Conditional Probability :-

Let A and B be any two events in a sample space S. The probability that B will occure given that A has already occure is called the conditional Probability and is denoted by $P(B|A)$

$$\therefore P(B|A) = \frac{P(A \cap B)}{P(A)}$$

Similarly the probability that A will occure, given that B has occured is called the conditional probability and it is denoted by $P(A|B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

If $n(S)$ = no. of elements in S

$n(B)$ = no. of elements in B

$n(A \cap B)$ = no. of elements in $A \cap B$

Probability of B $= P(B) = \frac{n(B)}{n(S)}$

Probability of $A \cap B = P(A \cap B) = \dfrac{n(A \cap B)}{n(S)}$

$\therefore$ Conditional probability of A given B is

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{n(A \cap B)}{n(S)} \times \frac{n(S)}{n(B)}$$

$$= \frac{n(A \cap B)}{n(B)}$$

$$= \frac{\text{no. of elements in } A \cap B}{\text{no. of elements in } B}$$

$$\therefore \quad P(A/B) = \frac{P(A \cap B)}{P(B)}$$

$$\Rightarrow \quad P(A \cap B) = P(B) \cdot P(A/B)$$

e.g. ① Two coins are tossed. The event of getting two tails given that there is at least one tail is a conditional event

Independent event :-

Let any two events are said to be independent events if the happening of the event is not effected by the happening of the other event

Two events A, B are said to be independent ($P(A) \neq 0, P(B) \neq 0$).

if $P(A|B) = P(A)$, $P(B) > 0$

$$P(B|A) = P(B), \quad P(A) > 0.$$

=x=

Independence and Exclusiveness

Independence and Exclusiveness of two events A & B are two entirely different concepts.

For exclusive ness the condition is $p(A \cap B) = 0$

For Independent the condition is $p(A \cap B) = p(A)\,p(B)$

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<u>Note:-1</u>

If A, B are exclusive events they can't be independent.

<u>Proof:-</u> If A, B are exclusive events

then $P(A \cap B) = 0$

But $P(A) \neq 0$, $P(B) \neq 0$

$$P(A \cap B) \neq P(A) \cdot P(B)$$

Hence A, B are not independant

<u>Note:-2</u>

If A & B are independent events they cann't be exclusive

<u>Proof:-</u> If A, B are independent events

then $P(A \cap B) = P(A) \cdot P(B)$

But $P(A) \neq 0$, $P(B) \neq 0$ then $P(A \cap B) \neq 0$

Hence A & B are not exclusive

=x=

Mutual Independence :-

If A, B, C are any three events then they are said to be mutually independent if they are pairwise independent and if

A is independent of $B \cap C$,

B is independent of $C \cap A$

C is independent of $A \cap B$.

thus, three events A, B, C are mutually independent

if $P(A \cap B) = P(A) \cdot P(B)$

$P(B \cap C) = P(B) \cdot P(C)$

$P(C \cap A) = P(C) \cdot P(A)$

$P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C)$

=

__Theorem:-__

If A, B are exclusive events, prove that

$P(A|B)=0$ and $P(B|A)=0$

__proof:-__ Here A, B are exclusive then $P(A\cap B)=0$

$$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{0}{P(B)} = 0$$

$$P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{0}{P(A)} = 0$$

__Theorem:-__ If $A\subset B$ then $P(B/A)=1$, $P(A/B)=\frac{P(A)}{P(B)}$

Given $A\subset B$, $A\cap B = A$.

(i) $$P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{P(A)}{P(A)} = 1$$

(ii) $$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{P(A)}{P(B)}$$

Theorem:- If A, B are any events

$$P(A/\overline{B}) = \frac{P(A) - P(A\cap B)}{1 - P(B)}$$

We know that

$$P(A|B) = \frac{P(A\cap B)}{P(B)}$$

$$P(A|\overline{B}) = \frac{P(A\cap\overline{B})}{P(\overline{B})}$$

$$= \frac{P(A) - P(A\cap B)}{1 - P(B)}$$

$(\because P(\overline{B}) = 1 - P(B)$

$P(A\cap\overline{B}) = P(A - P(A\cap B)$

$= \times =$

General Law of mutually independent events:

If $A_1, A_2 \ldots A_n$ are mutually indepen..

then

$$P(A_i \cap A_j) = P(A_i) \cdot P(A_j) \quad i \neq j$$

$$P(A_i \cap A_j \cap A_k) = P(A_i) \cdot P(A_j) P(A_k), \quad i \neq j \neq k.$$

$$P(A_1 \cap A_2 \cap A_3 \ldots A_n) = P(A_1) \cdot P(A_2) \cdots P(A_n)$$