Continuous Probability Distributions.

1. Continuous Uniform Distribution:-

A c.r.v X is said to have a uniform distribution over the interval $[a,b]$ if its p.d.f is given by

$$f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & \text{Otherwise} \end{cases}$$

and denoted by $X \sim U(a,b)$, read as X follows uniform distribution with parameter a and b.

Note:-

The CDF of X is given by

$$F(x) = P(X \le x) = \int_{-\infty}^{x} f(x)\,dx$$

$$= \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \le x \le b \\ 1, & x \ge b. \end{cases}$$

$= \times =$

Mean and Variance of the Uniform Distribution

$$\mu = E(x) = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$= \int_{-\infty}^{a} x f(x)\, dx + \int_{a}^{b} x f(x)\, dx + \int_{b}^{\infty} x f(x)\, dx$$

$$= \int_{a}^{b} x \cdot \left(\frac{1}{b-a}\right) dx$$

$$= \frac{1}{b-a}\left(\frac{x^2}{2}\right)_a^b$$

$$= \frac{1}{b-a}\left(\frac{b^2}{2} - \frac{a^2}{2}\right)$$

$$= \frac{1}{\cancel{b-a}}\left(\frac{\cancel{(b-a)}(b+a)}{2}\right)$$

$$= \frac{b+a}{2} = \frac{a+b}{2}$$

$$E(x^2) = \int_{a}^{b} x^2 f(x)\, dx$$

$$= \int_{a}^{b} x^2 \left(\frac{1}{b-a}\right) dx$$

$$= \frac{1}{b-a}\left(\frac{x^3}{3}\right)_a^b$$

$$= \frac{1}{b-a}\left(\frac{b^3-a^3}{3}\right)$$

$$= \frac{1}{\cancel{b-a}}\left(\frac{\cancel{(b-a)}(b^2+ab+a^2)}{3}\right)$$

$$= \frac{b^2+ab+a^2}{3}$$

$$V(x) = E(x^2) - [E(x)]^2$$

$$= \frac{b^2+ab+a^2}{3} - \left(\frac{a+b}{2}\right)^2$$

$$= \frac{b^2+ab+a^2}{3} - \frac{(a+b)^2}{4}$$

a^2+b^2+2ab

$$= \frac{4b^2+4ab+4a^2-3a^2-3b^2-6ab}{12}$$

$$= \frac{a^2+b^2-2ab}{12}$$

$$V(x) = \frac{(a-b)^2}{12} = \frac{(b-a)^2}{12}$$

$$\sigma^2 = V(x) = \frac{(b-a)^2}{12}$$

$$\sigma = \sqrt{V(x)} = S.d = \frac{(b-a)}{\sqrt{12}}$$

The time that a professor takes to grade a paper is uniformly distributed between 5 minutes and 10 minutes. Find the mean and variance of the time the professor takes to grade a paper.

Sol:- x = the time the professor takes to a grade a paper.

then $X \sim U(a,b)$

i.e $X \sim U(5,10)$

$$\text{mean} = \mu = E(x) = \frac{b+a}{2} = \frac{10+5}{2} = \frac{15}{2} = 7.5$$

$$\text{Variance} = \sigma^2 = V(X) = \frac{(b-a)^2}{12}$$

$$= \frac{(10-5)^2}{12} = \frac{25}{12} = 2.5$$

$= X =$

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