Assignment_4

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```
# 'Benchmarking' package is required for running the DEA functions.
#require(Benchmarking)
#install.packages("Benchmarking")
library(Benchmarking)
## Warning: package 'Benchmarking' was built under R version 4.1.3
## Loading required package: lpSolveAPI
## Warning: package 'lpSolveAPI' was built under R version 4.1.3
## Loading required package: ucminf
## Warning: package 'ucminf' was built under R version 4.1.3
## Loading required package: quadprog
##
## Loading Benchmarking version 0.30h, (Revision 244, 2022/05/05 16:31:31) ...
## Build 2022/05/05 16:31:40
Loading the Data
# Create matrix for the two inputs
X \leftarrow \text{matrix}(c(150, 400, 320, 520, 350, 320, 0.2, 0.7, 1.2, 2.0, 1.2, 0.7), \text{ncol} = 2)
# Create matrix for the two outputs
Y <- matrix(c(14000, 14000, 42000, 28000, 19000, 14000, 3500, 21000, 10500, 42000, 25000, 15000), ncol
# Name the columns of the inputs and outputs
colnames(X) <- c("Staff Hours per Day", "Supplies per Day")</pre>
colnames(Y) <- c("Reimburse Patient-Days", "Privately Paid Patient-Days")</pre>
# Returning the matrices
print(X)
        Staff Hours per Day Supplies per Day
## [1,]
                         150
                                           0.2
## [2,]
                                          0.7
                         400
## [3,]
                         320
                                          1.2
## [4,]
                         520
                                          2.0
## [5,]
```

1.2

0.7

350

320

[6,]

print(Y)

```
##
        Reimburse Patient-Days Privately Paid Patient-Days
## [1,]
                          14000
                                                          3500
## [2,]
                          14000
                                                         21000
## [3,]
                          42000
                                                         10500
## [4,]
                          28000
                                                         42000
## [5,]
                           19000
                                                         25000
## [6,]
                          14000
                                                         15000
```

1. Formulate and perform DEA analysis under all DEA assumptions of FDH, CRS, VRS, IRS, DRS, and FRH. 2. Determine the Peers and Lambdas under each of the above assumptions

The below code will return the results of DEA using FDH method.

```
# DEA code utilizing the FDH method
FDH <- rep("FDH", times = 6)
Not_Applicable <- rep(NA, times = 6)
DEA_FDH <- dea(X, Y, RTS = "FDH")
DEA_FDH_Peers <- peers(DEA_FDH)
DEA_FDH_Lambda <- lambda(DEA_FDH)
print(DEA_FDH)</pre>
```

[1] 1 1 1 1 1 1

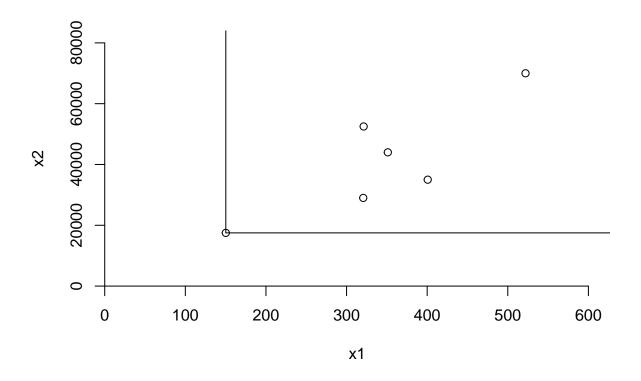
print(DEA_FDH_Peers)

```
## [1,] 1
## [2,] 2
## [3,] 3
## [4,] 4
## [5,] 5
## [6,] 6
```

print(DEA_FDH_Lambda)

```
## L1 L2 L3 L4 L5 L6
## [1,] 1 0 0 0 0 0 0
## [2,] 0 1 0 0 0 0
## [3,] 0 0 1 0 0 0
## [4,] 0 0 0 1 0 0
## [5,] 0 0 0 0 1 0
## [6,] 0 0 0 0 1 1
```

```
dea.plot.isoquant(X, Y, RTS= "FDH")
```



```
# Summarize the results for addition to a summary table
DEA_FDH_Peers <- cbind(DEA_FDH_Peers, Not_Applicable, Not_Applicable)</pre>
FDH_Summary <- cbind(FDH, DEA_FDH$eff, DEA_FDH_Peers, DEA_FDH_Lambda)
colnames(FDH_Summary) <- c("Method", "Eff", "P1", "P2", "P3", "L1", "L2", "L3", "L4", "L5", "L6")
print(FDH_Summary)
        Method Eff P1 P2 P3 L1 L2 L3 L4 L5
##
               "1" "1" NA NA "1" "0" "0" "0" "0" "0"
## [1,] "FDH"
  [2,] "FDH"
               "1" "2" NA NA "0" "1" "0" "0" "0" "0"
## [3,] "FDH"
               "1" "3" NA NA "0" "0" "1" "0" "0" "0"
               "1" "4" NA NA "0" "0" "0" "1" "0" "0"
## [4,] "FDH"
               "1" "5" NA NA "0" "0" "0" "0" "1" "0"
## [5,] "FDH"
```

The below code will return the results of DEA using CRS method.

[6,] "FDH"

"1" "6" NA NA "0" "0" "0" "0" "1"

```
# DEA code utilizing the CRS method
CRS <- rep("CRS", times = 6)
DEA_CRS <- dea(X, Y, RTS = "CRS")
DEA_CRS_Peers <- peers(DEA_CRS)
DEA_CRS_Lambda <- lambda(DEA_CRS)
print(DEA_CRS)</pre>
```

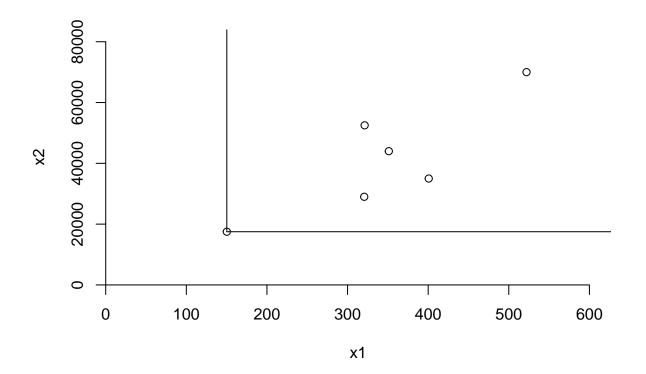
[1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675

print(DEA_CRS_Peers)

```
##
          peer1 peer2 peer3
## [1,]
               1
                     NA
                             NA
## [2,]
               2
                     NA
                             NA
## [3,]
               3
                     {\tt NA}
                             {\tt NA}
## [4,]
               4
                     {\tt NA}
                             NA
## [5,]
                      2
                              4
               1
## [6,]
               1
                       2
                              4
```

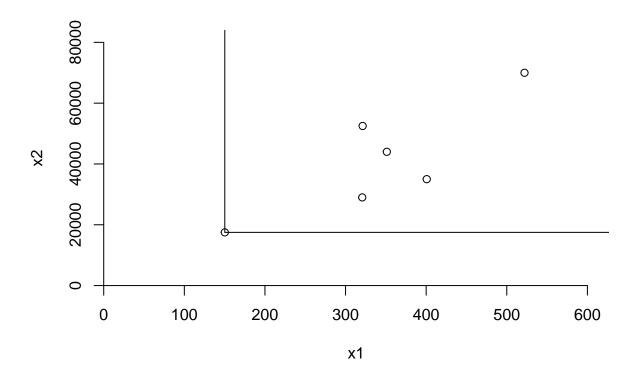
print(DEA_CRS_Lambda)

dea.plot.isoquant(X, Y, RTS= "CRS")



```
# Summarize the results for addition to a summary table
DEA_CRS_Lambda <- cbind(DEA_CRS_Lambda, Not_Applicable, Not_Applicable)</pre>
CRS Summary <- cbind(CRS, DEA CRS$eff, DEA CRS Peers, DEA CRS Lambda)
colnames(CRS_Summary) <- c("Method", "Eff", "P1", "P2", "P3", "L1", "L2", "L3", "L4", "L5", "L6")
CRS_Summary <- as.data.frame(CRS_Summary)</pre>
CRS Summary
##
    Method
                         Eff P1
                                  P2
                                       Р3
                                                         L1
                                                                            L2 L3
## 1
       CRS
                           1 1 <NA> <NA>
                                                          1
                                                                             0 0
## 2
        CRS
                           1 2 <NA> <NA>
                                                          0
                                                                             1 0
## 3
       CRS
                                                          0
                           1 3 <NA> <NA>
## 4
       CRS
                           1 4 <NA> <NA>
                                                          0
                                                                             0
                                                                               0
## 5
       CRS 0.977498691784406 1
                                   2
                                                        0.2 0.0804814233385661
## 6
       CRS 0.867452135493373 1
                                   2
                                        ##
                   L4 L5
                            L6
## 1
                    O <NA> <NA>
## 2
                    O <NA> <NA>
## 3
                    O <NA> <NA>
## 4
                     1 <NA> <NA>
## 5 0.538330716902146 <NA> <NA>
## 6 0.131075110456554 <NA> <NA>
The below code will return the results of DEA using VRS method.
# DEA code utilizing the VRS method
VRS <- rep("VRS", times = 6)</pre>
DEA VRS <- dea(X, Y, RTS = "VRS")
DEA_VRS_Peers <- peers(DEA_VRS)</pre>
DEA_VRS_Lambda <- lambda(DEA_VRS)</pre>
print(DEA_VRS)
## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
print(DEA_VRS_Peers)
##
       peer1 peer2 peer3
## [1,]
          1
                NA
## [2,]
           2
                NA
                      NA
## [3,]
           3
              NA
                      NA
## [4,]
            4
              NA
                      NA
## [5,]
           5
                NA
                      NA
                 2
## [6,]
           1
                       5
print(DEA_VRS_Lambda)
                        L2 L3 L4
##
              L1
                                        L5
## [1,] 1.0000000 0.0000000 0 0.0000000
## [2,] 0.0000000 1.0000000 0 0.0000000
## [3,] 0.0000000 0.0000000 1 0 0.0000000
## [4,] 0.0000000 0.0000000 0 1 0.0000000
## [5,] 0.0000000 0.0000000 0 0 1.0000000
## [6,] 0.4014399 0.3422606 0 0 0.2562995
```

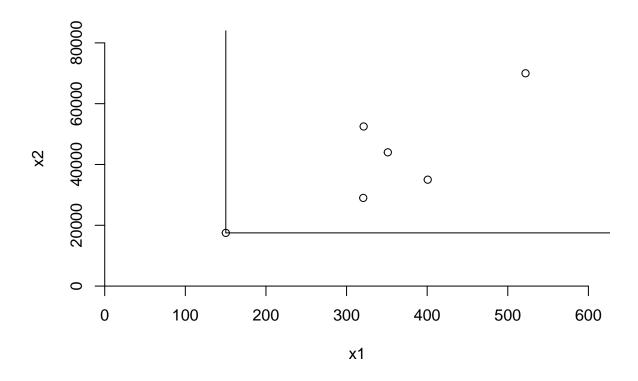
```
dea.plot.isoquant(X, Y, RTS= "VRS")
```



```
# Summarize the results for addition to a summary table
DEA_VRS_Lambda <- cbind(DEA_VRS_Lambda, Not_Applicable)</pre>
VRS_Summary <- cbind(VRS, DEA_VRS$eff, DEA_VRS_Peers, DEA_VRS_Lambda)</pre>
colnames(VRS_Summary) <- c("Method", "Eff", "P1", "P2", "P3", "L1", "L2", "L3", "L4", "L5", "L6")
VRS_Summary <- as.data.frame(VRS_Summary)</pre>
VRS_Summary
##
     Method
                           Eff P1
                                     P2
                                          Р3
                                                              L1
                                                                                 L2 L3
## 1
                                                                                  0
        VRS
                              1
                                 1 <NA> <NA>
                                                               1
                                                                                     0
## 2
        VRS
                                 2 <NA> <NA>
                                                               0
                              1
                                                                                  1
                                                                                     0
## 3
                                                               0
        VRS
                                 3 <NA> <NA>
                                                                                  0
                                                                                     1
                                 4 <NA> <NA>
                                                               0
## 4
        VRS
                              1
                                                                                  0
                                                                                     0
## 5
                                 5 <NA> <NA>
                                                               0
        VRS
                              1
                                                                                  0
                                                                                     0
## 6
        VRS 0.896328293736501
                                      2
                                           5 0.401439884809215 0.342260619150468 0
                                 1
##
     L4
                        L5
                              L6
## 1
     0
                         O <NA>
## 2
      0
                         O <NA>
## 3
      0
                         O <NA>
## 4
                         O <NA>
## 5
      0
                         1 <NA>
     0 0.256299496040317 <NA>
```

The below code will return the results of DEA using IRS method.

```
# DEA code utilizing the IRS method
IRS <- rep("IRS", times = 6)</pre>
DEA_IRS <- dea(X, Y, RTS = "IRS")</pre>
DEA_IRS_Peers <- peers(DEA_IRS)</pre>
DEA_IRS_Lambda <- lambda(DEA_IRS)</pre>
print(DEA_IRS)
## [1] 1.0000 1.0000 1.0000 1.0000 1.0000 0.8963
print(DEA_IRS_Peers)
##
       peer1 peer2 peer3
## [1,]
          1 NA
## [2,]
           2 NA
                      NA
          3 NA
## [3,]
                      NA
## [4,]
          4 NA
                      NA
## [5,]
           5 NA
                      NA
## [6,]
                 2
           1
                       5
print(DEA_IRS_Lambda)
##
              L1
                        L2 L3 L4
## [1,] 1.0000000 0.0000000 0 0 0.0000000
## [2,] 0.0000000 1.0000000 0 0.0000000
## [3,] 0.0000000 0.0000000 1 0 0.0000000
## [4,] 0.0000000 0.0000000 0 1 0.0000000
## [5,] 0.0000000 0.0000000 0 0 1.0000000
## [6,] 0.4014399 0.3422606 0 0 0.2562995
```

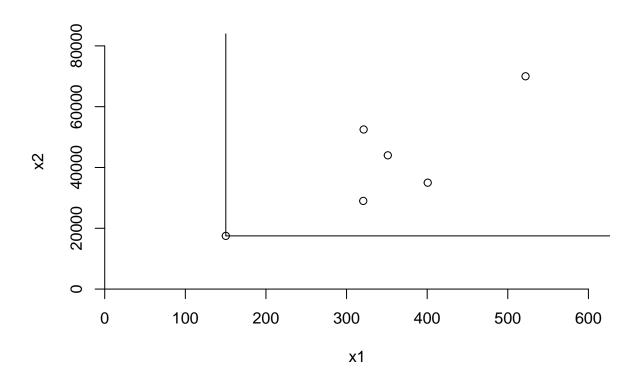


```
# Summarize the results for addition to a summary table
DEA_IRS_Lambda <- cbind(DEA_IRS_Lambda, Not_Applicable)</pre>
IRS_Summary <- cbind(IRS, DEA_IRS$eff, DEA_IRS_Peers, DEA_IRS_Lambda)</pre>
colnames(IRS_Summary) <- c("Method", "Eff", "P1", "P2", "P3", "L1", "L2", "L3", "L4", "L5", "L6")
IRS_Summary <- as.data.frame(IRS_Summary)</pre>
IRS_Summary
##
     Method
                            Eff P1
                                      P2
                                           РЗ
                                                               L1
                                                                                  L2 L3
## 1
        IRS
                              1
                                 1 <NA> <NA>
                                                                1
                                                                                   0
                                                                                      0
## 2
                                 2 <NA> <NA>
                                                                0
        IRS
                              1
                                                                                      0
## 3
        IRS
                                 3 <NA> <NA>
                                                                0
                              1
                                                                                   0
                                                                                      1
                                                                0
## 4
        IRS
                              1
                                 4 <NA> <NA>
                                                                                      0
                                                                                   0
                                                                0
## 5
        IRS
                              1
                                 5 <NA> <NA>
                                                                                   0
                                                                                      0
                                       2
## 6
        IRS 0.896328293736501
                                            5 0.401439884809215 0.342260619150468
##
     L4
                         L5
                              L6
## 1
                          O <NA>
## 2
      0
                          O <NA>
## 3
      0
                          O <NA>
## 4
      1
                          O <NA>
## 5
                          1 <NA>
## 6
      0 0.256299496040317 <NA>
```

The below code will return the results of DEA using DRS method.

```
# DEA code utilizing the DRS method
DRS <- rep("DRS", times = 6)
DEA_DRS <- dea(X, Y, RTS = "DRS")</pre>
DEA_DRS_Peers <- peers(DEA_DRS)</pre>
DEA_DRS_Lambda <- lambda(DEA_DRS)</pre>
print(DEA_DRS)
## [1] 1.0000 1.0000 1.0000 1.0000 0.9775 0.8675
print(DEA_DRS_Peers)
##
       peer1 peer2 peer3
## [1,]
          1 NA
## [2,]
          2 NA
                      NA
          3 NA
## [3,]
                      NA
## [4,]
          4 NA
                      NA
## [5,]
           1
                2
                      4
## [6,]
                 2
                       4
print(DEA_DRS_Lambda)
##
              L1
                         L2 L3
## [1,] 1.0000000 0.00000000 0 0.0000000
## [2,] 0.0000000 1.00000000 0 0.0000000
## [3,] 0.0000000 0.00000000 1 0.0000000
## [4,] 0.0000000 0.00000000 0 1.0000000
## [5,] 0.2000000 0.08048142 0 0.5383307
## [6,] 0.3428571 0.39499264 0 0.1310751
```

dea.plot.isoquant(X, Y, RTS= "DRS")



```
# Summarize the results for addition to a summary table
DEA_DRS_Lambda <- cbind(DEA_DRS_Lambda, Not_Applicable, Not_Applicable)</pre>
DRS_Summary <- cbind(DRS, DEA_DRS$eff, DEA_DRS_Peers, DEA_DRS_Lambda)</pre>
colnames(DRS_Summary) <- c("Method", "Eff", "P1", "P2", "P3", "L1", "L2", "L3", "L4", "L5", "L6")
DRS_Summary <- as.data.frame(DRS_Summary)</pre>
DRS_Summary
##
     Method
                           Eff P1
                                     P2
                                          РЗ
                                                              L1
                                                                                  L2 L3
## 1
        DRS
                             1
                                 1 <NA> <NA>
                                                               1
                                                                                   0
                                                                                      0
## 2
        DRS
                                 2 <NA> <NA>
                                                               0
                                                                                      0
                             1
                                                                                   1
## 3
        DRS
                                 3 <NA> <NA>
                                                               0
                                                                                   0
                             1
                                                                                      1
                                   <NA> <NA>
                                                               0
## 4
        DRS
                                                                                   0
                                                                                      0
                                 4
## 5
        DRS 0.977498691784406
                                1
                                      2
                                                            0.2 0.0804814233385655
## 6
                                      2
        DRS 0.867452135493373
                                1
                                           4 0.342857142857143 0.394992636229749
##
                     L4
                          L5
                                L6
## 1
                      O <NA> <NA>
## 2
                      O <NA> <NA>
## 3
                      O <NA> <NA>
## 4
                      1 <NA> <NA>
## 5 0.538330716902146 <NA> <NA>
## 6 0.131075110456554 <NA> <NA>
```

The below code will return the results of DEA using FRH/ADD method.

```
# DEA code utilizing the ADD method
ADD <- rep("ADD", times = 6)
DEA\_ADD \leftarrow dea(X, Y, RTS = "ADD")
DEA_ADD_Peers <- peers(DEA_ADD)</pre>
DEA ADD Lambda <- lambda(DEA ADD)</pre>
print(DEA_ADD)
## [1] 1 1 1 1 1 1
print(DEA_ADD_Peers)
##
       peer1
## [1,]
## [2,]
## [3,]
## [4,]
           4
## [5,]
           5
## [6,]
           6
print(DEA_ADD_Lambda)
       L1 L2 L3 L4 L5 L6
##
## [1,] 1 0 0 0 0 0
## [2,] 0 1 0 0 0 0
## [3,] 0 0 1 0 0 0
## [4,]
       0 0 0 1 0 0
## [5,]
       0 0 0 0 1 0
## [6,] 0 0 0 0 0 1
# Summarize the results for addition to a summary table
DEA_ADD_Peers <- cbind(DEA_ADD_Peers, Not_Applicable, Not_Applicable)</pre>
ADD_Summary <- cbind(ADD, DEA_ADD$eff, DEA_ADD_Peers, DEA_ADD_Lambda)
colnames(ADD_Summary) <- c("Method", "Eff", "P1", "P2", "P3", "L1", "L2", "L3", "L4", "L5", "L6")
ADD_Summary <- as.data.frame(ADD_Summary)</pre>
ADD_Summary
    Method Eff P1 P2 P3 L1 L2 L3 L4 L5 L6
## 1
       ADD 1 1 <NA> <NA> 1 0 0 0 0
## 2
       ADD
            1 2 <NA> <NA> 0 1 0 0 0
## 3
       ADD
            1 3 <NA> <NA> 0 0 1 0 0
## 4
       ADD
             1 4 <NA> <NA> 0 0 0 1 0 0
            1 5 <NA> <NA> 0 0 0 1 0
## 5
       ADD
## 6
       ADD
            1 6 <NA> <NA> 0 0 0 0 1
```

3. Summarize your results in a tabular format

```
# Combine all of the method summary tables into one large summary table for each method
Summary_Table <- rbind(FDH_Summary, CRS_Summary, VRS_Summary, IRS_Summary, DRS_Summary, ADD_Summary)
# Return the summary table for review
print(Summary_Table)</pre>
```

```
Eff P1 P2 P3
##
    Method
                                           L1
                                                                L2
## 1
                      1 1 <NA> <NA>
    FDH
                                                                  0
                                                 1
      FDH
## 2
                       1 2 <NA> <NA>
                                                0
                                                                  1
      FDH
                       1 3 <NA> <NA>
## 3
                                                 0
## 4
       FDH
                        1 4 <NA> <NA>
                                                  0
                                                                  0
    FDH
## 5
                       1 5 <NA> <NA>
                                                0
                                                                  0
## 6
    FDH
                       1 6 <NA> <NA>
     CRS
                       1 1 <NA> <NA>
                                                 1
## 7
                                                                  0
     CRS
## 8
                       1 2 <NA> <NA>
                                                 0
                                                                  1
## 9
      CRS
                       1 3 <NA> <NA>
                                                 0
## 10
     CRS
                       1 4 <NA> <NA>
                                        0.2 0.0804814233385661
       CRS 0.977498691784406 1 2 4
## 11
       CRS 0.867452135493373 1 2 4 0.342857142857143 0.39499263622975
## 12
## 13
       VRS 1 1 <NA> <NA>
                                     1
## 14
       VRS
                       1 2 <NA> <NA>
                                                 0
                                                                  1
## 15
       VRS
                       1 3 <NA> <NA>
                                                  0
                                                                  0
## 16
       VRS
                       1 4 <NA> <NA>
                                                 0
## 17
       VRS
                       1 5 <NA> <NA>
## 18
       VRS 0.896328293736501 1 2 5 0.401439884809215 0.342260619150468
## 19
       IRS 1 1 \langle NA \rangle \langle NA \rangle
                                                 1
                       1 2 <NA> <NA>
## 20
       IRS
                                                 Ω
                                                                  1
## 21
       IRS
                       1 3 <NA> <NA>
                       1 4 <NA> <NA>
## 22
       IRS
                                                 0
## 23
       IRS
                       1 5 <NA> <NA>
                                                 0
## 24
       IRS 0.896328293736501 1 2 5 0.401439884809215 0.342260619150468
## 25
       DRS 1 1 <NA> <NA>
                                       1
                                                        0
                       1 2 <NA> <NA>
## 26
       DRS
                                                 0
                                                                  1
## 27
       DRS
                       1 3 <NA> <NA>
                                                 0
## 28
       DRS
                       1 4 <NA> <NA>
                                                 0
       DRS 0.977498691784406 1 2 4 0.2 0.0804814233385655
## 29
       DRS 0.867452135493373 1 2 4 0.342857142857143 0.394992636229749
## 30
## 31
       ADD
           1 1 <NA> <NA>
                                         1
                                                                  0
## 32
       ADD
                       1 2 <NA> <NA>
                                                0
## 33
       ADD
                       1 3 <NA> <NA>
                                                0
                                                                  0
## 34
       ADD
                        1 4 <NA> <NA>
                                                 0
                                                                  0
## 35
       ADD
                       1 5 <NA> <NA>
                                                 0
                                                                  0
                      1 6 <NA> <NA>
## 36
       ADD
                                                 0
                                                                  0
## L3
                  L4
                                 L5 L6
## 1
    0
                    0
                                   0
                                       0
## 2 0
                    0
                                   0
## 3
                                  0
     1
                    0
## 4
                    1
                                   0
     0
## 5
                                  1
     0
                    0
## 6
                   0
                                  0 1
     0
## 7
                   0
                                 <NA> <NA>
     0
                                 <NA> <NA>
                    0
## 8
     0
## 9
     1
                                 <NA> <NA>
## 10 0
                                <NA> <NA>
                               <NA> <NA> <NA>
## 11 0 0.538330716902146
## 12
     0 0.131075110456554
## 13 0
                    0
                                 O <NA>
## 14 0
                    0
                                  O <NA>
## 15 1
                    0
                                   O <NA>
## 16 0
                    1
                                    O <NA>
```

| ## | 17 | 0 | 0 | 1 | <na></na> |
|----|----|---|-------------------|-------------------|-----------|
| ## | 18 | 0 | 0 | 0.256299496040317 | <na></na> |
| ## | 19 | 0 | 0 | 0 | <na></na> |
| ## | 20 | 0 | 0 | 0 | <na></na> |
| ## | 21 | 1 | 0 | 0 | <na></na> |
| ## | 22 | 0 | 1 | 0 | <na></na> |
| ## | 23 | 0 | 0 | 1 | <na></na> |
| ## | 24 | 0 | 0 | 0.256299496040317 | <na></na> |
| ## | 25 | 0 | 0 | <na></na> | <na></na> |
| ## | 26 | 0 | 0 | <na></na> | <na></na> |
| ## | 27 | 1 | 0 | <na></na> | <na></na> |
| ## | 28 | 0 | 1 | <na></na> | <na></na> |
| ## | 29 | 0 | 0.538330716902146 | <na></na> | <na></na> |
| ## | 30 | 0 | 0.131075110456554 | <na></na> | <na></na> |
| ## | 31 | 0 | 0 | 0 | 0 |
| ## | 32 | 0 | 0 | 0 | 0 |
| ## | 33 | 1 | 0 | 0 | 0 |
| ## | 34 | 0 | 1 | 0 | 0 |
| ## | 35 | 0 | 0 | 1 | 0 |
| ## | 36 | 0 | 0 | 0 | 1 |
| | | | | | |

4. Compare and contrast the above results

As shown in the summary table, all six DMU's return FRH and FDH efficiency values of 1.0, along with identical peer and lambda values.

In the CRS method, DMU[1:4] is efficient at 1.0.

In the VRS method DMU[1:5] is efficient at 1.0.

In the IRS method DMU[1:5] is efficient at 1.0, and

In the DRS method DMU[1:4] is efficient at 1.0.

All of these less efficient DMU's had a Peer[1] and Peer [2] value of 1 and 2, respectively;

Depending on the method, However, the Peer[3] value was either 4 or 5.

Furthermore, all methods were relatively close in terms of relative weights (lambdas).