Assignment_3

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1) Formulating Transportation using R

```
library(lpSolve)
```

```
## Warning: package 'lpSolve' was built under R version 4.1.3
```

```
library(lpSolveAPI)
```

```
## Warning: package 'lpSolveAPI' was built under R version 4.1.3
```

```
library(tinytex)
```

This lp problem has 5 constraints, 6 decision variables and it has minimisation function

```
#set the matrix
AEDs <- matrix(c(22,14,30,600,100,
                 16,20,24,625,120,
                 80,60,70,"-","-"),ncol=5,byrow=TRUE)

colnames(AEDs) <- c("Warehouse1", "Warehouse2", "Warehouse3","Production cost","Production Capacity")
rownames(AEDs) <- c("Plant A", "Plant B", "Monthly Demand")
AEDs <- as.table(AEDs)
AEDs
```

```
##                Warehouse1 Warehouse2 Warehouse3 Production cost
## Plant A        22         14         30         600
## Plant B        16         20         24         625
## Monthly Demand 80         60         70         -
##                Production Capacity
## Plant A        100
## Plant B        120
## Monthly Demand -
```

objective Function:

The above transportation problem can be formulated in the LP format as below:

$$\text{Miminize } TC = 622x_{A1} + 614x_{A2} + 630x_{A3} + 641x_{B1} + 645x_{B2} + 649x_{B3}$$

Subject to:

Supply constraints

$$x_{A1} + x_{A2} + x_{A3} \leq 100$$

$$x_{B1} + x_{B2} + x_{B3} \leq 120$$

Demand Constraints:

$$x_{A1} + x_{B1} \geq 80$$
$$x_{A2} + x_{B2} \geq 60$$
$$x_{A3} + x_{B3} \geq 70$$

Non-negativity of the variables:

$$x_{ij} \geq 0$$

$$Xij$$

- where represents the number of AEDs shipped from plant i to warehouse j.

$$i = A, B$$

and

$$j = 1, 2, 3$$

I have used R programming language to solve the above transpiration cost minimization problem. This transportation problem is unbalanced one (demand is not equal to supply), that is demand is less than supply by 10, so I create a dummy variable in column 4 with transportation cost zero and demand 10.

```
#Set up AEDs_Costs matrix
AEDs_Costs <- matrix(c(622,614,630,0,
                       641,645,649,0),ncol = 4,byrow=TRUE)
## Set the names of the rows (constraints) and columns (decision variables)
colnames(AEDs_Costs) <- c("Warehouse1", "Warehouse2", "Warehouse3","Dummy")
rownames(AEDs_Costs) <- c("Plant A", "Plant B")
AEDs_Costs
```

```
##         Warehouse1 Warehouse2 Warehouse3 Dummy
## Plant A        622        614        630     0
## Plant B        641        645        649     0
```

```
#setting up constraint signs and right-hand sides(supply side)
row.signs <- rep("<=",2)
row.rhs <- c(100,120)
#Supply function cannot be greater than the specified units

#Demand side constraints#
col.signs <- rep(">=",4)
col.rhs <- c(80,60,70,10)
#Demand function can be greater than the specified units
```

```r
#solve the model
lptrans <- lp.transport(AEDs_Costs, "min", row.signs, row.rhs, col.signs, col.rhs)
```

Next, we will return the values of the decision variables to decide how many units should be produced and shipped from each plant

```r
## Get the optimum decision variables (6)values
lptrans$solution
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0   60   40    0
## [2,]   80    0   30   10
```

Plant A Units Shipped to Warehouse 1: 0 units Plant A Units Shipped to Warehouse 2: 60 units Plant A Units Shipped to Warehouse 3: 40 units Plant B Units Shipped to Warehouse 1: 80 units Plant B Units Shipped to Warehouse 2: 0 units Plant B Units Shipped to Warehouse 3: 30 units and "10" shows up in the 4th variable it is a "throw-away variable"(dummy).

The function below will give the minimum value for the objective function

```r
lptrans$objval
```

```
## [1] 132790
```

The minimum combined shipping and production costs will be 132,790 dollars based on the given information and constraints.

```r
lptrans$duals
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
```

2)Formulating the dual of the transportation problem

The number of variables in primal is equal to the number of constants in dual.First, we need to determine the primal of the LP.Taking the minimization from the primal, we will maximize it in the dual.The dual problem can be solved with the variables m and n.

```r
AEDs_2 <- matrix(c(622,614,630,100,"m1",
641,645,649,120,"m2",
80,60,70,220,"-",
"n1","n2","n3","-","-"),ncol=5,nrow=4,byrow=TRUE)
colnames(AEDs_2) <- c("Warehouse1","Warehouse2","Warehouse3","Production Capacity","Supply (Dual)")
rownames(AEDs_2) <- c("PlantA","PlantB","Monthly Demand","Demand (Dual)")
AEDs_2 <- as.table(AEDs_2)
AEDs_2
```

```
##                Warehouse1 Warehouse2 Warehouse3 Production Capacity
## PlantA         622        614        630        100
## PlantB         641        645        649        120
```

```
## Monthly Demand 80         60         70         220
## Demand (Dual)  n1         n2         n3         -
##                Supply (Dual)
## PlantA         m1
## PlantB         m2
## Monthly Demand -
## Demand (Dual)  -
```

$$\text{Max } \; Z = 100m_1 + 120m_2 + 80n_1 + 60n_2 + 70n_3$$

Subject to the following constraints

$$m_1 + n_1 \leq 622$$

$$m_1 + n_2 \leq 614$$

$$m_1 + n_3 \leq 630$$

$$m_2 + n_1 \leq 641$$

$$m_2 + n_2 \leq 645$$

$$m_2 + n_3 \leq 649$$

Where n1 = Warehouse_1

n2 = Warehouse_2

n3 = Warehouse_3

m1 = Plant_1

m2 = Plant_2

The transposed matrix of the Primal of Linear Programming function provides these constants.You can check yourself by transposing f.con into the matrix and matching it to the constants above.where

$$m_k, n_l$$

where m= 1,2 and n=1,2,3

```r
#Objective function

f.obj <- c(100,120,80,60,70)

#transposed from the constraints matrix in the primal
f.con <- matrix(c(1,0,1,0,0,
                  1,0,0,1,0,
                  1,0,0,0,1,
                  0,1,1,0,0,
                  0,1,0,1,0,
                  0,1,0,0,1), nrow = 6, byrow = TRUE)

f.dir <- c("<=",
           "<=",
           "<=",
           "<=",
```

```
            "<=",
            "<=")

f.rhs <- c(622,614,630,641,645,649)
lp("max",f.obj,f.con,f.dir,f.rhs)
```

```
## Success: the objective function is 139120
```

```
lp("max",f.obj,f.con,f.dir,f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

So Z=139,120 dollars and variables are:

$$m_1 = 614$$

which represents Plant A

$$m_2 = 633$$

which represents Plant B

$$n_1 = 8$$

which represents Warehouse 1

$$n_3 = 16$$

which represents Warehouse 3

3) Economic Interpretation of the dual

Observations:

The maximum combined shipping and production costs will be 139,120 dollars based on the given information and constraints. There is a minimal Z=132790 (Primal) and a maximum Z=139120 (Dual).This problem is trying to find a maximum and a minimum.As a result, we discovered that we should not be shipping from Plant(A/B) to all three warehouses at the same time. We should be shipping from:

$$60x_{12}$$

which is 60 Units from Plant A to Warehouse 2.

$$40x_{13}$$

which is 40 Units from Plant A to Warehouse 3.

$$80x_{21}$$

which is 80 Units from Plant B to Warehouse 1.

$$30x_{23}$$

which is 30 Units from Plant B to Warehouse 3. We will Max the profit from each distribution to the respective capacity.

We have the following:

$$m_1^0 - n_1^0 \leq 622$$

then we subtract

$$n_1^0$$

to the other side to get

$$m_1^0 \leq 622 - n_1^0$$

To compute it would be $614 <= (-8+622) which is correct. we would continue to evaluate these equations:

$$m_1 \leq 622 - n_1 => 614 \leq 622 - 8 = 614 => correct$$

$$m_1 \leq 614 - n_2 => 614 \leq 614 - 0 = 614 => correct$$

$$m_1 \leq 630 - n_3 => 614 \leq 630 - 16 = 614 => correct$$

$$m_2 \leq 641 - n_1 => 633 \leq 614 - 8 = 633 => correct$$

$$m_2 \leq 645 - n_2 => 633 \leq 645 - 0 = 645 => Incorrect$$

$$m_2 \leq 649 - n_3 => 633 \leq 649 - 16 = 633 => correct$$

Learning from the Duality-and-Sensitivity, we can test for the shadow price by updating each of the columns.In our LP Transportation problem , we change 100 to 101 and 120 to 121.Here we can see it R.

```
row.rhs1 <- c(101,120)
row.signs1 <- rep("<=",2)
col.rhs1 <- c(80,60,70,10)
col.signs1 <- rep(">=",4)
row.rhs2 <- c(100,121)
row.signs2 <- rep("<=",2)
col.rhs2 <- c(80,60,70,10)
col.signs2 <- rep(">=",4)

lp.transport(AEDs_Costs,"min",row.signs,row.rhs,col.signs,col.rhs)
```

```
## Success: the objective function is 132790
```

```
lp.transport(AEDs_Costs,"min",row.signs1,row.rhs1,col.signs1,col.rhs1)
```

```
## Success: the objective function is 132771
```

```
lp.transport(AEDs_Costs,"min",row.signs2,row.rhs2,col.signs2,col.rhs2)
```

```
## Success: the objective function is 132790
```

By taking the minimum of this specific function, seeing the number go down by 19 means the shadow price is 19, which was calculated by adding 1 to every plant.The Plant B does not have a shadow price. We also

From the dual variable

$$n_2$$

where Marginal Revenue <= Marginal Cost. The equation was

$$m_2 \leq 645 - n_2 => 633 \leq 645 - 0 = 645 => Incorrect$$

and this was found by using

$$m_1^0 - n_1^0 \leq 622$$

then we subtract

$$n_1^0$$

to the other side to get

$$m_1^0 \leq 622 - n_1^0$$

```
lp("max", f.obj,f.con, f.dir,f.rhs)$solution
```

```
## [1] 614 633   8   0  16
```

$$n_2 = 0$$

.

The interpretation from above: from the primal:

$$60x_{12}$$

which is 60 Units from Plant A to Warehouse 2.

$$40x_{13}$$

which is 40 Units from Plant A to Warehouse 3.

$$80x_{21}$$

which is 80 Units from Plant B to Warehouse 1.

$$30x_{23}$$

which is 60 Units from Plant B to Warehouse 3.

from the dual

Our goal is to get MR=MC.In five of the six cases, MR<=MC.Only Plant B to Warehouse 2 fails to satisfy this requirement.From the primal, we can see that no AED devices will be shipped there.