Bays theorem :-

Statement :-

Suppose $E_1, E_2, \ldots E_n$ are mutually exclusive events of a sample space S $\ni$ $P(E_i) > 0$ for $i = 1, 2 \ldots n$ and A is any arbitrary event of S $\ni$ $P(A) > 0$ and $A \subseteq \bigcup_{i=1}^{n} E_i$

Then conditional probability of E_i given A is

$$P(E_i | A) = \frac{P(A|E_i) \cdot P(E_i)}{\sum_{i=1}^{n} P(A|E_i) \cdot P(E_i)}$$

Proof :-

Given $A \subseteq \bigcup_{i=1}^{n} E_i$

$$(A \subseteq B \text{ then } A \cap B = A)$$

$$A = A \cap \bigcup_{i=1}^{n} E_i$$

$$\boxed{A = \bigcup_{i=1}^{n} (A \cap E_i)}$$

$A \cap (E_1 \cup E_2 \cup \cdots E_n)$

$(A \cap E_1) \cup (A \cap E_2) \cup \cdots \cup (A \cap E_n)$

$E_1, E_2 \ldots E_n$ are mutually exclusive

then $A \cap E_1, A \cap E_2 \ldots A \cap E_n$ are also mutually exc.

$$(A \cap E_i) \cap (A \cap E_j) = \phi \quad \text{for } i \neq j$$

$$P(A) = P\left[\bigcup_{i=1}^{n} (A \cap E_i)\right]$$

$$= P\left[\underline{(A \cap E_1)} \cup \underline{(A \cap E_2)} \cup \cdots \cup \underline{(A \cap E_n)}\right]$$

$$= P(A \cap E_1) \underline{+} P(A \cap E_2) \underline{+} \cdots + \underline{P}(A \cap E_n)$$

$$P(A) = \sum_{i=1}^{n} P(\underline{A \cap E_i})$$

$$P(A \cap E_i) = P(A|E_i) \cdot P(E_i)$$

$$P(A) = \sum_{i=1}^{n} P(A|E_i) \cdot P(E_i)$$

$$P(E_i|A) = \frac{P(A \cap E_i)}{P(A)} = \frac{P(A|E_i) P(E_i)}{\sum_{i=1}^{n} P(A|E_i) \cdot P(E_i)}$$