RANDOM VARIABLE.

Random Variable:

Let S be the sample space associated with a random experiment. Let R be the set of real numbers. If $X: S \to R$ i.e X is a real valued function defined on the sample space, then X is known as a random variable.

In otherwords, random variable is a function which takes real values which are determined by the outcomes in the sample space.

eg: consider a random experiment consisting of tossing a coin twice.

The sample space $S = \{s_1, s_2, s_3, s_4\}$
$= \{HH, HT, TH, TT\}$

Define a function $X: S \to R$ by
$X(s) =$ "no. of heads"

then $X(s_1) = X(HH) = 2$

$X(s_2) = X(HT) = 1$

$X(s_3) = X(TH) = 1$

$X(s_4) = X(TT) = 0$

This show that X is random Variable

Range of $X = \{ X(s) \mid s \in S \}$

$= \{ X(s_1), X(s_2), X(s_3), X(s_4) \}$

$= \{ 2, 1, 1, 0 \}$

$= \{ 0, 1, 2 \}$

eg :-

Consider the random experiment of throwing a pair of dice and noting sum.

The sample space consisting of points

$S = \{ 36 \text{ elements} \}$

Define the function $X : S \to R$ by

$X(s) =$ sum of the numbers appearing on the faces of dice

i.e $X((i,j)) = i + j$

for $(i,j) \in S$

Then $X(\text{Sum}2) = X((1,1)) = 1$

$X(\text{Sum}3) = X((1,2)(2,1)) = 2$

$X(\text{Sum}4) = X((1,3)(3,1)(2,2)) = 3$

$X(\text{Sum}5) = X((1,4)(4,1)(2,3)(3,2)) = 4$

$X(\text{Sum}6) = X((1,5)(5,1)(2,4)(4,2),$
$(3,3)\}$

$= 5$

$X(\text{Sum}7) = X((1,6)(6,1)(2,5)(5,2)$
$(3,4)(4,3)\}$

$= 6$

$X(\text{Sum}8) = X((2,6)(6,2)(3,5)(5,3)$
$(4,4)\}$

$= 5$

$X(\text{Sum}9) = X((3,6)(6,3)(4,5)(5,4)\}$

$= 4$

$X(\text{Sum}10) = X((4,6)(6,4)(5,5))$
$= 3$

$x(\text{sum } 11) = x((5,6)(6,5)) = 2$

$x(\text{sum } 12) = x((6,6)) = 1$

This shows x is a random variable

Types of Random Variable

Random Variable is of two types.

i) Discrete Random Variable

ii) Continuous Random Variable

i) Discrete Random Variable:-

A random variable x which can take only a finite no. of discrete values in an interval of domain is called a discrete random variable.

In otherwords, if the random variable takes the values only on the set $\{0,1,2 \cdots n\}$ is called a Discrete Random Variable.

eg: Tossing a coin

Throwing a dice.

The no. of defectives in a sample of electric bulbs.

Probability function of DRV :-

If for a DRV x, the real valued function $p(x)$ is such that $P(X=x) = p(x)$. then $p(x)$ is called probability function or probability distribution function probability mass function of DRV X. probability function $p(x)$ gives the measure of probability for different values of X

Properties of a Probability Function

If $p(x)$ is a probability function of random variable X then it possess the following properties

1. $p(x) \geq 0 \ \forall x$.

2. $\sum p(x) = 1$, summation is taken over $\forall$ values of x

3. $p(x)$ cannot be negative for any values of x

Discrete Probability distribution:
(Probability Mass function)

Probability distribution of a random variable is the set of its possible values together with their respective probabilities. Suppose x is a discrete r.v with possible outcome (values) $x_1, x_2, \ldots, x_n$. The probabilities of each possible outcomes x_i is

$$P_i = P(X = x_i) = P(x_i)$$

for $i = 1, 2, 3, \ldots$

If the numbers $p(x_i)$, $i = 1, 2, 3 \ldots$ satisfy the two condition

i) $p(x_i) > 0 \ \forall$ values of i, $0 < P_i \leq 1$

ii) $\sum p(x_i) = 1$, $i = 1, 2, 3 \ldots$

then the function $p(x)$ is called PMF of the r.v x and the set $\{p(x_i)\}$ for $i = 1, 2, 3 \ldots$ is called DPD of the DRV X

The probability distribution of the r.v. X is given by mean of the following table

x	x_1	x_2	x_3	$\ldots$ x_i	x_n
$P(x)$	$P(x_1)$	$P(x_2)$	$P(x_3)$	$\ldots$	$P(x_n)$

Further

$$P(X < x_i) = P(x_1) + P(x_2) + \cdots + P(x_{i-1})$$

$$P(X \le x_i) = P(x_1) + P(x_2) + \cdots + P(x_i)$$

$$P(X > x_i) = 1 - P(X \le x_i)$$