General Multiplication Rule:-

If A and B are any events in S then $P(A \cap B) = P(A) \cdot P(B|A)$ if $P(A) \neq 0$

$$P(A \cap B) = P(B) \cdot P(A|B) \text{ if } P(B) \neq 0$$

=*=

Theorem:- If A, B are independent events then A and B^c are also indep.

Proof:- Given A and B are independent events

$$P(A \cap B) = P(A) \cdot P(B)$$

We have to prove that A, B^c are indep. events

$$P(A \cap B^c) = P(A) - P(A \cap B)$$

$$= P(A) - P(A) \cdot P(B)$$

$$= P(A)(1 - P(B))$$

$$= P(A) \cdot P(B^c)$$

Hence prove

1ly We can prove that

$$P(A^c \cap B) = P(A^c) \cdot P(B)$$

Theorem:- If A, B are indep// events then A^c, B^c are also indep//.

Proof Given that A, B are indep// events

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A^c \cap B^c) = P[(A \cup B)^c] \quad (\because \text{De Morgan Law})$$

$$= 1 - P(A \cup B)$$

$$= 1 - [P(A) + P(B) - P(A \cap B)]$$

$$= 1 - P(A) - P(B) + P(A) \cdot P(B)$$

$$= (1 - P(A)) - P(B)(1 - P(A))$$

$$= (1 - P(A))(1 - P(B))$$

$$= P(A^c) \cdot P(B^c)$$

Hence proved

Theorem: If A, B, C are mutually independent events, then $A \cup B$ and C are also independent.

Proof:- Given

$$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(B \cap C) = P(B) \cdot P(C)$$

$$P(C \cap A) = P(C) \cdot P(A)$$

We have prove that

$$P((A \cup B) \cap C) = P(A \cup B) \cdot P(C)$$

L.H.S $P((A \cup B) \cap C) = P((A \cap C) \cup (B \cap C))$

$$= P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$$

$$= P(A)\cdot P(C) + P(B)\cdot P(C) - P(A)\cdot P(B)\cdot P(C)$$

$$= P(C)\left[P(A) + P(B) - P(A)\cdot P(B)\right]$$

$$= P(C)\left[P(A) + P(B) - P(A \cap B)\right]$$

$$= P(C)\cdot P(A \cup B)$$

$$= R.H.S$$

Hence proved