Elementary theorems:

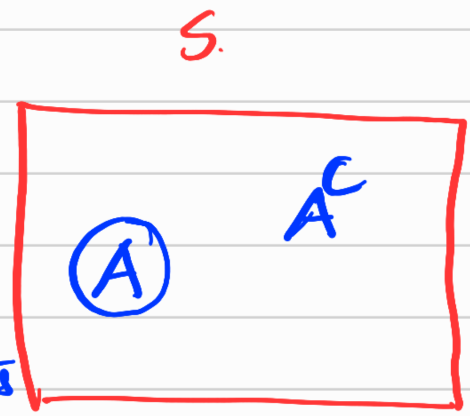
Theorem:-

Probability of complementary event A^c $= 1 - P(A)$

(or)

$$P(A^c) = 1 - P(A)$$

Proof:-



$S = A \cup A^c$ — (1)

A and A^c are disjoint sets

The events A, A^c are mutually exclusive

$\therefore \quad A \cap A^c = \phi$

$P(A \cap A^c) = 0$

From (1) $\quad P(S) = P(A \cup A^c)$

$\Rightarrow \quad 1 = P(A) + P(A^c)$

$\Rightarrow P(A^c) = 1 - P(A)$

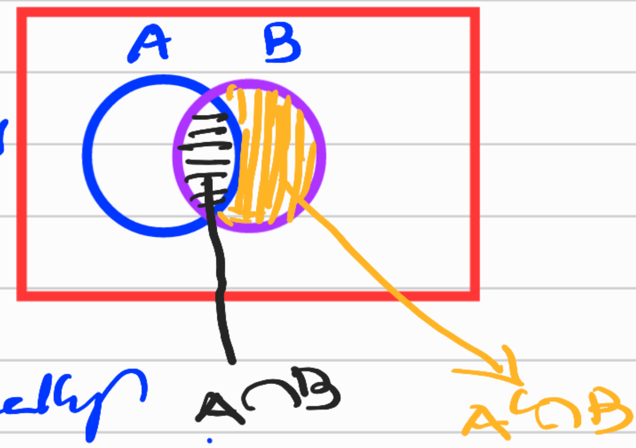
$$\therefore \quad \boxed{P(A^c) = 1 - P(A)}$$

Theorem:- for any two events A & B, then

i) $P(A^c \cap B) = P(B) - P(A \cap B)$

ii) $P(A \cap B^c) = P(A) - P(A \cap B)$

Proof:-



$A \cap B$ shown by Horizontal lines
$A^c \cap B$ shown by Vertical lines
$A \cap B$, $A^c \cap B$ are disjoint sets

Events $A \cap B$, $A^c \cap B$ are mutually Exclusive event

$$(A \cap B) \cap (A^c \cap B) = \phi$$

$$P((A \cap B) \cap (A^c \cap B)) = 0$$

$$(A \cap B) \cup (A^c \cap B) = \underline{B}$$

$$P((A \cap B) \cup (A^c \cap B)) = P(B)$$

$$P(A \cap B) + P(A^c \cap B) = P(B)$$

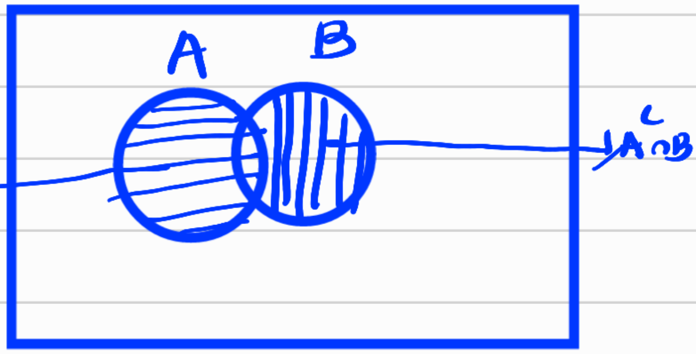
$$P(A^c \cap B) = P(B) - P(A \cap B)$$

IIly we prove that

$$P(A \cap B^c) = P(A) - P(A \cap B)$$

Theorem:- for any two events A and B then
$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

Proof:-



A shown by Horizontal lines A
$A^c\cap B$ " vertical "

A, $A^c\cap B$ are disjoint sets

Events A, $A^c\cap B$ are mutually exclusive
$$A\cap(A^c\cap B) = \phi \text{ then } P(A\cap(A^c\cap B)) = 0$$
$$P(A\cup(A^c\cap B)) = P(A) + P(A^c\cap B)$$

Now
$$A\cup(A^c\cap B) = (A\cup A^c)\cap(A\cup B)$$
$$= S\cap(A\cup B)$$
$$= A\cup B$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$
(from theorem 2)

$$\therefore P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

=x=

Theorem:- for any three events A, B, C

then $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

Proof:-

let $P(A \cup B \cup C) = P(D \cup C)$, where $D = A \cup B$

$= P(D) + P(C) - P(D \cap C)$

Replace D by $A \cup B$

$$P(A \cup B \cup C) = P(A \cup B) + P(C) - P((A \cup B) \cap C))$$

$$= P(A) + P(B) - P(A \cap B) + P(C) - P[(A \cap C) \cup (B \cap C)]$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - [P(A \cap C) + P(B \cap C) - P((A \cap C) \cap (B \cap C))]$$

$$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

Hence proved

Theorem:-

If $A \subseteq B$, then prove that i) $P(A^c \cap B) = P(B) - P(A)$

ii) $P(A) \leq P(B)$

iii) $P(A \cap B^c) = 0$

Proof:- i) we know that

$$P(A^c \cap B) = P(B) - P(A \cap B)$$

Given $A \subseteq B$ then $A \cap B = A$

$$P(A^c \cap B) = P(B) - P(A)$$

ii) we know that

$$P(A^c \cap B) = P(B) - P(A \cap B)$$

from i) $P(A^c \cap B) = P(B) - P(A) \geq 0$

$$P(B) - P(A) \geq 0$$

$$P(B) \geq P(A)$$

$$P(A) \leq P(B)$$

iii) we know that

$$P(A \cap B^c) = P(A) - P(A \cap B)$$

Given $A \subseteq B$ then $A \cap B = A$

$$= P(A) - P(A)$$

$$= 0$$

Theorem:

If A and B are mutually exclusive events then $P(A) \leq P(B^c)$

Proof:- Since A, B are mutually exclusive events

$A \cap B = \phi$ then $P(A \cap B) = 0$

We know that

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= P(A) + P(B) \quad (\because P(A \cap B) = 0)$$

$$\leq 1 \quad (\because \text{probability of an event is } \leq 1)$$

$$P(A) + P(B) \leq 1$$

$$P(A) \leq 1 - P(B)$$

$$P(A) \leq P(B^c)$$

$= x =$