reading on image registration see Goshtasby [2012]. Bronson and Costa [2009] is a good reference for additional reading on vectors and matrices. See Chapter 4 for a detailed treatment of the Fourier transform, and Chapters 6, 8, and 12 for details on other image transforms. See Montgomery and Runger [2011], and Ross [2014], for introductory material on probability and random variables. For details on the software aspects of many of the examples and projects in this chapter, see Gonzalez, Woods, and Eddins [2009].

Problems

Solutions to the problems marked with an asterisk (*) are in the DIP4E Student Support Package (consult the book website: www.ImageProcessingPlace.com)..

2.1* When discussing Fig. 2.2 we mentioned that the fovea is circular, with a diameter of approximately 1.5 mm. But, when discussing Fig. 2.3, we concluded that the height of the tree in the image reflected on the retina was $h = 2.5$ mm. Explain the discrepancy between these two numbers.

2.2 If you use a sheet of white paper to shield your eyes when looking directly at the sun, the side of the sheet facing you appears black. Which of the visual processes discussed in Section 2.1 is responsible for this?

2.3* Using the background information provided in Section 2.1, and thinking purely in geometrical terms, estimate the diameter of the smallest printed dot that the eye can discern if the page on which the dot is printed is 0.2 m away from the eyes. Assume for simplicity that the visual system ceases to detect the dot when the image of the dot on the fovea becomes smaller than the diameter of one receptor (cone) in that area of the retina. Assume further that the fovea can be modeled as a square array of dimension 1.5 mm on the side, and that the cones and spaces between the cones are distributed uniformly throughout this array.

2.4* When you enter a dark theater on a bright day, it takes an appreciable interval of time before you can see well enough to find an empty seat. Which of the visual processes explained in Section 2.1 causes this?

2.5 Although it is not shown in Fig. 2.10, alternating current is part of the electromagnetic spectrum. Commercial alternating current in the United States has a frequency of 60 Hz. What is the wavelength in kilometers of this component of the spectrum?

2.6 You are hired to design the front end of an imaging system for studying the shapes of cells, bacteria, viruses, and proteins. The front end consists in this case of the illumination source(s) and corresponding imaging camera(s). The diameters of circles required to fully enclose individual specimens in each of these categories are 50, 1, 0.1, and 0.01 μm, respectively. In order to perform automated analysis, the smallest detail discernible on a specimen must be 0.001 μm.

(a)* Can you solve the imaging aspects of this problem with a single sensor and camera? If your answer is yes, specify the illumination wavelength band and the type of camera needed. By "type," we mean the band of the electromagnetic spectrum to which the camera is most sensitive (e.g., infrared).

(b) If your answer in (a) is no, what type of illumination sources and corresponding imaging sensors would you recommend? Specify the light sources and cameras as requested in part (a). Use the minimum number of illumination sources and cameras needed to solve the problem. (*Hint:* From the discussion in Section 2.2, the illumination required to "see" an object must have a wavelength the same size or smaller than the object.)

2.7 You are preparing a report and have to insert in it an image of size 2048×2048 pixels.

(a)* Assuming no limitations on the printer, what would the resolution in line pairs per mm have to be for the image to fit in a space of size 5×5 cm?

(b) What would the resolution have to be in dpi for the image to fit in 2×2 inches?

2.8 A CCD camera chip of dimensions 7×7 mm and 1024×1024 sensing elements, is focused on a square, flat area, located 0.5 m away. The camera is equipped with a 35-mm lens. How many line pairs per mm will this camera be able to resolve? (*Hint:* Model the imaging process as in Fig. 2.3, with the focal length of the camera lens substituting for the focal length of the eye.)

2.9* Suppose that a given automated imaging application requires a minimum resolution of 5 line pairs per mm to be able to detect features of interest in objects viewed by the camera. The distance between the focal center of the camera lens and the area to be imaged is 1 m. The area being imaged is 0.5×0.5 m. You have available a 200 mm lens, and your job is to pick an appropriate CCD imaging chip. What is the minimum number of sensing elements and square size, $d \times d$, of the CCD chip that will meet the requirements of this application? (*Hint:* Model the imaging process as in Fig. 2.3, and assume for simplicity that the imaged area is square.)

2.10 An automobile manufacturer is automating the placement of certain components on the bumpers of a limited-edition line of sports cars. The components are color-coordinated, so the assembly robots need to know the color of each car in order to select the appropriate bumper component. Models come in only four colors: blue, green, red, and white. You are hired to propose a solution based on imaging. How would you solve the problem of determining the color of each car, keeping in mind that cost is the most important consideration in your choice of components?

2.11 A common measure of transmission for digital data is the *baud rate*, defined as symbols (bits in our case) per second. As a minimum, transmission is accomplished in packets consisting of a start bit, a byte (8 bits) of information, and a stop bit. Using these facts, answer the following:

(a)* How many seconds would it take to transmit a sequence of 500 images of size 1024×1024 pixels with 256 intensity levels using a 3 M-baud (10^6 bits/sec) baud modem? (This is a representative medium speed for a DSL (Digital Subscriber Line) residential line.

(b) What would the time be using a 30 G-baud (10^9 bits/sec) modem? (This is a representative medium speed for a commercial line.)

2.12* High-definition television (HDTV) generates images with 1125 horizontal TV lines interlaced (i.e., where every other line is "painted" on the screen in each of two fields, each field being 1/60th of a second in duration). The width-to-height aspect ratio of the images is 16:9. The fact that the number of horizontal lines is fixed determines the vertical resolution of the images. A company has designed a system that extracts digital images from HDTV video. The resolution of each horizontal line in their system is proportional to vertical resolution of HDTV, with the proportion being the width-to-height ratio of the images. Each pixel in the color image has 24 bits of intensity, 8 bits each for a red, a green, and a blue component image. These three "primary" images form a color image. How many bits would it take to store the images extracted from a two-hour HDTV movie?

2.13 When discussing linear indexing in Section 2.4, we arrived at the linear index in Eq. (2-14) by inspection. The same argument used there can be extended to a 3-D array with coordinates x, y, and z, and corresponding dimensions M, N, and P. The linear index for any (x, y, z) is

$$s = x + M(y + Nz)$$

Start with this expression and

(a)* Derive Eq. (2-15).

(b) Derive Eq. (2-16).

2.14* Suppose that a flat area with center at (x_0, y_0) is illuminated by a light source with intensity distribution

$$i(x,y) = Ke^{-[(x-x_0)^2 + (y-y_0)^2]}$$

Assume for simplicity that the reflectance of the area is constant and equal to 1.0, and let $K = 255$. If the intensity of the resulting image is quantized using k bits, and the eye can detect an abrupt change of eight intensity levels between adjacent pixels, what is the highest value of k that will cause visible false contouring?

2.15 Sketch the image in Problem 2.14 for $k = 2$.

2.16 Consider the two image subsets, S_1 and S_2 in the following figure. With reference to Section 2.5, and assuming that $V = \{1\}$, determine whether these two subsets are:

(a)* 4-adjacent.

(b) 8-adjacent.

(c) m-adjacent.

	S_1				S_2				
0	0	0	0	0	0	0	1	1	0
1	0	0	1	0	0	1	0	0	1
1	0	0	1	0	1	1	0	0	0
0	0	1	1	1	0	0	0	0	0
0	0	1	1	1	0	0	1	1	1

2.17* Develop an algorithm for converting a one-pixel-thick 8-path to a 4-path.

2.18 Develop an algorithm for converting a one-pixel-thick m-path to a 4-path.

2.19 Refer to the discussion toward the end of Section 2.5, where we defined the background of an image as $(R_u)^c$, the complement of the union of all the regions in the image. In some applications, it is advantageous to define the background as the subset of pixels of $(R_u)^c$ that are not *hole* pixels (informally, think of holes as sets of background pixels surrounded by foreground pixels). How would you modify the definition to exclude hole pixels from $(R_u)^c$? An answer such as "the background is the subset of pixels of $(R_u)^c$ that are not hole pixels" is not acceptable. (*Hint:* Use the concept of connectivity.)

2.20 Consider the image segment shown in the figure that follows.

(a)* As in Section 2.5, let $V = \{0,1\}$ be the set of intensity values used to define adjacency. Compute the lengths of the shortest 4-, 8-, and m-path between p and q in the following image. If a particular path does not exist between these two points, explain why.

	3	1	2	1 (q)
	2	2	0	2
	1	2	1	1
(p)	1	0	1	2

(b) Repeat (a) but using $V = \{1,2\}$.

2.21 Consider two points p and q.

(a)* State the condition(s) under which the D_4 distance between p and q is equal to the shortest 4-path between these points.

(b) Is this path unique?

2.22 Repeat problem 2.21 for the D_8 distance.

2.23 Consider two *one-dimensional* images f and g of the same size. What has to be true about the orientation of these images for the elementwise and matrix products discussed in Section 2.6 to make sense? Either of the two images can be first in forming the product.

2.24* In the next chapter, we will deal with operators whose function is to compute the sum of pixel values in a small subimage area, S_{xy}, as in Eq. (2-43). Show that these are linear operators.

2.25 Refer to Eq. (2-24) in answering the following:

(a)* Show that image summation is a linear operation.

(b) Show that image subtraction is a linear operation.

(c)* Show that image multiplication in a nonlinear operation.

(d) Show that image division is a nonlinear operation.

2.26 The median, ζ, of a set of numbers is such that half the values in the set are below ζ and the other half are above it. For example, the median of the set of values $\{2,3,8,20,21,25,31\}$ is 20. Show that an operator that computes the median of a subimage area, S, is nonlinear. (*Hint:* It is sufficient to show that ζ fails the linearity test for a simple numerical example.)

2.27* Show that image averaging can be done recursively. That is, show that if $a(k)$is the average of k images, then the average of $k+1$ images can be obtained from the already-computed average, $a(k)$, and the new image, f_{k+1}.

2.28 With reference to Example 2.5:

(a)* Prove the validity of Eq. (2-27).

(b) Prove the validity of Eq. (2-28).

For part (b) you will need the following facts from probability: (1) the variance of a constant times a

2.38 With reference to Table 2.3, provide single, composite transformation functions for performing the following operations:

(a)* Scaling and translation.

(b)* Scaling, translation, and rotation.

(c) Vertical shear, scaling, translation, and rotation.

(d) Does the order of multiplication of the individual matrices to produce a single transformations make a difference? Give an example based on a scaling/translation transformation to support your answer.

2.39 We know from Eq. (2-45) that an affine transformation of coordinates is given by

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \mathbf{A}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

where (x', y') are the transformed coordinates, (x, y) are the original coordinates, and the elements of $\mathbf{A}$ are given in Table 2.3 for various types of transformations. The inverse transformation, $\mathbf{A}^{-1}$, to go from the transformed back to the original coordinates is just as important for performing inverse mappings.

(a)* Find the inverse scaling transformation.

(b) Find the inverse translation transformation.

(c) Find the inverse vertical and horizontal shearing transformations.

(d)* Find the inverse rotation transformation.

(e)* Show a composite inverse translation/rotation transformation.

2.40 What are the equations, analogous to Eqs. (2-46) and (2-47), that would result from using triangular instead of quadrilateral regions?

2.41 Do the following.

(a)* Prove that the Fourier kernel in Eq. (2-59) is separable and symmetric.

(b) Repeat (a) for the kernel in Eq. (2-60).

2.42* Show that 2-D transforms with separable, symmetric kernels can be computed by: (1) computing 1-D transforms along the individual rows (columns) of the input image; and (2) computing 1-D transforms along the columns (rows) of the result from step (1).

2.43 Answer the following:

(a)* A day of the week is selected at random. What is the probability that it is a weekend day?

(b)* A single card is drawn from a deck of 52 cards. What is the probability that it is a king or a club?

(c) A single die is rolled. What is the probability that the outcome is a 2, a 5, or an even number?

(d) The probability of rain tomorrow is 0.7. The probability that it will snow is 0.5. The probability of both rain and snow is 0.3. What is the probability that it will either rain or snow tomorrow?

2.44* Derive Eq. (2-69) by applying Eq. (2-68) to combined events. (*Hint:* You will need to use the distributive law in Table 2.1.)

2.45 Demonstrate the validity of the product rule of probability for the case $n = 3$. Let the events be denoted by A, B, and C.

2.46 An experiment in particle physics is set up with an imaging system whose (processed) output consists of two types of images. Images of Type I have at least one particle collision present, while images of Type II are "blank" (i.e., they contain no particle collisions). The images are stored sequentially, as they come out of the experiment. The occurrence of collisions is random, and it is known that blank images occur 75% of the time. In a particular run, the experiment generates 1,000 images.

(a)* What is the probability that if we look at the first three images they will all be of Type I? (*Hint:* Consider using the product rule of probability.)

(b) Will the result be different if we look at the last three images?

2.47* Given sets, A, B, C, and D, list all the conditions for these sets to be mutually independent.

2.48 With reference to Eq. (2-74), show that the expression $P(A \cap B | C) = P(A|C)P(B|A \cap C)$ is true.