Problems based on Bay's theorem (2)

In a railway reservation office, two clerks are engaged in checking reservation forms. On an average, the first clerk 55% of the forms while second does the remaining. The first clerk has an error rate of 0.03 and second has an error rate of 0.02. A reservation form is selected at random from the total number of forms checking during a day, and is found to have an error. Find the probability that it was checked by

i) the first clerk
ii) the second clerk.

Solution:-

Let E_1 = The selected form is checked by clerk I

E_2 = The selected form is checked by clerk-II.

E = The selected form has an error.

$P(E_1) = 55\% = 0.55$ $\qquad$ $P(E/E_1) = 0.03$

$P(E_2) = 45\% = 0.45$ $\qquad$ $P(E|E_2) = 0.02$

$$P(\text{Error}) = P(E) = P(E_1)\cdot P(E|E_1) + P(E_2)\cdot P(E|E_2)$$

$$= (0.55)(0.03) + (0.45)(0.02)$$

$$= 0.0255$$

By Using Bayes theorem

$$P(E_1|E) = \frac{P(E_1)\cdot P(E|E_1)}{P(E)}$$

$$= \frac{(0.55)(0.03)}{0.0255}$$

$$= 0.647$$

$$P(E_2|E) = \frac{P(E_2)\cdot P(E|E_2)}{P(E)}$$

$$= \frac{(0.45)(0.02)}{0.0255}$$

$$= 0.353$$

The result of an investigation by an expert on a fire accident in a skyscraper are summarized below

(i) Prob (there could have been short circuit) = 0.8

(ii) Prob (LPG cylinder explosion) = 0.2

(iii) Chance of fire accident is 30% given a short circuit and 95% given an LPG explosion.

Based on these, what do you think is the most probable cause of fire

Solution:-

E_1 = short circuit $\Rightarrow P(E_1) = 0.8$

E_2 = LPG explosion $\Rightarrow P(E_2) = 0.2$

E = Fire accident

$P(E|E_1) = 30\% = 0.30$

$P(E|E_2) = 95\% = 0.95$

$$P(\text{Fire accident}) = P(E)$$

$$= P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)$$

$$= (0.8)(0.30) + (0.2)(0.95)$$

$$= 0.43$$

By Bay's theorem

$$P(E_1|E) = \frac{P(E_1)\cdot P(E|E_1)}{P(E)}$$

$$= \frac{(0.8)(0.30)}{0.43}$$

$$= 0.558$$

$$P(E_2|E) = \frac{P(E_2)\cdot P(E|E_1)}{P(E)}$$

$$= \frac{(0.2)(0.95)}{0.43}$$

$$= 0.44$$

$$P(E_1|E) > P(E_2|E)$$

short circuit is the most probable cause of fire

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There are three boxes I, II, III.

Box I contains 4 Red, 5 blue, 6 white

Box II contains 3 Red, 4 blue, 5 white

Box III contains 5 Red, 10 blue, 5 white

One box is chosen and one ball is drawn from it what is the probability that

(i) Red ball is Drawn

(ii) Red ball is Drawn from Box I

(iii) Red ball is Drawn from Box II

(iv) Red ball is Drawn from Box III.

Solution:-

E_1 = event that Box I

E_2 = event that Box II

E_3 = event that Box III

$\therefore P(E_1) = \frac{1}{3}, \quad P(E_2) = \frac{1}{3}, \quad P(E_3) = \frac{1}{3}$

Let E = event that the red ball is Selected

$P(E|E_1) = \frac{4}{15}$ ($\because$ the total 15 balls in box I)

$P(E|E_2) = \frac{3}{12}$ ($\because$ the total 12 balls in box II.

$P(E|E_3) = \frac{5}{20}$ ($\because$ the total 20 balls in box III)

$$P(E) = P(E_1) \cdot P(E|E_1) + P(E_2) \cdot P(E|E_2) + P(E_3) \cdot P(E|E_3)$$

$$= \frac{1}{3}\,\frac{4}{15} + \frac{1}{3}\,\frac{3}{12} + \frac{1}{3}\,\frac{5}{20}$$

$$= \frac{1}{3}\left[\frac{4}{15} + \frac{3}{12} + \frac{5}{20}\right]$$

$$= 0.253$$

By Bay's theorem

$$P(E_1|E) = \frac{P(E_1) \cdot P(E|E_1)}{P(E)}$$

$$= \frac{\frac{1}{3} \cdot \frac{4}{15}}{0.253} = 0.342$$

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