

Problems based on Baye's Theorem (1)

In a bolt factory machines A, B, C manufacture respectively 25%, 35% and 40% of the total. Of their outputs 5, 4, 2 percent are known to be defective bolts. A bolt is drawn from the product and is found to be defective. What are the probabilities that it was manufactured by

- Machine A
- Machine B or C.

Solution :-

Let E_1 = bolt is manufactured by machine A

E_2 = bolt is manufactured by machine B

E_3 = bolt is manufactured by machine C

$$\text{Given } P(E_1) = 25\% = \frac{25}{100} = 0.25$$

$$P(E_2) = 35\% = \frac{35}{100} = 0.35$$

$$P(E_3) = 40\% = \frac{40}{100} = 0.40$$

Let D = defective bolt.

$$P(D|E_1) = 5\% = \frac{5}{100} = 0.05$$

$$P(D|E_2) = 4\% = \frac{4}{100} = 0.04$$

$$P(D|E_3) = 2\% = \frac{2}{100} = 0.02$$

$$P(D) = P(E_1) \cdot P(D|E_1) + P(E_2) \cdot P(D|E_2) + P(E_3) \cdot P(D|E_3).$$

$$= (0.25)(0.05) + (0.35)(0.04) + (0.40)(0.02)$$

$$= 0.0345$$

i) The probability that a defective bolt chosen at random is manufactured by E_1
 By Baye's theorem

$$P(E_1|D) = \frac{P(E_1) \cdot P(D|E_1)}{\sum_{i=1}^3 P(E_i) \cdot P(D|E_i)}$$

$$= \frac{(0.25)(0.05)}{0.0345}$$

$$= 0.36.$$

(ii) the probability that a defective bolt chosen at random is manufactured by Machine B or C

$$\text{i.e. } P(E_2|D) + P(E_3|D)$$

$$\begin{aligned} \text{Now } P(E_2|D) &= \frac{P(E_2) \cdot P(D|E_2)}{\sum_{i=1}^3 P(E_i) P(D|E_i)} \\ &= \frac{(0.35)(0.04)}{0.0345} \\ &= 0.41 \end{aligned}$$

$$\begin{aligned} P(E_3|D) &= \frac{P(E_3) \cdot P(D|E_3)}{\sum_{i=1}^3 P(E_i) P(D|E_i)} \\ &= \frac{(0.40)(0.02)}{0.0345} \\ &= 0.23 \end{aligned}$$

$$\begin{aligned} \therefore P(E_2|D) + P(E_3|D) &= 0.41 + 0.23 \\ &= 0.64 \end{aligned}$$

$= x =$

In a group consisting of equal number men and women 10% of the men and 40% of the women are unemployed. If a person is selected at randomly from the group then.

- (i) Find the probability that the person is an employee.
- (ii) Find the probability that the person is an unemployed.
- (iii) Find the probability that the employee person is men
- (iv) Find the probability that the employee person is women
- (v) Find the probability that the unemployed person is men
- (vi) Find the probability that the unemployed person is women

solution

$$P(M) = P(W) = \frac{1}{2}$$

Let U = Unemployed person

E = employee person

Given $P(U|M) = 10\% = \frac{10}{100}$

$$P(E|M) = 1 - P(U|M)$$

$$= 1 - \frac{10}{100} = \frac{90}{100} = \frac{9}{10}$$

$$P(U|W) = 45\% = \frac{45}{100}$$

$$P(E|W) = 1 - P(U|W)$$

$$= 1 - \frac{45}{100} = \frac{55}{100}$$

(i) $P(E) = P(M) \cdot P(E|M) + P(W) \cdot P(E|W)$

$$= \frac{1}{2} \cdot \frac{9}{10} + \frac{1}{2} \cdot \frac{55}{100}$$

$$= \frac{1}{2} \left[\frac{90+55}{100} \right]$$

$$= \frac{145}{200} = 0.725$$

(ii) $P(U) = P(M) \cdot P(U|M) + P(W) \cdot P(U|W)$

$$= \frac{1}{2} \cdot \frac{10}{100} + \frac{1}{2} \cdot \frac{45}{100}$$

$$= \frac{1}{2} \left[\frac{55}{100} \right] = \frac{55}{200} = 0.275$$

$$\begin{aligned}
 \text{(iii) } P(M|E) &= \frac{P(M) \cdot P(E|M)}{P(M)P(E|M) + P(W) \cdot P(E|W)} \\
 &= \frac{\frac{1}{2} \frac{9}{10}}{0.725} = \frac{9}{20 \times 0.725} \\
 &= \frac{0.45}{0.725} \\
 &= 0.620
 \end{aligned}$$

$$\begin{aligned}
 \text{(iv) } P(W|E) &= \frac{P(W) \cdot P(E|W)}{P(M)P(E|M) + P(W)P(E|W)} \\
 &= \frac{\frac{1}{2} \frac{55}{100}}{0.725} = \frac{55}{200} \\
 &= \frac{0.275}{0.725} \\
 &= 0.379
 \end{aligned}$$

$$\begin{aligned}
 \text{(v) } P(M|U) &= \frac{P(M) \cdot P(U|M)}{P(M)P(U|M) + P(W)P(U|W)} \\
 &= \frac{\frac{1}{2} \frac{10}{100}}{0.725} \\
 &= \frac{\frac{1}{20}}{0.725} = \frac{0.05}{0.725} \\
 &= 0.068
 \end{aligned}$$

$$\begin{aligned}
 \text{(vi) } P(W|U) &= \frac{P(W) \cdot P(U|W)}{P(M) \cdot P(U|M) + P(W) \cdot P(U|W)} \\
 &= \frac{\frac{1}{2} \cdot \frac{45}{100}}{0.725} \\
 &= \frac{\frac{45}{200}}{0.725} = \frac{0.225}{0.725} \\
 &= 0.310.
 \end{aligned}$$

$\equiv x =$

- # Three machines I, II, III produce 40%, 30%, 30% of the total number of items of factory. The percentages of defective items of these machines are 4%, 2%, 3%. If an item is selected at random.
- Find the probability that the item is defective
 - Find the probability that the defective item is produced by Machine I
 - Find the probability that the defective item is produced by Machine II

(iv)

Find the probabilities that the defective items is produced by Machine III

$= x =$