Expectation, Mean, Variance and standard deviation of a Probability distribution :-

1) Expectation of DRV :-

Suppose a r.v X assumes the values $x_1, x_2, x_3 \dots x_n$ with respective probabilities $p_1, p_2, p_3 \dots p_n$. The the Mathematical expectation or mean or expected value of X, denoted by $E(X)$, is defined as the sum of product of different values of x and the corresponding probabilities.

$$\therefore E(X) = x_1 P_1 + x_2 P_2 + \dots + x_n P_n$$

$$= \sum_{i=1}^{n} x_i P_i \quad (\text{or}) \quad \sum_{i=1}^{n} x_i \, p(x_i)$$

$$E(X^r) = \sum_{i=1}^{n} x_i^r \, p(x_i).$$

Note:- Expected Values of X is a population mean

If population mean is μ then $E(X)=\mu$.

Properties of Expectations:-

i) If X is a random variable and K is a constant then

a) $E(X+K)=E(X)+K$

b) $E[KX]=KE[X]$.

ii) If X is a r.v and a, b are constat

then $E[aX\pm b]=aE[X]\pm b$.

iii) If X, Y are two R.V then

$E[X+Y]=E[X]+E[Y]$

provided $E(X)$, $E(Y)$ exists

$E[XY]=E(X)E(Y)$

if X, Y two independent R.V.

Note:- $E\left[\frac{1}{X}\right]$ and $\frac{1}{E(X)}$ are not same.

2) Mean:-

The mean value μ of the discrete distribution function is given by

$$\mu = \frac{\sum p_i x_i}{\sum p_i}$$

$$= \sum p_i x_i \quad (\because \sum p_i = 1)$$

$$= E[x]$$

$$\therefore E[x] = \mu$$

Note:- If $E(x) = \mu$, then

$$E[x - \mu] = E[x] - \mu$$
$$= \mu - \mu$$
$$= 0$$

3). Variance:-

Variance of the probability distribution of a r.v x is the Mathematical expectation of $[x - E(x)]^2$ then

$$Var(x) = E[(x - E(x))^2]$$

i.e $Var(x) = \sum_{i=1}^{n} \{[x_i - E(X)]^2 p(x_i)$

Another form of Variance:

If X is a r.v, then the mathematical expectation of $(x-\mu)^2$ is defined to be the variance of the random variable X. Then

$$\sigma^2 = Var(x) = E[(x-\mu)^2]$$

$$= \sum_{i=1}^{n} (x_i - \mu)^2 P_i \qquad \boxed{P_i = p(x_i)}$$

$$= \sum_{i=1}^{n} (x_i^2 + \mu^2 - 2x_i\mu) P_i$$

$$= \sum_{i=1}^{n} x_i^2 P_i + \mu^2 - 2\mu^2$$

$$= E[X^2] - \mu^2$$

$$= E[X^2] - [E(X)]^2$$

— x —

Note:- The Variance of a r.V X is also denoted by $V(X)$

4) Standard deviation:-

It is the +ve square root of the Variance

$$S.d. = \sigma = \sqrt{\text{Variance}}$$

$$= \sqrt{V(X)}$$

Properties of V(X):-

1) Variance of constant is zero

i.e $V(K) = 0$

2) If K is a constant, then

$$V(KX) = K^2 V(X).$$

3) If X is a R.V. and K is const. then $V(X+K) = V(X)$

4) If X is a DRV, then

$$V(aX+b) = a^2 V(X).$$

where $V(x)$ is a variance and a, b are constant

if $b=0$ then $V(ax) = a^2$

If $a=0$ then $V(b) = 0$

if $a=1$ then $V(x+b) = V(x)$

5) If x, y are two independent r.v then

$$V(x \pm y) = V(x) \pm V(y)$$

Note :-

Probability of events :-

If in a random experiment, the event corresponding to a number 'a' occurs, then the corresponding r.v x is said to assume the value a and probability of the event is denoted by $P(x=a)$

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→ The probability of the event x assuming any value between a and b is denoted by $P(a < x < b)$

→ The probability of the event x assumes any value smaller than or equal to c and denoted by
$$P(x \leq c$$

— 1. $P(x \leq c) + P(x > c) = P(-\infty < x < \infty)$

2. $P(x > c) = 1 - P(x \leq c)$

3. $P(x = a \text{ or } x = b)$
$$= P\{(x = a) \cup (x = b)\}$$

4. $P(x = a \text{ and } x = b)$
$$= P\{(x = a) \cap (x = b)\}$$