1. Back Savers is a company that produces backpacks primarily for students. They are considering offering some combination of two different models—the Collegiate and the Mini. Both are made out of the same rip-resistant nylon fabric. Back Savers has a long-term contract with a supplier of the nylon and receives a 5000 square-foot shipment of the material each week. Each Collegiate requires 3 square feet while each Mini requires 2 square feet. The sales forecasts indicate that at most 1000 Collegiates and 1200 Minis can be sold per week. Each Collegiate requires 45 minutes of labor to produce and generates a unit profit of $32. Each Mini requires 40 minutes of labor and generates a unit profit of $24. Back Savers has 35 laborers that each provides 40 hours of labor per week. Management wishes to know what quantity of each type of backpack to produce per week.
    a. Clearly define the decision variables
    b. What is the objective function?
    c. What are the constraints?
    d. Write down the full mathematical formulation for this LP problem.

Answer:

a)Decision Variables:

Here we have two Decision Variables:

1.Collegiate -X

2.Mini -Y

b)Objective Function:

Each Collegiate generates $32 profit and Each Mini Generates a profit of $24.So, The total profit/Maximum Profit is given by:

$$Z= \$(32X + 24 Y)$$

Each Collegiate requires 3 square feet and each Mini requires 2square feet.And the total nylon sheet material in Square-feet is 5000.

So,the total nylon sheet material required is

$$3X + 2 Y \leq 5000 \text{ Square-feet}$$

c)Constraints:

Number of collegiate sold per week is 1000.

Number of Mini sold per week is 1200.

So,

$X \leq 1000$ and

$Y \leq 1200$

Total labour required to produce X-collegiate and Y-mini backpack is

$$(45X+40Y) \text{ minutes}$$

Now,Available labour per week is 35 and each works 40 hours per week so,

$$= 35 * 40 * 60 = 84000 \text{ minutes}$$

Therefore, total labour required is $45X + 40 Y \leq 84000$

d)Mathematical Formulation for this LP problem:

Here,our problem is to ,

Maximize, $Z = 32X + 24 Y$

Subject to, $3X + 2 Y \leq 5000$

$X \leq 1000$ and

$Y \leq 1200$

$45X + 40 Y \leq 84000$

$X,Y \geq 0$

2. The Weigelt Corporation has three branch plants with excess production capacity. Fortunately, the corporation has a new product ready to begin production, and all three plants have this capability, so some of the excess capacity can be used in this way. This product can be made in three sizes--large, medium, and small--that yield a net unit profit of $420, $360, and $300, respectively. Plants 1, 2, and 3 have the excess capacity to produce 750, 900, and 450 units per day of this product, respectively, regardless of the size or combination of sizes involved.

   The amount of available in-process storage space also imposes a limitation on the production rates of the new product. Plants 1, 2, and 3 have 13,000, 12,000, and 5,000 square feet, respectively, of in-process storage space available for a day's production of this product. Each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

   Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

   At each plant, some employees will need to be laid off unless most of the plant's excess production capacity can be used to produce the new product. To avoid layoffs if possible, management has decided that the plants should use the same percentage of their excess capacity to produce the new product.

   Management wishes to know how much of each of the sizes should be produced by each of the plants to maximize profit.

   a. Define the decision variables

b. Formulate a linear programming model for this problem.

Answer:

a) Decision Variables:

Here, we have 9 decision variables.They are:

Number of large units produced per day at Plant 1 is xP1L

Number of medium units produced per day at Plant 1 is xP1M

Number of small units produced per day at Plant 1 is xP1S

Number of large units produced per day at Plant 2 is xP2L

Number of medium units produced per day at Plant 2 is xP2M

Number of small units produced per day at Plant 2 is xP2S

Number of large units produced per day at Plant 3 is xP3L

Number of medium units produced per day at Plant 3 is xP3M

Number of small units produced per day at Plant 3 is xP3S

b) Formulating a linear programming model

Also , Z denote the total net profit per day, the linear programming model is

Maximize Z = 420 xP1L + 360 xP1M + 300 xP1S + 420 xP2L + 360 xP2M + 300 xP2S + 420 xP3L + 360 xP3M + 300 xP3S.

subject to:

Constraints are:

xP1L + xP1M + xP1S ≤ 750

xP2L + xP2M + xP2S ≤ 900

xP3L + xP3M + xP3S ≤ 450

Production rates Constraints:

20 xP1L + 15 xP1M + 12 xP1S ≤ 13000

20 xP2L + 15 xP2M + 12 xP2S ≤ 12000

20 xP3L + 15 xP3M + 12 xP3S ≤ 5000

Storage Constraints:

xP1L + xP2L + xP3L ≤ 900

xP1M + xP2M + xP3M ≤ 1200

xP1S + xP2S + xP3S ≤ 750

1/750 (xP1L + xP1M + xP1S) – 1/900 (xP2L + xP2M + xP2S) = 0

1/750(xP1L + xP1M + xP1S) – 1/450(xP3L + xP3M + xP3S) = 0 and

xP1L ≥ 0, xP1M ≥ 0, xP1S ≥ 0, xP2L ≥ 0, xP2M ≥ 0, xP2S ≥ 0, xP3L ≥ 0, xP3M ≥ 0, xP3S ≥ 0.

From the above set of equality constraints we can also can include this constraint:

1/900 (xP2L + xP2M + xP2S)− 1/450 (xP3L + xP3M + xP3S) = 0.