

Assignment: Module 9- Goal Programming

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Solutions:

The problem has all the goals that are comparable in importance. Hence, I can say that it is an example of non-preemptive goal programming.

The Research and Development Division of the Exam Corporation has all three types of goals:

- 1) An upper, one-sided goal: Total profit
- 2) A two-sided goal: Employment Level
- 3) A lower, one-sided goal: Earnings next year

- As we can see there are three key factors. So let the decision variables be x_1 , x_2 and x_3

The maximize function can therefore be written as :

$$\text{Maximize } Z = 20x_1 + 15x_2 + 25x_3$$

Similarly, the other two decision factors can be expressed as:

$$6x_1 + 4x_2 + 5x_3 = 50 \text{ (Employment goal)}$$

$$8x_1 + 7x_2 + 5x_3 \geq 75 \text{ (Next year's Earnings goal)}$$

- Defining the auxiliary variables, we formulate y_1 and y_2

$$y_1 = y_{1p} - y_{1m}, \text{ where } y_{1p}, y_{1m} \geq 0$$

y_{1p} denotes the penalty for an employment level goal exceeding by 50 and y_{1m} is the penalty for an employment level goal decreasing below 50, as mentioned in the question.

$$y_2 = y_{2p} - y_{2m}, \text{ where } y_{2p}, y_{2m} \geq 0$$

Similarly, y_{2m} denotes the penalty for not achieving next year's earnings and y_{2p} represents the exceeding earnings next year.

$$y1 = 6x1 + 4x2 + 5x3 - 50$$

$$y2 = 8x1 + 7x2 + 5x3 - 75$$

This concludes that we are successfully able to express management's objective function in terms of $x1, x2, x3, y1+, y1-, y2+$ and $y2-$.

- Now, we can implement the auxiliary variables and easily write the overall management objective :

$$\text{Maximize } Z = 20x1 + 15x2 + 25x3 - 6y1p - 6y1m - 3y2m$$

$$\text{Subject to : } 6x1 + 4x2 + 5x3 - y1p + y1m = 50$$

$$8x1 + 7x2 + 5x3 - y2p + y2m \geq 75$$

Solving this problem in R

```
#Solving the Emax linear programming model using lpSolveAPI

#Loading the lpSolveAPI library
library(lpSolveAPI)
##entering the values manually
## setting up the problem with 7 decision variables, and 2 constraints.
lpprec <- make.lp(2, 7)

## Setting objective function
set.objfn(lpprec, c(20, 15, 25, -6, -6, 0, -3))
#Objective function is to maximise
lp.control(lpprec, sense='max')

## $anti.degen
## [1] "fixedvars" "stalling"
##
## $basis.crash
## [1] "none"
##
## $bb.depthlimit
## [1] -50
##
## $bb.floorfirst
## [1] "automatic"
##
## $bb.rule
## [1] "pseudononint" "greedy"          "dynamic"          "rcostfixing"
##
## $break.at.first
## [1] FALSE
```

```

##
## $break.at.value
## [1] 1e+30
##
## $epsilon
##      epsb      epsd      epsel      epsint  epsperturb  epspivot
##      1e-10      1e-09      1e-12      1e-07      1e-05      2e-07
##
## $improve
## [1] "dualfeas" "thetagap"
##
## $infinite
## [1] 1e+30
##
## $maxpivot
## [1] 250
##
## $mip.gap
## absolute relative
##      1e-11      1e-11
##
## $negrange
## [1] -1e+06
##
## $obj.in.basis
## [1] TRUE
##
## $pivoting
## [1] "devex"      "adaptive"
##
## $presolve
## [1] "none"
##
## $scalelimit
## [1] 5
##
## $scaling
## [1] "geometric"  "equilibrate" "integers"
##
## $sense
## [1] "maximize"
##
## $simplextype
## [1] "dual"      "primal"
##
## $timeout
## [1] 0
##
## $verbose
## [1] "neutral"

```

```

##Setting values for the rows
#Left hand side constraints
set.row(lprec, 1, c(6, 4, 5, -1, 1, 0, 0), indices = c(1, 2, 3, 4, 5, 6, 7))
set.row(lprec, 2, c(8, 7, 5, 0, 0, -1, 1), indices = c(1, 2, 3, 4, 5, 6, 7))
#Right hand side values
rhs <- c(50, 75)
set.rhs(lprec, rhs)

# Set constraint type and set variable types
set.constr.type(lprec, c("=", ">="))
set.bounds(lprec, lower = rep(0, 7))

#Wrting the decision variables names
lp.rownames <- c("EmploymentLevelGoal", "NextYearEarningsGoal")
lp.colnames <- c("x1", "x2", "x3", "y1p", "y1m", "y2p", "y2m")
dimnames(lprec) <- list(lp.rownames, lp.colnames)

# View the model
lprec

## Model name:
##
##          x1    x2    x3    y1p    y1m    y2p    y2m
## Maximize    20    15    25     -6     -6      0     -3
## EmploymentLevelGoal    6     4     5     -1      1      0      0    = 50
## NextYearEarningsGoal    8     7     5      0      0     -1      1   >= 75
## Kind          Std     Std     Std     Std     Std     Std     Std
## Type          Real    Real    Real    Real    Real    Real    Real
## Upper          Inf     Inf     Inf     Inf     Inf     Inf     Inf
## Lower          0       0       0       0       0       0       0

# Save this into a file
write.lp(lprec, filename = "emax.lp", type = "lp")

#solving the model
solve(lprec)

## [1] 0

#Show the value of objective function, variables, constraints
get.objective(lprec)

## [1] 225

get.variables(lprec)

## [1] 0 0 15 25 0 0 0

get.constraints(lprec)

## [1] 50 75

```

Findings and Observations:

- 1) After successfully implementing the model the values of the variables appear to be as follows:

$$x_1 = 0, x_2 = 0, x_3 = 15, y_{1p} = 25, y_{1m} = 0, y_{2p} = 0, y_{2m} = 0$$

- 2) We can observe that $y_1 = 25$ and $y_2 = 0$, which means the second goal of next year's earnings are fully satisfied.

But it can also be observed that the employment level goal of 50 is exceeded by 25 i.e. 2500 employees.

- 3) The objective function is 225. This means that the Exam Corporation needs to produce 15 units of product 3 (x_3) and none of products 1 and 2 to achieve a profit of 225 million.