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Answer Key 10

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P12

5 $\because \|f\|'' \leq \|f\|,\ \|f\|' \leq (b-a) * \|f\|$

$\therefore$ if in the sense of $\|\ \|\ f_n \to f$, then in the sense of $\|\ \|',\ \|\ \|''\ f_n \to f$

$\exists f_n \in C[a,b],\ f \in C[a,b]$, in the sense of $\|\ \|\ f_n \nrightarrow f$, but in the sense of $\|\ \|',\ \|\ \|''\ f_n \to f$ and norm $\|\ \|'$ is not equal to norm $\|\ \|''$

P18

4 (i) $\because$ integral is linear

$\therefore I$ is linear

(ii) for $f_n(x) = \begin{cases} 1 & -n < x < n \\ x+n+1 & -n-1 \leqslant x \leqslant -n \\ n+1-x & n \leqslant x \leqslant n+1 \\ 0 & \text{other} \end{cases}$

$f_n \in C_c(R),\ \|f_n\| = 1,\ \lim\limits_{n \to +\infty} I(f_n) = +\infty$

$\therefore I$ is not bounded

P26

5 Construct a discontinuous function $y_i = \begin{cases} \ln|\boldsymbol{x}|, \ \boldsymbol{x} \neq \mathbf{0} \\ 0, \ \boldsymbol{x} = \mathbf{0} \end{cases}$, $1 \leqslant i \leqslant m$ and evidently the preimage of a compact set is also compact

$\therefore$ negative $\square$