Automated Time Tabling for University Lectures

Efficient Algorithms from Simple-Minded Researchers

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The Basic Model

- There are n lectures $\{1, \ldots, n\}$ that have to be assigned to m time slots
- For each lecture i, we know the number of required time slots s_i
- For each pair of lectures i, j, we know the number $c_{i,j}$ of students taking both lectures
- The goal is to minimize the total number of conflicts, i.e. each pair of lectures i, j in the same time slot adds a conflict of $c_{i,j}$

Lemma

This problem is NP-hart

Example

Lecture	Slots	Programming	Algorithms	Theory	Math
Programming	2	-	20	3	23
Algorithms	2	20	-	12	7
Theory	1	3	12	-	8
Math	1	23	7	8	-

Slot	Lectures	Conflicts
1	Programming	0
2	Algorithms, Theory	12
3	Programming, Algorithms, Math	$20 + 23 + 7$
		62

Why Central Planning

Bachelor of Education

Subject 1 | Subject 2 | Didactics

Why central planning

There are about

$$\binom{22}{2} \approx 200$$

combinations of subjects.

Simplification of the “Kreuzer-Model”

- Currently the “Kreuzer-Model” (due to Prof. Dr. Alexander Kreuzer, Uni Hamburg) is used at the JGU
- Simplification of the basic model (in order to make the optimization problem tractable and other reasons):

Simlification

Each subject has exactly one lecture per semester taking 3 time slots each

Input Data and Result of Kreuzer-Model

Studierendenzahlen Mainz- Gesamt WS 2003/04 bis SS 08

	De	En	Fr	Spa	Ma	Geo	Che	Bio	Phy	Spo	Ges	EvR	KaR	Phi	Sk	La	Gr	Ita	Rus	BK	Mus		
De		2550	964	224	231	956	124	446	24	524	1761	430	487	769	1460	235	9	83	53	326	363	**12019**	De
En	2550		1159	673	322	1122	138	476	55	838	1309	280	199	381	805	171	0	86	21	183	106	**10874**	En
Fr	964	1159		432	240	469	49	116	16	209	949	61	93	162	115	56	0	96	0	83	82	**5351**	Fr
Spa	224	673	432		63	89	14	50	2	78	171	15	22	56	66	64	0	135	8	76	32	**2270**	Spa
Ma	231	322	240	63		494	611	197	1031	571	330	103	175	190	136	146	3	5	58	88	229	**5223**	Ma
Geo	956	1122	469	89	494		296	543	107	1242	345	97	156	86	396	50	0	28	46	99	151	**6772**	Geo
Che	124	138	49	14	611	296		468	156	137	135	10	33	20	91	45	0	11	13	21	23	**2395**	Che
Bio	446	476	116	50	197	543	468		24	360	140	64	41	66	85	48	0	5	1	63	47	**3240**	Bio
Phy	24	55	16	2	1031	107	156	24		101	91	1	11	49	33	13	0	4	1	1	26	**1746**	Phy
Spo	524	838	209	78	571	1242	137	360	101		315	83	119	36	280	47	10	14	16	42	38	**5060**	Spo
Ges	1761	1309	949	171	330	345	135	140	91	315		370	408	527	440	377	9	74	36	84	65	**7936**	Ges
EvR	430	280	61	15	103	97	10	64	1	83	370			78	72	81	0	9	15	12	53	**1834**	EvR
KaR	487	199	93	22	175	156	33	41	11	119	408			86	79	164	1	13	3	27	68	**2185**	KaR
Phi	769	381	162	56	190	86	20	66	49	36	527	78	86		106	87	8	36	27	135	11	**2916**	Phi
Sk	1460	805	115	66	136	396	91	85	33	280	440	72	79	106		24	0	4	4	61	23	**4280**	Sk
La	235	171	56	64	146	50	45	48	13	47	377	81	164	87	24		52	7	6	4	42	**1719**	La
Gr	9	0	0	0	3	0	0	0	0	10	9	0	1	8	0	52		0	0	0	6	**98**	Gr
Ita	83	86	96	135	5	28	11	5	4	14	74	9	13	36	4	7	0		16	22	9	**657**	Ita
Rus	53	21	0	8	58	46	13	1	1	16	36	15	3	27	4	6	0	16		6	2	**332**	Rus
BK	326	183	83	76	88	99	21	63	1	42	84	12	27	135	61	4	0	22	6		0	**1333**	BK
Mus	363	106	82	32	229	151	23	47	26	38	65	53	68	11	23	42	6	9	2	0		**1376**	Mus
	12019	**10874**	**5351**	**2270**	**5223**	**6772**	**2395**	**3240**	**1746**	**5060**	**7936**	**1834**	**2185**	**2916**	**4280**	**1719**	**98**	**657**	**332**	**1333**	**1376**	**39808**	
	De	En	Fr	Spa	Ma	Geo	Che	Bio	Phy	Spo	Ges	EvR	KaR	Phi	Sk	La	Gr	Ita	Rus	BK	Mus		

Überschneidungstypen:

alpha: keine Überschneidungen zwischen den Fächern, auch bei untersch. Studienjahren; max. 2 SWS Überschneidungen zwischen 1. und 3. Studienjahr

beta: wie alpa, zzgl. max. 2 SWS Überschneidungen zwischen 1. und 2. Fach in unterschiedlichen Studienjahren

gamma: wie beta, aber weniger Überschneidungen zwischen den Fächern in unterschiedlichen Studienjahren

delta: in der Regel mit 2 SWS Überschneidungen zwischen den Fächern desselben Studienjahres

Kombination kann in der Regel nur zeitversetzt studiert werden; führt in der Regel zu Studienzeitverlängerung von 1 Semester

Result of Prof. Dr. Alexander Kreuzer for the first Semester

1. Jahr										
Zeit	**Montag**		**Dienstag**		**Mittwoch**		**Donnerstag**		**Freitag**	
8 - 10	Soz/Ita	Mathe	Deu	Physik	Soz/Ita	Chem	Eng/Gr	Chem	Geo	Gesch
10 - 12	Span	Russ	Geo	Phil	Span	Physik	Bio	Gesch	Bio	Phil
12 - 14	Eng/Gr	Russ	Span	Mathe	Sport	Rel	Bio	Rel	Deu	Mathe
14 - 16	Geo	Latein	Fra	Latein	Sport	Latein	Soz/Ita	Russ	Sport	Gesch
16 - 18	Deu	Chem	Fra	Rel	BWS		Fra	Phil	Eng/Gr	Physik

- Determined roughly as follows:
  - Reduce the number of subjects to 16 by joining some of the small ones (e.g. Soz/Ita)
  - There are two times 8 non-overlapping slots, the others have an overlap of 1 time slot
  - Group the big subject (manually) with small overlap
  - Use complete enumeration to assign the other subjects

Extension: Compulsory Elective Lectures

- Each subject has compulsory elective lectures
  - Several lectures from which one is to be selected
  - Several equivalent exercise groups
- Mainz-Policy: It should be possible to select at least one of these lectures

Simlification

Each subject has exactly two compulsory elective lectures per semester taking 3 time slots each

Examples

Conflict:

Slot	1	2	3	4	5	6	7	8
Subject 1	W1		W1	K	K	W2		W2
Subject 2		W2	W2		W1	W1	K	K

No-Conflict:

Slot	1	2	3	4	5	6	7	8
Subject 1	W1		W1	K	K	W2		W2
Subject 2		W1	W2		W2	W1	K	K

Examples

Conflict: (e.g. Choice (1,1))

Slot	1	2	3	4	5	6	7	8
Subject 1	W1		W1	K	K	W2		W2
Subject 2		W2	W2		W1	W1	K	K

No-Conflict: Choice (1,1)

Slot	1	2	3	4	5	6	7	8
Subject 1	W1		W1	K	K	W2		W2
Subject 2		W1	W2		W2	W1	K	K

Result of Prof. Dr. Alexander Kreuzer for the first Semester

Wahlzeit 1

Zeit	Montag		Dienstag		Mittwoch		Donnerstag		Freitag	
8 - 10	Eng/Gr	Chem	Span	Mathe	Deu	Physik	Deu	Physik	Sport	Rel
10 - 12	Eng/Gr	Chem	Bio	Gesch	Soz/Ita	BWS	Sport	Rel	Sport	Rel
12 - 14	Deu	Physik	Soz/Ita	Russ	Fra	Latein	Fra	Latein	Soz/Ita	BWS
14 - 16	Bio	Gesch	Geo	Phil Russ	Geo	Phil	Eng/Gr	Chem	Fra	Latein
16 - 18	Span	Mathe	Geo	Phil	Kernzeit BWS		Bio	Gesch	Span	Mathe

Wahlzeit 2

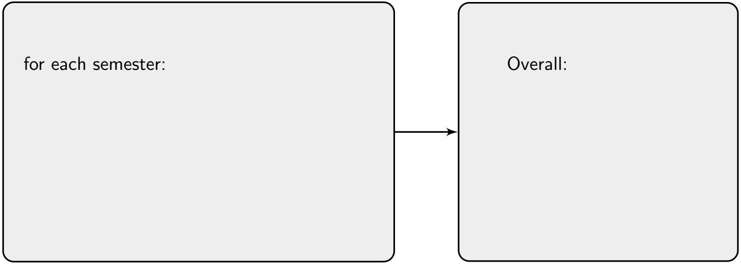
Zeit	Montag		Dienstag		Mittwoch		Donnerstag		Freitag	
8 - 10	Geo	Gesch	Sport	Latein	Bio	Rel	Bio	Rel	Deu	Mathe
10 - 12	Geo	Gesch	Eng/Gr	Physik	Fra Russ	Phil	Deu	Mathe	Deu	Mathe
12 - 14	Bio	Rel	Fra	Phil	Span	BWS	Span	Russ	Fra	Phil
14 - 16	Eng/Gr	Physik	Soz/Ita	Chem	Soz/Ita	Chem	Geo	Gesch	Span	BWS
16 - 18	Sport	Latein	Soz/Ita	Chem	Kernzeit BWS		Eng/Gr	Physik	Sport	Latein

Overview of our Approach

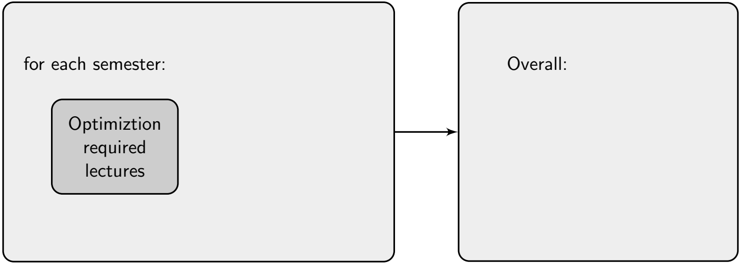
Overview of our Approach

for each semester:

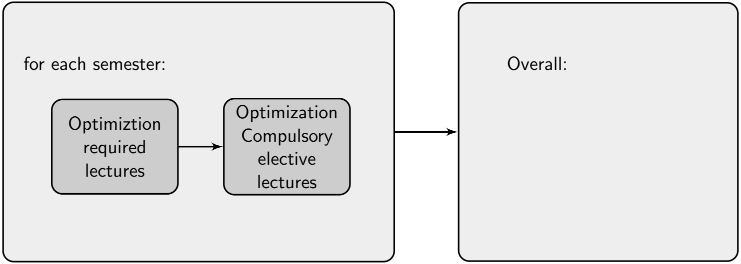
Overview of our Approach



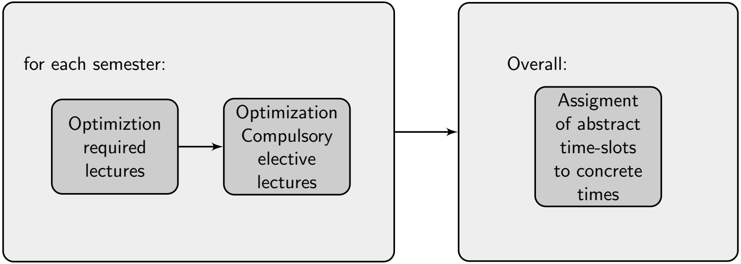
Overview of our Approach



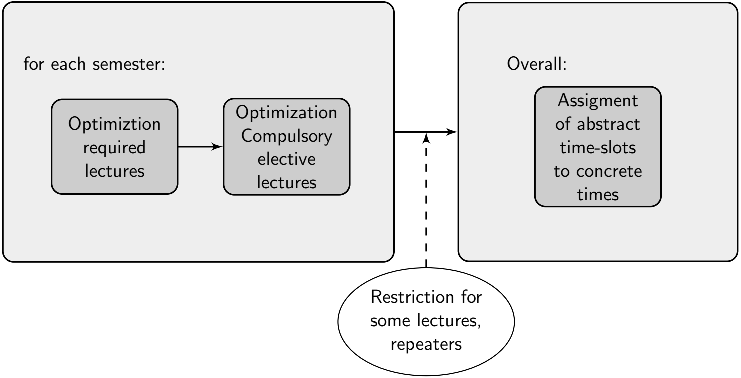
Overview of our Approach



Overview of our Approach



Overview of our Approach



Formulation as Quadratic Integer Program

Varialbes: $x_{ik} = 1 \Leftrightarrow$ Subject i has Time Slot k.

QP:

$$\begin{aligned} \text{min} \quad & \sum_{k=1}^{L} \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} \cdot x_{ik} \cdot x_{jk} \\ \text{wobei} \quad & \sum_{k=1}^{L} x_{ik} = 3 && i = 1, \ldots, N, \\ & x_{ik} \in \{0, 1\} && i = 1, \ldots, N, k = 1, \ldots, L, \end{aligned}$$

$L =$ Number of Time Slots, $N =$ Number of Subjects,

$c_{ij} =$ Number of Students with subjects (i, j),

Less Conflicts as with Kreuzer-Model

Model	Conflicts/Semester[1]
Kreuzer-Model	268
Optimization	175[2]

1 Conflicts within required lectures without arts and music, Total number of students in B.Ed. 2003 – 2008

2 With additional constraint that there is at most one overlapping slot for each pair of subjects

First Solution Approach: Linearization

$$\begin{aligned} \min \quad & \textstyle\sum_{i,j,k} c_{ij} \cdot x_{ik} \cdot x_{jk} \\ s.t. \quad & \textstyle\sum_{k=1}^{L} x_{ik} = 3 \end{aligned} \qquad \begin{aligned} \min \quad & \textstyle\sum_{i,j,k} c_{ij} x_{ijk} \\ s.t. \quad & \textstyle\sum_{k=1}^{L} x_{ik} = 3 \\ & x_{ijk} \geq x_{ik} + x_{jk} - 1 \end{aligned}$$

- Linearization:
  - Add "product-variable" $x_{ijk} = x_{ik} \cdot x_{jk}$
  - Objective Function is linear in these variables
  - Add constraints $x_{ij} \geq x_i + x_j - 1$
  - (constraints $x_{ij} \leq x_i$ are not necessary)
- Solve Integer Linear Program with branch-and-bound
- Problem:
  - Branch-and-bound works good, if the bound from the LP-relaxation is tight
  - Here the bound is 0 (set all variables to $3/L$)

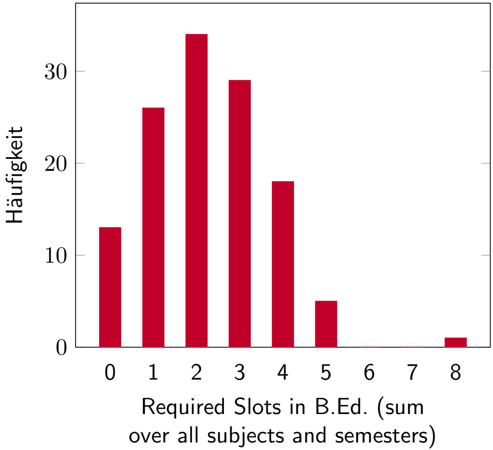
Second Solution Approach: Dantzig-Wolfe-Decomposition

- Dantzig-Wolfe-Decomposition
  - Use variables y_P, where $P \subseteq \{1, \ldots, n\}$ with
  - $y_P = 1$, iff the subjects in P are in one time slot
  - Integer Linear Program:

$$
\begin{aligned}
\max \qquad & \sum c_P y_P \\
& \sum y_P = L \\
& \sum_{P \mid i \in P} y_P \geq 3
\end{aligned}
$$

- Advantages:
  - Tight bounds from the LP-Relaxation
  - Elimination of symmetry
- Disadvantage:
  - Large number of variables
  - Using Kreuzer-simplification, we have about 1 million variables which is handable
  - Otherwise, we have to use dynamic column generation

Algorithmic Solution Allows More Flexible Planning



On average:
$\bar{x} = 2.27$ required slots per subject and semester

Standart deviation:
$\sigma = 1.41$

Kreuzer-Simplifiction:
Currently, we have exactly 3 slots per subject and semester

Results

Formalizing of Compulsory Elective Lectures

- Variables:
  - w^1_{ik} First elective lecture of subject i has time-slot k
  - w^2_{ik} Second elective lecture of subject i has time-slot k
- Constraints:
  - First/Second elective lecture of subject i has 3 time-slots
  - For each pair (i, j) of subjects there is a conflict-free choice
- A choice $a, b \in \{1, 2\}$ is conflict-free for a pair (i, j) of subjects is if
  - The slots w^a_i and w^b_j do not overlap
  - The slots w^a_i and x_j do not overlap
  - The slots w^b_j and x_i do not overlap
- Problem: “Or” in the constraints (there is a choice)
- Possible Solutions:
  - Trival: Add decision variables and formulate using “big-M-constraints”
  - Balas-Approach
  - “Efficient Algorithms from Simple-Minded” Researchers

Trivial Solution

- Formulation:
  - Add binary variables s_{ij}^{ab} $(i, j \in \{1, \ldots, N\}, k, \ell \in \{1, 2\})$ with $s_{ij}^{ab} = 1$, if elective lecture a of i and elective lecture b of j is used when subjects are i and j
  - For each pair (i, j) exactly one of the four variables has to be 1
  - Force $x_{ik} + w_{jk}^{b} \leq 1$ if $s_{ij}^{1b} + s_{ij}^{2b} = 1$
  - Force $w_{ik}^{a} + w_{jk}^{b} \leq 1$ if $s_{ij}^{ab} = 1$
  - Symmetric cases
- Discussion:
  - Easily formulated
  - Very weak bound

Balas-Solution

- If s_{ij}^{ab} would be known, we can easily formulate the constraints
- E.g. for $s_{ij}^{11} = 1$, we have $x_{ik} + w_{jk}^1 \leq 1$ and $w_{ik}^1 + w_{jk}^1 \leq 1$
- Balas-Solution is a generic method to formulate "$A^1 x \leq b^1$ or $A^2 x \leq b^2$ and works as follows:
  - Introduce two additional copies of the variables x^1 and x^2
  - Force $A^a x^a \leq b^a$ for $a \in \{1, 2\}$
  - Force x to be a convex-combination of x^1 and x^2,
    i.e. $x = \lambda x^1 + (1 - \lambda) x^2$ for $0 \leq \lambda \leq 1$
- Disadvantage: Numer of variables is doubled

Our Solution: Efficient Algorithms for Simple-Minded Researchers

- There are linear constraints describing the feasible solutions
- (e.g. by projecting out the s_{ij}^{ab} variables in the trivial solution)
- We are too simple-minded to find those linear constraints
- Our solution:
  - For a pair of subjects, enumerate all possible slot-assignments that are conflict free (only for variables relevant for these two subjects)
  - Use a software to compute the linear constraints that describe the convex-hull of these assignments
  - Use these linear constraints for an integer linear program

Results

- We find a conflict-free solution with the trivial approach
- ... even faster than with the Balas-Approach
- In our case it is trival to prove the optimality (no conflicts)
- Thinks can change, when we have conflicts (and hence rely on the LP-Bound)
- Our approach is still under development
- Combine these approaches with Dantzig-Wolfe-Decomposition

Outlook: Model

Simlification

- Each subject has exactly one lecture per semester taking 3 time slots each
- Each subject has exactly two compulsory elective lectures per semester taking 3 time slots each

- Advantages
  - Smaller problem size
  - Some flexibility for the subjects
- Disadvantages
  - Lectures that are shared between subjects
  - Different plans for start in summer or winter terms
  - Lectures that are shared with other (multi-topic) TODO: Studiengänge

We support the political decision process by providing "numbers“

Outlook

- Implementation
  - Implement dynamic column generation
  - Tune algorithms to scale to ”Uni-Mainz“-Instances (in particular, if simplifications are dropped)
- Development of Algorithms
  - Algorithms for the overall planning (over the semesters)