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Answer Key 10

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P12

5
- $\because \|f\|'' \leq \|f\|,\ \|f\|' \leq (b-a)\|f\|$

  $\therefore$ if in the sense of $\|\ \|$ $f_n \to f$, then in the sense of $\|\ \|'$, $\|\ \|''$ $f_n \to f$

- $\exists f_n \in C^k[a,b],\ f \in C^k[a,b]$, in the sense of $\|\ \|$ $f_n \nrightarrow f$, but in the sense of $\|\ \|'$, $\|\ \|''$ $f_n \to f$

- norm $\|\ \|'$ is not equal to norm $\|\ \|''$ $(k=0)$

P18

4 (i) $\because$ integral is linear

$\therefore I$ is linear

(ii) for $f_n(x) = \begin{cases} 1 & -n < x < n \\ x+n+1 & -n-1 \leqslant x \leqslant -n \\ n+1-x & n \leqslant x \leqslant n+1 \\ 0 & \text{other} \end{cases}$

$f_n \in C_c(R),\ \|f_n\| = 1,\ \lim\limits_{n\to+\infty} I(f_n) = +\infty$

$\therefore I$ is not bounded

P26

5 suppose $m = n = 1$ construct a discontinuous function $y_i = \begin{cases} \ln|x|, \ x \neq 0 \\ 0, \ x = 0 \end{cases}$ and evidently the preimage of a compact set is also compact

$\therefore$ negative $\square$