Problems based Discrete Probability Distribution:-

Discrete Uniform Distribution:-

1. If an unbiased die is thrown once and x is equal to number on the die.

Solution:- Here $x = 1, 2, 3, 4, 5, 6$ and

$P(x = i) = \frac{1}{6}$ for $i = 1, 2, 3, 4, 5, 6$ and

$x \sim U(6)$ ✓

$$\text{Mean} = \mu = E(x) = \frac{n+1}{2} = \frac{6+1}{2} = \frac{7}{2}$$

$$\text{Variance} = \sigma^2 = \frac{n^2-1}{12} = \frac{36-1}{12} = \frac{35}{12}$$

2. Four fair coins are tossed. If the outcomes are assumed to be independent, then find p.m.f, C.D.F of the number of heads obtained.

Solution:- Let x be the no. of heads in tossing 4 coin

Then $x \sim B(4, \frac{1}{2})$ where $p = P(\text{head}) = \frac{1}{2}$

$q = 1 - p = \frac{1}{2}$

$$p(x) = P(x = x) = {}^nC_x \, p^x \, q^{n-x}, \quad x = 0, 1, 2, 3, 4 \ldots n$$

$p(x) = P(X=x) = {}^4C_x \, p^x q^{4-x}, \quad x = 0,1,2,3,4$

Then $p(0) = {}^4C_0 \, p^0 q^{4-0} = \left(\frac{1}{2}\right)^4 = \frac{1}{16}$

$p(1) = 4C_1 \, p^1 q^{4-1} = \frac{4}{16}$

$p(2) = 4C_2 \, p^2 q^{4-2} = \frac{6}{16}$

$p(3) = 4C_3 \, p^3 q^{4-3} = \frac{4}{16}$

$p(4) = 4C_4 \, p^4 q^{4-4} = \frac{1}{16}$

Then p.m.f and CDF

x	p.m.f $p(x)$	C.D.F
0	$1/16$	$1/16$
1	$4/16$	$\frac{1}{16} + \frac{4}{16} = 5/16$
2	$6/16$	$\frac{5}{16} + \frac{6}{16} = 11/16$
3	$4/16$	$\frac{11}{16} + 4/16 = 15/16$
4	$1/16$	$15/16 + 1/16 = \frac{16}{16} = 1$

3. The mean and Variance of B.D are 4 and $\frac{4}{3}$ respectively. Find $p(x \geq 1)$

Sol:- Given mean $= np = 4$

Variance $= npq = \frac{4}{3}$

$\Rightarrow 4q = \frac{4}{3}$

$q = \frac{1}{3}$

$p = 1 - q = 1 - \frac{1}{3} = \frac{2}{3}$

$\therefore np = 4$

$n\left(\frac{2}{3}\right) = 4$

$n = 6$

$\therefore n = 6, p = \frac{2}{3}, q = \frac{1}{3}$

$P(x \geq 1) = 1 - p(x < 1)$

$= 1 - p(x = 0)$

$= 1 - 6C_0 \, p^0 q^{6-0}$

$= 1 - \left(\frac{1}{3}\right)^6$

$\underline{\underline{\quad}}$ x $=$

4. One hundred ball are tossed into 50 boxes. What is the expected (mean) number of balls in the tenth box.

Here $p = \frac{1}{50}$ $q = 1 - \frac{1}{50} = \frac{49}{50}$

X = the no. of balls that go into the tenth box

Then $X \sim B(100, \frac{1}{50})$

mean of the Binomial distribution $= np$

$$= 100 \times \frac{1}{50} = 2$$

$p = \frac{1}{50}$

$n = 100$

Mean $= np$

$= 100 \times \frac{1}{50}$

$= 2$

$X \sim B(100, \frac{1}{50})$

5. The probability of a man hitting a target is $\frac{1}{4}$

i) If he fires 7 times, what is the probability of his hitting the target at least twice?

ii) How many times must he fire so that the probability of hitting the target at least once is greater than $\frac{2}{3}$?

Solution: Let X = no. of times a man hitting the target in 7 fires.

$p = P$(man hitting the target) $= \frac{1}{4}$

$q = 1 - p = 1 - \frac{1}{4} = \frac{3}{4}$.

$n = 7$ then $X \sim B(7, \frac{1}{4})$

$p(x) = p(X=x) = {}^7C_x p^x q^{7-x}$, $x = 0, 1, 2, 3, 4, 5, 6, 7$

i) P (at least two hits)

$= p(X \geq 2)$

$= 1 - p(X < 2)$

$= 1 - [p(X=0) + p(X=1)]$

$= 1 - [{}^7C_0 (\frac{1}{4})^0 (\frac{3}{4})^{7-0} +$

${}^7C_1 (\frac{1}{4})^1 (\frac{3}{4})^6]$

$= \frac{4547}{8192} = 0.55$

ii) Find n such that $p(X \geq 1) > \frac{2}{3}$

$\Rightarrow 1 - p(X < 1) > \frac{2}{3}$

$\Rightarrow 1 - P(X=0) > \frac{2}{3}$

$\Rightarrow -1 + P(X=0) < -\frac{2}{3}$

$\Rightarrow P(X=0) < \frac{1}{3}$

$\Rightarrow {}^nC_0 (\frac{1}{4})^0 (\frac{3}{4})^{n-0} < \frac{1}{3}$

$$\Rightarrow \left(\frac{3}{4}\right)^n < \frac{1}{3}$$

$$\Rightarrow \log\left(\frac{3}{4}\right)^n < \log\left(\frac{1}{3}\right)$$

$$\Rightarrow n \log\left(\frac{3}{4}\right) < \log\left(\frac{1}{3}\right)$$

$$\Rightarrow n[\log 3 - \log 4] < \log 1 - \log 3$$

$$\Rightarrow n[\log 4 - \log 3] > \log 3$$

$$\Rightarrow n > \frac{\log 3}{\log 4 - \log 3} = 3.8$$

$\because$ n cannot be fractional.

$\therefore$ Required no. of shots is 4

=x=

6 Messages arrive at a switchboard in a poisson manner at an average rate of six per hour. Find the probability for each of the following events:

a) Exactly two messages arrive within one hour

b) No messages arrive within one hour

c) At least three messages arrive within one hour.

Solution: Let X = the no. of messages arriving at the switchboard within a one hour interval

Then $X \sim P(\lambda)$

$X \sim P(6)$

i.e $p(x) = P(X=x) = \dfrac{e^{-\lambda} \cdot \lambda^{x}}{x!}$

Given $\lambda = 6$

$= \dfrac{e^{-6} \cdot 6^{x}}{x!}$

i) $P(X=2) = \dfrac{e^{-6} \cdot 6^{2}}{2!} = \dfrac{\overset{18}{\cancel{36}}\, e^{-6}}{\cancel{2}} = 18e^{-6}$

ii) $P(X=0) = \dfrac{e^{-6} \cdot 6^{0}}{0!} = e^{-6}$

iii) $P(X \geq 3) = 1 - P(X<3)$

$= 1 - [P(X=0) + P(X=1) + P(X=2)]$

$= 1 - \left[\dfrac{e^{-6} \cdot 6^{0}}{0!} + \dfrac{e^{-6} \cdot 6^{1}}{1!} + \dfrac{e^{-6} \cdot 6^{2}}{2!}\right]$

$= 1 - e^{-6}[1 + 6 + 18]$

$= 1 - 25e^{-6}$

7. If X and Y are independent poisson variate such that $P(X=1)=P(X=2)$ and $P(Y=2)=P(Y=3)$. find the variance of $(X-2Y)$

Solution: $X \sim P(\lambda)$

$Y \sim P(\mu)$

then $$P(X=x)=\frac{e^{-\lambda}\cdot\lambda^{x}}{x!}, \quad x=0,1,2\cdots, \lambda>0$$

$$P(Y=y)=\frac{e^{-\mu}\cdot\mu^{y}}{y!}, \quad y=0,1,2\cdots, \mu>0$$

$\because$ $$P(X=1)=\frac{e^{-\lambda}\cdot\lambda^{1}}{1!}=e^{-\lambda}\cdot\lambda$$

$$P(X=2)=\frac{e^{-\lambda}\cdot\lambda^{2}}{2!}=\frac{e^{-\lambda}\lambda^{2}}{2}$$

Given $P(X=1)=P(X=2)$

$$\cancel{e^{-\lambda}}\cdot\cancel{\lambda}=\frac{\cancel{e^{-\lambda}}\cdot\lambda^{\cancel{2}}}{2!}$$

$\Rightarrow$ $\lambda=2$

$\because$ $P(Y=2)=P(Y=3)$

$$\Rightarrow \frac{\cancel{e^{-\mu}}\cdot\cancel{\mu^2}}{2!}=\frac{\cancel{e^{-\mu}}\cdot\cancel{\mu}^{3}}{3!}$$

$\Rightarrow \quad \therefore \mu = 3$

$(\because V(aX) = a^2 V(X))$

$V(X-2Y) = V(X) + (-2)^2 V(Y)$

$= 2 + 4(3)$ ($\because$ mean and varia are equal in poisson distribution)

$= 2 + 12$

$= 14$