## Problems (DRV):-

# A random variable X has the following probability function:

| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(x)$ | 0 | $K$ | $2K$ | $2K$ | $3K$ | $K^2$ | $2K^2$ | $7K^2+K$ |

(i) Determine K

(ii) $P(X<6)$

(iii) $P(X\geq 6)$

(iv) $P(0<X<5)$

(v) $P(0\leq X\leq 4)$

(vi) if $P(X\leq K)>\frac{1}{2}$, find the minimum value of K and

(vii) Determine the distribution function of X (or) CDF

(viii) Mean

(ix) Variance

## Solution:-

(i) We know that $\sum_{x=0}^{7} P(x)=1$

$\Rightarrow P(0)+P(1)+P(2)+P(3)+P(4)+P(5)+P(6)+P(7)=1$

$\Rightarrow 0+K+2K+2K+3K+K^2+2K^2+7K^2+K=1$

$\Rightarrow 10K^2+9K-1=0$

$K=-1,\quad K=\frac{1}{10}$

$\therefore \boxed{K=\frac{1}{10}}$

(ii) $P(x<6) = P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)+P(x=5)$

$= 0 + K + 2K + 2K + 3K + K^2$

$= 8K + K^2$

$= 8\cdot\frac{1}{10} + \left(\frac{1}{10}\right)^2 = \frac{8}{10} + \frac{1}{100} = \frac{81}{100} = 0.81$

(iii) $P(x \geq 6) = 1 - P(x<6)$

$= 1 - 0.81 = 0.19$

(iv) $P(0<x<5) = P(x=1)+P(x=2)+P(x=3)+P(x=4)$

$= K + 2K + 2K + 3K$

$= 8K = \frac{8}{10} = 0.8$

(v) $P(0 \leq x \leq 4) = P(x=0) + P(x=1) + P(x=2) + P(x=3) + P(x=4)$

$= 0 + K + 2K + 2K + 3K$

$= 8K = \frac{8}{10} = 0.8$

(vi) if $P(x \leq k) > \frac{1}{2}$, find the minimum value of $k$

put $k=2$ then $P(x \leq 2) = P(x=0)+P(x=1)+P(x=2)$

$= 0 + K + 2K$

$= 3K = \frac{3}{10} = 0.3 < \frac{1}{2}$

put $k=3$ then $P(x \leq 3) = P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$= 0 + K + 2K + 2K$

$= 5K = \frac{5}{10} = 0.5 = \frac{1}{2}$

put $k=4$ then $P(x \leq 4) = P(x=0)+P(x=1)+P(x=2)+P(x=3)+P(x=4)$

$= 8K = \frac{8}{10} = 0.8 > \frac{1}{2}$

$K = 4$

(viii) To find Mean

$$\mu = E[x] = \sum_{x=0}^{7} x\, p(x)$$

$$= 0\,p(0) + 1\,p(1) + 2p(2) + 3p(3) + 4p(4) + 5p(5) + 6p(6) + 7\,p(7)$$

$$= 0 + 1(K) + 2(2K) + 3(2K) + 4(3K) + 5(K^2) + 6(2K^2) + 7(7K^2 + K)$$

$$= 66K^2 + 30K$$

$$= 66 \cdot \frac{1}{100} + 30\,\frac{1}{10} \quad (\because K = \tfrac{1}{10})$$

$$\mu = \frac{366}{100} = 3.66$$

$$E[x^2] = \sum_{x=0}^{7} x^2\, p(x)$$

$$= 0^2 p(0) + 1^2 p(1) + 2^2 p(2) + 3^2 p(3) + 4^2 p(4) + 5^2 p(5) + 6^2 p(6) + 7^2 p(7)$$

$$= 0 + 1(K) + 4(2K) + 9(2K) + 16(3K) + 25(K^2) + 36(2K^2) + 49(7K^2 + K)$$

$$= 440K^2 + 124K$$

$$= 440\frac{1}{100} + \frac{124}{10} = 4.40 + 12.4$$

$$= 16.80$$

ix) Variance $V(x) = \sigma^2 = E[x^2] - [E(x)]^2$

$$= 16.80 - (3.66)^2 = \underline{\underline{3.4044}}$$

vii) To find CDF

| $x$ | $P(x)$ | CDF $F(x)=P(X\le x)$ |
|---|---|---|
| 0 | 0 | 0 |
| 1 | $K=\frac{1}{10}$ | $0+\frac{1}{10} = \frac{1}{10}$ |
| 2 | $2K=\frac{2}{10}$ | $\frac{1}{10}+\frac{2}{10} = \frac{3}{10}$ |
| 3 | $2K=\frac{2}{10}$ | $\frac{3}{10}+\frac{2}{10} = \frac{5}{10}$ |
| 4 | $3K=\frac{3}{10}$ | $\frac{5}{10}+\frac{3}{10} = \frac{8}{10}$ |
| 5 | $K^2=\frac{1}{100}$ | $\frac{8}{10}+\frac{1}{100} = \frac{81}{100}$ |
| 6 | $2K^2=\frac{2}{100}$ | $\frac{81}{100}+\frac{2}{100} = \frac{83}{100}$ |
| 7 | $7K^2+K= \frac{7}{100}+\frac{1}{10}=\frac{17}{100}$ | $\frac{83}{100}+\frac{17}{100} = \frac{100}{100}=1$ |

# A r.v X has the following probability distribution

| X: | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| P(X): | K | 2K | 3K | 4K | 5K | 6K | 7K | 8K |

Find the values of

i) $K$

ii) $P(X\le 2)$

iii) $P(2\le X\le 5)$

iv) mean

v) Variance.

vi) $F(x)$ (or) CDF.

# A sample of 4 items is selected at random from a box containing 12 items of which 5 are defective. Find the expected number E of defective items

**Solution:-** Let $x$ = no. of defective items.

$= 0, 1, 2, 3, 4$

Here No. of good items $= 7$

No. of defective items $= 5$

$P(x=0) = P(\text{no defective})$

$$= \frac{{}^7C_4}{{}^{12}C_4} = \frac{35}{495} = \frac{7}{99}$$

$P(x=1) = P(\text{one defective \& 3 non defective})$

$$= \frac{{}^5C_3 \cdot {}^7C_1}{{}^{12}C_4} = \frac{175}{495} = \frac{35}{99}$$

$P(x=2) = P(\text{2 defective \& 2 non defective})$

$$= \frac{{}^5C_2 \cdot {}^7C_2}{{}^{12}C_4} = \frac{210}{495} = \frac{42}{99}$$

$P(x=3) = P(\text{3 defectives \& 1 non defective})$

$$= \frac{{}^5C_3 \cdot {}^7C_1}{{}^{12}C_4} = \frac{70}{495} = \frac{14}{99}$$

$P(x=4) = P(\text{All are defective})$

$$= \frac{{}^5C_4}{{}^{12}C_4} = \frac{5}{495} = \frac{1}{99}$$

Probability distribution

| X | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| P(x) | $\frac{7}{99}$ | $\frac{35}{99}$ | $\frac{42}{99}$ | $\frac{14}{99}$ | $\frac{1}{19}$ |

mean $= \mu =$ expected value $= \sum_{x=0}^{4} x\,p(x)$

$= 0\,p(0) + 1\,P(1) + 2P(2) + 3\,P(3) + 4P(4)$

$= 0 + 1\left(\frac{35}{99}\right) + 2\left(\frac{42}{99}\right) + 3\left(\frac{14}{99}\right) + 4\left(\frac{1}{99}\right)$

$= \frac{165}{199}$

$\overline{\phantom{x}}x =$