# :- Probability Distribution :-

## * Discrete Probability Distribution:

### 1. Discrete Uniform Distribution :-

A r.v $X$ is said to have a discrete uniform distribution over the range $[1, n]$, if its p.m.f is given by

$$p(x) = P(X=x) = \begin{cases} \frac{1}{n}, & x = 1, 2, 3 \ldots \ldots n \\ 0, & \text{otherwise} \end{cases}$$

It is denoted by $X \sim U(n)$, read as $X$ follows discrete uniform distribution with parameter $n$.

**Ex:-1** If an unbiased die is thrown once and $X$ is equal to number on the die. Then $x = 1, 2, 3, 4, 5, 6$ and

$$P(x=i) = \frac{1}{6} \text{ for } i = 1, 2, 3, 4, 5, 6 \text{ and}$$

$X \sim U(6)$.

| $X$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $P(x)$ | $1/6$ | $1/6$ | $1/6$ | $1/6$ | $1/6$ | $1/6$ |

## General form of UPD :-

| $x$ | 1 | 2 | 3 | 4 | $\cdots$ | $n$ |
|---|---|---|---|---|---|---|
| $p(x)$ | $1/n$ | $1/n$ | $1/n$ | $1/n$ | $\cdots$ | $1/n$ |

$$\text{Mean} = E(x) = \sum x\,p(x)$$

$$= 1\cdot\left(\frac{1}{n}\right) + 2\left(\frac{1}{n}\right) + \cdots + n\left(\frac{1}{n}\right)$$

$$= \frac{1}{n}\left[1+2+3+\cdots+n\right]$$

$$= \frac{1}{\cancel{n}}\,\frac{\cancel{n}(n+1)}{2} = \frac{n+1}{2}$$

$$E(x^2) = 1^2\left(\frac{1}{n}\right) + 2^2\left(\frac{1}{n}\right) + \cdots + n^2\left(\frac{1}{n}\right)$$

$$= \frac{1}{n}\left(1^2+2^2+3^2+\cdots+n^2\right)$$

$$= \frac{1}{\cancel{n}}\,\frac{\cancel{n}(n+1)(2n+1)}{6} = \frac{(n+1)(2n+1)}{6}$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= \frac{(n+1)(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{2(n+1)(2n+1) - 3(n+1)^2}{12}$$

$$= \frac{(n+1)\left[2(2n+1) - 3(n+1)\right]}{12}$$

$$= \frac{(n+1)\left[4n+2-3n-3\right]}{12}$$

$$= \frac{(n+1)(n-1)}{12}$$

$$= \frac{(n^2-1)}{12}$$

2. **Bernoulli distribution:-**

* **Bernoulli Experiment:-** A random experiment whose outcomes are of two types Success (S), Failure (F), occuring with probabilities $p$ and $q$ $(=1-p)$ respectively, is called a Bernoulli experiment

**Bernoulli Distribution:-** In a Bernoulli experiment, if a r.v $X$ is defined such that

it takes value '1' with probability $p$ when S occurs
it takes value '0' with probability $q$ when F occurs
then we say that X follows Bernoulli distribution
and p.m.f is given by

$$P(x) = P(X=x) = \begin{cases} p^{x} q^{1-x}, & x=0,1 \\ 0, & \text{otherwise} \end{cases}$$

**Note:-**

$\mu$ = Mean of the Bernoulli distribution = $p$
$\sigma^2$ = Variance " " = $pq$
$\sigma$ = S.d of " " = $\sqrt{pq}$

## 3. Binomial distribution:-

A r.v X is said to follow a B.D with parameters $n, p$ if its p.m.f is given by

$$p(x) = P(X=x) = \begin{cases} {}^{n}C_{x}\, p^{x} q^{n-x}, & x=0,1,2\cdots n \\ 0, & \text{otherwise} \end{cases}$$

and it is denoted by $X \sim B(n,p)$, read as X follows Binomial distribution with parameters $n, p$.

eg:- 1. Number of heads in n tosses of a coin
2. Number of boys in a family of n children
3. Number of times hitting a target in n attempts

## Note :-

1. Mean of the Bernoulli's Distribution $= p$.
2. Variance of the " " $= pq$
3. $\mu =$ Mean of the Binomial Distribution $= np$
4. Variance of the " " $= npq$
5. S.d of the " " $= \sqrt{npq}$

=x=

## Theorem :-

If $n \to \infty$ then Binomial distribution approaches Poisson distribution

**Proof:-** Binomial Distribution

$$P(X=x) = B(x, n, p) = {}^nC_x p^x q^{n-x}, \quad x = 0, 1, 2 \dots n$$

where $p$ is the probability of getting $x$ success among $n$ trails

$$B(x, n, p) = {}^nC_x\, p^x q^{n-x}$$

$$= \frac{n!}{(n-x)!\,x!} p^x (1-p)^{n-x} \qquad \left(\because p+q=1,\ q = 1-p\right)$$

$$= \frac{n(n-1)(n-2)\dots(n-(x-1))\cancel{(n-x)!}}{\cancel{(n-x)!}\,x!} p^x (1-p)^{n-x}$$

$$= \frac{np\,(np-p)(np-2p)\dots(np-(x-1)p)}{x!} (1-p)^{n-x}$$

$$= \frac{\mu\left(\mu-\frac{\mu}{n}\right)\left(\mu-\frac{2\mu}{n}\right)\cdots\left(\mu-\frac{(x-1)\mu}{n}\right)\left(1-\frac{\mu}{n}\right)^{n-x}}{x!}$$

$$= \frac{\mu^{x}\left(1-\frac{1}{n}\right)\left(1-\frac{2}{n}\right)\cdots\left(1-\frac{(x-1)}{n}\right)}{x!}\frac{\left(1-\frac{\mu}{n}\right)^{n}}{\left(1-\frac{\mu}{n}\right)^{x}}$$

take $n \to \infty$ as $\frac{1}{n} \to 0$

$$= \frac{\mu^{x} e^{-\mu}}{x!} = \frac{e^{-\mu} \cdot \mu^{x}}{x!}$$

$$= p(x, \mu)$$

= poisson Distribution

## 4. Poisson Distribution :-

A r.v X is said to follow a Poisson distribution with parameter $\lambda$, if its P.m.f is given by

$$p(x) = P(X=x) = \begin{cases} \dfrac{e^{-\lambda} \cdot \lambda^{x}}{x!}, & x = 0, 1, 2 \cdots \quad \lambda > 0 \\ 0, & \text{otherwise} \end{cases}$$

and denoted by $X \sim P(\lambda)$ read as $X$ follows poisson distribution with parameters $\lambda$.

eg:- 1. No. of defectives in a packet of 100 blades

2. No. of print mistakes in a page of book

3. No. of air accidents in some unit of time.