

Assignment 3 (Module 6)

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The transportation problem can be formulated as:

- Objective function: Minimize the combined cost of production and shipping

$$\text{Min} = 22x_{11} + 14x_{12} + 30x_{13} + 16x_{21} + 20x_{22} + 24x_{23}$$

- Subject to :

$$x_{11} + x_{12} + x_{13} \leq 100 \text{ (For Plant A)}$$

$$x_{21} + x_{22} + x_{23} \geq 120 \text{ (For Plant B)}$$

- Constraints :

Demand Constraints

$$\text{Warehouse 1: } x_{11} + x_{21} \geq 80$$

$$\text{Warehouse 2: } x_{12} + x_{22} \geq 60$$

$$\text{Warehouse 3: } x_{13} + x_{23} \geq 70$$

1. Formulating and solving transportation problem using R:

As we can see this transportation problem is unbalanced. Since demand < supply by 10.

A problem is said an unbalanced problem when either Demand < Supply or Demand > Supply. Before moving forward, it should be first created into a balanced problem.

Balanced Equation- Demand = Supply

Thus, we can solve the above equation using two methods:

- 1) Using inequalities
- 2) Using Dummy Variables

I am using the dummy variable method. Where I created a dummy variable in column 4 with the transportation cost = 0 and demand = 10.

```

#solving transportation problem
library(lpSolve)
#writing down the costs
#in other words setting up cost matrix
costs <- matrix(c(622,614,630,0,
                  641,645,649,0), ncol = 4, byrow = TRUE)

#setting up cost for all the three variables
colnames(costs) <- c("Warehouse1", "Warehouse2", "WAREHOUSE3", "Dummy")

#setting up cost for the two plants
row.names(costs) <- c("PlantA", "PLantB")
costs <- as.table(costs)
costs

##           Warehouse1 Warehouse2 WAREHOUSE3 Dummy
## PlantA           622         614         630      0
## PLantB           641         645         649      0

#supply side
row.signs <- rep("<=", 2)
row.rhs <- c(100,120)

#demand side
col.signs <- rep(">=", 4)
col.rhs <- c(80,60,70,10)

lptrans <- lp.transport(costs, "min", row.signs, row.rhs, col.signs, col.rhs)
#values of all variables
lptrans$solution

##           [,1] [,2] [,3] [,4]
## [1,]         0  60  40      0
## [2,]        80   0  30  10

#values of the objective function
lptrans$objval

## [1] 132790

#the answer is 132790

```

The objective function = 132790

The values of the variables are :

x12 = 60

$$x_{13} = 40$$

$$x_{21} = 80$$

$$x_{23} = 30$$

$$x_{24} = 10$$

Answer: The minimum combined cost for production and shipping is 132790. Therefore, to minimize the cost Plant A should produce 100 units, 60 units for warehouse 2 and 40 units for warehouse 3. On the other hand, Plant B should produce 110 units, 80 units for warehouse 1 and 30 units for warehouse 3.

2. Dual of the transportation problem

Here we have to maximize the objective

- $\text{Max} = (80 P_1 + 60 P_2 + 70 P_3) - (100 P_1^\circ - 120 P_2^\circ)$

- Subject to :

Plant A constraints: $P_1 - P_1^\circ \geq 22$

$$P_2 - P_1^\circ \geq 14$$

$$P_3 - P_1^\circ \geq 30$$

Plant B constraints: $P_1 - P_2^\circ \geq 16$

$$P_2 - P_2^\circ \geq 20$$

$$P_3 - P_2^\circ \geq 24$$

3. Implementation in R

```
#economic interpretation of the dual
lptrans$duals
```

```
##      [,1] [,2] [,3] [,4]
## [1,]    0    0    0    0
## [2,]    0    0    0    0
```

Conclusion: As we know, the dual is the shadow price of the primal. According to the dual solution to this issue, the fundamental problem's shadow prices are all equal to zero. It indicates there is no chance of increasing revenue or lowering costs when allocating resources. As a result, we might say that the outcome is a viable solution.

When marginal cost and marginal revenue are equal, the company's output level is at its peak. At this point, efficiency is at its highest point and profits are at their highest point.

According to economic theory, a company should increase production until marginal cost and marginal revenue are equal.