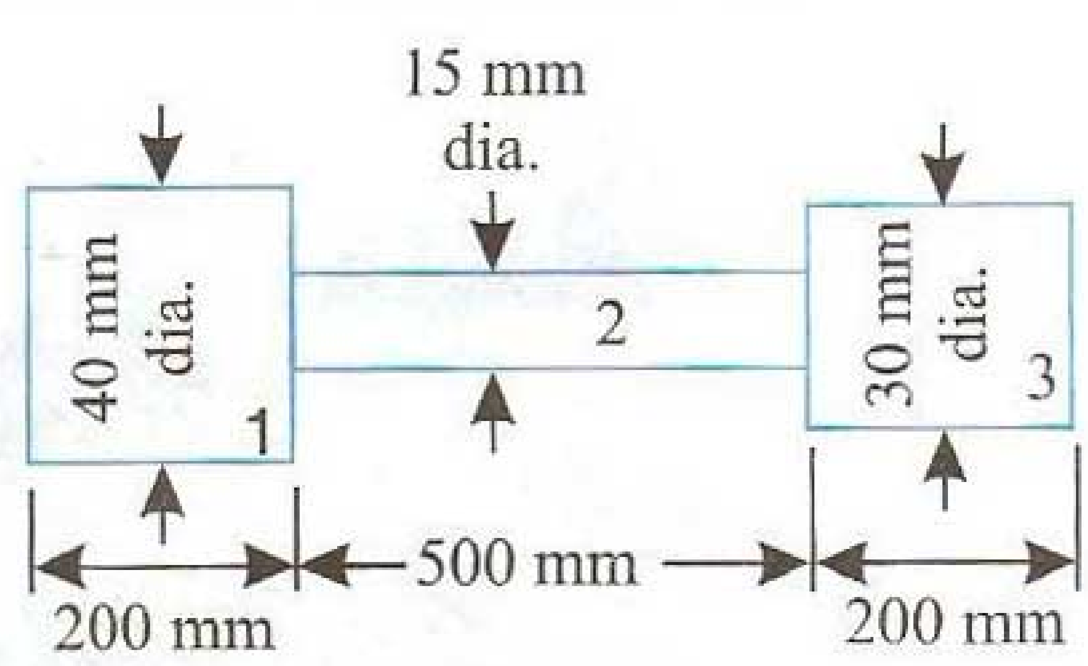


Fig. 1.9

Load, $P = 15$ kN

Area, $A_1 = \frac{\pi}{4} \times 40^2 = 1256.6\ mm^2 = 0.001256\ m^2$

Area, $A_2 = \frac{\pi}{4} \times 15^2 = 176.7\ mm^2 = 0.0001767\ m^2$

Area, $A_3 = \frac{\pi}{4} \times 30^2 = 706.8\ mm^2 = 0.0007068\ m^2$

Lengths : $l_1 = 200$ mm $= 0.2$ m, $l_2 = 500$ mm $= 0.5$ m and $l_3 = 200$ mm $= 0.2$ m

Total extension of the bar :

Let δl_1, δl_2 and δl_3 be the extensions in the parts 1, 2 and 3 of the steel bar respectively.

Then, $\delta l_1 = \frac{Pl_1}{A_1 E}$, $\delta l_2 = \frac{Pl_2}{A_2 E}$, $\delta l_3 = \frac{Pl_3}{A_3 E}$ $\quad \left[\because E = \frac{\sigma}{e} = \frac{P/A}{\delta l/l} = \frac{P \cdot l}{A \cdot \delta l} \text{ or } \delta l = \frac{Pl}{AE}\right]$

Total extension of the bar,

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3$$

$$= \frac{Pl_1}{A_1 E} + \frac{Pl_2}{A_2 E} + \frac{Pl_3}{A_3 E} = \frac{P}{E}\left[\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3}\right]$$

$$= \frac{15 \times 10^3}{200 \times 10^9}\left[\frac{0.20}{0.001256} + \frac{0.50}{0.0001767} + \frac{0.20}{0.0007068}\right]$$

$$= 0.0002454 \text{ m} = 0.2454 \text{ mm}$$

Hence, total extension of the steel bar $= \mathbf{0.2454\ mm}$ **(Ans.)**

Example 1.6. *The bar shown in Fig. 1.10 is subjected to a tensile load of 50 kN. Find the diameter of the middle portion if the stress is limited to 130 MN/m². Find also the length of the middle portion if the total elongation of the bar is 0.15 mm. Take E = 200 GN/m².*

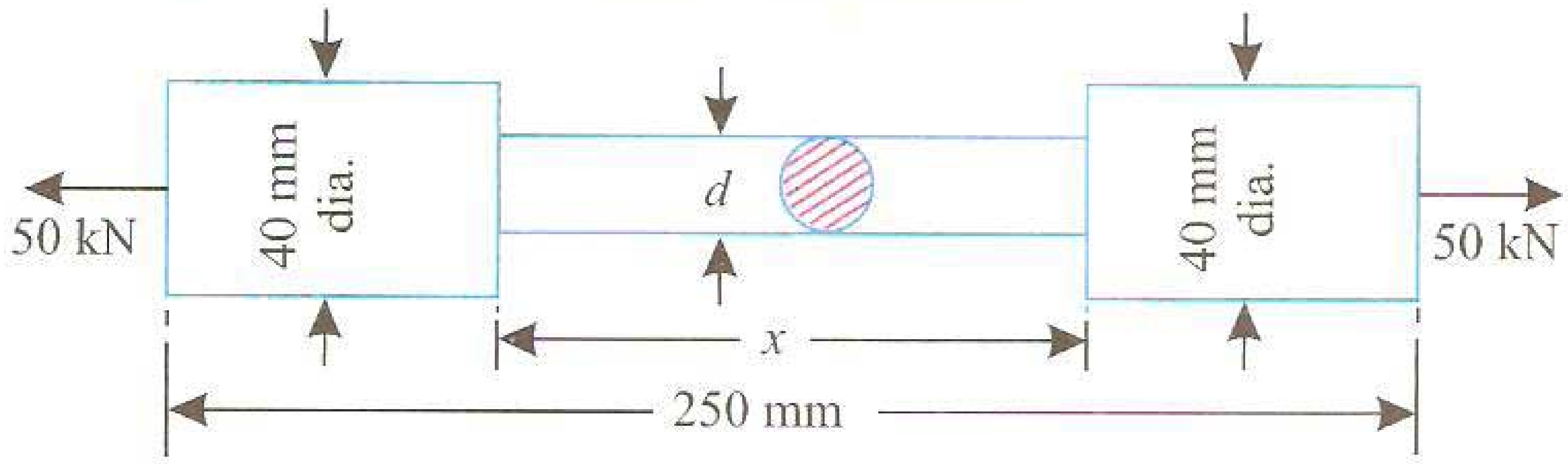


Fig. 1.10

Solution. Magnitude of tensile load, $P = 50$ kN

Stress in the middle portion, $\sigma = 130$ MN/m²

Total elongation of the bar, $\delta l = 0.15 \text{ mm} = 0.15 \times 10^{-3}$ m

Modulus of elasticity, $E = 200$ GN/m²

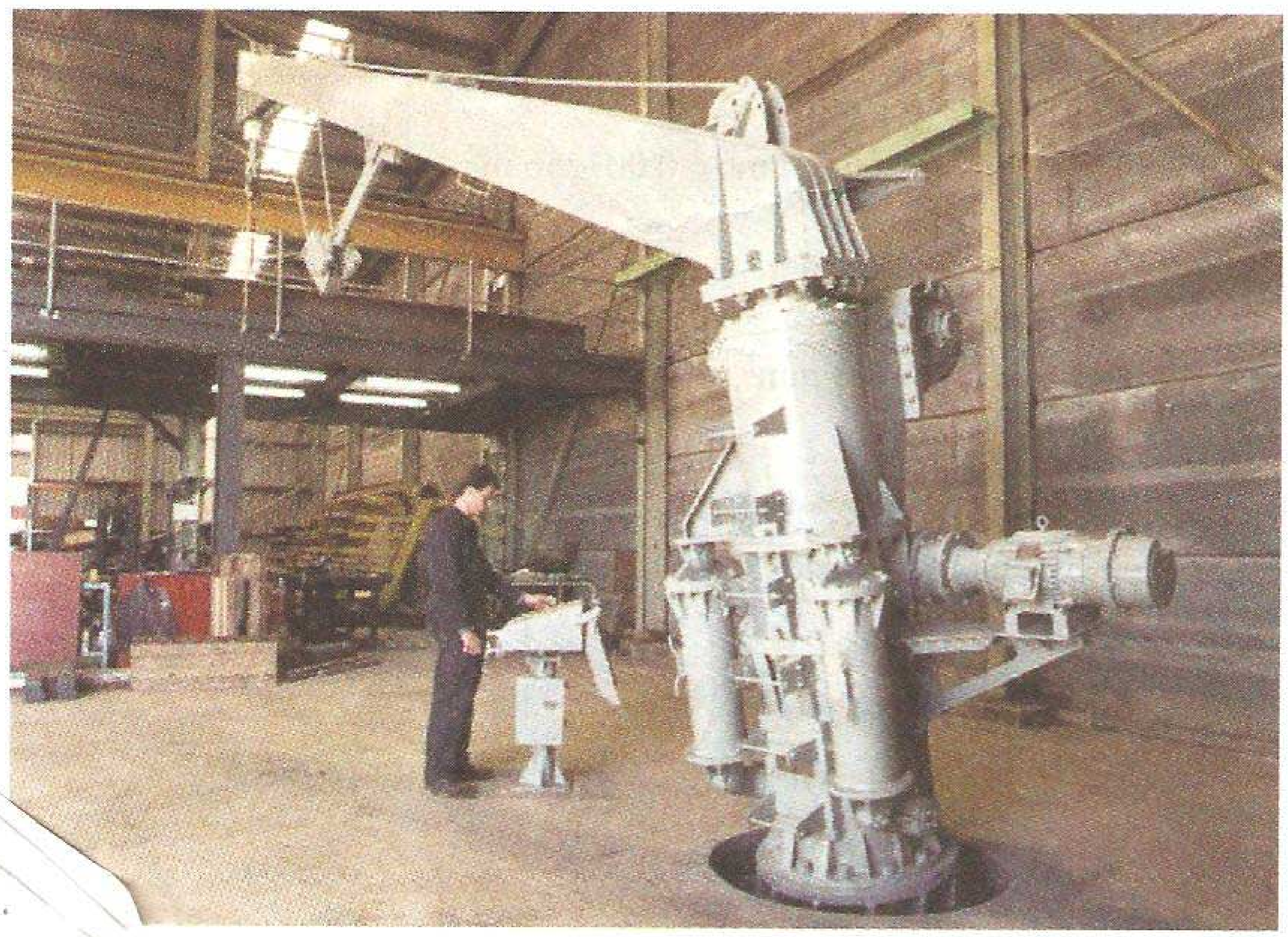
Diameter of the middle portion, *d* :

Now, stress in the middle portion, $$\sigma = \frac{P}{A} = \frac{50 \times 1000}{(\pi/4)\, d^2} = 130 \times 10^6$$

$$\therefore \quad d = \left[\frac{50 \times 1000}{(\pi/4) \times 130 \times 10^6}\right]^{1/2}$$

$$= 0.0221 \text{ m or } 22.1 \text{ mm}$$

Hence, diameter of the middle portion $= \mathbf{22.1\ mm}$ **(Ans.)**



Marine crane under testing.

Length of the middle portion :

Let the length of the middle portion $= x$ metre

Stress in the end portions, $\sigma' = \dfrac{50 \times 1000}{\pi/4 \times \left(\dfrac{40}{1000}\right)^2}$

$= 39.79 \times 10^6 \text{ N/m}^2$

$\therefore$ Elongation of the end portion $= \sigma' \times \dfrac{(0.25 - x)}{E}$

Also, elongation of the end portions + extension of the middle portion $= 0.15 \times 10^{-3}$

$$\frac{39.79 \times 10^6 \times (0.25 - x)}{200 \times 10^9} + \frac{130 \times 10^6 \times x}{200 \times 10^9} = 0.15 \times 10^{-3}$$

$$39.79 \times 10^6 \times (0.25 - x) + 130 \times 10^6 \times x = 200 \times 10^9 \times 0.15 \times 10^{-3}$$

Dividing both sides by 39.79×10^6 , we get

$$0.25 - x + 3.267\, x = 0.754$$

$\therefore$ $x = 0.222$ m or 222 mm

Hence, length of the middle portion **= 222 mm (Ans.)**

Example 1.7. *A steel tie rod 50 mm in diameter and 2.5 m long is subjected to a pull of 100 kN. To what length the rod should be bored centrally so that the total extension will increase by 15 per cent under the same pull, the bore being 25 mm diameter? Take $E = 200$ GN / m².*

Solution. Refer to Fig. 1.11 (*a*, *b*).

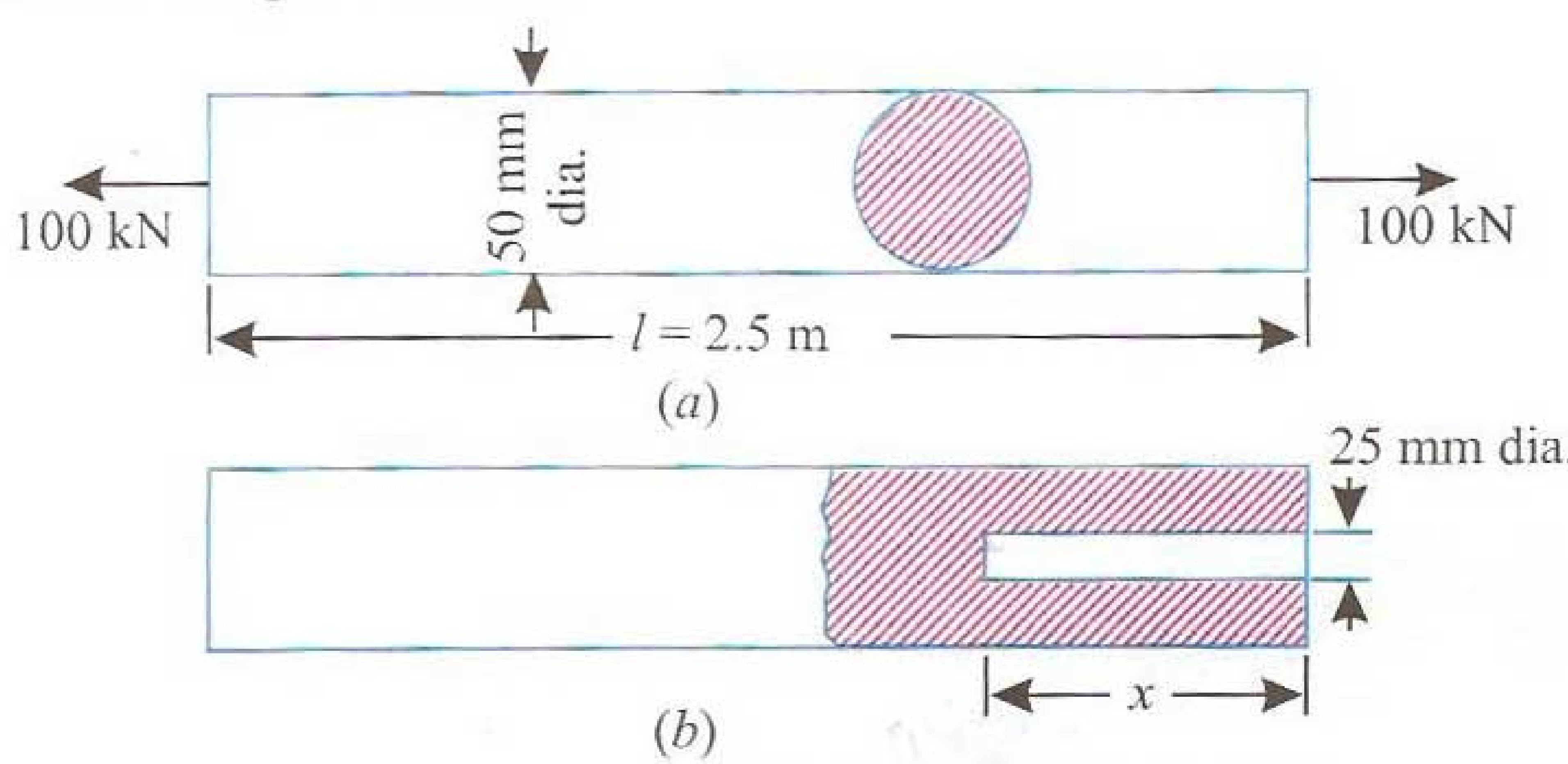


Fig. 1.11

Diameter of the steel tie rod $= 50$ mm $= 0.05$ m

Length of the steel rod, $l = 2.5$ m

Magnitude of the pull, $P = 100$ kN

Diameter of the bore $= 25$ mm $= 0.025$ m

Modulus of elasticity, $E = 200 \times 10^9 \text{ N/m}^2$

Length of the bore, x :

Stress in the solid rod, $\sigma = \dfrac{P}{A} = \dfrac{100 \times 1000}{(\pi/4) \times (0.05)^2} = 50.92 \times 10^6 \text{ N/m}^2$

Elongation of the solid rod, $\delta l = \dfrac{\sigma l}{E} = \dfrac{50.92 \times 10^6 \times 2.5}{200 \times 10^9}$

$= 0.000636$ m or 0.636 mm

Elongation after the rod is bored $= 1.15 \times 0.636 = 0.731$ mm

Area at the reduced section $= \dfrac{\pi}{4}(0.05^2 - 0.025^2) = 0.001472 \text{ m}^2$

Stress in the reduced section, $\sigma' = \dfrac{100 \times 1000}{0.001472} = 67.9 \times 10^6 \text{ N/m}^2$

$\therefore$ Elongation of the rod $= \dfrac{\sigma(2.5 - x)}{E} + \dfrac{\sigma'.x}{E} = 0.731 \times 10^{-3}$

or, $= \dfrac{50.92 \times 10^6 (2.5 - x)}{200 \times 10^9} + \dfrac{67.9 \times 10^6 x}{200 \times 10^9} = 0.731 \times 10^{-3}$

or, $= 50.92 \times 10^6 (2.5 - x) + 67.9 \times 10^6 x$

$= 200 \times 10^9 \times 0.731 \times 10^{-3}$

or, $= (2.5 - x) + 1.33\,x = 2.87$

or, $x = 1.12$ m

Hence, length of the bore **= 1.12 m (Ans.)**

Example 1.8. *A brass bar having cross-sectional area of 1000 mm² is subjected to axial forces shown in the Fig. 1.12. Find the total elongation of the bar. Modulus of elasticity of brass = 100 GN/m².*

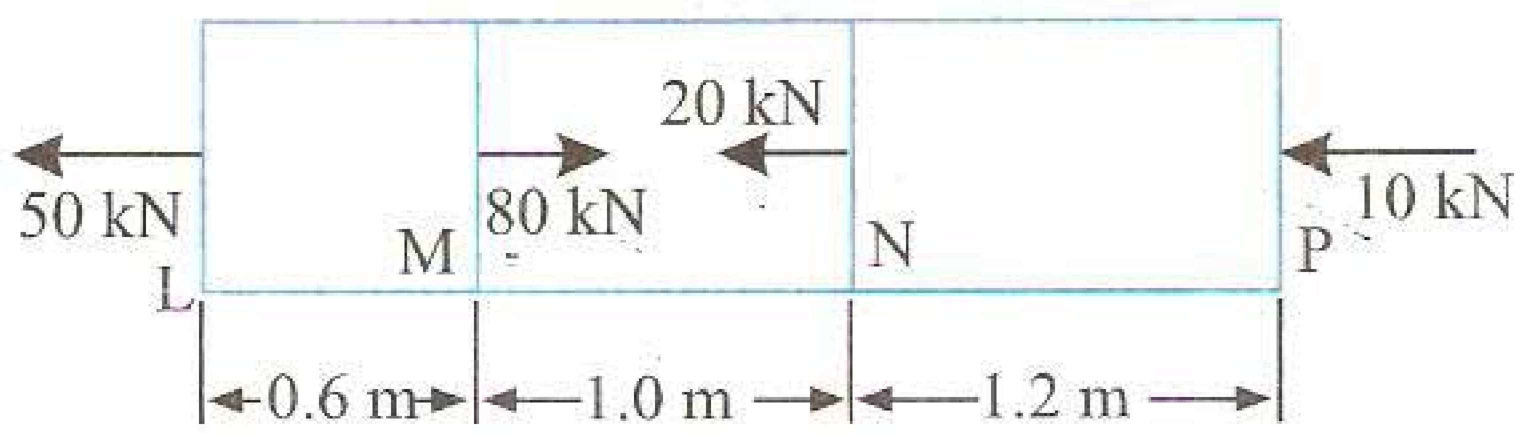


Fig. 1.12

Solution. Refer to Fig. 1.12. The loading of three different portions *LM*, *MN* and *NP* is shown separately in the Fig. 1.13. It may be noted that portion *LM* is under the tensile force 50 kN to the left, and to the right of it, there is again an effective force 50 kN which is a resultant of three forces to its right

i.e. (80 – 20 – 10) = 50 kN.

A metal fabrication unit.

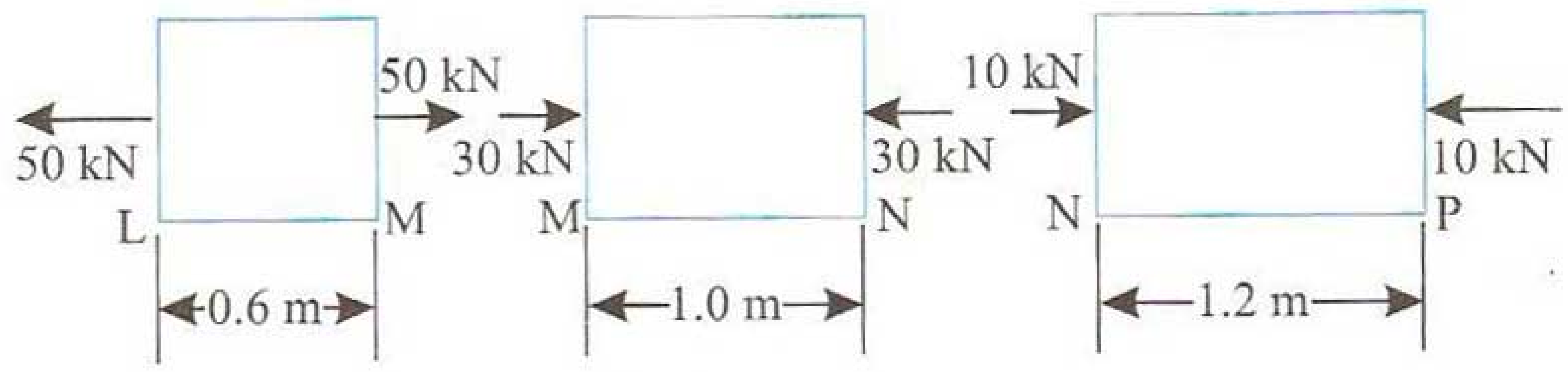


Fig. 1.13

Similarly, in portion *MN*, the compressive force on the left is 30 kN (*i.e.*, 80 – 50) and 30 kN on the right (*i.e.*, 20 + 10). In *NP*, the compressive load is 10 kN, (*i.e.*, 80 – 50 – 20) and on the right, there is already a compressive load of 10 kN. So, we observe that the bar is in equilibrium under the action of these forces.

Total elongation of the bar :

Let δl_1, δl_2 and δl_3 be the changes in length *LM, MN* and *NP* respectively.

Then,
$$\delta l_1 = \frac{P_1 l_1}{AE} \text{ .... increase } (+)$$
$$\delta l_2 = \frac{P_2 l_2}{AE} \text{ .... decrease } (-)$$
$$\delta l_3 = \frac{P_3 l_3}{AE} \text{ .... decrease } (-)$$

∴ Net change in length,
$$\delta l = \delta l_1 - \delta l_2 - \delta l_3$$
$$= \frac{P_1 l_1}{AE} - \frac{P_2 l_2}{AE} - \frac{P_3 l_3}{AE} = \frac{1}{AE}(P_1 l_1 - P_2 l_2 - P_3 l_3)$$
$$= \frac{10^3}{1000 \times 10^{-6} \times 100 \times 10^9}(50 \times 0.6 - 30 \times 1 - 10 \times 1.2)$$
$$= \frac{1}{10^5}(30 - 30 - 12) = \frac{1}{10^5} \times (-12)$$
$$= -0.00012 \text{ m} = -0.12 \text{ mm}$$

Negative sign indicates that the bar is **shortened by 0.12 mm (Ans.)**

Example 1.9. *A member LMNP is subjected to point loads as shown in Fig. 1.14.*
Calculate :
(*i*) *Force P necessary for equilibrium.*
(*ii*) *Total elongation of the bar.*
Take $E = 210\ GN/m^2$

Solution. Refer to Fig. 1.14

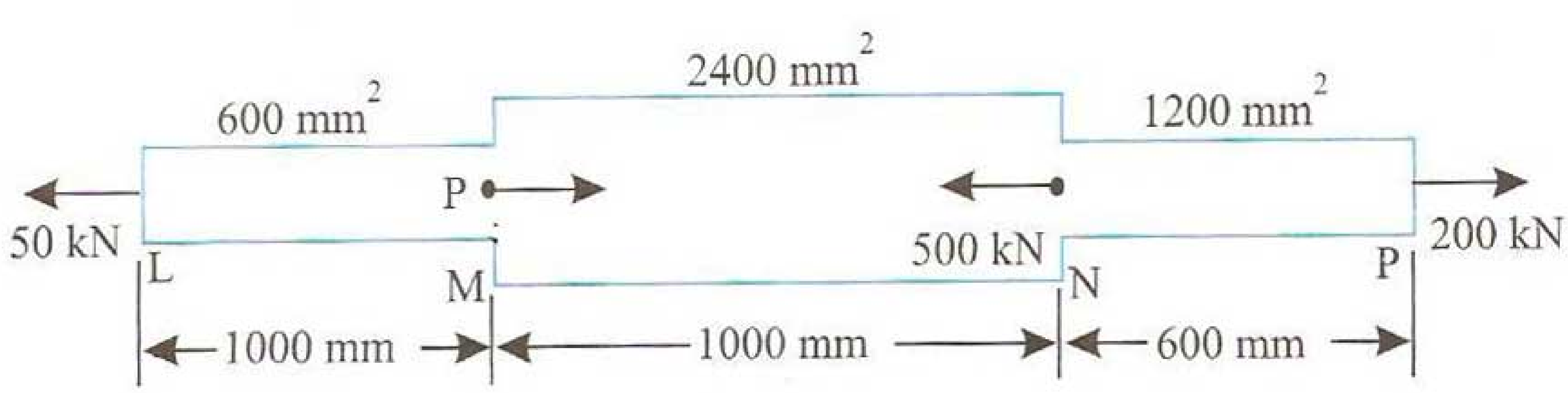


Fig. 1.14

(i) Force P necessary for equilibrium :

Resolving the forces on the rod along its axis, we get

$$50 + 500 = P + 200$$

$$\mathbf{P = 350\ kN} \quad \textbf{(Ans.)}$$

(ii) Total elongation of the bar :

Let δl_1, δl_2 and δl_3 be the changes in lengths LM, MN and NP respectively.

Then,
$$\delta l_1 = \frac{P_1 l_1}{A_1 E} = \frac{50 \times 1000 \times 1}{600 \times 10^{-6} \times 210 \times 10^9} = 3.97 \times 10^{-4} \text{ m ....increase } (+)$$

$$\delta l_2 = \frac{P_2 l_2}{A_2 E} = \frac{300 \times 1000 \times 1}{2400 \times 10^{-6} \times 210 \times 10^9} = 5.95 \times 10^{-4} \text{ m ....decrease } (-)$$

$$\delta l_3 = \frac{P_3 l_3}{A_3 E} = \frac{200 \times 1000 \times 0.6}{1200 \times 10^{-6} \times 210 \times 10^9} = 4.76 \times 10^{-4} \text{ m ....increase } (+)$$

$\therefore$ Total elongation, $\delta l = \delta l_1 - \delta l_2 + \delta l_3 = 3.97 \times 10^{-4} - 5.95 \times 10^{-4} + 4.76 \times 10^{-4}$

$$= 10^{-4}(3.97 - 5.95 + 4.76) = 2.78 \times 10^{-4} \text{ m, or, } 0.278 \text{ mm}$$

Hence, total elongation of the bar = **0.278 mm (Ans.)**

Example 1.10. *In Fig. 1.15 is shown a steel bar of cross-sectional area 250 mm² held firmly by the end supports and loaded by an axial force of 25 kN.*

Determine :

(*i*) *Reactions at L and M.*

(*ii*) *Extension of the left portion.*

E = 200 GN/m².

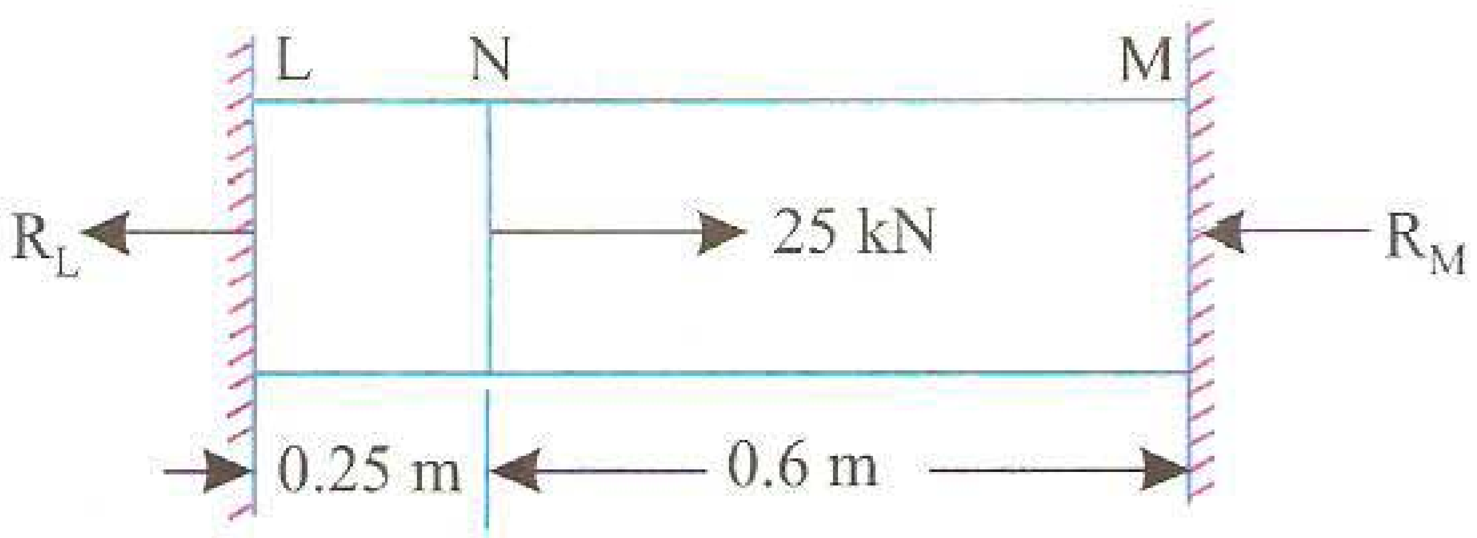


Fig. 1.15

Solution. Refer to Fig. 1.15

(i) Reactions at L and M :

As the bar is in equilibrium.

$$\therefore \quad R_L + R_M = 25 \text{ kN} \qquad ...(i)$$

Also, since total length of the bar remains unchanged,

$\therefore$ Extension in LN = contraction in MN

$$\frac{R_L \times 0.25}{A \times E} = \frac{R_M \times 0.6}{A \times E}$$

$$R_L \times 0.25 = R_M \times 0.6$$

$$R_L = \frac{R_M \times 0.6}{0.25} = 2.4\ R_M$$

Substituting the value of R_L in (*i*), we get

$$2.4\ R_M + R_M = 25$$

From which $\quad \mathbf{R_M = 7.353\ kN}$ **(Ans.)**

$\therefore \quad R_L = 25 - 7.353 = \mathbf{17.647\ kN}$ **(Ans.)**

(ii) Elongation of left portion :

Elongation of left portion

$$= \frac{R_L \times 0.25}{A \times E} = \frac{17.647 \times 10^3 \times 0.25}{250 \times 10^{-6} \times 200 \times 10^9} = 0.0000882 \text{ m}$$

$$= \mathbf{0.0882\ mm} \quad \textbf{(Ans.)}$$

Example 1.11. *A straight uniform bar AD is clamped at both ends and loaded as shown in Fig. 1.16. Initially the bar is stress free. Determine the stresses in all the three parts (AB, BC, CD) of the bar if the cross-sectional area of bar is 1000 mm².*

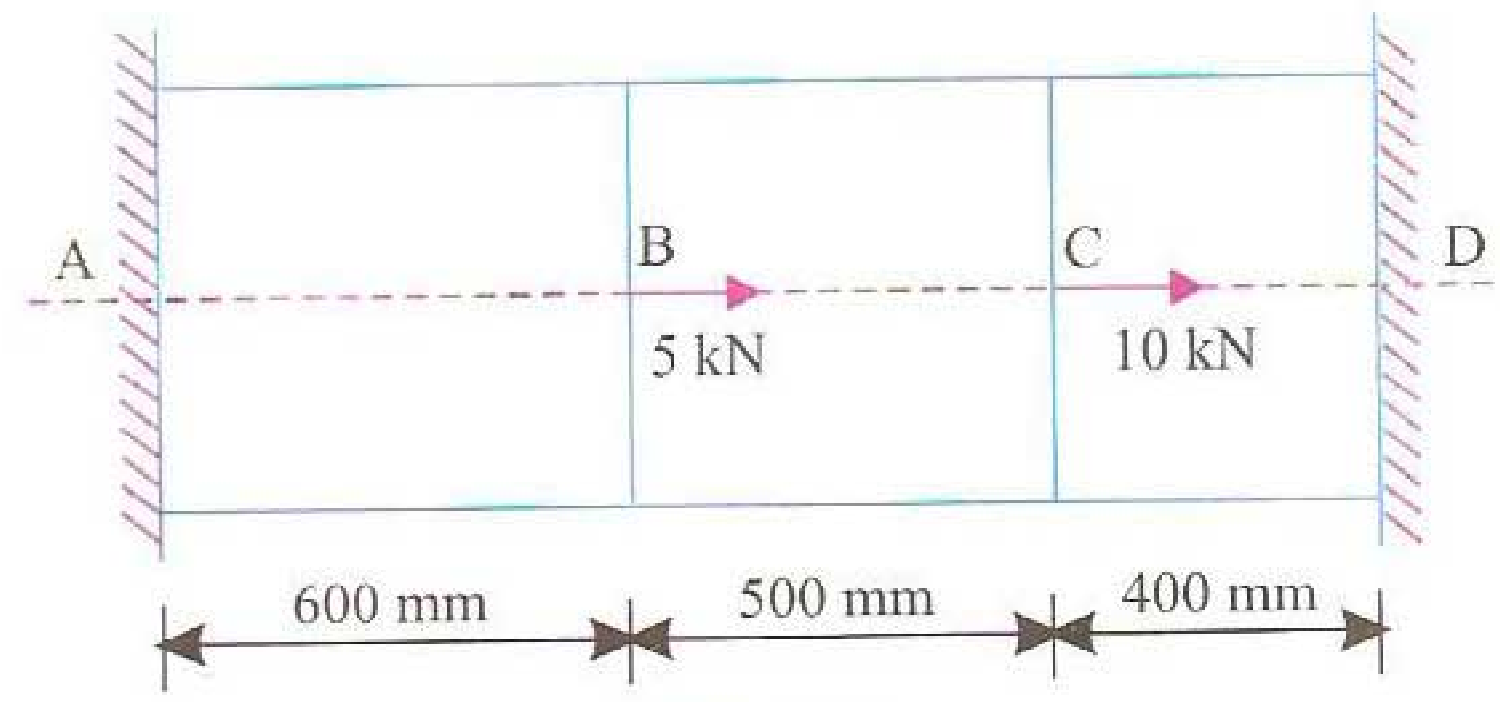


Fig. 1.16

Solution. Refer to Figs. 1.16 and 1.17

Let R_A and R_D be the reactions at the supports A and D respectively. For equilibrium of the bar AD, these reactions must act towards the left.

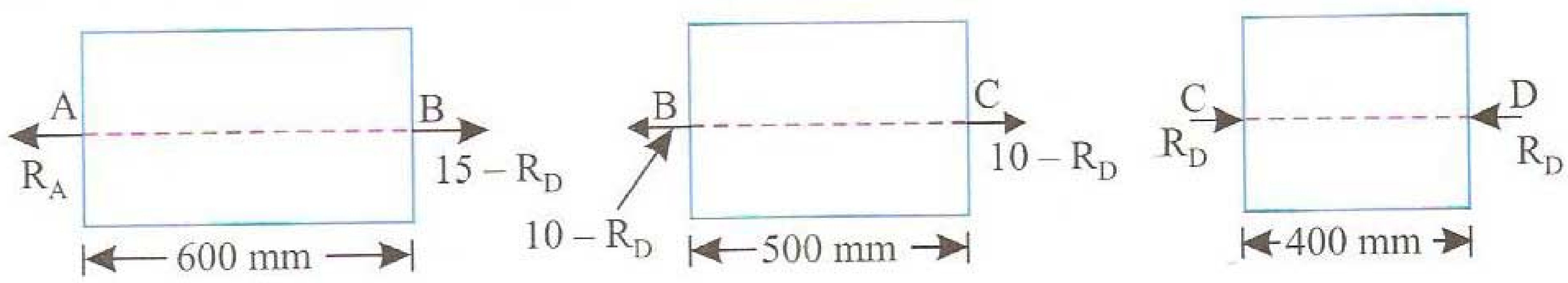


Fig. 1.17

Hence $\qquad R_A + R_D = 5 + 10 = 15$ kN $\qquad$ ...(*i*)

Fig. 1.17 shows the free-body diagram for the parts AB, BC and CD.

Let δl_{AB}, δl_{BC} and δl_{CD} be the extensions in the parts AB, BC and CD respectively.

Then
$$\delta l_{AB} = \frac{(15 - R_D) \times l_{AB}}{A_{AB} \times E} \qquad \text{...... (extension)}$$

$$\delta l_{BC} = \frac{(10 - R_D) \times l_{BC}}{A_{BC} \times E} \qquad \text{...... (extension)}$$

$$\delta l_{CD} = \frac{R_D \times l_{CD}}{A_{CD} \times E} \qquad \text{...... (compression)}$$

Since the *supports are rigid*, therefore elongation of AB and BC shall be *equal to* the compressions of CD.

Hence, $\qquad \delta l_{AB} + \delta l_{BC} = \delta l_{CD}$

Substituting the values, we have

$$\frac{(15 - R_D) \times 600}{1000 \times E} + \frac{(10 - R_D) \times 500}{1000 \times E} = \frac{R_D \times 400}{1000 \times E}$$

$$(\because A_{AB} = A_{BC} = A_{CD} = 1000 \text{ mm}^2 \text{ .... given})$$

or, $\quad 6\,(15 - R_D) + 5\,(10 - R_D) = 4\,R_D$

or, $\quad 90 - 6\,R_D + 50 - 5\,R_D = 4\,R_D$

or, $\quad 15\,R_D = 140$

$$\therefore \qquad R_D = \frac{140}{15} = 9.33 \text{ kN}$$

From eqn. (*ii*), we have

$$R_A + 9.33 = 15$$

or, $\quad R_A = 15 - 9.33 = 5.67$ kN

Stress in part AB, $\quad \sigma_{AB} = \dfrac{\text{Load in part } AB}{\text{Cross-sectional area of part } AB}$

$$= \frac{(15 - R_D) \times 1000}{1000} \text{ N/mm}^2 = \frac{15 - 9.33}{1000} \times 1000$$

$$\mathbf{= 5.67\ N/mm^2\ (tensile)}\quad \textbf{(Ans.)}$$

Stress in part BC, $$\sigma_{BC} = \frac{(10 - R_D) \times 1000}{1000} \text{ N/mm}^2 = \frac{10 - 9.33}{1000} \times 1000$$

$$\mathbf{= 0.67\ N/m^2\ (tensile)}\quad \textbf{(Ans.)}$$

Stress in part CD, $$\sigma_{CD} = \frac{R_D \times 1000}{1000} \text{ N/mm}^2 = \mathbf{9.33\ N/mm^2}\quad \textbf{(Ans.)}$$

Example 1.12. *For the bar shown in Fig. 1.18, calculate the reaction produced by the lower support on the bar. Take $E = 200\ GN/m^2$. Find also the stresses in the bars.*

Solution. Refer to Fig. 1.18.

Let, R_1 = Reaction at the upper support ;

R_2 = Reaction at the lower support when the bar touches it.

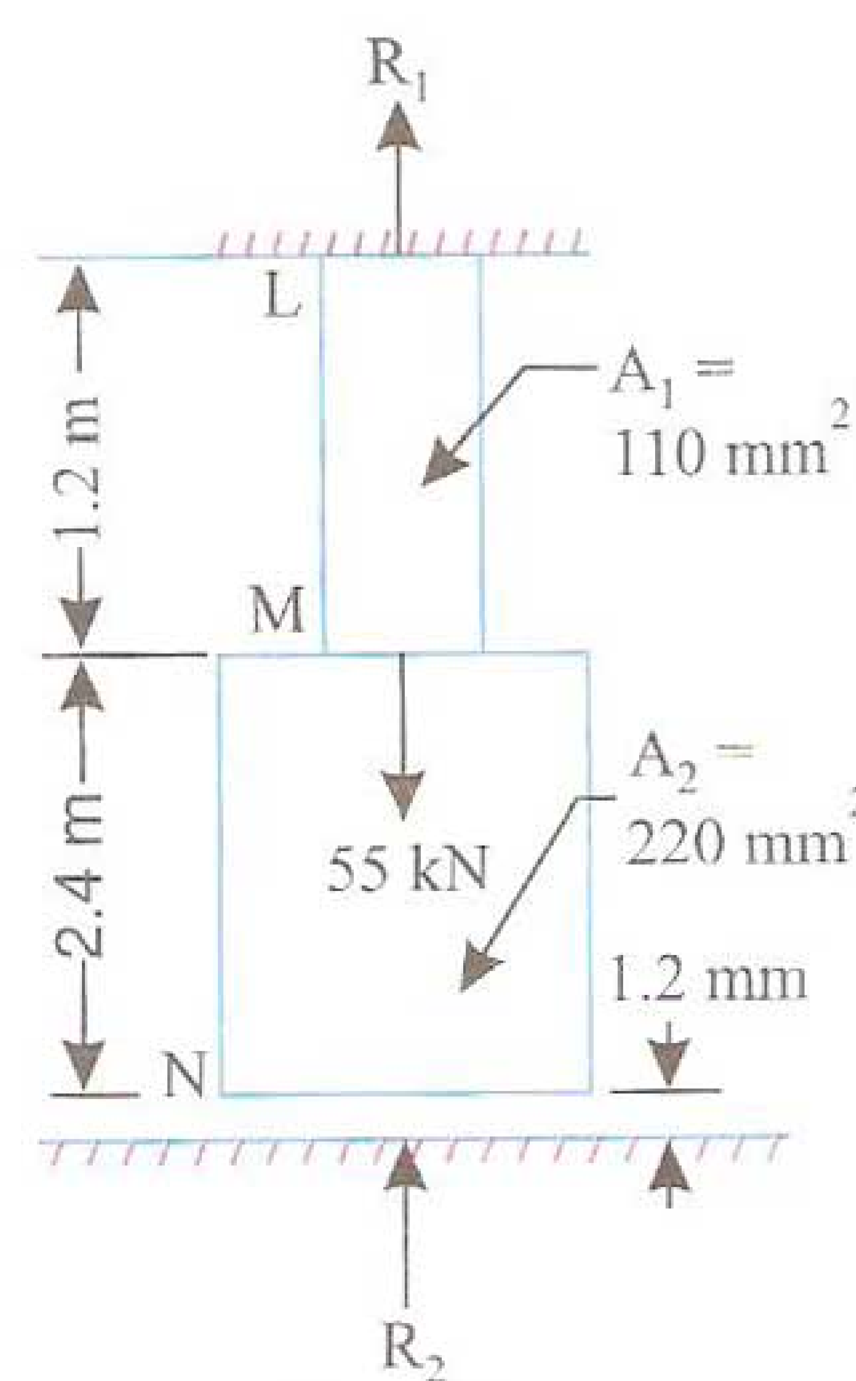


Fig. 1.18

If the bar MN finally rests on the lower support, we have

$$R_1 + R_2 = 55 \text{ kN} = 55000 \text{ N}$$

For bar LM, the total force

$$= R_1 = 55000 - R_2 \text{ (tensile)}$$

For bar MN, the total force = R_2 (compressive)

$\therefore$ δl_1 = Extension of LM

$$= \frac{(55000 - R_2) \times 1.2}{(110 \times 10^{-6}) \times 200 \times 10^9} \text{ m}$$

and, δl_2 = Contraction of MN

$$= \frac{R_2 \times 2.4}{220 \times 10^{-6} \times 200 \times 10^9}$$

In order that N rests on the lower support, we have from compatibility equation,

$$\delta l_1 - \delta l_2 = 1.2/1000 = 0.0012 \text{ m}$$

or, $$\frac{(55000 - R_2) \times 1.2}{(110 \times 10^{-6}) \times 200 \times 10^9} - \frac{R_2 \times 2.4}{220 \times 10^{-6} \times 200 \times 10^9} = 0.0012$$

or, $$2\,(55000 - R_2) \times 1.2 - 2.4\,R_2 = (220 \times 10^{-6}) \times 200 \times 10^9 \times 0.0012$$

or, $$132000 - 2.4\,R_2 - 2.4\,R_2 = 52800$$

or, $$4.8\,R_2 = 79200$$

$\therefore$ $R_2 = 16500$ N or **16.5 kN** **(Ans.)**

and, $R_1 = 55 - 16.5 =$ **38.5 kN** **(Ans.)**

Stress in *LM*

$$= \frac{R_1}{A_1} = \frac{38.5}{110 \times 10^{-6}} = 0.350 \times 10^6 \text{ kN/m}^2$$

$$\mathbf{= 350\ MN/m^2}\quad \textbf{(Ans.)}$$

Tractors are especially built to run on rough and bumpy terrains. The engine provides high torque at low speeds.

Stress in MN

$$= \frac{R_2}{A_2} = \frac{16.5}{220 \times 10^{-6}} = 0.075 \times 10^6 \text{ kN/m}^2$$

$$= \mathbf{75\ MN/m^2} \quad \textbf{(Ans.)}$$

Example 1.13. *A 700 mm length of aluminium alloy bar is suspended from the ceiling so as to provide a clearance of 0.3 mm between it and a 250 mm length of steel bar as shown in Fig. 1.19.* $A_{al} = 1250\ mm^2$, $E_{al} = 70\ GN/m^2$, $A_s = 2500\ mm^2$, $E_s = 210\ GN/m^2$. *Determine the stress in the aluminium and in the steel due to a 300 kN load applied 500 mm from the ceiling.*

Solution. Refer to Fig. 1.19. On application of load of 300 kN at Q, the portion LQ will move forward and come in contact with N so that QM and NP will both be under compression. LQ will elongate, while QM and NP will contract and the net elongation will be equal to gap of 0.3 mm between M and N.

Let, σ_1 = Tensile stress in LQ,

σ_2 = Compressive stress in QM, and

σ_3 = Compressive stress in NP.

Elongation of $LQ = \dfrac{\sigma_1 \times 0.5}{70 \times 10^9}$ m (+)

Contraction of $QM = \dfrac{\sigma_2 \times 0.2}{70 \times 10^9}$ m (–)

Contraction of $NP = \dfrac{\sigma_3 \times 0.25}{210 \times 10^9}$ m (–)

But, force in QM = force in NP

$$\therefore \quad \sigma_2 \times 1250 \times 10^{-6} = \sigma_3 \times 2500 \times 10^{-6}$$

$$\therefore \quad \sigma_3 = \frac{\sigma_2}{2}$$

$$\therefore \text{ Contraction of } NP = \frac{\sigma_2 \times 0.25}{2 \times 210 \times 10^9} \text{ m}$$

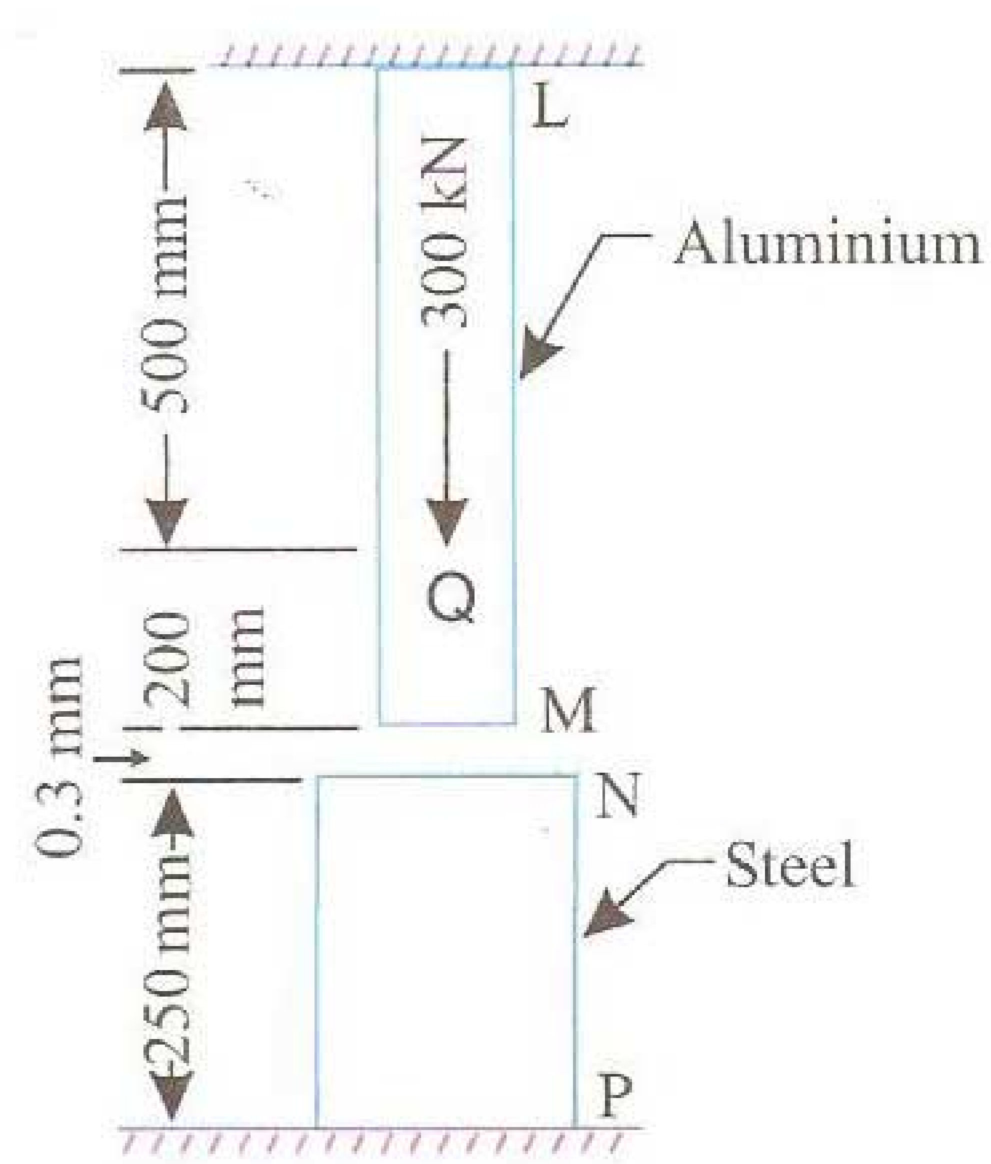


Fig. 1.19

$\therefore$ Extension in $dy = \dfrac{9.81\,\rho y}{3E}.dy$

and total extension in the bar,

$$\delta l = \int_0^l \frac{9.81\,\rho y}{3E}.dy = \frac{9.81\,\rho}{3E}\int_0^l y\,dy$$

$$= \frac{9.81\,\rho}{3E}\left|\frac{y^2}{2}\right|_0^l = \frac{9.81\,\rho l^2}{6E}$$

$$\therefore \quad \delta l = \frac{9.81\,\rho l^2}{6E} \qquad \text{...(1.16)}$$

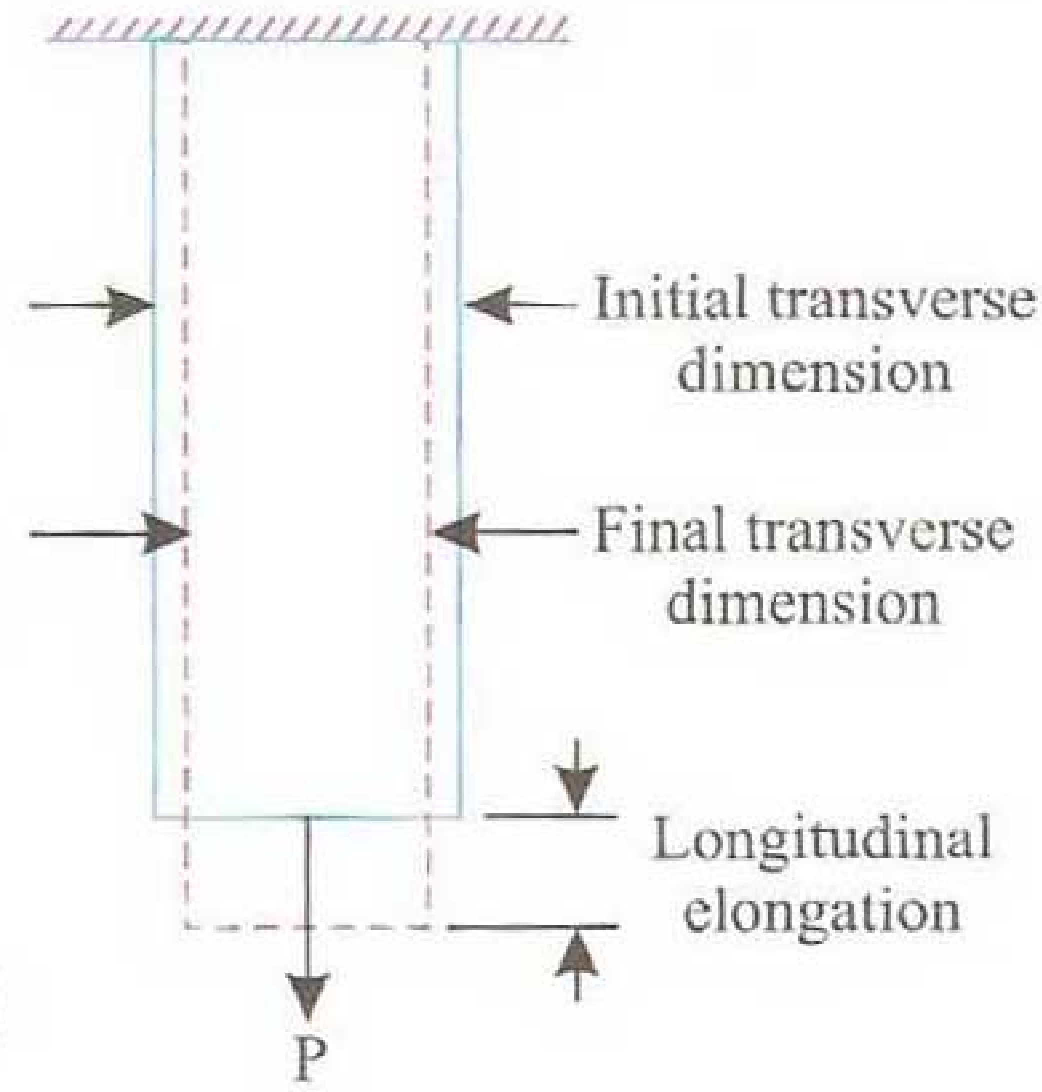


Fig. 1.28

1.10. POISSON'S RATIO

If a body is subjected to a load, its length changes; ratio of this change in length to the original length is known as *linear or primary strain*. Due to this load, the dimensions of the body change; in all directions at right angles to its line of application the strains thus produced are called *lateral or secondary or transverse strains* and are of *nature opposite to that of primary strains*. For example, if the load is tensile, there will be an increase in length and a corresponding decrease in cross-sectional area of the body (Fig. 1.28). In this case, linear or primary strain will be tensile and secondary or lateral or transverse strain compressive.

The ratio of lateral strain to linear strain is known as **Poisson's ratio**.

i.e., Poisson's ratio,

$$\mu = \frac{\text{Lateral strain or transverse strain}}{\text{Linear or primary strain}} = \frac{1}{m}$$

where m is a constant and its value varies between 3 and 4 for different materials.

Table 1.1 gives the average values of Poisson's ratio for common materials.

TABLE 1.1. Poisson's Ratio for Some of the Common Materials

S. No.	*Material*	*Poisson's ratio*
1.	Aluminium	0.330
2.	Brass	0.340
3.	Bronze	0.350
4.	Cast iron	0.270
5.	Concrete	0.200
6.	Copper	0.355
7.	Steel	0.288
8.	Stainless steel	0.305
9.	Wrought iron	0.278

1.11. RELATIONS BETWEEN THE ELASTIC MODULII

Relations exist between the elastic constants for any specific material and these relations hold good for all materials within the elastic range. The relations result from the fact that the application of any particular type of stress necessarily produces other types of stress at other places in the material. Further, each of the stresses produces its corresponding strain and all the strains produced must be consistent.

$$e_v = \frac{\delta V}{V}$$

But, $e_v = e_x + e_y + e_z$

$= 4.85 \times 10^{-5} + 1.389 \times 10^{-4} + 2.292 \times 10^{-4}$

$= 10^{-4}(0.485 + 1.389 + 2.292) = 4.166 \times 10^{-4}$

$\therefore \quad \delta V = e_v \times V = 4.166 \times 10^{-4} \times (60 \times 60 \times 180)\ \text{mm}^3$

$= \mathbf{269.9\ mm^3}$ **(Ans.)**

1.12. STRESSES INDUCED IN COMPOUND TIES OR STRUTS

Frequently ties consist of two materials, fastened together to prevent uneven straining of the two materials. In these cases, it is interesting to calculate the distribution of the load between the materials: It will be assumed that the two materials also are symmetrically distributed about the axis of the bar, as with a cylindrical rod encased in a tube (Fig. 1.32).

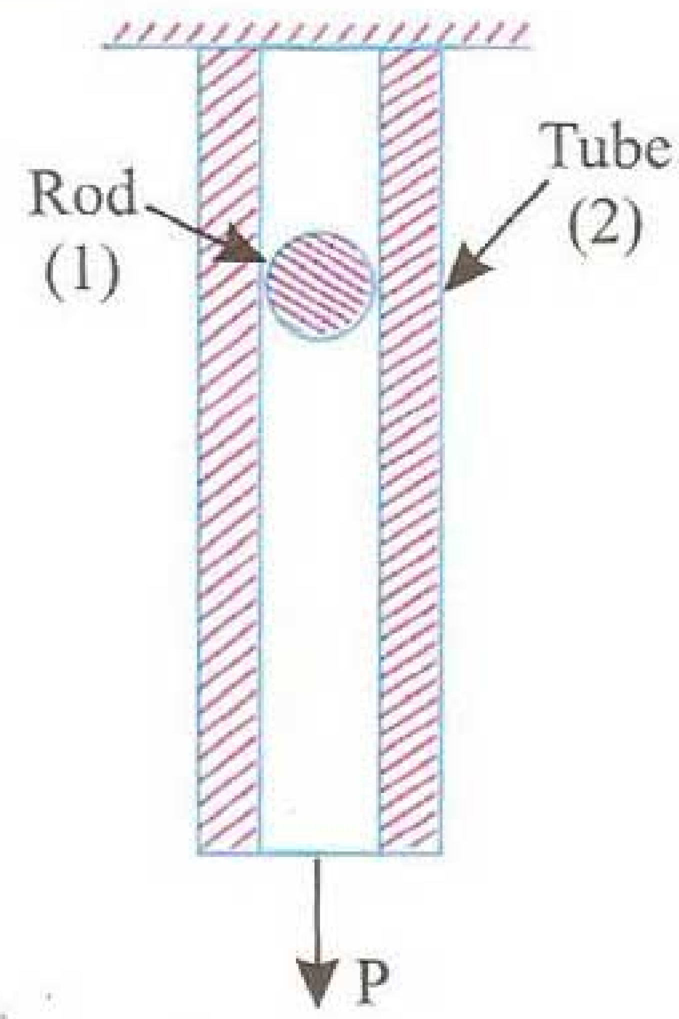


Fig. 1.32

If an axial load P is applied to the bar,

$$P = \sigma_1 A_1 + \sigma_2 A_2 \quad \text{...(1.20)}$$

where, σ_1 and σ_2 are the stresses induced and A_1 and A_2 are the cross-sectional areas of the materials.

The strains produced, e_1 and e_2 are equal.

$\therefore \quad e_1 = e_2$

$$\therefore \quad \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \Rightarrow \frac{\sigma_1}{\sigma_2} = \frac{E_1}{E_2} \quad \text{...[1.20 }(a)]$$

Example 1.25. *A concrete column of cross-sectional area 400 mm × 400 mm is reinforced by four longitudinal 50 mm diameter round steel bars placed at each corner. If the column carries a compressive load of 300 kN, determine :*

(*i*) *Loads carried;*

(*ii*) *The compressive stress produced in the concrete and steel bars.*

Young's modulus of elasticity of steel is 15 times that of concrete.

Solution. Refer to Fig. 1.33.

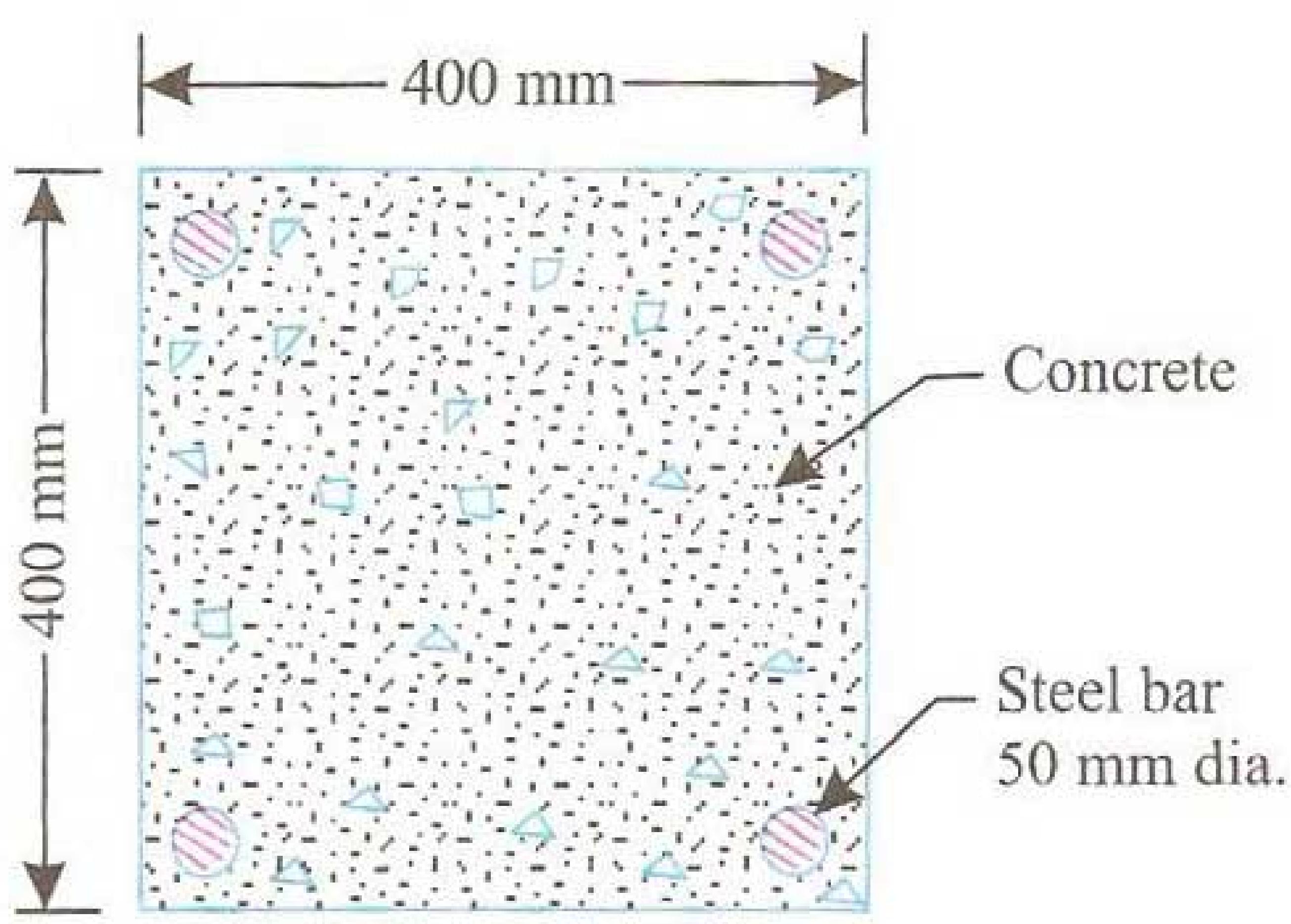


Fig. 1.33

Cross-sectional area of the column

$= 0.4 \times 0.4 = 0.16\ \text{m}^2$

Area of steel bars,

$$A_s = 4 \times \frac{\pi}{4} \times (0.05)^2 = 0.00785\ \text{m}^2$$

$\therefore$ Area of concrete,

$$A_c = 0.16 - 0.00785 = 0.1521\ \text{m}^2$$

Since the steel bars and concrete shorten by the *same amount* under the compressive load,

$\therefore$ Strain in steel bars = Strain in concrete

or, $\quad e_s = e_c$

$$\frac{\sigma_s}{E_s} = \frac{\sigma_c}{E_c} \quad \text{or,} \quad \sigma_s = \sigma_c \cdot \frac{E_s}{E_c} = 15\,\sigma_c \quad (\because E_s = 15\,E_c)$$

Also, load shared by steel bars + load shared by concrete = 300000 N

or, $P_s + P_c = 300000$ N

or, $\sigma_s \times A_s + \sigma_c \times A_c = 300000$

$15\,\sigma_c \times 0.00785 + \sigma_c \times 0.1521 = 300000$

$\sigma_c\,(15 \times 0.00785 + 0.1521) = 300000$

or, compressive stress in concrete,

$$\sigma_c = 1.11 \times 10^6 \text{ N/m}^2 = \mathbf{1.11\ MN/m^2}\ \textbf{(Ans.)}$$

and, compressive stress in steel bars,

$$\sigma_s = 15\,\sigma_c = 15 \times 1.11 \times 10^6$$
$$= 16.65 \times 10^6 \text{ N/m}^2 = \mathbf{16.65\ MN/m^2}\ \textbf{(Ans.)}$$

Load carried by steel bars, $P_s = \sigma_s \times A_s = 16.65 \times 10^6 \times 0.00785$
$= 130.7$ kN $\simeq$ **131 kN (Ans.)**

Load carried by concrete, $P_c = 1.11 \times 10^6 \times 0.1521$
$= 168.9 \simeq$ **169 kN (Ans.)**

Example 1.26. *A copper rod of 40 mm diameter is surrounded tightly by a cast-iron tube of 80 mm external diameter, the ends being firmly fastened together. When put to a compressive load of 30 kN, what load will be shared by each ? Also determine the amount by which the compound bar shortens if it is 2 m long.*

Take: $E_{c.i} = 175$ GN/m^2,

and $E_c = 75$ GN/m^2.

Solution. Refer to Fig. 1.34.

Diameter of the copper rod = 0.04 m

∴ Area of copper rod,

$$A_c = \frac{\pi}{4} \times 0.04^2$$
$$= 0.0004\ \pi\text{m}^2$$

External diameter of cast iron tube = 0.08 m

∴ Area of cast iron tube,

$$A_{ci} = \frac{\pi}{4}\,(0.08^2 - 0.04^2)$$
$$= 0.0012\ \pi \text{ m}^2$$

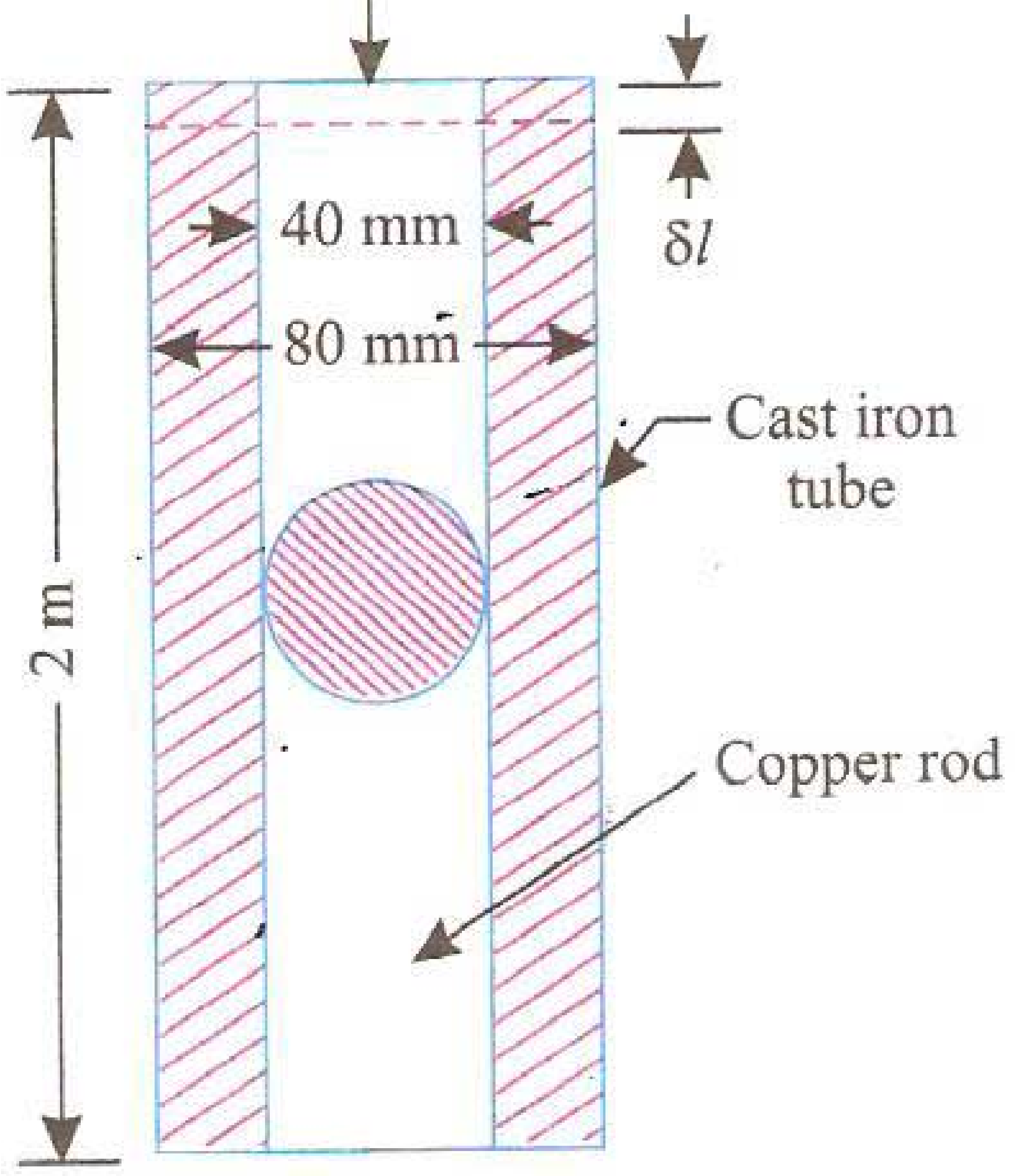


Fig. 1.34

We know that,

Strain in cast-iron tube = Strain in the copper rod

$$\frac{\sigma_{c.i}}{E_{c.i}} = \frac{\sigma_c}{E_c}$$

or, $$\frac{\sigma_{c.i}}{\sigma_c} = \frac{E_{c.i}}{E_c} = \frac{175 \times 10^9}{75 \times 10^9} = 2.33$$

∴ $$\sigma_{c.i.} = 2.33\,\sigma_c \quad \text{...}(i)$$

Also, total load = Load shared by cast-iron tube + load shared by copper rod.

or, $$P = P_{c.i.} + P_c$$
$$30 = \sigma_{c.i.} \cdot A_{c.i} + \sigma_c \cdot A_c$$

or, $$30 = \sigma_{c.i.} \times 0.0012\pi + \sigma_c \times 0.0004\,\pi \quad \text{...}(ii)$$

Substituting the value of $\sigma_{c.i.}$ from (*i*) in (*ii*), we get

$$30 = 2.33\,\sigma_c \times 0.0012\,\pi + \sigma_c \times 0.0004\,\pi$$

$$= \sigma_c (0.008785 + 0.001257)$$
$$= 0.010042\, \sigma_c$$
$$\sigma_c = \frac{30}{0.010042} = 2987.5 \text{ kN/m}^2$$

And, from equation (*i*),

$$\sigma_{c.i.} = 2.33\, \sigma_c = 2.33 \times 2987.5$$
$$= 6960.8 \text{ kN/m}^2$$

Load shared by the copper rod,

$$P_c = \sigma_c A_c = 2987.5 \times 0.0004\, \pi$$
$$= \mathbf{3.75\ kN} \quad \textbf{(Ans.)}$$

Load shared by cast iron tube,

$$P_{ci} = \mathbf{30 - 3.75 = 26.25\ kN} \quad \textbf{(Ans.)}$$

Strain $= \dfrac{\sigma_c}{E_c}$

or, $\dfrac{\sigma_c}{E_c} = \dfrac{\text{Decrease in length}}{\text{Original length}} = \dfrac{\delta l}{l}$

$$\therefore \quad \delta l = \frac{\sigma_c}{E_c} \times l = \frac{2987.5 \times 10^3}{75 \times 10^9} \times 2$$
$$= 0.0000796 \text{ m} \quad \text{or} \quad 0.0796 \text{ mm}$$

Hence, decrease in length = **0.0796 mm (Ans.)**

Bicycle brakes are the forerunners of the modern automobile brakes.

Example 1.27. *A solid steel cylinder 500 mm long and 70 mm diameter is placed inside an aluminium cylinder having 75 mm inside diameter and 100 mm outside diameter. The aluminium cylinder is 0.16 mm longer than the steel cylinder. An axial load of 500 kN is applied to the bar and cylinder through rigid cover plates as shown in Fig. 1.35. Find the stress developed in the steel cylinder and aluminium tube. Assume for steel, E = 220 GN/m² and for aluminium, E = 70 GN/m².*

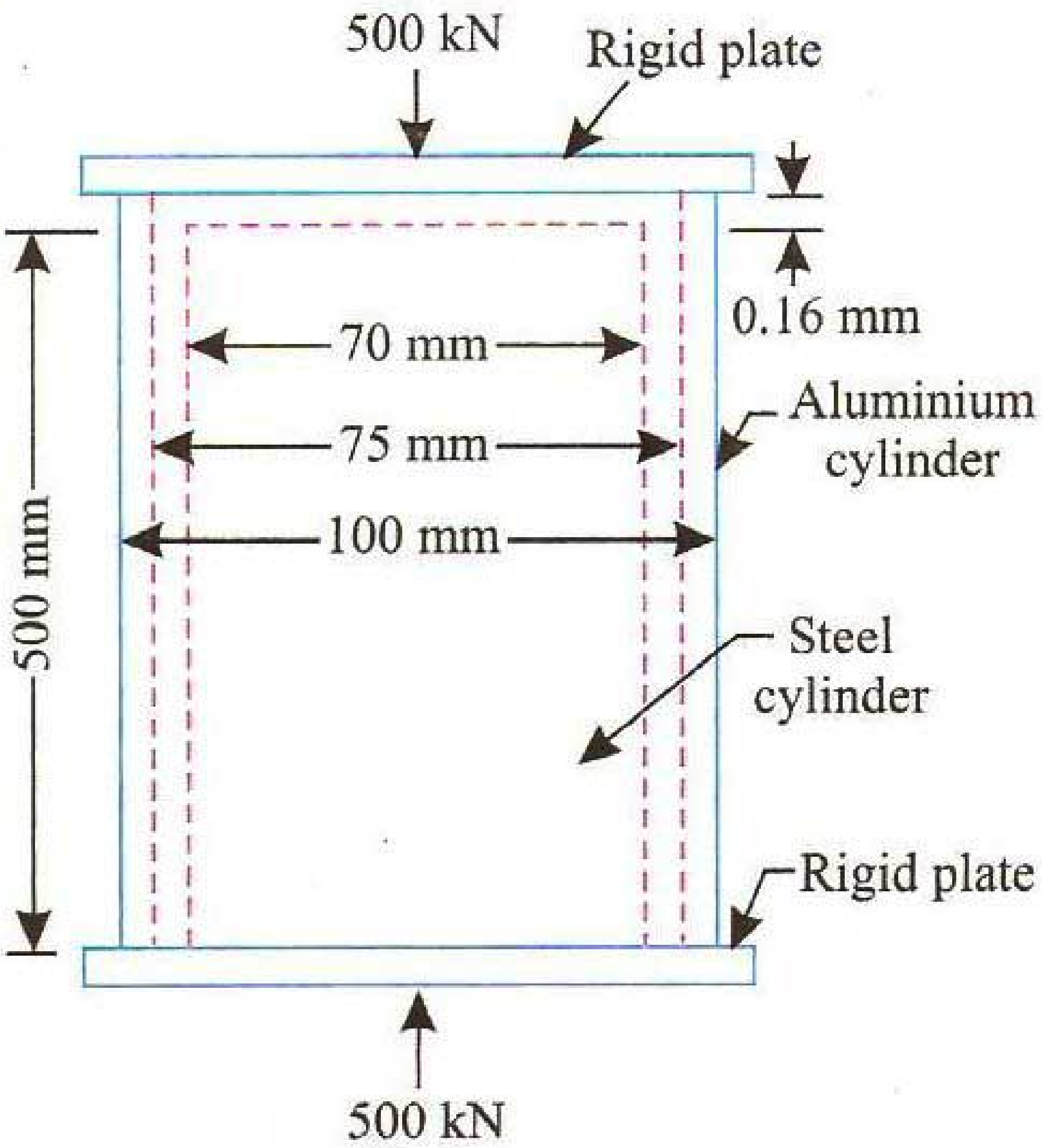


Fig. 1.35

Solution. Refer to Fig. 1.35.

Since the aluminium cylinder is 0.16 mm longer than the steel cylinder, the load required to compress this cylinder by 0.16 mm will be found as follows :

$$E = \frac{\text{Stress}}{\text{Strain}} = \frac{P/A}{\delta l/l} = \frac{Pl}{A.\delta l}$$

or, $$P = \frac{E.A.\delta l}{l}$$

$$= \frac{70 \times 10^9 \times \frac{\pi}{4}(0.1^2 - 0.075^2) \times 0.00016}{0.50016} = 76944 \text{ N}$$

When the aluminium cylinder is compressed by its extra length 0.16 mm, the load then shared by both aluminium as well as steel cylinder will be,

$$500000 - 76944 = 423056 \text{ N}$$

Let, e_s = Strain in steel cylinder,

e_a = Strain in aluminium cylinder,

σ_s = Stress produced in steel cylinder, and

σ_a = Stress produced in aluminium cylinder.

E_s = 220 GN/m^2

E_a = 70 GN/m^2

As both the cylinders are of the same length and are compressed by the same amount,

$$\therefore \quad e_s = e_a$$

or,
$$\frac{\sigma_s}{E_s} = \frac{\sigma_a}{E_a}$$

or,
$$\sigma_s = \frac{E_s}{E_a}.\sigma_a$$

$$= \frac{220 \times 10^9}{70 \times 10^9} \times \sigma_a = \frac{22}{7}\sigma_a$$

Also, $P_s + P_a = P$

or, $\sigma_s . A_s + \sigma_a . A_a = 423056$

$$\frac{22}{7}\sigma_a \times A_s + \sigma_a \times A_a = 423056$$

$$\therefore \quad \sigma_a = \frac{423056}{\frac{22}{7}A_s + A_a} = \frac{423056}{\frac{22}{7} \times \frac{\pi}{4} \times 0.07^2 + \frac{\pi}{4}(0.1^2 - 0.075^2)}$$

$$= \frac{423056}{0.012095 + 0.003436} = 27.24 \times 10^6 \text{ N/m}^2 = 27.24 \text{ MN/m}^2$$

and,
$$\sigma_s = \frac{22}{7} \times 27.24 = 85.61 \text{ MN/m}^2$$

Stress in the aluminium cylinder due to load 76944 N

$$= \frac{76944}{\frac{\pi}{4}(0.1^2 - 0.075^2)} = 22.39 \times 10^6 \text{ N/m}^2 = 22.39 \text{ MN/m}^2$$

$\therefore$ *Total stress in* **aluminium cylinder**

= 27.24 + 22.39

= 49.63 MN/m^2 (Ans.)

and, *stress in* **steel cylinder = 85.61 MN/m^2 (Ans.)**

Example 1.28. *Figure 1.36 shows a round steel rod supported in a recess and surrounded by co-axial brass tube. The upper end of the rod is 0.1mm below that of the tube and an axial load is applied to a rigid plate resting on the top of the tube.*

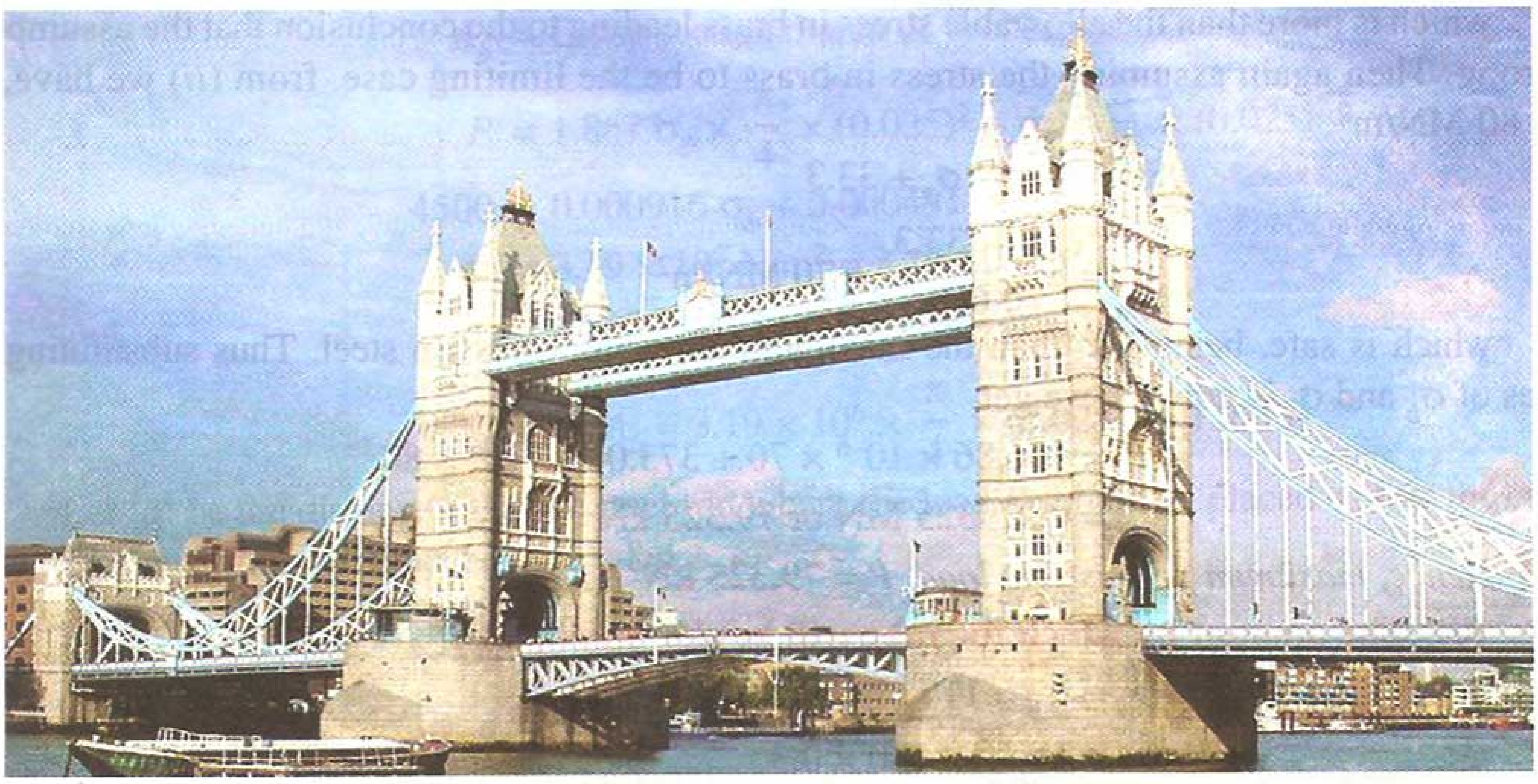
(*i*) *Determine the magnitude of the maximum permissible load if the compressive stress in the rod is not to exceed 110 MN/m^2 and that in the tube is not to exceed 80 MN/m^2.*

(*ii*) *Find the amount by which the tube will be shortened by the load if the compressive stress in the tube is the same as that in the rod.*

Take: $E_s = 200$ *GN/m^2;*

$E_b = 100$ *GN/m^2.*

(Panjab University)



Tower bridge, London. In bridges different members bear different magnitudes of tension, compression and shear stresses.

Solution. Refer to Fig. 1.36.

(*i*) Maximum permissible load, *P* :

Area of steel rod,

$$A_s = \frac{\pi}{4} \times \left(\frac{30}{1000}\right)^2 = 706.86 \times 10^{-6}\ \text{m}^2$$

Area of brass tube,

$$A_b = \frac{\pi}{4} \times \left[\left(\frac{50}{1000}\right)^2 - \left(\frac{45}{1000}\right)^2\right] = 373.06 \times 10^{-6}\ \text{m}^2$$

Let σ_s and σ_b MN/m² be the stresses induced in the steel rod and brass tube respectively.

Since the sum of the loads carried by the steel and brass is equal to the total load,

∴ Total load, $P = \sigma_s A_s + \sigma_b A_b = 706.86 \times 10^{-6}\,\sigma_s + 373.06 \times 10^{-6}\,\sigma_b$ ...(*i*)

When the top plate makes contact with steel and compresses it, the compression of the brass will exceed that of steel by 0.1 mm.

i.e., $\delta l_b = \delta l_s + 0.0001$ m

or, $$\frac{\sigma_b\, l_b}{E_b} = \frac{\sigma_s\, l_s}{E_s} + 0.0001$$

or, $$\frac{\sigma_b \times 0.3}{100 \times 10^3} = \frac{\sigma_s \times 0.4}{200 \times 10^3} + 0.0001$$

or, $\sigma_b = 0.667\,\sigma_s + 33.3$ ...(*ii*)

The maximum stress of 110 MN/m² in the rod and 80 MN/m² in the tube will not occur simultaneously, rather the magnitudes of induced stresses in the two materials will be related to each other by eqn. (*ii*). Thus if it is assumed that the stress in steel is the limiting case, then from eqn. (*ii*) for σ_s = 110 MN/m²,

$$\sigma_b = 0.667 \times 110 + 33.3$$
$$= 106.7\ \text{MN/m}^2$$

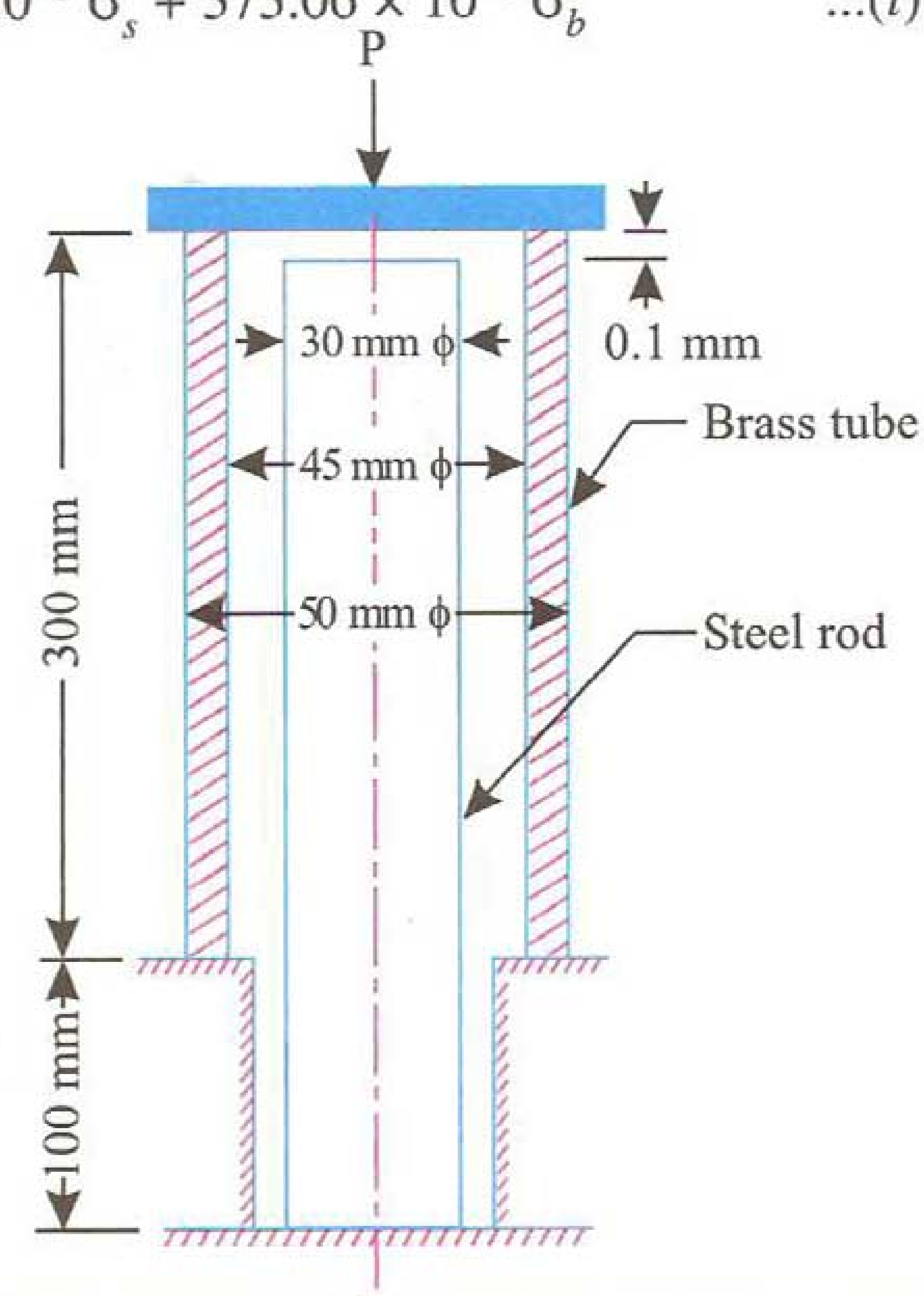


Fig. 1.36

which is more than the allowable stress in brass leading to the conclusion that the assumption is wrong. Then again assuming the stress in brass to be the limiting case, from (*ii*) we have, for $\sigma_b = 80$ MN/m^2,

$$80 = 0.667\,\sigma_s + 33.3$$

$$\therefore \quad \sigma_s = \frac{80 - 33.3}{0.667} = 70 \text{ MN/m}^2$$

which is safe, being less than the maximum allowable stress in steel. Thus substituting the values of σ_b and σ_s in eqn. (*i*), we get

$$P = 706.86 \times 10^{-6} \times 70 + 373.06 \times 10^{-6} \times 80$$
$$= 0.079325 \text{ MN or } 79.325 \text{ kN}$$

Hence, maximum permissible load, $P =$ **79.325 kN (Ans.)**

(*ii*) The amount by which the tube will be shortened, δl_b :

In this case $\quad \sigma_s = \sigma_b = 110$ MN/m^2 (*Given*)

$$\delta l_b = \frac{\sigma_b l_b}{E_b} = \frac{110 \times 0.3}{100 \times 10^3} = 0.00033 \text{ m or } 0.33 \text{ mm}$$

Hence, $\quad \delta l_b =$ **0.33 mm (Ans.)**

Example 1.29. *Two vertical rods, one of steel and the other of bronze, are rigidly fastened at upper ends at a horizontal distance of 760 mm apart. Each rod is 3 m long and 25 mm in diameter. A horizontal cross-piece connects the lower ends of the bars. Where should a load of 4.5 kN be placed on the cross-piece so that it remains horizontal after being loaded?*

Determine the stresses in each rod.

Given: $\quad E_s = 210$ GN/m^2;

$\quad E_b = 112.5$ GN/m^2.

Solution. Refer to Fig. 1.37.

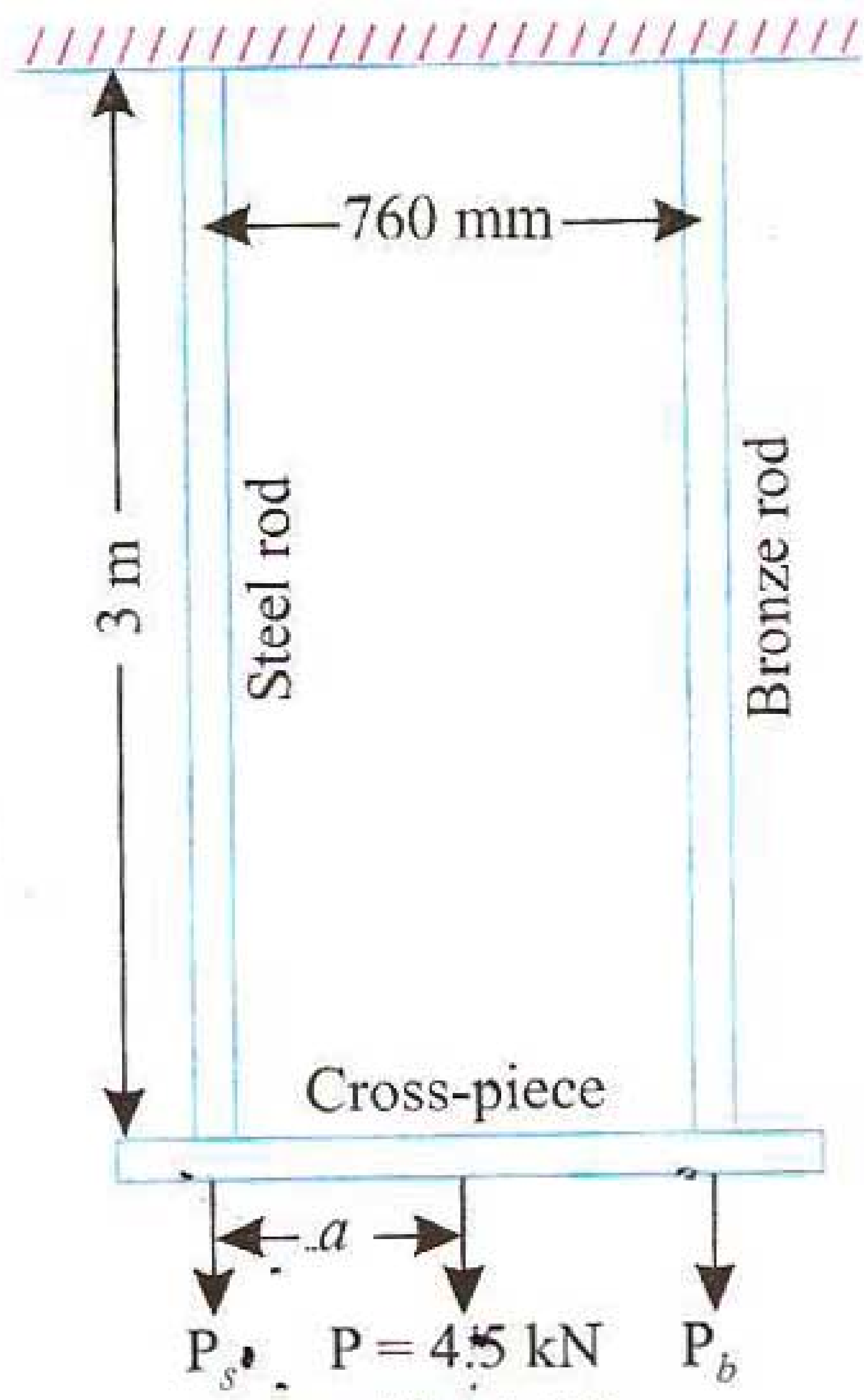


Fig. 1.37

Length of each rod $= 3$ m

Diameter of each rod $= 25$ mm $= 0.025$ m

Load, $\quad P = 4.5$ kN

Let, $\quad \sigma_s$ = Stress in steel rod,

σ_b = Stress in brass rod,

e_s = Strain in steel rod, and

e_b = Strain in bronze rod.

Since the load is placed in such a manner that the cross-piece remains horizontal,

$$\therefore \quad e_s = e_b$$

$$\text{or,} \quad \frac{\sigma_s}{E_s} = \frac{\sigma_b}{E_b}$$

$$\text{i.e.,} \quad \sigma_s = \sigma_b \cdot \frac{E_s}{E_b} = \sigma_b \cdot \frac{210 \times 10^9}{112.5 \times 10^9}$$

$$= 1.867\,\sigma_b \qquad \ldots(i)$$

Let P_s and P_b be the loads shared by the steel and bronze rods respectively.

Then, $\quad P = P_s + P_b$

or, $\quad P = \sigma_s \cdot A_s + \sigma_b \cdot A_b \qquad \ldots (ii)$

where, $\quad A_s$ = Cross-sectional area of steel rod, and

A_b = Cross-sectional area of bronze rod.

$$\therefore \quad P = 1.867\,\sigma_b \times \frac{\pi}{4} \times (0.025)^2 + \sigma_b \times \frac{\pi}{4} \times (0.025)^2$$

$$4500 = 0.000916\,\sigma_b + 0.000491\,\sigma_b$$

$$\sigma_b = 3.19 \times 10^6 \text{ N/m}^2 = 3.19 \text{ MN/m}^2$$

and, $$\sigma_s = 1.867 \times 3.19 = 5.956 \text{ MN/m}^2$$

$$\therefore \quad P_b = \sigma_b A_b = 3.19 \times 10^6 \times \frac{\pi}{4} \times (0.025)^2 = 1566 \text{ N}$$

Let 'a' be the distance from the steel rod where the load P should be placed so that the cross - piece remains horizontal after being loaded.

Then, $$P_b \times 760 = P \times a$$

$$\therefore \quad a = \frac{P_b \times 760}{P} = \frac{1566 \times 760}{4500}$$

$$= \mathbf{265\ mm} \quad \textbf{(Ans.)}$$

Example 1.30. *A beam weighing 450 N is held in a horizontal position by three vertical wires, one attached to each end of the beam, one to the middle of its length. The outer wires are of brass of diameter 1.25 mm and the central wire is of steel of diameter 0.625 mm. If the beam is rigid and wires of the same length and unstressed before the beam is attached, estimate the stresses induced in the wires. Take Young's modulus for brass as 86 GN/m² and for steel 210 GN/m².*

Solution. Refer to Fig. 1.38.

Let, P_b = Load taken by the brass wire, and

P_s = Load taken by the steel wire.

Then, $2P_b + P_s = P$ ...(*i*)

Since the beam is *horizontal*, all wires will extend by the *same amount*.

i.e., $e_b = e_s$

(∵ Length of each wire is same.)

where, e_b = Strain in brass wire, and

e_s = Strain in steel wire.

$$\frac{\sigma_b}{E_b} = \frac{\sigma_s}{E_s}$$

$$\frac{P_b}{A_b \cdot E_b} = \frac{P_s}{A_s \cdot E_s}$$

or, $$P_s = \frac{P_b\,A_s\,E_s}{A_b\,E_b}$$

Bronze flanges.

$$= \frac{P_b \times \frac{\pi}{4} \times (0.625 \times 10^{-3})^2 \times 210 \times 10^9}{\frac{\pi}{4} \times (1.25 \times 10^{-3})^2 \times 86 \times 10^9}$$

or, $P_s = 0.61\,P_b$ ...(*ii*)

Substituting the value of P_s in eqn. (*i*), we get

$$2\,P_b + 0.61\,P_b = P$$

$$2.61\,P_b = 450$$

$P_b = 172.4$ N

and, $P_s = 0.61 \times 172.4 = 105.2$ N

Now, *stress, induced in the brass wire,*

$$\sigma_b = \frac{P_b}{A_b} = \frac{172.4}{\frac{\pi}{4} \times (1.25 \times 10^{-3})^2} = 1.40 \times 10^8 \text{ N/m}^2$$

$$= \mathbf{140\ MN/m^2} \quad \textbf{(Ans.)}$$

and, *stress induced in a steel wire,*

$$\sigma_s = \frac{105.2}{\frac{\pi}{4} \times (0.625 \times 10^{-3})^2} = 3.429 \times 10^8 \text{ N/m}^2$$

$$= \mathbf{342.9\ MN/m^2} \quad \textbf{(Ans.)}$$

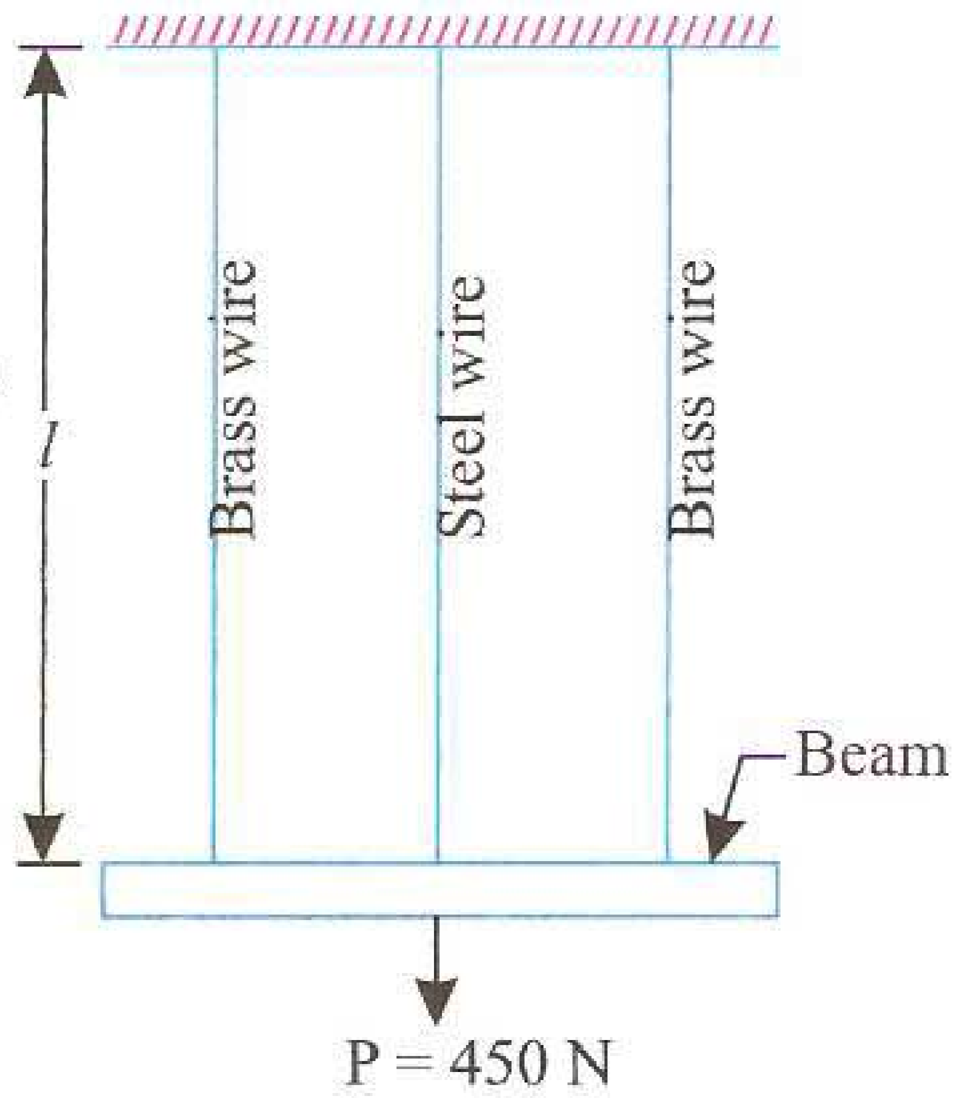


Fig. 1.38

Example 1.31. *A rigid bar is supported by three rods in the same vertical plane and equidistant. The outer rods are of brass and of length 600 mm and diameter 30 mm. The central rod is of steel of 900 mm length and of 37.5 mm diameter. Calculate the forces in the bars due to an applied force P, if the bar remains horizontal after the load has been applied. Take* $\frac{E_s}{E_b} = 2$.

Solution. Refer to Fig. 1.39. Let suffix 1 be used for brass and 2 for steel.

$$A_1 = \frac{\pi}{4} \times (30)^2 = 225\ \pi \text{ mm}^2$$

$$A_2 = \frac{\pi}{4} \times (37.5)^2 = 351.56\ \pi \text{ mm}^2$$

From statical equilibrium, we have

$$2P_1 + P_2 = P \quad \text{...}(i)$$

From compatibility equation, we have

$$\delta l_1 = \delta l_2$$

$$\frac{P_1 l_1}{A_1 E_1} = \frac{P_2 l_2}{A_2 E_2}$$

or, $$P_2 = \frac{P_1 A_2 E_2 l_1}{A_1 E_1 l_2}$$

$$= P_1 \times \frac{351.56\ \pi}{225\ \pi} \times \frac{2}{1} \times \frac{600}{900}$$

$$= 2.08\ P_1 \quad \text{...}(ii)$$

Substituting in (*i*), we get

$$2P_1 + 2.08\ P_1 = P$$

or, $P_1 = \mathbf{0.245\ P}$ **(Ans.)**

and, $P_2 = 2.08 \times 0.245\ P = \mathbf{0.51\ P}$ **(Ans.)**

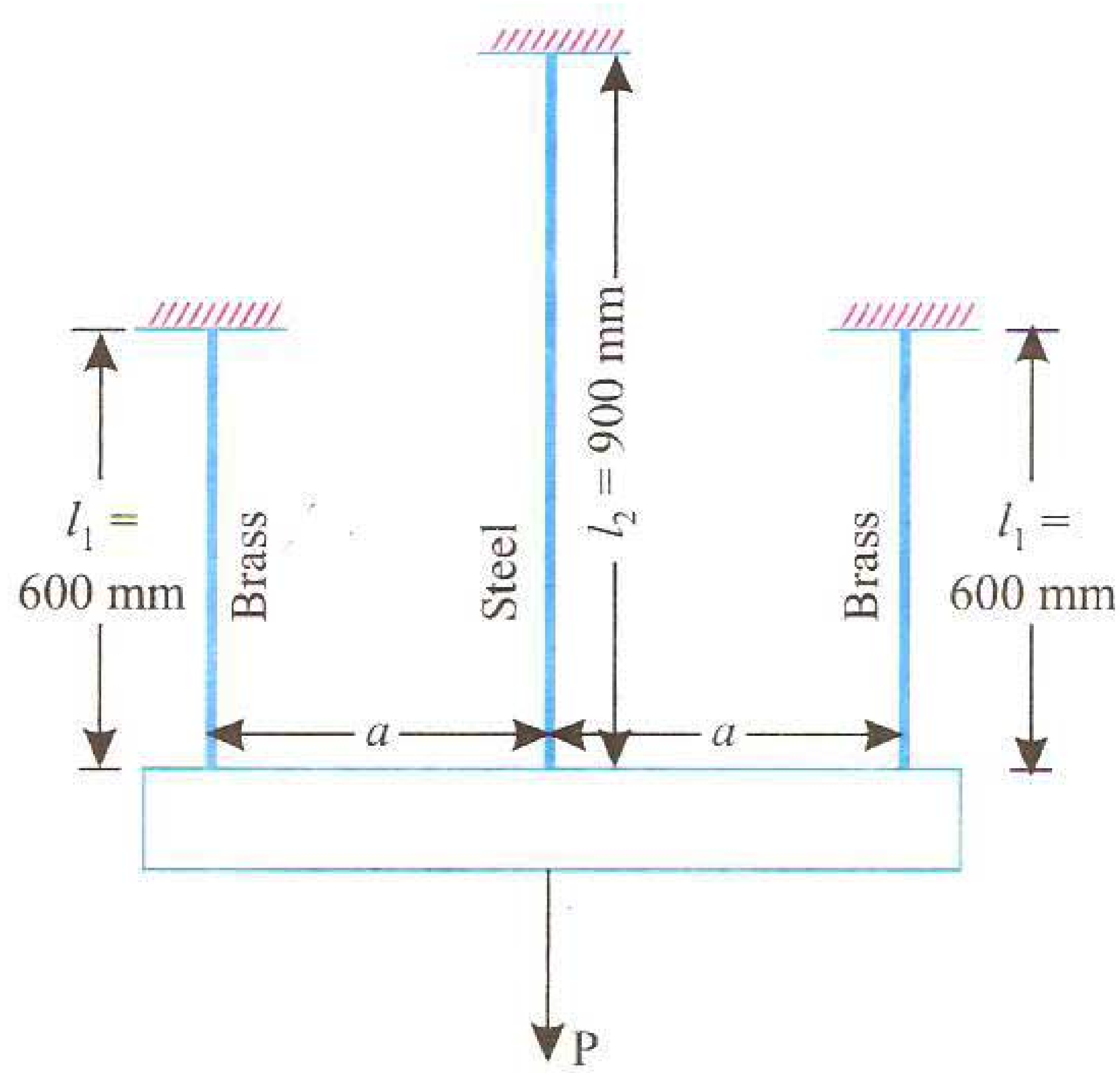


Fig. 1.39

Example 1.32. *Two copper rods and one steel rod together support a load as shown in Fig. 1.40. If the stresses in copper and steel are not to exceed* 60×10^6 *N/m²* *and* 120×10^6 *N/m², respectively, find the safe load that can be supported. Young's modulus for steel is twice that of copper.*

Solution. Refer to Fig. 1.40.

Each rod will be compressed to the same extent. Let each rod contract by δ metre, then $\delta = e_s \times l_s = e_c \times l_c$ (e_s and e_c being strains in steel and copper respectively)

$$\therefore \quad \frac{e_s}{e_c} = \frac{l_c}{l_s} = \frac{0.15}{0.20} = 0.75$$

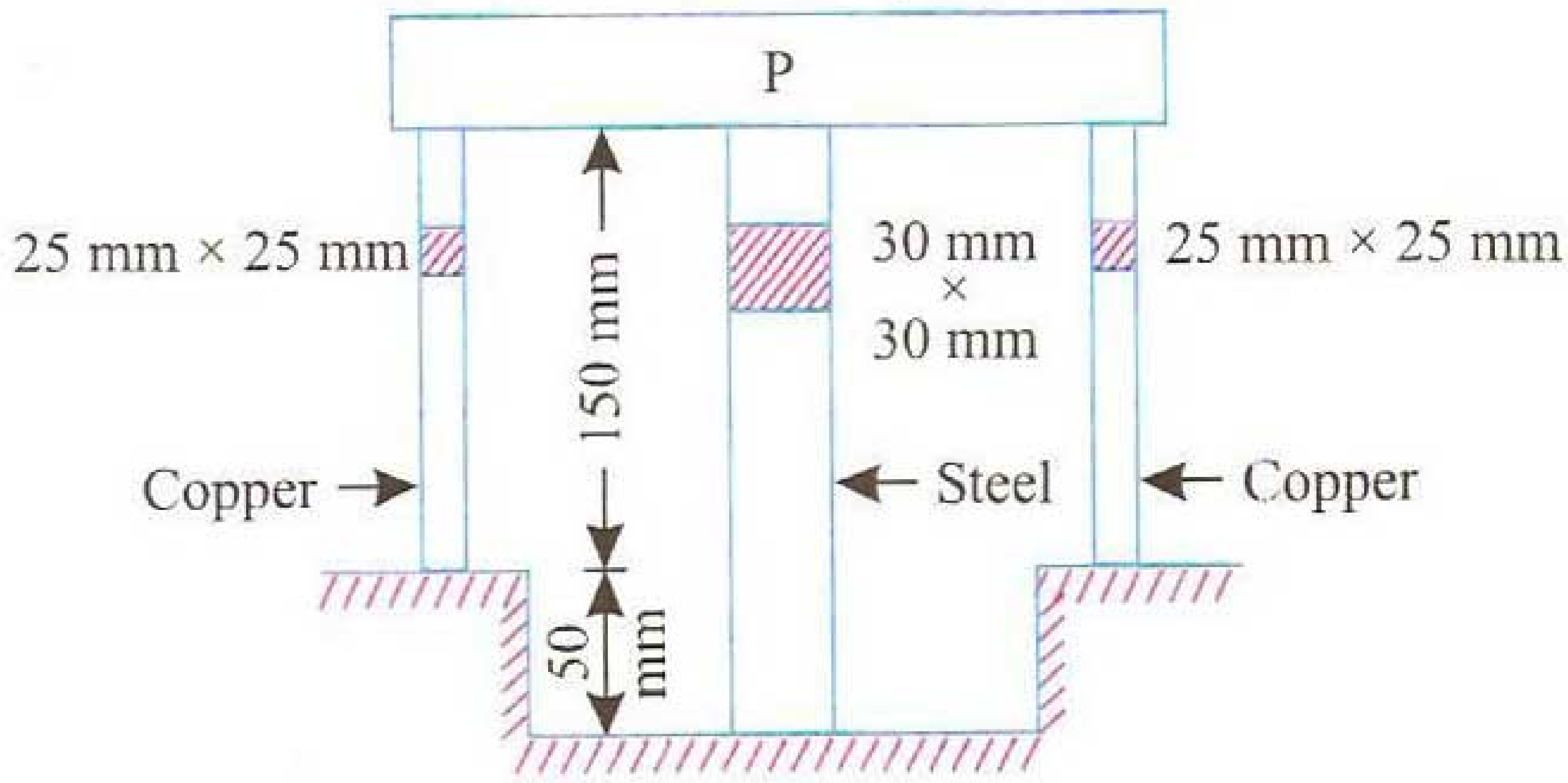


Fig. 1.40

Let the stresses in steel and copper be σ_s and σ_c respectively.

Then, $\sigma_s = e_s \times E_s$ and, $\sigma_c = e_c \times E_c$

$$\therefore \quad \frac{\sigma_s}{\sigma_c} = \frac{e_s}{e_c} \times \frac{E_s}{E_c} = 0.75 \times 2 = 1.5$$

$$\therefore \quad \sigma_s = 1.5\,\sigma_c$$

If the steel is permitted to reach its safe stress of 120×10^6 N / m^2, the corresponding stress in copper will be $\frac{120 \times 10^6}{1.5} = 80 \times 10^6$ N/m^2 for copper which exceeds the safe stress of 60×10^6 N/m^2. Therefore, let copper be allowed to reach its safe stress of 60×10^6 N/m^2, the corresponding stress in steel will be $60 \times 10^6 \times 1.5 = 90 \times 10^6$ N/m^2.

Bushings.

∴ *Total load,P* = Load on steel + load on copper

$= P_s + P_c = \sigma_s \times A_s + \sigma_c \times A_c$

$= 90 \times 10^6 \times 0.03 \times 0.03 + 2 \times 60 \times 10^6 \times 0.025 \times 0.025$

= 156000 N or **156 kN (Ans.)**

Example 1.33. *A steel bolt and sleeve assembly is shown in Fig. 1.41. The nut is tightened up on the tube through the rigid end blocks until the tensile force in the bolt is 40 kN. If an external load 30 kN is then applied to the end blocks, tending to pull them apart, estimate the resulting force in the bolt and sleeve.* **(AMIE Winter, 2001)**

Solution. Area of steel bolt, $A_b = \frac{\pi}{4} \times \left(\frac{25}{1000}\right)^2 = 4.908 \times 10^{-4}$ m^2

Area of steel sleeve, $A_s = \frac{\pi}{4} \times \left[\left(\frac{62.5}{1000}\right)^2 - \left(\frac{50}{1000}\right)^2\right] = 1.104 \times 10^{-3}$ m^2

Forces in the bolt and sleeve:

(i) Stresses due to tightening the nut:

Let, σ_b = Stress developed in steel bolt due to tightening the nut, and

σ_s = Stress developed in steel sleeve due to tightening the nut.