Negative Binomial (or Pascal) Distribution

Let x denote the no. of failures before the r^{th} success in the sequence of Bernoulli trails. Then the no. of trails required is $X+r$

A r.v X is said to follow a NBD with parameters r and p if its p.m.f is given by

$$p(x) = P(X=x) = \begin{cases} {}^{x+r-1}C_{r-1}\, p^{r} q^{x}, & x = 0,1,2\cdots \\ 0 & \text{otherwise} \end{cases}$$

and denoted by $X \sim NB(r, p)$

Note:-

$$\left({}^{x+r-1}C_{r-1}\right) = {}^{x+r-1}C_{x} \quad (\because {}^{n}C_{r} = {}^{n}C_{n-r})$$

$$= (-1)^{r} \left({}^{-r}C_{x}\right)$$

=x=

Another definition of NBD:-

$$p(x) = P(X=x) = \begin{cases} {}^{-r}C_{x}\, p^{r} (-q)^{x}, & x = 0,1,2\ldots \\ 0 & \text{otherwise} \end{cases}$$

Ex:-

1. No. of tails before third head.
2. No. of girls before the second son
3. No. of boys before the second daughter
4. No. of non-defective before the third defective

= x =

* Mean of the NBD is $\mu = \frac{rq}{p}$
* Variance of the NBD is $\sigma^2 = \frac{rq}{p^2}$

Note:-

1. $$\frac{\mu}{\sigma^2} = \frac{\frac{rq}{p}}{\frac{rq}{p^2}} = p < 1.$$

$\Rightarrow$ mean is smaller then variance in NBD

2. Let Y = no. of trails required to get rth success
$= x + r.$

$$P(Y=y) = P(x+r=y)$$
$$= P(x = y-r)$$
$$= {}^{y-1}C_{r-1}\, p^r q^{y-r}$$
for $y = r, r+1 \ldots$

$$E[y] = E[x+r]$$
$$= E[x] + r$$
$$= \frac{rq}{p} + r$$

$$V[Y] = V[x+r] \quad (\because V(x+b) = V(x))$$
$$= V(x)$$
$$= \frac{rq}{p^2}$$

$$= x =$$

① Find the probability that there are two daughters before the second son in a family when probability of son is 0.5.

Solution:-

Let X = the no. of daughters before second son.

$= 2$

By the definition of NBD.

$$P(X=x) = {}^{x+r-1}C_{r-1}\, p^r q^x$$

$$= {}^{x+2-1}C_{2-1}\, p^2 q^x.$$

$$P(X=2) = {}^{2+2-1}C_{2-1} \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^2$$

$$= {}^{3}C_{1} \left(\tfrac{1}{2}\right)^4$$

$$= 3\,\frac{1}{2^4} = \frac{3}{16}.$$

$= \times =$

Geometric distribution

Let x denote the no. of failurs before the first success in a sequence of Bernoulli trails. then the required no. of trails is $X+1$.

A r.v x is said to follow a G.D with parameter p if its p.m.f is given by

$$P(x) = P(X=x) = \begin{cases} p\,q^{x} & \text{for } x = 0,1,2\dots \\ 0 & \text{otherwise} \end{cases}$$

and it is denoted by $X \sim GD(P)$

* 1. Mean $= \mu = \dfrac{q}{p}$

  2. Variance $= \sigma^2 = \dfrac{q}{p^2}$

$=x=$

1. Find the probability that there are two daughters before the 1^{st} son in a family. when $P(\text{son}) = 0.5$

Sol:-

Let x = no. of daughters before the 1^{st} son

$= 2$

$$P(x) = P(X = x) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^x$$

$$= \left(\frac{1}{2}\right)^{x+1}$$

$$P(X=2) = \left(\frac{1}{2}\right)^{2+1}$$

$$= \left(\frac{1}{2}\right)^3$$

$$= \frac{1}{8}.$$

$= \times =$