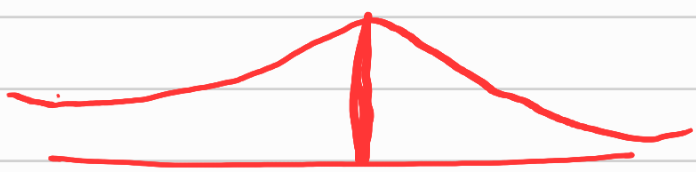
Area under the Normal curve (Normal Probability Integral):-

Conversion into standard Normal form:

By taking $z = \frac{x-\mu}{\sigma}$, then

the standard normal curve is formed. the total area under this curve is 1. The area under the curve is divided into two equal parts by $z=0$.



$z=0$

Left hand side area and right hand side area to $z=0$ is 0.5.

The area between the ordinate $z=0$ and any other ordinate can be noted from the table

The probability that r.v X will between $x=\mu$ and $X=x_1$ given by

$$P(\mu < X < x_1) = \int_{\mu}^{x_1} f(x)\,dx$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_{\mu}^{x_1} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

$$= \frac{1}{\cancel{\sigma}\sqrt{2\pi}} \int_{0}^{z_1} e^{-\frac{z^2}{2}} \cancel{\sigma}\, dz$$

$$\left(\text{put } z = \frac{x-\mu}{\sigma},\ \frac{x_1-\mu}{\sigma} = z_1\right)$$

Then

$$P(\mu < X < x_1) = P(0 < z < z_1) = \frac{1}{\sqrt{2\pi}} \int_{0}^{z_1} e^{-\frac{z^2}{2}} dz$$

$$= A(z_1) \text{ say}$$

i.e $A(z_1) = \frac{1}{\sqrt{2\pi}} \int_{0}^{z_1} e^{-\frac{z^2}{2}} dz$

The function $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$ is the probability function of standard normal variate

The definite integral $\int_0^{z_1} e^{-\frac{z^2}{2}} dz$ is known as the normal probability integral and gives the under standard normal curve between $z=0$ and $z=z_1$

In Perticular

The Probability of a r.v x lies in the interval $(\mu-\sigma, \mu+\sigma)$ is given by

$$P(\mu-\sigma, \mu+\sigma) = \int_{\mu-\sigma}^{\mu+\sigma} f(x)\, dx$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma} \int_{\mu-\sigma}^{\mu+\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx.$$

put $z = \frac{x-\mu}{\sigma}$

$$z_1 = \frac{\mu-\sigma-\mu}{\sigma} = \frac{-\sigma}{\sigma} = -1$$

$$z_2 = \frac{\mu+\sigma-\mu}{\sigma} = \frac{\sigma}{\sigma} = 1.$$

$$\therefore P(-1 < Z < 1) = \frac{1}{\sqrt{2\pi}} \int_{-1}^{1} e^{-\frac{z^2}{2}} dz$$

$$= \frac{2}{\sqrt{2\pi}} \int_0^1 e^{-\frac{z^2}{2}} dz$$

$$= 2\int_0^1 \phi(z)\,dz.$$

where $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}}$

$= 2(0.3413)$ (from table).

$= 0.6826$

$=$

Note:- $\because$ the integrand is an even function it follows that $A(-z_1) = A(z_1)$.

Illy $P(\mu - 2\sigma < x < \mu + 2\sigma)$

$= P(-2 < x < 2)$

$$= \int_{-2}^{2} \phi(z)\,dz$$

$$= 2\int_0^2 \phi(z)\,dz$$

$= 2(0.4772)$ (from table)

$= 0.9544.$

$=$

How to find Probability Density of Normal curve :-

The probability that the normal variate x with mean μ and standard deviation σ lies between two specific values x_1, x_2 with $x_1 \le x_2$ can be obtained using area under the standard normal curve as follows.

step:-1 Perform the change scale $z = \frac{x-\mu}{\sigma}$

and find z_1 and z_2 corresponding to the values of x_1 and x_2 respectively.

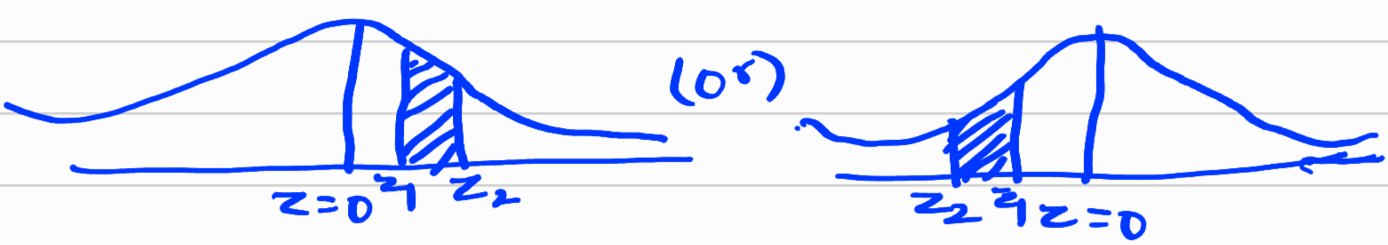
step-2 (a)

To find $P(x_1 \le x \le x_2) = P(z_1 \le Z \le z_2)$

Case:-1 : If both z_1 and z_2 are +ve (or both −ve)

Then $P(x_1 \le x \le x_2) = A(z_2) - A(z_1)$

$=$ (Area under the normal curve from 0 to z_2) − (Area under the normal curve from 0 to z_1)



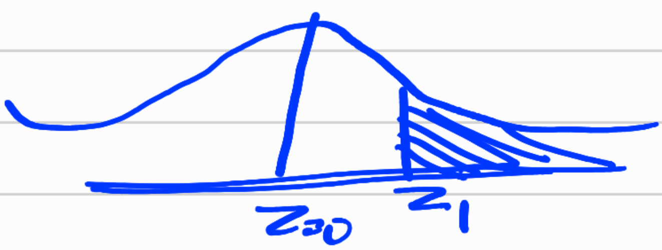
Case:-2 If both $z_1 < 0$, $z_2 > 0$ then

$$P(x_1 \le X \le x_2) = A(z_2) + A(z_1)$$

Step:- 2(b):

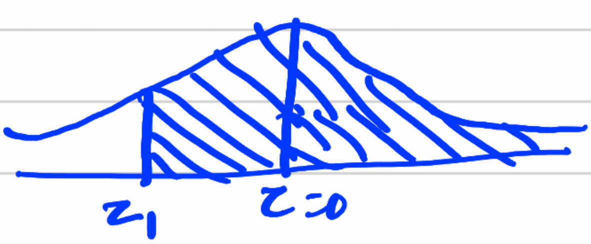
To find $P(z > z_1)$:-

Case:-1 If $z_1 > 0$ then



$$P(Z > z_1) = 0.5 - A(z_1)$$

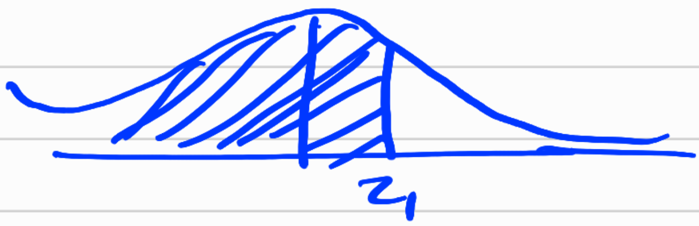
Case:-2 If $z_1 < 0$ then



$$P(Z > z_1) = 0.5 + A(z_1)$$

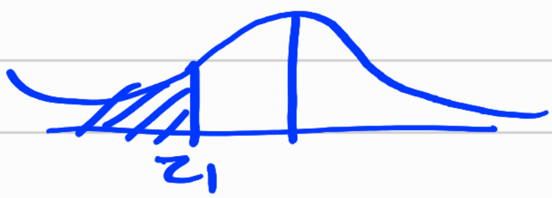
Step:- 2(c) To find $P(Z < z_1)$

Case:-1 If $z_1 > 0$.



$P(Z < z_1) = 0.5 + A(z_1)$

Case:-2: If $z_1 < 0$.



$P(Z < z_1) = 0.5 - A(z_1)$