Precalculus HN

7.5 Parametric Equations

Homework: p. 469 # 1-39 odds Due___________

#27-31 odds: you don't need to graph.

Warm up: Solve $e^x - 4e^{-x} = 0$.

$e^x = t > 0$

$e^{-x} = \frac{1}{e^x} = \frac{1}{t}$

$t - 4\frac{1}{t} = 0$

$t = \frac{4}{t}$ $t^2 = 4$

$t = \cancel{\pm}2$

$t = 2$

$e^x = 2$ $\boxed{\ln 2 = x}$

If f and g are continuous functions of t on the interval I, then the set of ordered pairs (f(t), g(t)) represent a **parametric curve**.The equations

x=f(t) and y=g(t)

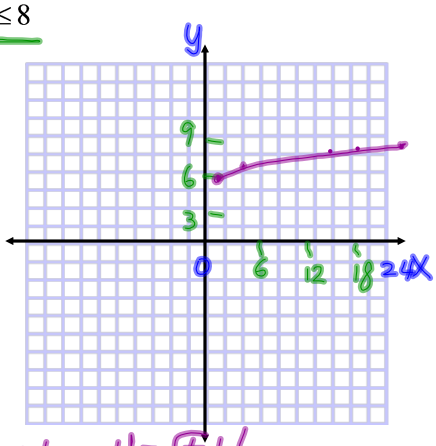
are parametric equations for this curve, t is a parameter and I is a parametric interval.

~~**Parametric Equations**~~
~~If a curve is defined by the equation y=f(x) and f is a function, y=f(x), then the parametric equation of the curve is ________________, ____________________, where t is the domain of f.~~

Ex 1. Sketch the curve given by each pair of parametric equations over the given interval. Write the rectangular equation of the curve. Then, find the domain and range.

a) $x = 3t, y = \sqrt{t} + 6; 0 \le t \le 8$

t	x	y
0	0	6
1	3	7
2	6	7.4
3	9	$\sqrt{3}+6 \approx 7.5$
4	12	$\sqrt{4}+6 \approx 8$
5	15	$\sqrt{5}+6 \approx 8.2$
6	18	$\sqrt{6}+6 \approx 8.4$
7	21	$\sqrt{7}+6 \approx 8.6$
8	24	$\sqrt{8}+6 \approx 8.8$



$x = 3t$ $\quad y = \sqrt{t}+6$

$t = \frac{x}{3}$

$$\boxed{y = \sqrt{\frac{x}{3}} + 6}$$

$0 \le t \le 8$

$0 \le x = 3t \le 24$

D: $0 \le x \le 24$, $[0, 24]$

R: $6 \le y \le \sqrt{8}+6$,

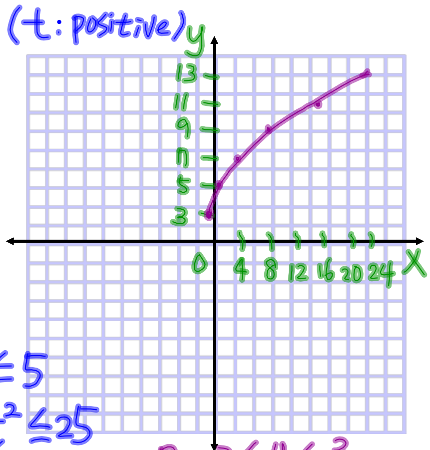
R: $[6, \sqrt{8}+6]$

b) $x = t^2, y = 2t + 3; 0 \le t \le 5$

$t = \pm\sqrt{x} \rightarrow t = \sqrt{x}$ (t: positive)

$y = 2\sqrt{x} + 3$

t	x	y
0	0	3
1	1	5
2	4	7
3	9	9
4	16	11
5	25	13



$0 \le t \le 5$

$0 \le x = t^2 \le 25$

D: $[0, 25]$

R: $3 \le y \le 3$

$0 \le t \le 5$

$y = 2t + 3$

$0 \le 2t \le 10$
$+3 \quad +3 \quad +3$

$3 \le 2t+3 \le 13$

$$\boxed{\therefore R: [3, 13]}$$

Ex 2. Write each in rectangular form and state the domain.

a) $x = t^2 - 5, y = 4t$

$t = \frac{y}{4}$

$x = \left(\frac{y}{4}\right)^2 - 5$

$x = \frac{y^2}{16} - 5$

$\frac{y^2}{16} = x + 5$

$y^2 = 16(x+5)$

$y = \pm\sqrt{16(x+5)}$

$\boxed{y = \pm 4\sqrt{x+5}}$

$x \geq -5$

D: $[-5, \infty)$

b) $x = -3t, y = t^2 + 2$

$t = \frac{x}{-3}$

$y = \left(-\frac{x}{3}\right)^2 + 2$

$\boxed{y = \frac{x^2}{9} + 2 \quad D: (-\infty, \infty)}$

c) $x = \sqrt{t+4}, y = \frac{1}{t}$

$x^2 = t + 4$

$t = x^2 - 4$

$x^2 - 4 \neq 0$

$\boxed{y = \frac{1}{x^2 - 4} \quad D: (-\infty, -2), (-2, 2), (2, \infty)}$

Ex 3. Write $x = 3\sin\theta, y = 8\cos\theta$ in rectangular form and sketch the graph.

$\sin^2\theta + \cos^2\theta = 1$

$x = 3\sin\theta \rightarrow \sin\theta = \frac{x}{3}$

$y = 8\cos\theta \rightarrow \cos\theta = \frac{y}{8}$

$\boxed{\therefore \left(\frac{x}{3}\right)^2 + \left(\frac{y}{8}\right)^2 = 1}$

(elipse)

Ex 4. Use each parameter to determine the parametric equations that can represent $x = 6 - y^2$.

a) t=x+1

$x = t-1$ $\quad x = 6-y^2$ $\quad t-1 = 6-y^2,$ $\quad t-7 = -y^2$ $\quad y^2 = 7-t$ $\quad \therefore \boxed{y = \pm\sqrt{7-t}}$

b) t=3x

$x = \frac{t}{3}$ $\quad \frac{t}{3} = 6-y^2$ $\quad -y^2 = \frac{t}{3}-6$ $\quad y^2 = 6-\frac{t}{3}$ $\quad \boxed{y = \pm\sqrt{6-\frac{t}{3}}}$

c) t=4-2x

$t-4 = -2x$ $\quad x = \frac{t-4}{-2}$ $\quad x = 6-y^2$ $\quad \frac{t-4}{-2} = 6-y^2$

$-y^2 = -\frac{t-4}{2} - 6$

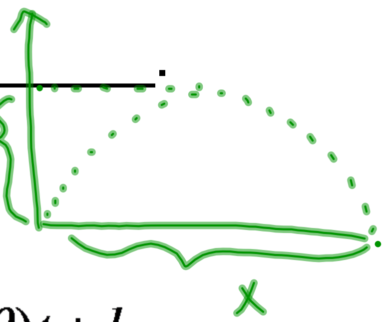
$y^2 = \frac{t-4}{2} + 6 = \frac{t-4+12}{2}$

$y^2 = \frac{t+8}{2}$

$\boxed{y = \pm\sqrt{\frac{t+8}{2}}}$

When an object is propelled upward at an inclination θ to the horizontal with initial speed v_0, the resulting motion is called

projetile motion.



The **parametric equations** are

initial velocity

$$x = (v_0\cos\theta)t,\ y = -\frac{1}{2}gt^2 + (v_0\sin\theta)t + h$$

distance $\qquad$ height $\qquad$ initial height

where θ is an inclination to the horizontal, t is the time, and g is the constant acceleration due to gravity (32 ft/sec/sec or 9.8m/sec/sec)

Ex 5. Suppose that Jim hit a golf ball with an initial velocity of 150 feet per second at an angle of 30 degrees to the horizontal.
(a) Find parametric equations that describe the position of the ball as a function of time. $x=(v_0\cos\theta)t,\ y=-\frac{1}{2}gt^2+(v_0\sin\theta)t+h$

$V_0=150 \quad \theta=30^\circ \quad h=0 \quad g=32$

$x=150\cos 30^\circ t=150\cdot\frac{\sqrt{3}}{2}t=75\sqrt{3}t \quad \boxed{\therefore x=75\sqrt{3}t}$

$y=-\frac{1}{2}\cdot 32\cdot t^2+150\sin 30^\circ t+0$

$=-16t^2+150\cdot\frac{1}{2}t=-16t^2+75t \quad \boxed{\therefore y=-16t^2+75t}$

(b) How long is the golf ball in the air?

$y=0=-16t^2+75t$

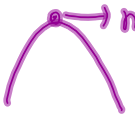
$-t(16t-75)=0$

$t=0,\ t=\frac{75}{16}=4.6875$

$\boxed{t=4.688\text{ sec}}$

(c) When is the ball at its maximum height?
Determine the maximum height of the ball. $y=ax^2+bx+c$

$y=-16t^2+75t$ max at $V\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$

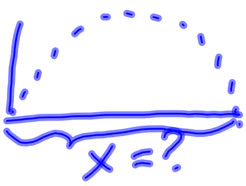


$t=-\frac{75}{2(-16)}=\frac{75}{32}$

$y\Big|_{t=\frac{75}{32}}=-16\left(\frac{75}{32}\right)^2+75\left(\frac{75}{32}\right)$

$\boxed{\approx 87.891\text{ ft}}$

(d) Determine the distance that the ball traveled.



$x=75\sqrt{3}t$

$=75\sqrt{3}(4.6875)$ from b)

$\boxed{x\approx 608.924\text{ ft}}$