

Week 3 作業

Cobb-Douglas 生產函數

$$Q = f(L, K) = L^\alpha K^\beta, \quad \alpha, \beta > 0$$

產出彈性 (勞動)

$$AP_L = \frac{Q}{L} = \frac{L^\alpha K^\beta}{L} = L^{\alpha-1} K^\beta$$

$$MP_L = \frac{dQ}{dL} = \alpha L^{\alpha-1} K^\beta$$

$$\epsilon^L = \frac{MP_L}{AP_L} = \frac{\alpha L^{\alpha-1} K^\beta}{L^{\alpha-1} K^\beta} = \alpha$$

(資本)

$$AP_K = \frac{Q}{K} = \frac{L^\alpha K^\beta}{K} = L^\alpha K^{\beta-1}$$

$$MP_K = \frac{dQ}{dK} = \beta L^\alpha K^{\beta-1}$$

$$\epsilon^K = \frac{MP_K}{AP_K} = \frac{\beta L^\alpha K^{\beta-1}}{L^\alpha K^{\beta-1}} = \beta$$

生產力彈性

$$Q = f(\phi L, \phi K) = \phi^{\alpha+\beta} L^\alpha K^\beta$$

$$\epsilon^\phi = \frac{\frac{dQ}{d\phi}}{\frac{Q}{\phi}} = \frac{(\alpha+\beta)\phi^{\alpha+\beta-1} L^\alpha K^\beta}{\phi^{\alpha+\beta-1} L^\alpha K^\beta} = \alpha + \beta$$

替代彈性 (邊際技術替代率)

$$MRTS = \frac{MP_L}{MP_K} = \frac{\alpha L^{\alpha-1} K^\beta}{\beta L^\alpha K^{\beta-1}} = \frac{\alpha L^{-1}}{\beta K^{-1}} = \frac{\alpha}{\beta} \cdot \frac{K}{L}$$

替代彈性

$$\epsilon_{LK} = \frac{d \ln \left(\frac{K}{L} \right)}{d \ln (MRTS)} = \frac{d \ln \left(\frac{K}{L} \right)}{d \ln \left(\frac{\alpha}{\beta} \right) + d \ln \left(\frac{K}{L} \right)} = 1$$

$$Q = 3K + 2L$$

若 K & L 加倍

$$xQ = x(3K + 2L)$$

$$xQ = x(3K + 2L) \quad \text{, 固定规模報酬}$$

- ① 函數呈現固定規模報酬 (正確)
- ② 函數呈現 MPK 与 MPL 遞減 (不正確)
- ③ 函數呈現固定技術替代率 (正確)