2. Exponential Distribution:-

A c.r.v X is said to follow exponential distribution with parameter λ, if its p.d.f is given by
$$f(x) = \begin{cases} \lambda e^{-\lambda x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$$

and it is denoted by $X \sim E(\lambda)$, read as X follows exponential distribution with parameter λ.

eg:- The time between two successive arrivals at any service facility.

CDF of Exponential Distribution:-

$$F(x) = P(X \leq x) = \int_0^x f(x)\,dx$$
$$= \lambda \int_0^x e^{-\lambda x}\,dx$$
$$= \cancel{\lambda}\left(\frac{e^{-\lambda x}}{-\cancel{\lambda}}\right)_0^x$$
$$= 1 - e^{-\lambda x}$$

* Mean of the exponential Distribution $= \frac{1}{\lambda}$
* Variance of the exponential Distribution $= \frac{1}{\lambda^2}$

Assume that the length of phone calls made at a particular telephone booth is exponentially distributed with a mean of 3 min. If you arrive at the telephone booth just as Ramu was about to make a call, find the following

(i) The probability that you will wait more than 5 minutes before Ramu is done with call

(ii) The probability that Ramu's call will last between 2 minutes and 6 minutes.

Sol:- Let X = the length of calls made at the telephone booth

mean length of calls $= \frac{1}{\lambda} = 3$

$$\Rightarrow \lambda = \frac{1}{3}$$

Then the p.d.f $f(x) = \lambda e^{-\lambda x}$

$$= \frac{1}{3} e^{-\frac{1}{3}x}$$

(i) $$P(X > 5) = \int_5^\infty f(x)\,dx$$

$$= \frac{1}{3} \int_5^\infty e^{-\frac{1}{3}x} dx$$

$$= \frac{1}{3} \left[\frac{e^{-\frac{1}{3}x}}{-\frac{1}{3}} \right]_5^\infty$$

$$= \frac{1}{\cancel{3}} \times \frac{-\cancel{3}}{1}\left[e^{-\frac{1}{3}x}\right]_5^{\infty}$$

$$= e^{-5/3}$$

(ii) $P[2 \leq x \leq 6] = \int_2^6 f(x)\,dx$

$$= \frac{1}{3}\int_2^6 e^{-\frac{1}{3}x}\,dx$$

$$= \frac{1}{3}\left[\frac{e^{-\frac{1}{3}x}}{-\frac{1}{3}}\right]_2^6$$

$$= \frac{1}{\cancel{3}} \times \frac{-\cancel{3}}{1}\left[e^{-\frac{1}{3}x}\right]_2^6$$

$$= e^{-\frac{2}{3}} - e^{-\frac{6}{3}}$$

$$= e^{-\frac{2}{3}} - e^{-2}$$