Peking University

Answer Key 11

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P65

5 (i) $|f(\boldsymbol{x}) - f(\boldsymbol{y})| = 0 \Rightarrow |\boldsymbol{x} - \boldsymbol{y}| = 0$

(ii) $|\,\mathrm{D}f \cdot \mathrm{d}\boldsymbol{x}| = |\,\mathrm{d}f(\boldsymbol{x})| \geqslant c|\,\mathrm{d}\boldsymbol{x}|$

$\therefore$ the absolute values of $\mathrm{D}f$'s eigenvalues are not smaller than c

(iii) $\because |f(\boldsymbol{x}) - f(\mathbf{0})| \geqslant c|\boldsymbol{x}|$

$\therefore \lim\limits_{|\boldsymbol{x}|\to+\infty} |f(\boldsymbol{x})| = +\infty$

$\therefore \forall \boldsymbol{\xi} \in \mathbb{R}^n,\ \exists \boldsymbol{x}_0 \in \mathbb{R}^n$ (minimum point) s.t. $\mathrm{d}|f(\boldsymbol{x}) - \boldsymbol{\xi}|^2\Big|_{\boldsymbol{x}=\boldsymbol{x}_0} = 2[f(\boldsymbol{x}_0) - \boldsymbol{\xi}]^{\mathrm{T}} \cdot \mathrm{D}f(\boldsymbol{x}_0) \cdot \mathrm{d}\boldsymbol{x} = 0$

$\because \mathrm{D}f \neq 0$

$\therefore f(\boldsymbol{x}_0) = \boldsymbol{\xi}$ $\square$

P79

3 S is bounded

for $\forall \varepsilon > 0$, the accumulation points can be covered by a finite set of closed rectangles with total volume $V_0 < \frac{\varepsilon}{2}$

$\because$ if there are infinite points left in S, there will be an arbitrarily small region containing infinite points in S outside previous rectangles, which contradicts to the fact that all accumulation points are already covered.

$\therefore$ the left points could be covered by another finite set of closed rectangles with total volume $V_1 < \frac{\varepsilon}{2}$ $\square$

P88

4 Negative

$f_n(x) = \frac{1}{x^2 + \frac{1}{n}}$ is integrable on $[0, 1]$

$\lim\limits_{n \to \infty} f_n(x) = \frac{1}{x^2} = f(x)$ is not integrable on $[0, 1]$ $\square$