Problems based on Normal Distribution:-

1). If x is a normally distributed with mean 12 and s.d $= 4$ then

a) Find the probability of the following.

i) $x \geq 20$

ii) $x \leq 20$

iii) $0 \leq x \leq 12$.

b) Find x when $p(x > x) = 0.24$

c) Find x_1 and x_2, when $p(x_1 < x < x_2) = 0.5$ and $p(x > x_2) = 0.25$.

Solution:-

Given $\mu = 12$

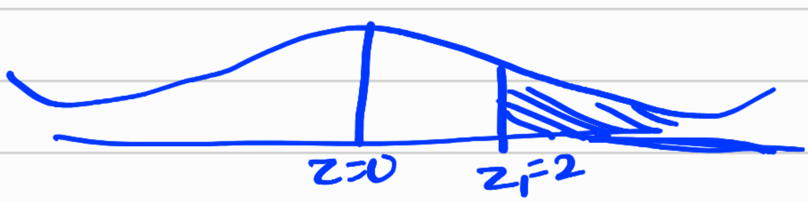
$\sigma = 4 \Rightarrow \sigma^2 = 16$

i.e $X \sim N(12, 16)$

i) Let $z = \frac{x-\mu}{\sigma}$

then $p(x \geq 20) = P\left(\frac{x-12}{4} \geq \frac{20-12}{4}\right)$

$= P(Z \geq 2)$



$= 0.5 - A(z_1)$

$= 0.5 - A(2)$

$= 0.5 - 0.4772$ (table)

$= 0.0228$

ii) $P(X \le 20) = 1 - P(X \ge 20)$

$= 1 - 0.0228$

$= 0.9772$

(iii) $P(0 \le X \le 12) = P\left(\frac{0-12}{4} \le \frac{X-12}{4} \le \frac{12-12}{4}\right)$

$= P(-3 \le Z \le 0)$

$= P(0 \le Z \le 3)$ (by symmetry)

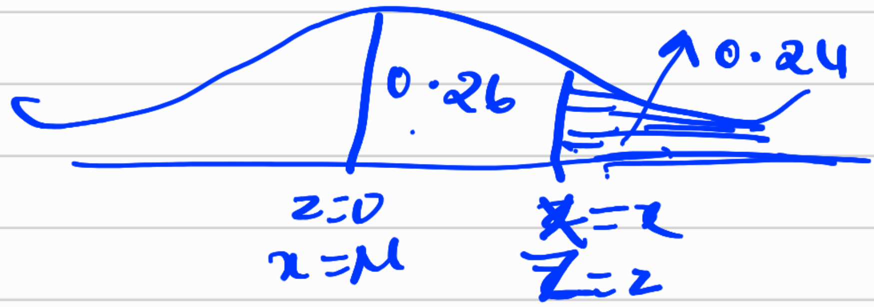
$= A(3)$

$= 0.4986$

b) $P(X > x) = 0.24$

$P\left(\frac{X-12}{4} > \frac{x-12}{4}\right) = 0.24$

$P(Z > z) = 0.24$



∴ From normal tables,
corresponding probability 0.26, values of $z = 0.71$ (approximately)

Hence $z = \frac{x-\mu}{\sigma}$

$$0.71 = \frac{x-12}{4}$$

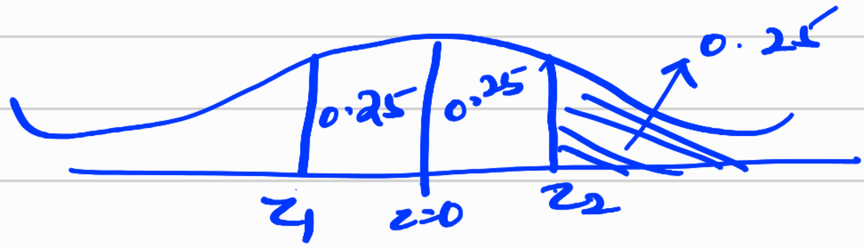
$$\Rightarrow x = 4(0.71)+12$$

$$= 14.84$$

c)

We are given $P(x_1 < x < x_2) = 0.5$, $P(x > x_2) = 0.25$

$\Rightarrow P(z_1 < z < z_2) = 0.5$, $P(z > z_2) = 0.25$



By symmetric of normal curve,

$$z_1 = -z_2$$

Find z_2 such that $P(0 < z < z_2) = 0.25$

Corresponding to probability 0.25 from the normal table, we have $z_2 = 0.67$

$$z_1 = -z_2$$
$$= -0.67$$

$$z_1 = \frac{x_1 - \mu}{\sigma} \qquad z_2 = \frac{x_2 - \mu}{\sigma}$$

$$-0.67 = \frac{x_1 - 12}{4} \qquad 0.67 = \frac{x_2 - 12}{4}$$

$$x_1 = 4(-0.67) + 12, \qquad x_2 = 4(0.67) + 12$$

$$x_1 = 9.32 \qquad x_2 = 14.68$$

— x —

The local authorities in a certain city install 10,000 electric lamps in the streets of the city. If these lamps have an average life of 1,000 burning hours with a s.d of 200 hours, assuming normality, what number of lamps might be expected to fail.

i) in the first 800 burning hours.

ii) between 800 and 1200 burning hours?

After what period of burning hours would you expect that

a) 10% of the lamps would fail?

b) 10% of the lamps would be still burning.?

Solution:-

Let X = Life of a bulb in burning hours.

Here $\mu = 1000$, $\sigma = 200 \Rightarrow \sigma^2 = 40000$

i.e $X \sim N(\mu, \sigma^2)$

$X \sim N(1000, 40,000)$

i) $P(X<800) = P\left(\frac{X-1000}{200} < \frac{800-1000}{200}\right)$

$= P(Z < -1)$

$= 0.5 - A(z_1)$

$= 0.5 - 0.3413$ (by symmetric)

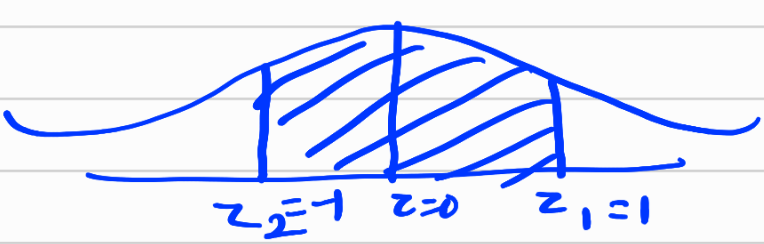
$= 0.1587$

$\therefore$ Out of 10,000 bulbs, number of bulbs which fail in the first 800 hours is $10{,}000 \times 0.1587 = 1{,}587$

ii) $P(800 < X < 1200)$

$= P\left(\frac{800-1000}{200} < \frac{X-1000}{200} < \frac{1200-1000}{200}\right)$

$= P(-1 < Z < 1)$



$= 2A(z_1) = 2P(0 < Z < 1)$

$= 2 \times 0.3413 = 0.6826.$

Hence, the expected number of bulbs with life between 800 and 1200 hours of burning life is

$$10\,000 \times 0.6826 = 6,826$$

a) Let 10% of the bulbs fail after x_1 hours of burning life.

$$P(X < x_1) = 0.1$$

$$P(Z < z_1) = 0.1$$

$z_1 = -1.28$ (from table)

$$\frac{x_1 - \mu}{\sigma} = -1.28$$

$$\frac{x_1 - 1000}{200} = -1.28$$

$$x_1 = -(1.28)200 + 1000$$

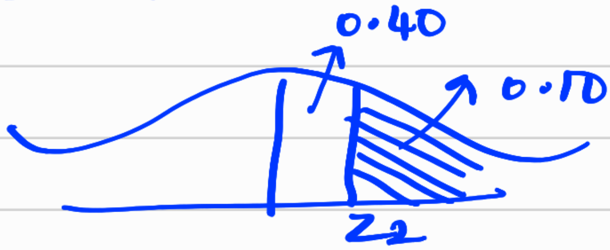
$$= 744.$$

$\therefore$ After 744 hours of burning life, 10% of the bulbs will fail

b) Let 10% of the bulbs be still burning after x_2 hours of burning life.

$P(X > x_2) = 0.10$

$P(Z > z_2) = 0.10$



Here $Z_2 = 1.28$

$$\frac{x_2 - \mu}{\sigma} = 1.28$$

$$\frac{x_2 - 1000}{200} = 1.28$$

$$x_2 = (1.28)200 + 1000$$

$$= 1256.$$

$\therefore$ after 1,256 hours of burning life,
10% of the bulbs will be still burning