Theorem:-

If A_1 and A_2 are independent events, then $P(A_1 \cup A_2) = 1 - P(\bar{A}_1) \cdot P(\bar{A}_2)$

Proof:-

$$R.H.S = 1 - P(\bar{A}_1) \cdot P(\bar{A}_2)$$

$$= 1 - [(1 - P(A_1))(1 - P(A_2))]$$

$$= 1 - [1 - P(A_2) - P(A_1) + P(A_1) \cdot P(A_2)]$$

$$= \cancel{1} - \cancel{1} + P(A_2) + P(A_1) - P(A_1) \cdot P(A_2)$$

$$= P(A_1) + P(A_2) - P(A_1 \cap A_2)$$

$$= P(A_1 \cup A_2)$$

$$\therefore P(A_1 \cup A_2) = 1 - P(\bar{A}_1) \cdot P(\bar{A}_2)$$

$= \times =$

Problems

A problem in probability is given to three students A, B, and C whose chances of solving it are $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ respectively. Find the probability that the problem will solved if they all try independently.

Solution:-

Let A = event that the problem is solved by A

B = event that the problem is solved by B

C = event that the problem is solved by C

$$P(A) = \frac{1}{3}, \Rightarrow P(\bar{A}) = 1 - \frac{1}{3} = \frac{2}{3}$$

$$P(B) = \frac{1}{4} \Rightarrow P(\bar{B}) = 1 - \frac{1}{4} = \frac{3}{4}$$

$$P(C) = \frac{1}{5} \Rightarrow P(\bar{C}) = 1 - \frac{1}{5} = \frac{4}{5}$$

The problem is solved if at least one of them is able to solve it

$$P(A \cup B \cup C) = 1 - P(\bar{A})\,P(\bar{B})\,P(\bar{C})$$

$$= 1 - \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5}$$

$$= 1 - \frac{2}{5} = \frac{3}{5}$$