Problems based on Elementary theorems

If $P(A)=\frac{1}{5}$, $P(B)=\frac{2}{3}$, $P(A\cap B)=\frac{1}{15}$

Find (i) $P(A\cup B)$

(ii) $P(A^c\cap B)$

(iii) $P(A\cap B^c)$

(iv) $P(A^c\cap B^c)$

(v) $P(A^c\cup B^c)$

Solution:-

(i) $P(A\cup B) = P(A)+P(B)-P(A\cap B)$

$$= \frac{1}{5}+\frac{2}{3}-\frac{1}{15}$$

$$= \frac{3+10-1}{15} = \frac{12}{15} = \frac{4}{5}.$$

(ii) $P(A^c\cap B)$

we know that

$$P(A^c\cap B) = P(B)-P(A\cap B)$$

$$= \frac{2}{3}-\frac{1}{15}$$

$$= \frac{10-1}{15} = \frac{9}{15} = \frac{3}{5}$$

(iii) $P(A \cap B^c)$

We know that

$$P(A \cap B^c) = P(A) - P(A \cap B)$$

$$= \frac{1}{5} - \frac{1}{15}$$

$$= \frac{3-1}{15} = \frac{2}{15}$$

(iv) $P(A^c \cap B^c)$

We know that

$$A^c \cap B^c = (A \cup B)^c$$

($\because$ De Morgan Laws)

$$P(A^c \cap B^c) = P((A \cup B)^c)$$

$$= 1 - P(A \cup B)$$

$$= 1 - \frac{4}{5}$$

$$= \frac{5-4}{5} = \frac{1}{5}.$$

v) $P(A^c \cup B^c)$

We know that

$$A^c \cup B^c = (A \cap B)^c$$

(De Morgan Laws)

$$P(A^c \cup B^c) = P((A \cap B)^c)$$

$$= 1 - P(A \cap B)$$

$$= 1 - \frac{1}{15}$$

$$= \frac{15-1}{15}$$

$$= \frac{14}{15}$$

=x=

If $P(A \cup B) = \frac{4}{5}$, $P(B^c) = \frac{1}{3}$ and $P(A \cap B) = \frac{1}{5}$ then find

(i) $P(B)$

(ii) $P(A)$

(iii) $P(A^c \cap B)$

Hint: (i) $P(B^c) = 1 - P(B)$

$P(B) = 1 - P(B^c)$

(ii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

(iii) $P(A^c \cap B) = P(B) - P(A \cap B)$

=x=

If $B \subseteq A$ and A and B are two events such that $P(A) = 3P(B)$ and $A \cup B = S$. Find $P(B) = ?$

solution:-

$\because B \subseteq A, \quad A \cap B = B$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\Rightarrow P(S) = P(A) + \cancel{P(B)} - \cancel{P(B)}$$

$$\Rightarrow 1 = 3P(B)$$

$$P(B) = \frac{1}{3}$$

=x=

If $P(A) = a$, $P(B) = b$ and $P(A \cap B) = c$, express the following probabilities interms a, b, c

(i) $P(A^c \cup B^c)$

(ii) $P(A^c \cap B^c)$

(iii) $P(A^c \cap B)$

(iv) $P(A \cap B^c)$

=x=

If two dice are thrown, what is the probability that the sum is

(i) greater than 8

(ii) neither 7 nor 11

(iii) an even number on the first die or a total of 8

<u>Solution:-</u>

The sample space

$$S = \{ (1,1)(1,2)(1,3)(1,4)(1,5)(1,6)$$
$$(2,1)(2,2)(2,3)(2,4)(2,5)(2,6)$$
$$(3,1)(3,2)(3,3)(3,4)(3,5)(3,6)$$
$$(4,1)(4,2)(4,3)(4,4)(4,5)(4,6)$$
$$(5,1)(5,2)(5,3)(5,4)(5,5)(5,6)$$
$$(6,1)(6,2)(6,3)(6,4)(6,5)(6,6) \}$$

$$n(S) = 36$$

(i) let A = getting sum 9

$$= \{(3,6)(6,3)(4,5)(5,4)\}$$

B = getting sum 10

$$= \{(4,6)(6,4)(5,5)\}$$

C = getting sum 11

$= \{(5,6)\ (6,5)\}$

D = getting sum 12

$= \{(6,6)\}$

let T = event of getting the sum is greater than 8

= sum 9 or sum 10 or sum 11 or sum 12

$= A \cup B \cup C \cup D$

$= \{(3,6)(6,3)(4,5)(5,4)$
$(4,6)(6,4)(5,5)\ (5,6)(6,5)$
$(6,6)\}$

$n(T) = 10$

$\therefore P(\text{greater than } 8) = \dfrac{n(T)}{n(S)}$

$= \dfrac{10}{36}$

$= \dfrac{5}{18}$

(ii) let A = event of getting sum 7

$= \{(1,6)\ (6,1)\ (2,5)\ (5,2)$
$(3,4)\ (4,3)\}$

$n(A) = 6$

$$P(A) = \frac{n(A)}{n(S)} = \frac{6}{36} = \frac{1}{6}$$

Let B = event of getting sum 11

$= \{(6,5)\ (5,6)\}$

$$n(B) = 2$$

$$P(B) = \frac{n(B)}{n(S)} = \frac{2}{36} = \frac{1}{18}$$

$$A \cup B = \{(1,6)(6,1)(2,5)(5,2)(3,4)(4,3)(6,5)(5,6)\}$$

$$n(A \cup B) = 8$$

$$P(A \cup B) = \frac{n(A \cup B)}{n(S)}$$

$$= \frac{8}{36} = \frac{2}{9}$$

P(neither 7 nor 11)

$$= P(A^c \cap B^c)$$

$$= P((A \cup B)^c)$$

$$= 1 - P(A \cup B)$$

$$= 1 - \frac{2}{9}$$

$$= \frac{7}{9}$$

(iii) Let A = event of getting an even number on the first die

$= \{(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$n(A) = 18$

$P(A) = \frac{18}{36}$

B = event of getting Total of 8

$= \{(2,6)\ (6,2)$
$(3,5)\ (5,3)$
$(4,4)$

$n(B) = 5$

$P(B) = \frac{5}{36}$

$A \cap B = \{(2,6)\ (6,2)\ (4,4)\}$

$n(A \cap B) = 3$

$P(A \cap B) = \frac{3}{36}$

Required Probability

$P(A \text{ or } B)$.

$= P(A \cup B)$

$= P(A) + P(B) - P(A \cap B)$

$= \frac{18}{36} + \frac{5}{36} - \frac{3}{36} = \frac{20}{36} = \frac{5}{9}$

$= \times =$

A card is drawn from a pack of 52 cards. Find the probability of getting a king or a heart or a red card.

<u>solution:-</u>

Let A = event of getting the card drawn is a king

B = event of getting the card drawn is a heart

C = event of getting the card drawn is a red card

A, B, C are not mutually exclusive event

$n(A) = 4 \Rightarrow P(A) = 4/52$

$n(B) = 13 \Rightarrow P(B) = 13/52$

$n(C) = 26 \Rightarrow P(C) = 26/52$

$n(A \cap B) = 1 \Rightarrow P(A \cap B) = 1/52$

$n(A \cap C) = 2 \Rightarrow P(A \cap C) = 2/52$

$n(B \cap C) = 13 \Rightarrow P(B \cap C) = 13/52$

$n(A \cap B \cap C) = 1 \Rightarrow P(A \cap B \cap C) = 1/52$

$P(A \text{ or } B \text{ or } C) = P(\text{King or heart or red card})$

$= P(A \cup B \cup C)$

$= P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{2}{52} - \frac{13}{52} + \frac{1}{52}$

$$= \frac{28}{52} = \frac{7}{13}$$

=x=

Among 150 students 80 are studying Maths, 40 are studying physics and 30 are studying Maths and physics. If a student is chosen at random. find the probability that the student

(i) studying Maths or physics

$(P(A \cup B) = ?$

$P(A) = \frac{80}{150}$

$P(B) = \frac{40}{150}$

$P(A \cap B) = \frac{30}{150}$

(ii) student studying neither Maths nor physics

[Hint $P(A^c \cap B^c)$]