

Assignment: Module 2 - The LP Model

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Solution 1:

Writing all the important information from the problem:

Given

- Two different models - Collegiate and the Mini.
Let us denote Collegiate Model = **X_1** and Mini Model = **X_2**
- Back savers have a long-term contract
5000 sq. foot shipment of material each week.
- Each Collegiate requires 3 square feet = **$3X_1$**
Each Mini requires 2 square feet = **$2X_2$**

Therefore, the total nylon fabric required can be written as:

$$3X_1 + 2X_2 \leq 5000$$

- The sales forecasts indicate that at most 1000 collegiates can be sold per week
 $X_1 \leq 1000$
Also, 1200 mini backpacks can be sold per week
 $X_2 \leq 1200$
- Each collegiate requires 45 minutes of labor to produce = **$45X_1$**
Each mini backpack requires 40 minutes of labor to produce = **$40X_2$**

Which can be written as :

$$\text{Total labor time required by collegiate and mini backpacks} = (45X_1 + 40X_2)$$

- Back Savers has 35 laborers that each provide 40 hours of labor per week.

Therefore total available labor time is = No of employees x hours per week x 60 mins

$$35 \times 40 \times 60 = 84000 \text{ minutes}$$

Which can be written as :

$$45X_1 + 40X_2 \leq 84000$$

- Profit generated
By collegiate = \$ 32 per unit
By mini backpack = \$ 24 per unit

We will denote profit by Z

$$\text{This can be written as : } Z = \$ (32X_1 + 24X_2)$$

ANSWERS:

a. Clearly define the decision variables

Decision variables are :

Collegiate Model = X_1

Mini Model = X_2

b. What is the objective function?

(Maximise Profit) $Z = \$ (32X_1 + 24X_2)$

c. What are the constraints?

Constraints :

Subject to,

$3X_1 + 2X_2 \leq 5000$ (material usage)

$X_1 \leq 1000$ (quantity sold)

$X_2 \leq 1200$ (quantity sold)

$45X_1 + 40X_2 \leq 84000$ (labor time)

And $X_1, X_2 \geq 0$ (non-negative)

Solution 2:

Writing all the important information from the problem:

- There are total of three plants : Plant 1, Plant 2, Plant 3
Each plant produce large, medium and small size produce

Therefore, let us denote Plant 1 by P_1 and the sizes by l (Large), m (Medium), s (Small)

The total number of variables are:

For Plant 1 : P_{1l}, P_{1m}, P_{1s}

For Plant 2: P_{2l}, P_{2m}, P_{2s}

For Plant 3: P_{3l}, P_{3m}, P_{3s}

- Product can be made in three sizes--large, medium, and small--that yield a net unit profit of \$420, \$360, and \$300, respectively.
- Plant 1 has the excess capacity to produce 750units per day of this product.
 $P_{1l} + P_{1m} + P_{1s} \leq 750$

Plant 2 has the excess capacity to produce 900 units per day of this product.

$P_{2l} + P_{2m} + P_{2s} \leq 900$

Plant 3 has the excess capacity to produce 450 units per day of this product.

$P_{3l} + P_{3m} + P_{3s} \leq 450$

- Available storage for each plant

note: each unit of the large, medium, and small sizes produced per day requires 20, 15, and 12 square feet, respectively.

Plant 1 has 13000 sq. feet of in-process storage available for a day's production

$$20P1l + 15P1m + 12P1s \leq 13000$$

Plant 2 has 12000 sq. feet of in-process storage available for a day's production

$$20P2l + 15P2m + 12P2s \leq 12000$$

Plant 3 has 5000 sq. feet of in-process storage available for a day's production

$$20P3l + 15P3m + 12P3s \leq 5000$$

- Sales forecasts indicate that if available, 900, 1,200, and 750 units of the large, medium, and small sizes, respectively, would be sold per day.

Capacity utilisation of each plant :

$$P1l + P2l + P3l \leq 900 \text{ (Large)}$$

$$P1m + P2m + P3m \leq 1200 \text{ (Medium)}$$

$$P1s + P2s + P3s \leq 750 \text{ (Small)}$$

$$\frac{1}{750}(P1l + P1m + P1s) - \frac{1}{900}(P2l + P2m + P3s) = 0$$

$$\frac{1}{750}(P1l + P1m + P1s) - \frac{1}{450}(P2l + P2m + P3s) = 0$$

ANSWERS:

a. Clearly define the decision variables

The total number of decision variables are “nine”

For Plant 1 : P1l, P1m, P1s

For Plant 2: P2l, P2m, P2s

For Plant 3: P3l, P3m, P3s

b. What is the objective function?

Maximise

$$Z = \underbrace{(420P1l + 360P1m + 300P1s)}_{\text{Plant 1}} + \underbrace{(420P2l + 360P2m + 300P2s)}_{\text{Plant 2}} + \underbrace{(420P3l + 360P3m + 300P3s)}_{\text{Plant 3}}$$

Profit is denoted by Z

c. What are the constraints?

Constraints :

Subject to :

$$P1l + P1m + P1s \leq 750$$

$$P2l + P2m + P2s \leq 900$$

$$P3l + P3m + P3s \leq 450$$

$$20P1l + 15P1m + 12P1s \leq 13000$$

$$20P2l + 15P2m + 12P2s \leq 12000$$

$$20P3l + 15P3m + 12P3s \leq 5000$$

$$P1l + P2l + P3l \leq 900 \text{ (Large)}$$

$$P1m + P2m + P3m \leq 1200 \text{ (Medium)}$$

$$P1s + P2s + P3s \leq 750 \text{ (Small)}$$

$$\frac{1}{750}(P1l + P1m + P1s) - \frac{1}{900} (P2l + P2m + P3s) = 0$$

$$\frac{1}{750}(P1l + P1m + P1s) - \frac{1}{450} (P2l + P2m + P3s) = 0$$

$$\text{Non-negativity: } Z \geq 0, P1l \geq 0, P1m \geq 0, P1s \geq 0, P2l \geq 0, P2m \geq 0, P2s \geq 0, P3l \geq 0, P3m \geq 0, P3s \geq 0$$